



INVESTMENT SERIES

FIXED INCOME ANALYSIS

SECOND EDITION



Frank J. Fabozzi, CFA

Foreword by MARTIN L. LEIBOWITZ

FIXED INCOME ANALYSIS

Second Edition

Frank J. Fabozzi, PhD, CFA, CPA

with contributions from

Mark J.P. Anson, PhD, CFA, CPA, Esq.

Kenneth B. Dunn, PhD

J. Hank Lynch, CFA

Jack Malvey, CFA

Mark Pitts, PhD

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FIXED INCOME ANALYSIS

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FOREWORD

There is an argument that an understanding of any financial market must incorporate an appreciation of the functioning of the bond market as a vital source of liquidity. This argument is true in today's financial markets more than ever before, because of the central role that debt plays in virtually every facet of our modern financial markets. Thus, anyone who wants to be a serious student or practitioner in finance should at least become familiar with the current spectrum of fixed income securities and their associated derivatives and structural products.

This book is a fully revised and updated edition of two volumes used earlier in preparation for the Chartered Financial Analysts (CFA®) program. However, in its current form, it goes beyond the original CFA role and provides an extraordinarily comprehensive, and yet quite readable, treatment of the key topics in fixed income analysis. This breadth and quality of its contents has been recognized by its inclusion as a basic text in the finance curriculum of major universities. Anyone who reads this book, either thoroughly or by dipping into the portions that are relevant at the moment, will surely reach new planes of knowledgeability about debt instruments and the liquidity they provide throughout the global financial markets.

I first began studying the bond market back in the 1960s. At that time, bonds were thought to be dull and uninteresting. I often encountered expressions of sympathy about having been misguided into one of the more moribund backwaters in finance. Indeed, a designer of one of the early bond market indexes (not me) gave a talk that started with a declaration that bonds were “dull, dull, dull!”

In those early days, the bond market consisted of debt issued by US Treasury, agencies, municipalities, or high grade corporations. The structure of these securities was generally quite “plain vanilla”: fixed coupons, specified maturities, straightforward call features, and some sinking funds. There was very little trading in the secondary market. New issues of tax exempt bonds were purchased by banks and individuals, while the taxable offerings were taken down by insurance companies and pension funds. And even though the total outstanding footings were quite sizeable relative to the equity market, the secondary market trading in bonds was miniscule relative to stocks.

Bonds were, for the most part, locked away in frozen portfolios. The coupons were still—literally—“clipped,” and submitted to receive interest payments (at that time, scissors were one of the key tools of bond portfolio management). This state of affairs reflected the environment of the day—the bond-buying institutions were quite traditional in their culture (the term “crusty” may be only slightly too harsh), bonds were viewed basically as a source of income rather than an opportunity for short-term return generation, and the high transaction costs in the corporate and municipal sectors dampened any prospective benefit from trading.

However, times change, and there is no area of finance that has witnessed a more rapid evolution—perhaps revolution would be more apt—than the fixed income markets. Interest rates have swept up and down across a range of values that was previously thought to be unimaginable. New instruments were introduced, shaped into standard formats, and then

exploded to huge markets in their own right, both in terms of outstanding footings and the magnitude of daily trading. Structuring, swaps, and a variety of options have become integral components of the many forms of risk transfer that make today's vibrant debt market possible.

In stark contrast to the plodding pace of bonds in the 1960s, this book takes the reader on an exciting tour of today's modern debt market. The book begins with descriptions of the current tableau of debt securities. After this broad overview, which I recommend to everyone, the second chapter delves immediately into the fundamental question associated with any investment vehicle: What are the risks? Bonds have historically been viewed as a lower risk instrument relative to other markets such as equities and real estate. However, in today's fixed income world, the derivative and structuring processes have spawned a veritable smorgasbord of investment opportunities, with returns and risks that range across an extremely wide spectrum.

The completion of the Treasury yield curve has given a new clarity to term structure and maturity risk. In turn, this has sharpened the identification of minimum risk investments for specific time periods. The Treasury curve's more precisely defined term structure can then help in analyzing the spread behavior of non-Treasury securities. The non-Treasury market consists of corporate, agency, mortgage, municipal, and international credits. Its total now far exceeds the total supply of Treasury debt. To understand the credit/liquidity relationships across the various market segments, one must come to grips with the constellation of yield spreads that affects their pricing. Only then can one begin to understand how a given debt security is valued and to appreciate the many-dimensional determinants of debt return and risk.

As one delves deeper into the multiple layers of fixed income valuation, it becomes evident that these same factors form the basis for analyzing all forms of investments, not just bonds. In every market, there are spot rates, forward rates, as well as the more aggregated yield measures. In the past, this structural approach may have been relegated to the domain of the arcane or the academic. In the current market, these more sophisticated approaches to capital structure and term effects are applied daily in the valuation process.

Whole new forms of securitized fixed income instruments have come into existence and grown to enormous size in the past few decades, for example, the mortgage backed, asset backed, and structured-loan sectors. These sectors have become critical to the flow of liquidity to households and to the global economy at large. To trace how liquidity and credit availability find their way through various channels to the ultimate demanders, it is critical to understand how these assets are structured, how they behave, and why various sources of funds are attracted to them.

Credit analysis is another area that has undergone radical evolution in the past few years. The simplistic standard ratio measures of yesteryear have been supplemented by market oriented analyses based upon option theory as well as new approaches to capital structure.

The active management of bond portfolios has become a huge business where sizeable funds are invested in an effort to garner returns in excess of standard benchmark indices. The fixed income markets are comprised of far more individual securities than the equity market. However, these securities are embedded in term structure/spread matrix that leads to much tighter and more reliable correlations. The fixed income manager can take advantage of these tighter correlations to construct compact portfolios to control the overall benchmark risk and still have ample room to pursue opportunistic positive alphas in terms of sector selection, yield curve placement, or credit spreads. There is a widespread belief that exploitable inefficiencies persist within the fixed income market because of the regulatory and/or functional constraints

placed upon many of the major market participants. In more and more instances, these so-called alpha returns from active bond management are being “ported” via derivative overlays, possibly in a leveraged fashion, to any position in a fund’s asset allocation structure.

In terms of managing credit spreads and credit exposure, the development of credit default swaps (CDS) and other types of credit derivatives has grown at such an incredible pace that it now constitutes an important market in its own right. By facilitating the redistribution and diversification of credit risk, the CDS explosion has played a critical role in providing ongoing liquidity throughout the economy. These structure products and derivatives may have evolved from the fixed income market, but their role now reaches far afield, e.g., credit default swaps are being used by some equity managers as efficient alternative vehicles for hedging certain types of equity risks.

The worldwide maturing of pension funds in conjunction with a more stringent accounting/regulatory environment has created new management approaches such as surplus management, asset/liability management (ALM), or liability driven investment (LDI). These techniques incorporate the use of very long duration portfolios, various types of swaps and derivatives, as well as older forms of cash matching and immunization to reduce the fund’s exposure to fluctuations in nominal and/or real interest rates. With pension fund assets of both defined benefit and defined contribution variety amounting to over \$14 trillion in the United States alone, it is imperative for any student of finance to understand these liabilities and their relationship to various fixed income vehicles.

With its long history as the primary organization in educating and credentialing finance professionals, CFA Institute is the ideal sponsor to provide a balanced and objective overview of this subject. Drawing upon its unique professional network, CFA Institute has been able to call upon the most authoritative sources in the field to develop, review, and update each chapter. The primary author and editor, Frank Fabozzi, is recognized as one of the most knowledgeable and prolific scholars across the entire spectrum of fixed income topics. Dr. Fabozzi has held positions at MIT, Yale, and the University of Pennsylvania, and has written articles in collaboration with Franco Modigliani, Harry Markowitz, Gifford Fong, Jack Malvey, Mark Anson, and many other noted authorities in fixed income. One could not hope for a better combination of editor/author and sponsor. It is no wonder that they have managed to produce such a valuable guide into the modern world of fixed income.

Over the past three decades, the changes in the debt market have been arguably far more revolutionary than that seen in equities or perhaps in any other financial market. Unfortunately, the broader development of this market and its extension into so many different arenas and forms has made it more difficult to achieve a reasonable level of knowledgeability. However, this highly readable, authoritative and comprehensive volume goes a long way towards this goal by enabling individuals to learn about this most fundamental of all markets. The more specialized sections will also prove to be a resource that practitioners will repeatedly dip into as the need arises in the course of their careers.

Martin L. Leibowitz
Managing Director
Morgan Stanley

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I would like to acknowledge the following individuals for their assistance.

First Edition (Reprinted from First Edition)

Dr. Robert R. Johnson, CFA, Senior Vice President of AIMR, reviewed more than a dozen of the books published by Frank J. Fabozzi Associates. Based on his review, he provided me with an extensive list of chapters for the first edition that contained material that would be useful to CFA candidates for all three levels. Rather than simply put these chapters together into a book, he suggested that I use the material in them to author a book based on explicit content guidelines. His influence on the substance and organization of this book was substantial.

My day-to-day correspondence with AIMR regarding the development of the material and related issues was with Dr. Donald L. Tuttle, CFA, Vice President. It would seem fitting that he would serve as one of my mentors in this project because the book he co-edited, *Managing Investment Portfolios: A Dynamic Process* (first published in 1983), has played an important role in shaping my thoughts on the investment management process; it also has been the cornerstone for portfolio management in the CFA curriculum for almost two decades. The contribution of his books and other publications to the advancement of the CFA body of knowledge, coupled with his leadership role in several key educational projects, recently earned him AIMR's highly prestigious C. Stewart Sheppard Award.

Before any chapters were sent to Don for his review, the first few drafts were sent to Amy F. Lipton, CFA of Lipton Financial Analytics, who was a consultant to AIMR for this project. Amy is currently a member of the Executive Advisory Board of the Candidate Curriculum Committee (CCC). Prior to that she was a member of the Executive Committee of the CCC, the Level I Coordinator for the CCC, and the Chair of the Fixed Income Topic Area of the CCC. Consequently, she was familiar with the topics that should be included in a fixed income analysis book for the CFA Program. Moreover, given her experience in the money management industry (Aetna, First Boston, Greenwich, and Bankers Trust), she was familiar with the material. Amy reviewed and made detailed comments on all aspects of the material. She recommended the deletion or insertion of material, identified topics that required further explanation, and noted material that was too detailed and showed how it should be shortened. Amy not only directed me on content, but she checked every calculation, provided me with spreadsheets of all calculations, and highlighted discrepancies between the solutions in a chapter and those she obtained. On a number of occasions, Amy added material that improved the exposition; she also contributed several end-of-chapter questions. Amy has been accepted into the doctoral program in finance at both Columbia University and Lehigh University, and will begin her studies in Fall of 2000.

After the chapters were approved by Amy and Don, they were then sent to reviewers selected by AIMR. The reviewers provided comments that were the basis for further revisions. I am especially appreciative of the extensive reviews provided by Richard O. Applebach, Jr., CFA and Dr. George H. Troughton, CFA. I am also grateful to the following reviewers: Dr. Philip Fanara, Jr., CFA; Brian S. Heimsoth, CFA; Michael J. Karpik, CFA; Daniel E. Lenhard, CFA; Michael J. Lombardi, CFA; James M. Meeth, CFA; and C. Ronald Sprecher, PhD, CFA.

I engaged William McLellan to review all of the chapter drafts. Bill has completed the Level III examination and is now accumulating enough experience to be awarded the CFA designation. Because he took the examinations recently, he reviewed the material as if he were a CFA candidate. He pointed out statements that might be confusing and suggested ways to eliminate ambiguities. Bill checked all the calculations and provided me with his spreadsheet results.

Martin Fridson, CFA and Cecilia Fok provided invaluable insight and direction for the chapter on credit analysis (Chapter 9 of Level II). Dr. Steven V. Mann and Dr. Michael Ferri reviewed several chapters in this book. Dr. Sylvan Feldstein reviewed the sections dealing with municipal bonds in Chapter 3 of Level I and Chapter 9 of Level II. George Kelger reviewed the discussion on agency debentures in Chapter 3 of Level I.

Helen K. Modiri of AIMR provided valuable administrative assistance in coordinating between my office and AIMR.

Megan Orem of Frank J. Fabozzi Associates typeset the entire book and provided editorial assistance on various aspects of this project.

Second Edition

Dennis McLeavey, CFA was my contact person at CFA Institute for the second edition. He suggested how I could improve the contents of each chapter from the first edition and read several drafts of all the chapters. The inclusion of new topics were discussed with him. Dennis is an experienced author, having written several books published by CFA Institute for the CFA program. Dennis shared his insights with me and I credit him with the improvement in the exposition in the second edition.

The following individuals reviewed chapters:

Stephen L. Avard, CFA
Marcus A. Ingram, CFA
Muhammad J. Iqbal, CFA
William L. Randolph, CFA
Gerald R. Root, CFA
Richard J. Skolnik, CFA
R. Bruce Swensen, CFA
Lavone Whitmer, CFA

Larry D. Guin, CFA consolidated the individual reviews, as well as reviewed chapters 1–7. David M. Smith, CFA did the same for Chapters 8–15.

The final proofreaders were Richard O. Applebach, CFA, Dorothy C. Kelly, CFA, Louis J. James, CFA and Lavone Whitmer.

Wanda Lauziere of CFA Institute coordinated the reviews. Helen Weaver of CFA Institute assembled, summarized, and coordinated the final reviewer comments.

Jon Fougner, an economics major at Yale, provided helpful comments on Chapters 1 and 2.

Finally, CFA Candidates provided helpful comments and identified errors in the first edition.

INTRODUCTION

CFA Institute is pleased to provide you with this Investment Series covering major areas in the field of investments. These texts are thoroughly grounded in the highly regarded CFA Program Candidate Body of Knowledge (CBOK®) that draws upon hundreds of practicing investment professionals and serves as the anchor for the three levels of the CFA Examinations. In the year this series is being launched, more than 120,000 aspiring investment professionals will each devote over 250 hours of study to master this material as well as other elements of the Candidate Body of Knowledge in order to obtain the coveted CFA charter. We provide these materials for the same reason we have been chartering investment professionals for over 40 years: to improve the competency and ethical character of those serving the capital markets.

PARENTAGE

One of the valuable attributes of this series derives from its parentage. In the 1940s, a handful of societies had risen to form communities that revolved around common interests and work in what we now think of as the investment industry.

Understand that the idea of purchasing common stock as an investment—as opposed to casino speculation—was only a couple of decades old at most. We were only 10 years past the creation of the U.S. Securities and Exchange Commission and laws that attempted to level the playing field after robber baron and stock market panic episodes.

In January 1945, in what is today CFA Institute *Financial Analysts Journal*, a fundamentally driven professor and practitioner from Columbia University and Graham-Newman Corporation wrote an article making the case that people who research and manage portfolios should have some sort of credential to demonstrate competence and ethical behavior. This person was none other than Benjamin Graham, the father of security analysis and future mentor to a well-known modern investor, Warren Buffett.

The idea of creating a credential took a mere 16 years to drive to execution but by 1963, 284 brave souls, all over the age of 45, took an exam and launched the CFA credential. What many do not fully understand was that this effort had at its root a desire to create a profession where its practitioners were professionals who provided investing services to individuals in need. In so doing, a fairer and more productive capital market would result.

A profession—whether it be medicine, law, or other—has certain hallmark characteristics. These characteristics are part of what attracts serious individuals to devote the energy of their life's work to the investment endeavor. First, and tightly connected to this Series, there must be a body of knowledge. Second, there needs to be some entry requirements such as those required to achieve the CFA credential. Third, there must be a commitment to continuing education. Fourth, a profession must serve a purpose beyond one's direct selfish interest. In this case, by properly conducting one's affairs and putting client interests first, the investment

professional can work as a fair-minded cog in the wheel of the incredibly productive global capital markets. This encourages the citizenry to part with their hard-earned savings to be redeployed in fair and productive pursuit.

As C. Stewart Sheppard, founding executive director of the Institute of Chartered Financial Analysts said, “Society demands more from a profession and its members than it does from a professional craftsman in trade, arts, or business. In return for status, prestige, and autonomy, a profession extends a public warranty that it has established and maintains conditions of entry, standards of fair practice, disciplinary procedures, and continuing education for its particular constituency. Much is expected from members of a profession, but over time, more is given.”

“The Standards for Educational and Psychological Testing,” put forth by the American Psychological Association, the American Educational Research Association, and the National Council on Measurement in Education, state that the validity of professional credentialing examinations should be demonstrated primarily by verifying that the content of the examination accurately represents professional practice. In addition, a practice analysis study, which confirms the knowledge and skills required for the competent professional, should be the basis for establishing content validity.

For more than 40 years, hundreds upon hundreds of practitioners and academics have served on CFA Institute curriculum committees sifting through and winnowing all the many investment concepts and ideas to create a body of knowledge and the CFA curriculum. One of the hallmarks of curriculum development at CFA Institute is its extensive use of practitioners in all phases of the process.

CFA Institute has followed a formal practice analysis process since 1995. The effort involves special practice analysis forums held, most recently, at 20 locations around the world. Results of the forums were put forth to 70,000 CFA charterholders for verification and confirmation of the body of knowledge so derived.

What this means for the reader is that the concepts contained in these texts were driven by practicing professionals in the field who understand the responsibilities and knowledge that practitioners in the industry need to be successful. We are pleased to put this extensive effort to work for the benefit of the readers of the Investment Series.

BENEFITS

This series will prove useful both to the new student of capital markets, who is seriously contemplating entry into the extremely competitive field of investment management, and to the more seasoned professional who is looking for a user-friendly way to keep one’s knowledge current. All chapters include extensive references for those who would like to dig deeper into a given concept. The workbooks provide a summary of each chapter’s key points to help organize your thoughts, as well as sample questions and answers to test yourself on your progress.

For the new student, the essential concepts that any investment professional needs to master are presented in a time-tested fashion. This material, in addition to university study and reading the financial press, will help you better understand the investment field. I believe that the general public seriously underestimates the disciplined processes needed for the best investment firms and individuals to prosper. These texts lay the basic groundwork for many of the processes that successful firms use. Without this base level of understanding and an appreciation for how the capital markets work to properly price securities, you may

not find competitive success. Furthermore, the concepts herein give a genuine sense of the kind of work that is to be found day to day managing portfolios, doing research, or related endeavors.

The investment profession, despite its relatively lucrative compensation, is not for everyone. It takes a special kind of individual to fundamentally understand and absorb the teachings from this body of work and then convert that into application in the practitioner world. In fact, most individuals who enter the field do not survive in the longer run. The aspiring professional should think long and hard about whether this is the field for him- or herself. There is no better way to make such a critical decision than to be prepared by reading and evaluating the gospel of the profession.

The more experienced professional understands that the nature of the capital markets requires a commitment to continuous learning. Markets evolve as quickly as smart minds can find new ways to create an exposure, to attract capital, or to manage risk. A number of the concepts in these pages were not present a decade or two ago when many of us were starting out in the business. Hedge funds, derivatives, alternative investment concepts, and behavioral finance are examples of new applications and concepts that have altered the capital markets in recent years. As markets invent and reinvent themselves, a best-in-class foundation investment series is of great value.

Those of us who have been at this business for a while know that we must continuously hone our skills and knowledge if we are to compete with the young talent that constantly emerges. In fact, as we talk to major employers about their training needs, we are often told that one of the biggest challenges they face is how to help the experienced professional, laboring under heavy time pressure, keep up with the state of the art and the more recently educated associates. This series can be part of that answer.

CONVENTIONAL WISDOM

It doesn't take long for the astute investment professional to realize two common characteristics of markets. First, prices are set by conventional wisdom, or a function of the many variables in the market. Truth in markets is, at its essence, what the market believes it is and how it assesses pricing credits or debits on those beliefs. Second, as conventional wisdom is a product of the evolution of general theory and learning, by definition conventional wisdom is often wrong or at the least subject to material change.

When I first entered this industry in the mid-1970s, conventional wisdom held that the concepts examined in these texts were a bit too academic to be heavily employed in the competitive marketplace. Many of those considered to be the best investment firms at the time were led by men who had an eclectic style, an intuitive sense of markets, and a great track record. In the rough-and-tumble world of the practitioner, some of these concepts were considered to be of no use. Could conventional wisdom have been more wrong? If so, I'm not sure when.

During the years of my tenure in the profession, the practitioner investment management firms that evolved successfully were full of determined, intelligent, intellectually curious investment professionals who endeavored to apply these concepts in a serious and disciplined manner. Today, the best firms are run by those who carefully form investment hypotheses and test them rigorously in the marketplace, whether it be in a quant strategy, in comparative shopping for stocks within an industry, or in many hedge fund strategies. Their goal is to create investment processes that can be replicated with some statistical reliability. I believe

those who embraced the so-called academic side of the learning equation have been much more successful as real-world investment managers.

THE TEXTS

Approximately 35 percent of the Candidate Body of Knowledge is represented in the initial four texts of the series. Additional texts on corporate finance and international financial statement analysis are in development, and more topics may be forthcoming.

One of the most prominent texts over the years in the investment management industry has been Maginn and Tuttle's *Managing Investment Portfolios: A Dynamic Process*. The third edition updates key concepts from the 1990 second edition. Some of the more experienced members of our community, like myself, own the prior two editions and will add this to our library. Not only does this tome take the concepts from the other readings and put them in a portfolio context, it also updates the concepts of alternative investments, performance presentation standards, portfolio execution and, very importantly, managing individual investor portfolios. To direct attention, long focused on institutional portfolios, toward the individual will make this edition an important improvement over the past.

Quantitative Investment Analysis focuses on some key tools that are needed for today's professional investor. In addition to classic time value of money, discounted cash flow applications, and probability material, there are two aspects that can be of value over traditional thinking.

First are the chapters dealing with correlation and regression that ultimately figure into the formation of hypotheses for purposes of testing. This gets to a critical skill that many professionals are challenged by: the ability to sift out the wheat from the chaff. For most investment researchers and managers, their analysis is not solely the result of newly created data and tests that they perform. Rather, they synthesize and analyze primary research done by others. Without a rigorous manner by which to understand quality research, not only can you not understand good research, you really have no basis by which to evaluate less rigorous research. What is often put forth in the applied world as good quantitative research lacks rigor and validity.

Second, the last chapter on portfolio concepts moves the reader beyond the traditional capital asset pricing model (CAPM) type of tools and into the more practical world of multifactor models and to arbitrage pricing theory. Many have felt that there has been a CAPM bias to the work put forth in the past, and this chapter helps move beyond that point.

Equity Asset Valuation is a particularly cogent and important read for anyone involved in estimating the value of securities and understanding security pricing. A well-informed professional would know that the common forms of equity valuation—dividend discount modeling, free cash flow modeling, price/earnings models, and residual income models (often known by trade names)—can all be reconciled to one another under certain assumptions. With a deep understanding of the underlying assumptions, the professional investor can better understand what other investors assume when calculating their valuation estimates. In my prior life as the head of an equity investment team, this knowledge would give us an edge over other investors.

Fixed Income Analysis has been at the frontier of new concepts in recent years, greatly expanding horizons over the past. This text is probably the one with the most new material for the seasoned professional who is not a fixed-income specialist. The application of option and derivative technology to the once staid province of fixed income has helped contribute to an

explosion of thought in this area. And not only does that challenge the professional to stay up to speed with credit derivatives, swaptions, collateralized mortgage securities, mortgage backs, and others, but it also puts a strain on the world's central banks to provide oversight and the risk of a correlated event. Armed with a thorough grasp of the new exposures, the professional investor is much better able to anticipate and understand the challenges our central bankers and markets face.

I hope you find this new series helpful in your efforts to grow your investment knowledge, whether you are a relatively new entrant or a grizzled veteran ethically bound to keep up to date in the ever-changing market environment. CFA Institute, as a long-term committed participant of the investment profession and a not-for-profit association, is pleased to give you this opportunity.

JEFF DIERMEIER, CFA
President and Chief Executive Officer
CFA Institute
September 2006

NOTE ON ROUNDING DIFFERENCES

It is important to recognize in working through the numerical examples and illustrations in this book that because of rounding differences you may not be able to reproduce some of the results precisely. The two individuals who verified solutions and I used a spreadsheet to compute the solution to all numerical illustrations and examples. For some of the more involved illustrations and examples, there were slight differences in our results.

Moreover, numerical values produced in interim calculations may have been rounded off when produced in a table and as a result when an operation is performed on the values shown in a table, the result may appear to be off. Just be aware of this. Here is an example of a common situation that you may encounter when attempting to replicate results.

Suppose that a portfolio has four securities and that the market value of these four securities are as shown below:

Security	Market value
1	8,890,100
2	15,215,063
3	18,219,404
4	12,173,200
	54,497,767

Assume further that we want to calculate the duration of this portfolio. This value is found by computing the weighted average of the duration of the four securities. This involves three steps. First, compute the percentage of each security in the portfolio. Second, multiply the percentage of each security in the portfolio by its duration. Third, sum up the products computed in the second step.

Let's do this with our hypothetical portfolio. We will assume that the duration for each of the securities in the portfolio is as shown below:

Security	Duration
1	9
2	5
3	8
4	2

Using an Excel spreadsheet the following would be computed specifying that the percentage shown in Column (3) below be shown to seven decimal places:

(1)	(2)	(3)	(4)	(5)
Security	Market value	Percent of portfolio	Duration	Percent $\times$ duration
1	8,890,100	0.1631278	9	1.46815
2	15,215,063	0.2791869	5	1.395935
3	18,219,404	0.3343147	8	2.674518
4	12,173,200	0.2233706	2	0.446741
Total	54,497,767	1.0000000		5.985343

I simply cut and paste the spreadsheet from Excel to reproduce the table above. The portfolio duration is shown in the last row of Column (5). Rounding this value (5.985343) to two decimal places gives a portfolio duration of 5.99.

There are instances in the book where it was necessary to save space when I cut and paste a large spreadsheet. For example, suppose that in the spreadsheet I specified that Column (3) be shown to only two decimal places rather than seven decimal places. The following table would then be shown:

(1)	(2)	(3)	(4)	(5)
Security	Market value	Percent of portfolio	Duration	Percent $\times$ duration
1	8,890,100	0.16	9	1.46815
2	15,215,063	0.28	5	1.395935
3	18,219,404	0.33	8	2.674518
4	12,173,200	0.22	2	0.446741
	54,497,767	1.00		5.985343

Excel would do the computations based on the precise percent of the portfolio and would report the results as shown in Column (5) above. Of course, this is the same value of 5.985343 as before. However, if you calculated for any of the securities the percent of the portfolio in Column (3) multiplied by the duration in Column (4), you do not get the values in Column (5). For example, for Security 1, 0.16 multiplied by 9 gives a value of 1.44, not 1.46815 as shown in the table above.

Suppose instead that the computations were done with a hand-held calculator rather than on a spreadsheet and that the percentage of each security in the portfolio, Column (3), and the product of the percent and duration, Column (5), are computed to two decimal places. The following table would then be computed:

(1)	(2)	(3)	(4)	(5)
Security	Market value	Percent of portfolio	Duration	Percent $\times$ duration
1	8,890,100	0.16	9	1.44
2	15,215,063	0.28	5	1.40
3	18,219,404	0.33	8	2.64
4	12,173,200	0.22	2	0.44
Total	54,497,767	1.00		5.92

Note the following. First, the total in Column (3) is really 0.99 (99%) if one adds the value in the columns but is rounded to 1 in the table. Second, the portfolio duration shown in Column (5) is 5.92. This differs from the spreadsheet result earlier of 5.99.

Suppose that you decided to make sure that the total in Column (3) actually totals to 100%. Which security's percent would you round up to do so? If security 3 is rounded up to 34%, then the results would be reported as follows:

(1)	(2)	(3)	(4)	(5)
Security	Market value	Percent of portfolio	Duration	Percent $\times$ duration
1	8,890,100	0.16	9	1.44
2	15,215,063	0.28	5	1.40
3	18,219,404	0.34	8	2.72
4	12,173,200	0.22	2	0.44
	54,497,767	1.000		6.00

In this case, the result of the calculation from a hand-held calculator when rounding security 3 to 34% would produce a portfolio duration of 6.

Another reason why the result shown in the book may differ from your calculations is that you may use certain built-in features of spreadsheets that we did not use. For example, you will see in this book how the price of a bond is computed. In some of the illustrations in this book, the price of one or more bonds must be computed as an interim calculation to obtain a solution. If you use a spreadsheet's built-in feature for computing a bond's price (if the feature is available to you), you might observe slightly different results.

Please keep these rounding issues in mind. You are not making computations for sending a rocket to the moon, wherein slight differences could cause you to miss your target. Rather, what is important is that you understand the procedure or methodology for computing the values requested.

In addition, there are exhibits in the book that are reproduced from published research. Those exhibits were not corrected to reduce rounding error.

CHAPTER 1

FEATURES OF DEBT SECURITIES

I. INTRODUCTION

In investment management, the most important decision made is the allocation of funds among asset classes. The two major asset classes are equities and fixed income securities. Other asset classes such as real estate, private equity, hedge funds, and commodities are referred to as “alternative asset classes.” Our focus in this book is on one of the two major asset classes: fixed income securities.

While many people are intrigued by the exciting stories sometimes found with equities—who has not heard of someone who invested in the common stock of a small company and earned enough to retire at a young age?—we will find in our study of fixed income securities that the multitude of possible structures opens a fascinating field of study. While frequently overshadowed by the media prominence of the equity market, fixed income securities play a critical role in the portfolios of individual and institutional investors.

In its simplest form, a fixed income security is a financial obligation of an entity that promises to pay a specified sum of money at specified future dates. The entity that promises to make the payment is called the **issuer** of the security. Some examples of issuers are central governments such as the U.S. government and the French government, government-related agencies of a central government such as Fannie Mae and Freddie Mac in the United States, a municipal government such as the state of New York in the United States and the city of Rio de Janeiro in Brazil, a corporation such as Coca-Cola in the United States and Yorkshire Water in the United Kingdom, and supranational governments such as the World Bank.

Fixed income securities fall into two general categories: debt obligations and preferred stock. In the case of a debt obligation, the issuer is called the **borrower**. The investor who purchases such a fixed income security is said to be the **lender** or **creditor**. The promised payments that the issuer agrees to make at the specified dates consist of two components: interest and principal (principal represents repayment of funds borrowed) payments. Fixed income securities that are debt obligations include **bonds**, **mortgage-backed securities**, **asset-backed securities**, and **bank loans**.

In contrast to a fixed income security that represents a debt obligation, **preferred stock** represents an ownership interest in a corporation. Dividend payments are made to the preferred stockholder and represent a distribution of the corporation’s profit. Unlike investors who own a corporation’s common stock, investors who own the preferred stock can only realize a contractually fixed dividend payment. Moreover, the payments that must be made to preferred stockholders have priority over the payments that a corporation pays to common

stockholders. In the case of the bankruptcy of a corporation, preferred stockholders are given preference over common stockholders. Consequently, preferred stock is a form of equity that has characteristics similar to bonds.

Prior to the 1980s, fixed income securities were simple investment products. Holding aside default by the issuer, the investor knew how long interest would be received and when the amount borrowed would be repaid. Moreover, most investors purchased these securities with the intent of holding them to their maturity date. Beginning in the 1980s, the fixed income world changed. First, fixed income securities became more complex. There are features in many fixed income securities that make it difficult to determine when the amount borrowed will be repaid and for how long interest will be received. For some securities it is difficult to determine the amount of interest that will be received. Second, the hold-to-maturity investor has been replaced by institutional investors who actively trades fixed income securities.

We will frequently use the terms “fixed income securities” and “bonds” interchangeably. In addition, we will use the term bonds generically at times to refer collectively to mortgage-backed securities, asset-backed securities, and bank loans.

In this chapter we will look at the various features of fixed income securities and in the next chapter we explain how those features affect the risks associated with investing in fixed income securities. The majority of our illustrations throughout this book use fixed income securities issued in the United States. While the U.S. fixed income market is the largest fixed income market in the world with a diversity of issuers and features, in recent years there has been significant growth in the fixed income markets of other countries as borrowers have shifted from funding via bank loans to the issuance of fixed income securities. This is a trend that is expected to continue.

II. INDENTURE AND COVENANTS

The promises of the issuer and the rights of the bondholders are set forth in great detail in a bond’s **indenture**. Bondholders would have great difficulty in determining from time to time whether the issuer was keeping all the promises made in the indenture. This problem is resolved for the most part by bringing in a trustee as a third party to the bond or debt contract. The indenture identifies the trustee as a representative of the interests of the bondholders.

As part of the indenture, there are **affirmative covenants** and **negative covenants**. Affirmative covenants set forth activities that the borrower promises to do. The most common affirmative covenants are (1) to pay interest and principal on a timely basis, (2) to pay all taxes and other claims when due, (3) to maintain all properties used and useful in the borrower’s business in good condition and working order, and (4) to submit periodic reports to a trustee stating that the borrower is in compliance with the loan agreement. Negative covenants set forth certain limitations and restrictions on the borrower’s activities. The more common restrictive covenants are those that impose limitations on the borrower’s ability to incur additional debt unless certain tests are satisfied.

III. MATURITY

The **term to maturity** of a bond is the number of years the debt is outstanding or the number of years remaining prior to final principal payment. The **maturity date** of a bond refers to the date that the debt will cease to exist, at which time the issuer will redeem the bond by paying

the outstanding balance. The maturity date of a bond is always identified when describing a bond. For example, a description of a bond might state “due 12/1/2020.”

The practice in the bond market is to refer to the “term to maturity” of a bond as simply its “maturity” or “term.” As we explain below, there may be provisions in the indenture that allow either the issuer or bondholder to alter a bond’s term to maturity.

Some market participants view bonds with a maturity between 1 and 5 years as “short-term.” Bonds with a maturity between 5 and 12 years are viewed as “intermediate-term,” and “long-term” bonds are those with a maturity of more than 12 years.

There are bonds of every maturity. Typically, the longest maturity is 30 years. However, Walt Disney Co. issued bonds in July 1993 with a maturity date of 7/15/2093, making them 100-year bonds at the time of issuance. In December 1993, the Tennessee Valley Authority issued bonds that mature on 12/15/2043, making them 50-year bonds at the time of issuance.

There are three reasons why the term to maturity of a bond is important:

Reason 1: Term to maturity indicates the time period over which the bondholder can expect to receive interest payments and the number of years before the principal will be paid in full.

Reason 2: The yield offered on a bond depends on the term to maturity. The relationship between the yield on a bond and maturity is called the **yield curve** and will be discussed in Chapter 4.

Reason 3: The price of a bond will fluctuate over its life as interest rates in the market change. The price volatility of a bond is a function of its maturity (among other variables). More specifically, as explained in Chapter 7, all other factors constant, the longer the maturity of a bond, the greater the price volatility resulting from a change in interest rates.

IV. PAR VALUE

The **par value** of a bond is the amount that the issuer agrees to repay the bondholder at or by the maturity date. This amount is also referred to as the **principal value**, **face value**, **redemption value**, and **maturity value**. Bonds can have any par value.

Because bonds can have a different par value, the practice is to quote the price of a bond as a percentage of its par value. A value of “100” means 100% of par value. So, for example, if a bond has a par value of \$1,000 and the issue is selling for \$900, this bond would be said to be selling at 90. If a bond with a par value of \$5,000 is selling for \$5,500, the bond is said to be selling for 110.

When computing the dollar price of a bond in the United States, the bond must first be converted into a price per US\$1 of par value. Then the price per \$1 of par value is multiplied by the par value to get the dollar price. Here are examples of what the dollar price of a bond is, given the price quoted for the bond in the market, and the par amount involved in the transaction:¹

Quoted price	Price per \$1 of par value (rounded)	Par value	Dollar price
$90\frac{1}{2}$	0.9050	\$1,000	905.00
$102\frac{3}{4}$	1.0275	\$5,000	5,137.50
$70\frac{5}{8}$	0.7063	\$10,000	7,062.50
$113\frac{11}{32}$	1.1334	\$100,000	113,343.75

¹See the preface to this book regarding rounding.

Notice that a bond may trade below or above its par value. When a bond trades below its par value, it is said to be **trading at a discount**. When a bond trades above its par value, it is said to be **trading at a premium**. The reason why a bond sells above or below its par value will be explained in Chapter 2.

V. COUPON RATE

The **coupon rate**, also called the **nominal rate**, is the interest rate that the issuer agrees to pay each year. The annual amount of the interest payment made to bondholders during the term of the bond is called the **coupon**. The coupon is determined by multiplying the coupon rate by the par value of the bond. That is,

$$\text{coupon} = \text{coupon rate} \times \text{par value}$$

For example, a bond with an 8% coupon rate and a par value of \$1,000 will pay annual interest of \$80 ($= \$1,000 \times 0.08$).

When describing a bond of an issuer, the coupon rate is indicated along with the maturity date. For example, the expression “6s of 12/1/2020” means a bond with a 6% coupon rate maturing on 12/1/2020. The “s” after the coupon rate indicates “coupon series.” In our example, it means the “6% coupon series.”

In the United States, the usual practice is for the issuer to pay the coupon in two semiannual installments. Mortgage-backed securities and asset-backed securities typically pay interest monthly. For bonds issued in some markets outside the United States, coupon payments are made only once per year.

The coupon rate also affects the bond’s price sensitivity to changes in market interest rates. As illustrated in Chapter 2, all other factors constant, the higher the coupon rate, the less the price will change in response to a change in market interest rates.

A. Zero-Coupon Bonds

Not all bonds make periodic coupon payments. Bonds that are not contracted to make periodic coupon payments are called **zero-coupon bonds**. The holder of a zero-coupon bond realizes interest by buying the bond substantially below its par value (i.e., buying the bond at a discount). Interest is then paid at the maturity date, with the interest being the difference between the par value and the price paid for the bond. So, for example, if an investor purchases a zero-coupon bond for 70, the interest is 30. This is the difference between the par value (100) and the price paid (70). The reason behind the issuance of zero-coupon bonds is explained in Chapter 2.

B. Step-Up Notes

There are securities that have a coupon rate that increases over time. These securities are called **step-up notes** because the coupon rate “steps up” over time. For example, a 5-year step-up note might have a coupon rate that is 5% for the first two years and 6% for the last three years. Or, the step-up note could call for a 5% coupon rate for the first two years, 5.5% for the third and fourth years, and 6% for the fifth year. When there is only one change (or step up), as in our first example, the issue is referred to as a **single step-up note**. When there is more than one change, as in our second example, the issue is referred to as a **multiple step-up note**.

An example of an actual multiple step-up note is a 5-year issue of the Student Loan Marketing Association (Sallie Mae) issued in May 1994. The coupon schedule is as follows:

6.05%	from	5/3/94	to	5/2/95
6.50%	from	5/3/95	to	5/2/96
7.00%	from	5/3/96	to	5/2/97
7.75%	from	5/3/97	to	5/2/98
8.50%	from	5/3/98	to	5/2/99

C. Deferred Coupon Bonds

There are bonds whose interest payments are deferred for a specified number of years. That is, there are no interest payments during for the deferred period. At the end of the deferred period, the issuer makes periodic interest payments until the bond matures. The interest payments that are made after the deferred period are higher than the interest payments that would have been made if the issuer had paid interest from the time the bond was issued. The higher interest payments after the deferred period are to compensate the bondholder for the lack of interest payments during the deferred period. These bonds are called **deferred coupon bonds**.

D. Floating-Rate Securities

The coupon rate on a bond need not be fixed over the bond's life. **Floating-rate securities**, sometimes called **variable-rate securities**, have coupon payments that reset periodically according to some reference rate. The typical formula (called the **coupon formula**) on certain determination dates when the coupon rate is reset is as follows:

$$\text{coupon rate} = \text{reference rate} + \text{quoted margin}$$

The **quoted margin** is the additional amount that the issuer agrees to pay above the reference rate. For example, suppose that the reference rate is the 1-month London interbank offered rate (LIBOR).² Suppose that the quoted margin is 100 basis points.³ Then the coupon formula is:

$$\text{coupon rate} = \text{1-month LIBOR} + 100 \text{ basis points}$$

So, if 1-month LIBOR on the coupon reset date is 5%, the coupon rate is reset for that period at 6% (5% plus 100 basis points).

The quoted margin need not be a positive value. The quoted margin could be subtracted from the reference rate. For example, the reference rate could be the yield on a 5-year Treasury security and the coupon rate could reset every six months based on the following coupon formula:

$$\text{coupon rate} = \text{5-year Treasury yield} - 90 \text{ basis points}$$

²LIBOR is the interest rate which major international banks offer each other on Eurodollar certificates of deposit.

³In the fixed income market, market participants refer to changes in interest rates or differences in interest rates in terms of basis points. A **basis point** is defined as 0.0001, or equivalently, 0.01%. Consequently, 100 basis points are equal to 1%. (In our example the coupon formula can be expressed as 1-month LIBOR + 1%.) A change in interest rates from, say, 5.0% to 6.2% means that there is a 1.2% change in rates or 120 basis points.

So, if the 5-year Treasury yield is 7% on the coupon reset date, the coupon rate is 6.1% (7% minus 90 basis points).

It is important to understand the mechanics for the payment and the setting of the coupon rate. Suppose that a floater pays interest semiannually and further assume that the coupon reset date is today. Then, the coupon rate is determined via the coupon formula and this is the interest rate that the issuer agrees to pay at the next interest payment date six months from now.

A floater may have a restriction on the maximum coupon rate that will be paid at any reset date. The maximum coupon rate is called a **cap**. For example, suppose for a floater whose coupon formula is the 3-month Treasury bill rate plus 50 basis points, there is a cap of 9%. If the 3-month Treasury bill rate is 9% at a coupon reset date, then the coupon formula would give a coupon rate of 9.5%. However, the cap restricts the coupon rate to 9%. Thus, for our hypothetical floater, once the 3-month Treasury bill rate exceeds 8.5%, the coupon rate is capped at 9%. Because a cap restricts the coupon rate from increasing, a cap is an unattractive feature for the investor. In contrast, there could be a minimum coupon rate specified for a floater. The minimum coupon rate is called a **floor**. If the coupon formula produces a coupon rate that is below the floor, the floor rate is paid instead. Thus, a floor is an attractive feature for the investor. As we explain in Section X, caps and floors are effectively embedded options.

While the reference rate for most floaters is an interest rate or an interest rate index, a wide variety of reference rates appear in coupon formulas. The coupon for a floater could be indexed to movements in foreign exchange rates, the price of a commodity (e.g., crude oil), the return on an equity index (e.g., the S&P 500), or movements in a bond index. In fact, through financial engineering, issuers have been able to structure floaters with almost any reference rate. In several countries, there are government bonds whose coupon formula is tied to an inflation index.

The U.S. Department of the Treasury in January 1997 began issuing inflation-adjusted securities. These issues are referred to as **Treasury Inflation Protection Securities (TIPS)**. The reference rate for the coupon formula is the rate of inflation as measured by the Consumer Price Index for All Urban Consumers (i.e., CPI-U). (The mechanics of the payment of the coupon will be explained in Chapter 3 where these securities are discussed.) Corporations and agencies in the United States issue **inflation-linked** (or **inflation-indexed**) **bonds**. For example, in February 1997, J. P. Morgan & Company issued a 15-year bond that pays the CPI plus 400 basis points. In the same month, the Federal Home Loan Bank issued a 5-year bond with a coupon rate equal to the CPI plus 315 basis points and a 10-year bond with a coupon rate equal to the CPI plus 337 basis points.

Typically, the coupon formula for a floater is such that the coupon rate increases when the reference rate increases, and decreases when the reference rate decreases. There are issues whose coupon rate moves in the opposite direction from the change in the reference rate. Such issues are called **inverse floaters** or **reverse floaters**.⁴ It is not too difficult to understand why an investor would be interested in an inverse floater. It gives an investor who believes interest rates will decline the opportunity to obtain a higher coupon interest rate. The issuer isn't necessarily taking the opposite view because it can hedge the risk that interest rates will decline.⁵

⁴In the agency, corporate, and municipal markets, inverse floaters are created as structured notes. We discuss structured notes in Chapter 3. Inverse floaters in the mortgage-backed securities market are common and are created through a process that will be discussed in Chapter 10.

⁵The issuer hedges by using financial instruments known as derivatives, which we cover in later chapters.

The coupon formula for an inverse floater is:

$$\text{coupon rate} = K - L \times (\text{reference rate})$$

where K and L are values specified in the prospectus for the issue.

For example, suppose that for a particular inverse floater, K is 20% and L is 2. Then the coupon reset formula would be:

$$\text{coupon rate} = 20\% - 2 \times (\text{reference rate})$$

Suppose that the reference rate is the 3-month Treasury bill rate, then the coupon formula would be

$$\text{coupon rate} = 20\% - 2 \times (\text{3-month Treasury bill rate})$$

If at the coupon reset date the 3-month Treasury bill rate is 6%, the coupon rate for the next period is:

$$\text{coupon rate} = 20\% - 2 \times 6\% = 8\%$$

If at the next reset date the 3-month Treasury bill rate declines to 5%, the coupon rate increases to:

$$\text{coupon rate} = 20\% - 2 \times 5\% = 10\%$$

Notice that if the 3-month Treasury bill rate exceeds 10%, then the coupon formula would produce a negative coupon rate. To prevent this, there is a floor imposed on the coupon rate. There is also a cap on the inverse floater. This occurs if the 3-month Treasury bill rate is zero. In that unlikely event, the maximum coupon rate is 20% for our hypothetical inverse floater.

There is a wide range of coupon formulas that we will encounter in our study of fixed income securities.⁶ These are discussed below. The reason why issuers have been able to create floating-rate securities with offbeat coupon formulas is due to derivative instruments. It is too early in our study of fixed income analysis and portfolio management to appreciate why some of these offbeat coupon formulas exist in the bond market. Suffice it to say that some of these offbeat coupon formulas allow the investor to take a view on either the movement of some interest rate (i.e., for speculating on an interest rate movement) or to reduce exposure to the risk of some interest rate movement (i.e., for interest rate risk management). The advantage to the issuer is that it can lower its cost of borrowing by creating offbeat coupon formulas for investors.⁷ While it may seem that the issuer is taking the opposite position to the investor, this is not the case. What in fact happens is that the issuer can hedge its risk exposure by using derivative instruments so as to obtain the type of financing it seeks (i.e., fixed rate borrowing or floating rate borrowing). These offbeat coupon formulas are typically found in “structured notes,” a form of medium-term note that will be discussed in Chapter 3.

⁶In Chapter 3, we will describe other types of floating-rate securities.

⁷These offbeat coupon bond formulas are actually created as a result of inquiries from clients of dealer firms. That is, a salesperson will be approached by fixed income portfolio managers requesting a structure be created that provides the exposure sought. The dealer firm will then notify the investment banking group of the dealer firm to contact potential issuers.

E. Accrued Interest

Bond issuers do not disburse coupon interest payments every day. Instead, typically in the United States coupon interest is paid every six months. In some countries, interest is paid annually. For mortgage-backed and asset-backed securities, interest is usually paid monthly. The coupon payment is made to the bondholder of record. Thus, if an investor sells a bond between coupon payments and the buyer holds it until the next coupon payment, then the entire coupon interest earned for the period will be paid to the buyer of the bond since the buyer will be the holder of record. The seller of the bond gives up the interest from the time of the last coupon payment to the time until the bond is sold. The amount of interest over this period that will be received by the buyer even though it was earned by the seller is called **accrued interest**. We will see how to calculate accrued interest in Chapter 5.

In the United States and in many countries, the bond buyer must pay the bond seller the accrued interest. The amount that the buyer pays the seller is the agreed upon price for the bond plus accrued interest. This amount is called the **full price**. (Some market participants refer to this as the **dirty price**.) The agreed upon bond price without accrued interest is simply referred to as the **price**. (Some refer to it as the **clean price**.)

A bond in which the buyer must pay the seller accrued interest is said to be trading *cum-coupon* (“with coupon”). If the buyer forgoes the next coupon payment, the bond is said to be trading *ex-coupon* (“without coupon”). In the United States, bonds are always traded *cum-coupon*. There are bond markets outside the United States where bonds are traded *ex-coupon* for a certain period before the coupon payment date.

There are exceptions to the rule that the bond buyer must pay the bond seller accrued interest. The most important exception is when the issuer has not fulfilled its promise to make the periodic interest payments. In this case, the issuer is said to be in default. In such instances, the bond is sold without accrued interest and is said to be **traded flat**.

VI. PROVISIONS FOR PAYING OFF BONDS

The issuer of a bond agrees to pay the principal by the stated maturity date. The issuer can agree to pay the entire amount borrowed in one lump sum payment at the maturity date. That is, the issuer is not required to make any principal repayments prior to the maturity date. Such bonds are said to have a **bullet maturity**. The bullet maturity structure has become the most common structure in the United States and Europe for both corporate and government issuers.

Fixed income securities backed by pools of loans (mortgage-backed securities and asset-backed securities) often have a schedule of partial principal payments. Such fixed income securities are said to be **amortizing securities**. For many loans, the payments are structured so that when the last loan payment is made, the entire amount owed is fully paid.

Another example of an amortizing feature is a bond that has a **sinking fund provision**. This provision for repayment of a bond may be designed to pay all of an issue by the maturity date, or it may be arranged to repay only a part of the total by the maturity date. We discuss this provision later in this section.

An issue may have a **call provision** granting the issuer an option to retire all or part of the issue prior to the stated maturity date. Some issues specify that the issuer must retire a predetermined amount of the issue periodically. Various types of call provisions are discussed in the following pages.

A. Call and Refunding Provisions

An issuer generally wants the right to retire a bond issue prior to the stated maturity date. The issuer recognizes that at some time in the future interest rates may fall sufficiently below the issue's coupon rate so that redeeming the issue and replacing it with another lower coupon rate issue would be economically beneficial. This right is a disadvantage to the bondholder since proceeds received must be reinvested in the lower interest rate issue. As a result, an issuer who wants to include this right as part of a bond offering must compensate the bondholder when the issue is sold by offering a higher coupon rate, or equivalently, accepting a lower price than if the right is not included.

The right of the issuer to retire the issue prior to the stated maturity date is referred to as a **call provision**. If an issuer exercises this right, the issuer is said to “call the bond.” The price which the issuer must pay to retire the issue is referred to as the **call price** or **redemption price**.

When a bond is issued, typically the issuer may not call the bond for a number of years. That is, the issue is said to have a **deferred call**. The date at which the bond may first be called is referred to as the **first call date**. The first call date for the Walt Disney 7.55s due 7/15/2093 (the 100-year bonds) is 7/15/2023. For the 50-year Tennessee Valley Authority 6 $\frac{7}{8}$ s due 12/15/2043, the first call date is 12/15/2003.

Bonds can be called in whole (the entire issue) or in part (only a portion). When less than the entire issue is called, the certificates to be called are either selected randomly or on a **pro rata basis**. When bonds are selected randomly, a computer program is used to select the serial number of the bond certificates called. The serial numbers are then published in *The Wall Street Journal* and major metropolitan dailies. Pro rata redemption means that all bondholders of the issue will have the same percentage of their holdings redeemed (subject to the restrictions imposed on minimum denominations). Pro rata redemption is rare for publicly issued debt but is common for debt issues directly or privately placed with borrowers.

A bond issue that permits the issuer to call an issue prior to the stated maturity date is referred to as a **callable bond**. At one time, the callable bond structure was common for corporate bonds issued in the United States. However, since the mid-1990s, there has been significantly less issuance of callable bonds by corporate issuers of high credit quality. Instead, as noted above, the most popular structure is the bullet bond. In contrast, corporate issuers of low credit quality continue to issue callable bonds.⁸ In Europe, historically the callable bond structure has not been as popular as in the United States.

1. Call (Redemption) Price When the issuer exercises an option to call an issue, the call price can be either (1) fixed regardless of the call date, (2) based on a price specified in the call schedule, or (3) based on a make-whole premium provision. We will use various debt issues of Anheuser-Busch Companies to illustrate these three ways by which the call price is specified.

a. Single Call Price Regardless of Call Date On 6/10/97, Anheuser-Busch Companies issued \$250 million of notes with a coupon rate of 7.1% due June 15, 2007. The prospectus stated that:

⁸As explained in Chapter 2, high credit quality issuers are referred to as “investment grade” issuers and low credit quality issuers are referred to as “non-investment grade” issuers. The reason why high credit quality issuers have reduced their issuance of callable bonds while it is still the more popular structure for low credit quality issuers is explained later.

... The Notes will be redeemable at the option of the Company at any time on or after June 15, 2004, as set forth herein.

The Notes will be redeemable at the option of the Company at any time on or after June 15, 2004, in whole or in part, upon not fewer than 30 days' nor more than 60 days' notice, at a Redemption Price equal to 100% of the principal amount thereof, together with accrued interest to the date fixed for redemption.

This issue had a deferred call of seven years at issuance and a first call date of June 15, 2004. Regardless of the call date, the call price is par plus accrued interest.

b. Call Price Based on Call Schedule With a **call schedule**, the call price depends on when the issuer calls the issue. As an example of an issue with a call schedule, in July 1997 Anheuser-Busch Companies issued \$250 million of debentures with a coupon rate of $7\frac{1}{8}\%$ due July 1, 2017. (We will see what a debt instrument referred to as a “debenture” is in Chapter 3.) The provision dealing with the call feature of this issue states:

The Debentures will be redeemable at the option of the Company at any time on or after July 1, 2007, in whole or in part, upon not fewer than 30 days' nor more than 60 days' notice, at Redemption Prices equal to the percentages set forth below of the principal amount to be redeemed for the respective 12-month periods beginning July 1 of the years indicated, together in each case with accrued interest to the Redemption Date:

12 months beginning July 1	Redemption price	12 months beginning July 1	Redemption price
2007	103.026%	2012	101.513%
2008	102.723%	2013	101.210%
2009	102.421%	2014	100.908%
2010	102.118%	2015	100.605%
2011	101.816%	2016	100.303%

This issue had a deferred call of 10 years from the date of issuance, and the call price begins at a premium above par value and declines over time toward par value. Notice that regardless of when the issue is called, the issuer pays a premium above par value.

A second example of a call schedule is provided by the \$150 million Anheuser-Busch Companies $8\frac{5}{8}\%$ s due 12/1/2016 issued November 20, 1986. This issue had a 10-year deferred call (the first call date was December 1, 1996) and the following call schedule:

If redeemed during the 12 months beginning December 1:	Call price	If redeemed during the 12 months beginning December 1:	Call price
1996	104.313	2002	101.725
1997	103.881	2003	101.294
1998	103.450	2004	100.863
1999	103.019	2005	100.431
2000	102.588	2006 and thereafter	100.000
2001	102.156		

Notice that for this issue the call price begins at a premium but after 2006 the call price declines to par value. The first date at which an issue can be called at par value is the **first par call date**.

c. Call Price Based on Make-Whole Premium A **make-whole premium provision**, also called a **yield-maintenance premium provision**, provides a formula for determining the premium that an issuer must pay to call an issue. The purpose of the make-whole premium is to protect the yield of those investors who purchased the issue at issuance. A make-whole premium does so by setting an amount for the premium, such that when added to the principal amount and reinvested at the redemption date in U.S. Treasury securities having the same remaining life, it would provide a yield equal to the original issue's yield. The premium plus the principal at which the issue is called is referred to as the **make-whole redemption price**.

We can use an Anheuser-Busch Companies issue to illustrate a make-whole premium provision—the \$250 million 6% debentures due 11/1/2041 issued on 1/5/2001. The prospectus for this issue states:

We may redeem the Debentures, in whole or in part, at our option at any time at a redemption price equal to the greater of (i) 100% of the principal amount of such Debentures and (ii) as determined by a Quotation Agent (as defined below), the sum of the present values of the remaining scheduled payments of principal and interest thereon (not including any portion of such payments of interest accrued as of the date of redemption) discounted to the date of redemption on a semi-annual basis (assuming a 360-day year consisting of twelve 30-day months) at the Adjusted Treasury Rate (as defined below) plus 25 basis points plus, in each case, accrued interest thereon to the date of redemption.

The prospectus defined what is meant by a “Quotation Agent” and the “Adjusted Treasury Rate.” For our purposes here, it is not necessary to go into the definitions, only that there is some mechanism for determining a call price that reflects current market conditions as measured by the yield on Treasury securities. (Treasury securities are explained in Chapter 3.)

2. Noncallable versus Nonrefundable Bonds If a bond issue does not have any protection against early call, then it is said to be a **currently callable** issue. But most new bond issues, even if currently callable, usually have some restrictions against certain types of early redemption. The most common restriction is that of prohibiting the refunding of the bonds for a certain number of years or for the issue's life. Bonds that are noncallable for the issue's life are more common than bonds which are nonrefundable for life but otherwise callable.

Many investors are confused by the terms **noncallable** and **nonrefundable**. Call protection is much more robust than refunding protection. While there may be certain exceptions to absolute or complete call protection in some cases (such as sinking funds and the redemption of debt under certain mandatory provisions discussed later), call protection still provides greater assurance against premature and unwanted redemption than refunding protection. Refunding protection merely prevents redemption from certain sources, namely the proceeds of other debt issues sold at a lower cost of money. The holder is protected only if interest rates decline and the borrower can obtain lower-cost money to pay off the debt.

For example, Anheuser-Busch Companies issued on 6/23/88 10% coupon bonds due 7/1/2018. The issue was immediately callable. However, the prospectus specified in the call schedule that

prior to July 1, 1998, the Company may not redeem any of the Debentures pursuant to such option, directly or indirectly, from or in anticipation of the proceeds of the issuance of any indebtedness for money borrowed having an interest cost of less than 10% per annum.

Thus, this Anheuser-Busch bond issue could not be redeemed prior to July 2, 1998 if the company raised the money from a new issue with an interest cost lower than 10%. There is

nothing to prevent the company from calling the bonds within the 10-year refunding protected period from debt sold at a higher rate (although the company normally wouldn't do so) or from money obtained through other means. And that is exactly what Anheuser-Busch did. Between December 1993 and June 1994, it called \$68.8 million of these relatively high-coupon bonds at 107.5% of par value (the call price) with funds from its general operations. This was permitted because funds from the company's general operations are viewed as more expensive than the interest cost of indebtedness. Thus, Anheuser-Busch was allowed to call this issue prior to July 1, 1998.

3. **Regular versus Special Redemption Prices** The call prices for the various issues cited above are called the **regular redemption prices** or **general redemption prices**. Notice that the regular redemption prices are above par until the first par call date. There are also **special redemption prices** for bonds redeemed through the sinking fund and through other provisions, and the proceeds from the confiscation of property through the right of eminent domain or the forced sale or transfer of assets due to deregulation. The special redemption price is usually par value. Thus, there is an advantage to the issuer of being able to redeem an issue prior to the first par call date at the special redemption price (usually par) rather than at the regular redemption price.

A concern of an investor is that an issuer will use all means possible to maneuver a call so that the special redemption price applies. This is referred to as the **par call problem**. There have been ample examples, and subsequent litigation, where corporations have used the special redemption price and bondholders have challenged the use by the issuer.

B. Prepayments

For amortizing securities that are backed by loans that have a schedule of principal payments, individual borrowers typically have the option to pay off all or part of their loan prior to a scheduled principal payment date. Any principal payment prior to a scheduled principal payment date is called a **prepayment**. The right of borrowers to prepay principal is called a **prepayment option**.

Basically, the prepayment option is the same as a call option. However, unlike a call option, there is not a call price that depends on when the borrower pays off the issue. Typically, the price at which a loan is prepaid is par value. Prepayments will be discussed when mortgage-backed and asset-backed securities are discussed.

C. Sinking Fund Provision

An indenture may require the issuer to retire a specified portion of the issue each year. This is referred to as a **sinking fund requirement**. The alleged purpose of the sinking fund provision is to reduce credit risk (discussed in the next chapter). This kind of provision for debt payment may be designed to retire all of a bond issue by the maturity date, or it may be designed to pay only a portion of the total indebtedness by the end of the term. If only a portion is paid, the remaining principal is called a **balloon maturity**.

An example of an issue with a sinking fund requirement that pays the entire principal by the maturity date is the \$150 million Ingersoll Rand 7.20s issue due 6/1/2025. This bond, issued on 6/5/1995, has a sinking fund schedule that begins on 6/1/2006. Each year the issuer must retire \$7.5 million.

Generally, the issuer may satisfy the sinking fund requirement by either (1) making a cash payment to the trustee equal to the par value of the bonds to be retired; the trustee then calls

the bonds for redemption using a lottery, or (2) delivering to the trustee bonds purchased in the open market that have a total par value equal to the amount to be retired. If the bonds are retired using the first method, interest payments stop at the redemption date.

Usually, the periodic payments required for a sinking fund requirement are the same for each period. Selected issues may permit variable periodic payments, where payments change according to certain prescribed conditions set forth in the indenture. Many bond issue indentures include a provision that grants the issuer the option to retire more than the sinking fund requirement. This is referred to as an **accelerated sinking fund provision**. For example, the Anheuser-Busch 8 $\frac{5}{8}$ s due 12/1/2016, whose call schedule was presented earlier, has a sinking fund requirement of \$7.5 million each year beginning on 12/01/1997. The issuer is permitted to retire up to \$15 million each year.

Usually the sinking fund call price is the par value if the bonds were originally sold at par. When issued at a premium, the call price generally starts at the issuance price and scales down to par as the issue approaches maturity.

VII. CONVERSION PRIVILEGE

A **convertible bond** is an issue that grants the bondholder the right to convert the bond for a specified number of shares of common stock. Such a feature allows the bondholder to take advantage of favorable movements in the price of the issuer's common stock. An **exchangeable bond** allows the bondholder to exchange the issue for a specified number of shares of common stock of a corporation different from the issuer of the bond. These bonds are discussed later where a framework for analyzing them is also provided.

VIII. PUT PROVISION

An issue with a **put provision** included in the indenture grants the bondholder the right to sell the issue back to the issuer at a specified price on designated dates. The specified price is called the **put price**. Typically, a bond is puttable at par if it is issued at or close to par value. For a zero-coupon bond, the put price is below par.

The advantage of a put provision to the bondholder is that if, after the issuance date, market rates rise above the issue's coupon rate, the bondholder can force the issuer to redeem the bond at the put price and then reinvest the put bond proceeds at the prevailing higher rate.

IX. CURRENCY DENOMINATION

The payments that the issuer makes to the bondholder can be in any currency. For bonds issued in the United States, the issuer typically makes coupon payments and principal repayments in U.S. dollars. However, there is nothing that forces the issuer to make payments in U.S. dollars. The indenture can specify that the issuer may make payments in some other specified currency.

An issue in which payments to bondholders are in U.S. dollars is called a **dollar-denominated issue**. A **nondollar-denominated issue** is one in which payments are not denominated in U.S. dollars. There are some issues whose coupon payments are in one currency and whose principal payment is in another currency. An issue with this characteristic is called a **dual-currency issue**.

X. EMBEDDED OPTIONS

As we have seen, it is common for a bond issue to include a provision in the indenture that gives the issuer and/or the bondholder an option to take some action against the other party. These options are referred to as **embedded options** to distinguish them from stand alone options (i.e., options that can be purchased on an exchange or in the over-the-counter market). They are referred to as embedded options because the option is embedded in the issue. In fact, there may be more than one embedded option in an issue.

A. Embedded Options Granted to Issuers

The most common embedded options that are granted to issuers or borrowers discussed in the previous section include:

- the right to call the issue
- the right of the underlying borrowers in a pool of loans to prepay principal above the scheduled principal payment
- the accelerated sinking fund provision
- the cap on a floater

The accelerated sinking fund provision is an embedded option because the issuer can call more than is necessary to meet the sinking fund requirement. An issuer usually takes this action when interest rates decline below the issue's coupon rate even if there are other restrictions in the issue that prevent the issue from being called.

The cap of a floater can be thought of as an option requiring no action by the issuer to take advantage of a rise in interest rates. Effectively, the bondholder has granted to the issuer the right not to pay more than the cap.

Notice that whether or not the first three options are exercised by the issuer or borrower depends on the level of interest rates prevailing in the market relative to the issue's coupon rate or the borrowing rate of the underlying loans (in the case of mortgage-backed and asset-backed securities). These options become more valuable when interest rates fall. The cap of a floater also depends on the prevailing level of rates. But here the option becomes more valuable when interest rates rise.

B. Embedded Options Granted to Bondholders

The most common embedded options granted to bondholders are:

- conversion privilege
- the right to put the issue
- floor on a floater

The value of the conversion privilege depends on the market price of the stock relative to the embedded purchase price held by the bondholder when exercising the conversion option. The put privilege benefits the bondholder if interest rates rise above the issue's coupon rate. While a cap on a floater benefits the issuer if interest rates rise, a floor benefits the bondholder if interest rates fall since it fixes a minimum coupon rate payable.

C. Importance of Understanding Embedded Options

At the outset of this chapter, we stated that fixed income securities have become more complex. One reason for this increased complexity is that embedded options make it more difficult to project the cash flows of a security. The cash flow for a fixed income security is defined as its interest and the principal payments.

To value a fixed income security with embedded options, it is necessary to:

1. model the factors that determine whether or not an embedded option will be exercised over the life of the security, and
2. in the case of options granted to the issuer/borrower, model the behavior of issuers and borrowers to determine the conditions necessary for them to exercise an embedded option.

For example, consider a callable bond issued by a corporation. Projecting the cash flow requires (1) modeling interest rates (over the life of the security) at which the issuer can refund an issue and (2) developing a rule for determining the economic conditions necessary for the issuer to benefit from calling the issue. In the case of mortgage-backed or asset-backed securities, again it is necessary to model how interest rates will influence borrowers to refinance their loan over the life of the security. Models for valuing bonds with embedded options will be covered in Chapter 9.

It cannot be overemphasized that embedded options affect not only the value of a bond but also the total return of a bond. In the next chapter, the risks associated with the presence of an embedded option will be explained. What is critical to understand is that due to the presence of embedded options it is necessary to develop models of interest rate movements and rules for exercising embedded options. Any analysis of securities with embedded options exposes an investor to **modeling risk**. Modeling risk is the risk that the model analyzing embedded options produces the wrong value because the assumptions are not correct or the assumptions were not realized. This risk will become clearer when we describe models for valuing bonds with embedded options.

XI. BORROWING FUNDS TO PURCHASE BONDS

In later chapters, we will discuss investment strategies an investor uses to borrow funds to purchase securities. The expectation of the investor is that the return earned by investing in the securities purchased with the borrowed funds will exceed the borrowing cost. There are several sources of funds available to an investor when borrowing funds. When securities are purchased with borrowed funds, the most common practice is to use the securities as collateral for the loan. In such instances, the transaction is referred to as a **collateralized loan**. Two collateralized borrowing arrangements are used by investors—margin buying and repurchase agreements.

A. Margin Buying

In a **margin buying arrangement**, the funds borrowed to buy the securities are provided by the broker and the broker gets the money from a bank. The interest rate banks charge brokers for these transactions is called the call money rate (or broker loan rate). The broker charges

the investor the call money rate plus a service charge. The broker is not free to lend as much as it wishes to the investor to buy securities. In the United States, the Securities and Exchange Act of 1934 prohibits brokers from lending more than a specified percentage of the market value of the securities. The 1934 Act gives the Board of Governors of the Federal Reserve the responsibility to set initial margin requirements, which it does under Regulations T and U. While margin buying is the most common collateralized borrowing arrangement for common stock investors (both retail investors and institutional investors) and retail bond investors (i.e., individual investors), it is not the common for institutional bond investors.

B. Repurchase Agreement

The collateralized borrowing arrangement used by institutional investors in the bond market is the repurchase agreement. We will discuss this arrangement in more detail later. However, it is important to understand the basics of the repurchase agreement because it affects how some bonds in the market are valued.

A **repurchase agreement** is the sale of a security with a commitment by the seller to buy the same security back from the purchaser at a specified price at a designated future date. The **repurchase price** is the price at which the seller and the buyer agree that the seller will repurchase the security on a specified future date called the repurchase date. The difference between the repurchase price and the sale price is the dollar interest cost of the loan; based on the dollar interest cost, the sales price, and the length of the repurchase agreement, an implied interest rate can be computed. This implied interest rate is called the **repo rate**. The advantage to the investor of using this borrowing arrangement is that the interest rate is less than the cost of bank financing. When the term of the loan is one day, it is called an **overnight repo** (or overnight RP); a loan for more than one day is called a **term repo** (or term RP). As will be explained, there is not one repo rate. The rate varies from transaction to transaction depending on a variety of factors.

RISKS ASSOCIATED WITH INVESTING IN BONDS

I. INTRODUCTION

Armed with an understanding of the basic features of bonds, we now turn to the risks associated with investing in bonds. These risks include:

- interest rate risk
- call and prepayment risk
- yield curve risk
- reinvestment risk
- credit risk
- liquidity risk
- exchange-rate risk
- volatility risk
- inflation or purchasing power risk
- event risk
- sovereign risk

We will see how features of a bond that we described in Chapter 1—coupon rate, maturity, embedded options, and currency denomination—affect several of these risks.

II. INTEREST RATE RISK

As we will demonstrate in Chapter 5, the price of a typical bond will change in the opposite direction to the change in interest rates or yields.¹ That is, when interest rates rise, a bond's price will fall; when interest rates fall, a bond's price will rise. For example, consider a 6% 20-year bond. If the yield investors require to buy this bond is 6%, the price of this bond would be \$100. However, if the required yield increased to 6.5%, the price of this bond would decline to \$94.4479. Thus, for a 50 basis point increase in yield, the bond's price declines by 5.55%. If, instead, the yield declines from 6% to 5.5%, the bond's price will rise by 6.02% to \$106.0195.

¹At this stage, we will use the terms interest rate and yield interchangeably. We'll see in Chapter 6 how to compute a bond's yield.

Since the price of a bond fluctuates with market interest rates, the risk that an investor faces is that the price of a bond held in a portfolio will decline if market interest rates rise. This risk is referred to as **interest rate risk** and is the major risk faced by investors in the bond market.

A. Reason for the Inverse Relationship between Changes in Interest Rates and Price

The reason for this inverse relationship between a bond's price change and the change in interest rates (or change in market yields) is as follows. Suppose investor X purchases our hypothetical 6% coupon 20-year bond at a price equal to par (100). As explained in Chapter 6, the yield for this bond is 6%. Suppose that immediately after the purchase of this bond two things happen. First, market interest rates rise to 6.50% so that if a bond issuer wishes to sell a bond priced at par, it will require a 6.50% coupon rate to attract investors to purchase the bond. Second, suppose investor X wants to sell the bond with a 6% coupon rate. In attempting to sell the bond, investor X would not find an investor who would be willing to pay par value for a bond with a coupon rate of 6%. The reason is that any investor who wanted to purchase this bond could obtain a similar 20-year bond with a coupon rate 50 basis points higher, 6.5%.

What can the investor do? The investor cannot force the issuer to change the coupon rate to 6.5%. Nor can the investor force the issuer to shorten the maturity of the bond to a point where a new investor might be willing to accept a 6% coupon rate. The only thing that the investor can do is adjust the price of the bond to a new price where a buyer would realize a yield of 6.5%. This means that the price would have to be adjusted down to a price below par. It turns out, the new price must be 94.4479.² While we assumed in our illustration an initial price of par value, the principle holds for any purchase price. Regardless of the price that an investor pays for a bond, an instantaneous increase in market interest rates will result in a decline in a bond's price.

Suppose that instead of a rise in market interest rates to 6.5%, interest rates decline to 5.5%. Investors would be more than happy to purchase the 6% coupon 20-year bond at par. However, investor X realizes that the market is only offering investors the opportunity to buy a similar bond at par with a coupon rate of 5.5%. Consequently, investor X will increase the price of the bond until it offers a yield of 5.5%. That price turns out to be 106.0195.

Let's summarize the important relationships suggested by our example.

1. A bond will trade at a price equal to par when the coupon rate is equal to the yield required by market. That is,³

$$\text{coupon rate} = \text{yield required by market} \rightarrow \text{price} = \text{par value}$$

2. A bond will trade at a price below par (sell at a discount) or above par (sell at a premium) if the coupon rate is different from the yield required by the market. Specifically,

$$\text{coupon rate} < \text{yield required by market} \rightarrow \text{price} < \text{par value (discount)}$$

$$\text{coupon rate} > \text{yield required by market} \rightarrow \text{price} > \text{par value (premium)}$$

²We'll see how to compute the price of a bond in Chapter 5.

³The arrow symbol in the expressions means "therefore."

3. The price of a bond changes in the opposite direction to the change in interest rates. So, for an instantaneous change in interest rates the following relationship holds:

if interest rates increase $\rightarrow$ price of a bond decreases

if interest rates decrease $\rightarrow$ price of a bond increases

B. Bond Features that Affect Interest Rate Risk

A bond's price sensitivity to changes in market interest rates (i.e., a bond's interest rate risk) depends on various features of the issue, such as maturity, coupon rate, and embedded options.⁴ While we discuss these features in more detail in Chapter 7, we provide a brief discussion below.

1. The Impact of Maturity All other factors constant, *the longer the bond's maturity, the greater the bond's price sensitivity to changes in interest rates*. For example, we know that for a 6% 20-year bond selling to yield 6%, a rise in the yield required by investors to 6.5% will cause the bond's price to decline from 100 to 94.4479, a 5.55% price decline. Similarly for a 6% 5-year bond selling to yield 6%, the price is 100. A rise in the yield required by investors from 6% to 6.5% would decrease the price to 97.8944. The decline in the bond's price is only 2.11%.

2. The Impact of Coupon Rate A property of a bond is that all other factors constant, *the lower the coupon rate, the greater the bond's price sensitivity to changes in interest rates*. For example, consider a 9% 20-year bond selling to yield 6%. The price of this bond would be 134.6722. If the yield required by investors increases by 50 basis points to 6.5%, the price of this bond would fall by 5.13% to 127.7605. This decline is less than the 5.55% decline for the 6% 20-year bond selling to yield 6% discussed above.

An implication is that zero-coupon bonds have greater price sensitivity to interest rate changes than same-maturity bonds bearing a coupon rate and trading at the same yield.

3. The Impact of Embedded Options In Chapter 1, we discussed the various embedded options that may be included in a bond issue. As we continue our study of fixed income analysis, we will see that the value of a bond with embedded options will change depending on how the value of the embedded options changes when interest rates change. For example, we will see that as interest rates decline, the price of a callable bond may not increase as much as an otherwise option-free bond (that is, a bond with no embedded options).

For now, to understand why, let's decompose the price of a callable bond into two components, as shown below:

$$\text{price of callable bond} = \text{price of option-free bond} - \text{price of embedded call option}$$

The reason for subtracting the price of the embedded call option from the price of the option-free bond is that the call option is a benefit to the issuer and a disadvantage to the bondholder. This reduces the price of a callable bond relative to an option-free bond.

⁴Recall from Chapter 1 that an embedded option is the feature in a bond issue that grants either the issuer or the investor an option. Examples include call option, put option, and conversion option.

Now, when interest rates decline, the price of an option-free bond increases. However, the price of the embedded call option in a callable bond also increases because the call option becomes more valuable to the issuer. So, when interest rates decline both price components increase in value, *but* the change in the price of the callable bond depends on the relative price change between the two components. Typically, a decline in interest rates will result in an increase in the price of the callable bond but not by as much as the price change of an otherwise comparable option-free bond.

Similarly, when interest rates rise, the price of a callable bond will not fall as much as an otherwise option-free bond. The reason is that the price of the embedded call option declines. So, when interest rates rise, the price of the option-free bond declines but this is partially offset by the decrease in the price of the embedded call option component.

C. The Impact of the Yield Level

Because of credit risk (discussed later), different bonds trade at different yields, even if they have the same coupon rate, maturity, and embedded options. How, then, holding other factors constant, does the level of interest rates affect a bond's price sensitivity to changes in interest rates? As it turns out, the higher a bond's yield, the lower the price sensitivity.

To see this, we compare a 6% 20-year bond initially selling at a yield of 6%, and a 6% 20-year bond initially selling at a yield of 10%. The former is initially at a price of 100, and the latter 65.68. Now, if the yield for both bonds increases by 100 basis points, the first bond trades down by 10.68 points (10.68%) to a price of 89.32. The second bond will trade down to a price of 59.88, for a price decline of only 5.80 points (or 8.83%). Thus, we see that the bond that trades at a lower yield is more volatile in both percentage price change and absolute price change, as long as the other bond characteristics are the same. An implication of this is that, for a given change in interest rates, price sensitivity is lower when the level of interest rates in the market is high, and price sensitivity is higher when the level of interest rates is low.

D. Interest Rate Risk for Floating-Rate Securities

The change in the price of a fixed-rate coupon bond when market interest rates change is due to the fact that the bond's coupon rate differs from the prevailing market interest rate. For a floating-rate security, the coupon rate is reset periodically based on the prevailing market interest rate used as the reference rate plus a quoted margin. The quoted margin is set for the life of the security. The price of a floating-rate security will fluctuate depending on three factors.

First, the longer the time to the next coupon reset date, the greater the potential price fluctuation.⁵ For example, consider a floating-rate security whose coupon resets every six months and suppose the coupon formula is the 6-month Treasury rate plus 20 basis points. Suppose that on the coupon reset date the 6-month Treasury rate is 5.8%. If on the day after the coupon reset date, the 6-month Treasury rate rises to 6.1%, this security is paying a 6-month coupon rate that is less than the prevailing 6-month rate for the next six months. The price of the security must decline to reflect this lower coupon rate. Suppose instead that the coupon resets every month at the 1-month Treasury rate and that this rate rises immediately

⁵As explained in Chapter 1, the coupon reset formula is set at the reset date at the beginning of the period but is not paid until the end of the period.

after the coupon rate is reset. In this case, while the investor would be realizing a sub-market 1-month coupon rate, it is only for one month. The one month coupon bond's price decline will be less than the six month coupon bond's price decline.

The second reason why a floating-rate security's price will fluctuate is that the required margin that investors demand in the market changes. For example, consider once again the security whose coupon formula is the 6-month Treasury rate plus 20 basis points. If market conditions change such that investors want a margin of 30 basis points rather than 20 basis points, this security would be offering a coupon rate that is 10 basis points below the market rate. As a result, the security's price will decline.

Finally, a floating-rate security will typically have a cap. Once the coupon rate as specified by the coupon reset formula rises above the cap rate, the coupon will be set at the cap rate and the security will then offer a below-market coupon rate and its price will decline. In fact, once the cap is reached, the security's price will react much the same way to changes in market interest rates as that of a fixed-rate coupon security. This risk for a floating-rate security is called **cap risk**.

E. Measuring Interest Rate Risk

Investors are interested in estimating the price sensitivity of a bond to changes in market interest rates. We will spend a good deal of time looking at how to quantify a bond's interest rate risk in Chapter 7, as well as other chapters. For now, let's see how we can get a rough idea of how to quantify the interest rate risk of a bond.

What we are interested in is a first approximation of how a bond's price will change when interest rates change. We can look at the price change in terms of (1) the percentage price change from the initial price or (2) the dollar price change from the initial price.

1. Approximate Percentage Price Change The most straightforward way to calculate the percentage price change is to average the percentage price change resulting from an increase and a decrease in interest rates of the same number of basis points. For example, suppose that we are trying to estimate the sensitivity of the price of bond ABC that is currently selling for 90 to yield 6%. Now, suppose that interest rates increase by 25 basis points from 6% to 6.25%. The change in yield of 25 basis points is referred to as the "rate shock." The question is, how much will the price of bond ABC change due to this rate shock? To determine what the new price will be if the yield increases to 6.25%, *it is necessary to have a valuation model*. A valuation model provides an estimate of what the value of a bond will be for a given yield level. We will discuss the various models for valuing simple bonds and complex bonds with embedded options in later chapters.

For now, we will assume that the valuation model tells us that the price of bond ABC will be 88 if the yield is 6.25%. This means that the price will decline by 2 points or 2.22% of the initial price of 90. If we divide the 2.22% by 25 basis points, the resulting number tells us that the price will decline by 0.0889% per 1 basis point change in yield.

Now suppose that the valuation model tells us that if yields decline from 6% to 5.75%, the price will increase to 92.7. This means that the price increases by 2.7 points or 3.00% of the initial price of 90. Dividing the 3.00% by 25 basis points indicates that the price will change by 0.1200% per 1 basis point change in yield.

We can average the two percentage price changes for a 1 basis point change in yield up and down. The average percentage price change is 0.1044% [= (0.0889% + 0.1200%)/2].

This means that for a 100 basis point change in yield, the average percentage price change is 10.44% (100 times 0.1044%).

A formula for estimating the *approximate percentage price change for a 100 basis point change in yield* is:

$$\frac{\text{price if yields decline} - \text{price if yields rise}}{2 \times (\text{initial price}) \times (\text{change in yield in decimal})}$$

In our illustration,

price if yields decline by 25 basis points = 92.7

price if yields rise by 25 basis points = 88.0

initial price = 90

change in yield in decimal = 0.0025

Substituting these values into the formula we obtain the approximate percentage price change for a 100 basis point change in yield to be:

$$\frac{92.7 - 88.0}{2 \times (90) \times (0.0025)} = 10.44$$

There is a special name given to this estimate of the percentage price change for a 100 basis point change in yield. It is called **duration**. As can be seen, duration is a measure of the price sensitivity of a bond to a change in yield. So, for example, if the duration of a bond is 10.44, this means that the approximate percentage price change if yields change by 100 basis points is 10.44%. For a 50 basis point change in yields, the approximate percentage price change is 5.22% (10.44% divided by 2). For a 25 basis point change in yield, the approximate percentage price change is 2.61% (10.44% divided by 4).

Notice that the approximate percentage is assumed to be the same for a rise and decline in yield. When we discuss the properties of the price volatility of a bond to changes in yield in Chapter 7, we will see that the percentage price change is not symmetric and we will discuss the implication for using duration as a measure of interest rate risk. *It is important to note that the computed duration of a bond is only as good as the valuation model used to get the prices when the yield is shocked up and down. If the valuation model is unreliable, then the duration is a poor measure of the bond's price sensitivity to changes in yield.*

2. Approximating the Dollar Price Change It is simple to move from duration, which measures the approximate percentage price change, to the approximate dollar price change of a position in a bond given the market value of the position and its duration. For example, consider again bond ABC with a duration of 10.44. Suppose that the market value of this bond is \$5 million. Then for a 100 basis point change in yield, the approximate dollar price change is equal to 10.44% times \$5 million, or \$522,000. For a 50 basis point change in yield, the approximate dollar price change is \$261,000; for a 25 basis point change in yield the approximate dollar price change is \$130,500.

The approximate dollar price change for a 100 basis point change in yield is sometimes referred to as the **dollar duration**.

III. YIELD CURVE RISK

We know that if interest rates or yields in the market change, the price of a bond will change. One of the factors that will affect how sensitive a bond's price is to changes in yield is the bond's maturity. A portfolio of bonds is a collection of bond issues typically with different maturities. So, when interest rates change, the price of each bond issue in the portfolio will change and the portfolio's value will change.

As you will see in Chapter 4, there is not one interest rate or yield in the economy. There is a structure of interest rates. One important structure is the relationship between yield and maturity. The graphical depiction of this relationship is called the **yield curve**. As we will see in Chapter 4, when interest rates change, they typically do not change by an equal number of basis points for all maturities.

For example, suppose that a \$65 million portfolio contains the four bonds shown in Exhibit 1. All bonds are trading at a price equal to par value.

If we want to know how much the value of the portfolio changes if interest rates change, typically it is assumed that all yields change by the same number of basis points. Thus, if we wanted to know how sensitive the portfolio's value is to a 25 basis point change in yields, we would increase the yield of the four bond issues by 25 basis points, determine the new price of each bond, the market value of each bond, and the new value of the portfolio. Panel (a) of Exhibit 2 illustrates the 25 basis point increase in yield. For our hypothetical portfolio, the value of each bond issue changes as shown in panel (a) of Exhibit 1. The portfolio's value decreases by \$1,759,003 from \$65 million to \$63,240,997.

Suppose that, instead of an equal basis point change in the yield for all maturities, the 20-year yield changes by 25 basis points, but the yields for the other maturities change as follows: (1) 2-year maturity changes by 10 basis points (from 5% to 5.1%), (2) 5-year maturity changes by 20 basis points (from 5.25% to 5.45%), and (3) 30-year maturity changes by 45 basis points (from 5.75% to 6.2%). Panel (b) of Exhibit 2 illustrates these yield changes. We will see in later chapters that this type of movement (or shift) in the yield curve is referred to as a "steepening of the yield curve." For this type of yield curve shift, the portfolio's value is shown in panel (b) of Exhibit 1. The decline in the portfolio's value is \$2,514,375 (from \$65 million to \$62,485,625).

Suppose, instead, that if the 20-year yield changes by 25 basis points, the yields for the other three maturities change as follows: (1) 2-year maturity changes by 5 basis points (from 5% to 5.05%), (2) 5-year maturity changes by 15 basis points (from 5.25% to 5.40%), and (3) 30-year maturity changes by 35 basis points (from 5.75% to 6.1%). Panel (c) of Exhibit 2 illustrates this shift in yields. The new value for the portfolio based on this yield curve shift is shown in panel (c) of Exhibit 1. The decline in the portfolio's value is \$2,096,926 (from \$65 million to \$62,903,074). The yield curve shift in the third illustration does not steepen as much as in the second, when the yield curve steepens considerably.

The point here is that portfolios have different exposures to how the yield curve shifts. This risk exposure is called **yield curve risk**. The implication is that any measure of interest rate risk that assumes that the interest rates change by an equal number of basis points for all maturities (referred to as a "parallel yield curve shift") is only an approximation.

This applies to the duration concept that we discussed above. We stated that the duration for an individual bond is the approximate percentage change in price for a 100 basis point change in yield. A duration for a portfolio has the same meaning: it is the approximate percentage change in the portfolio's value for a 100 basis point change in the yield *for all maturities*.

EXHIBIT 1 Illustration of Yield Curve Risk

Composition of the Portfolio

Bond	Coupon (%)	Maturity (years)	Yield (%)	Par value (\$)
A	5.00	2	5.00	5,000,000
B	5.25	5	5.25	10,000,000
C	5.50	20	5.50	20,000,000
D	5.75	30	5.75	30,000,000
Total				65,000,000

a. Parallel Shift in Yield Curve of + 25 Basis Points

Bond	Coupon (%)	Maturity (years)	Original yield (%)	Par value (\$)	New yield (%)	New bond price	Value
A	5.00	2	5.00	5,000,000	5.25	99.5312	4,976,558
B	5.25	5	5.25	10,000,000	5.50	98.9200	9,891,999
C	5.50	20	5.50	20,000,000	5.75	97.0514	19,410,274
D	5.75	30	5.75	30,000,000	6.00	96.5406	28,962,166
Total				65,000,000			63,240,997

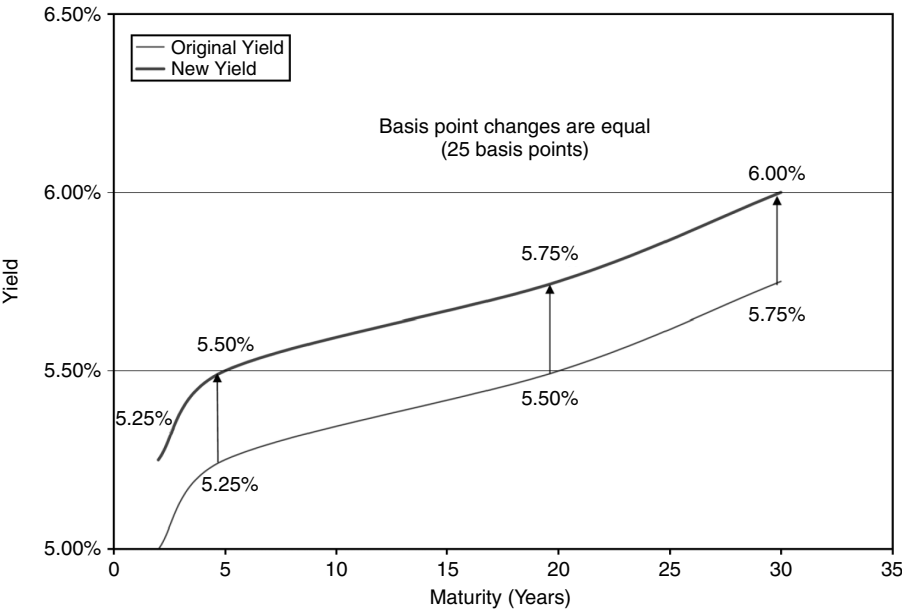
b. Nonparallel Shift of the Yield Curve

Bond	Coupon (%)	Maturity (years)	Original yield (%)	Par value (\$)	New yield (%)	New bond price	Value
A	5.00	2	5.00	5,000,000	5.10	99.8121	4,990,606
B	5.25	5	5.25	10,000,000	5.45	99.1349	9,913,488
C	5.50	20	5.50	20,000,000	5.75	97.0514	19,410,274
D	5.75	30	5.75	30,000,000	6.20	93.9042	28,171,257
Total				65,000,000			62,485,625

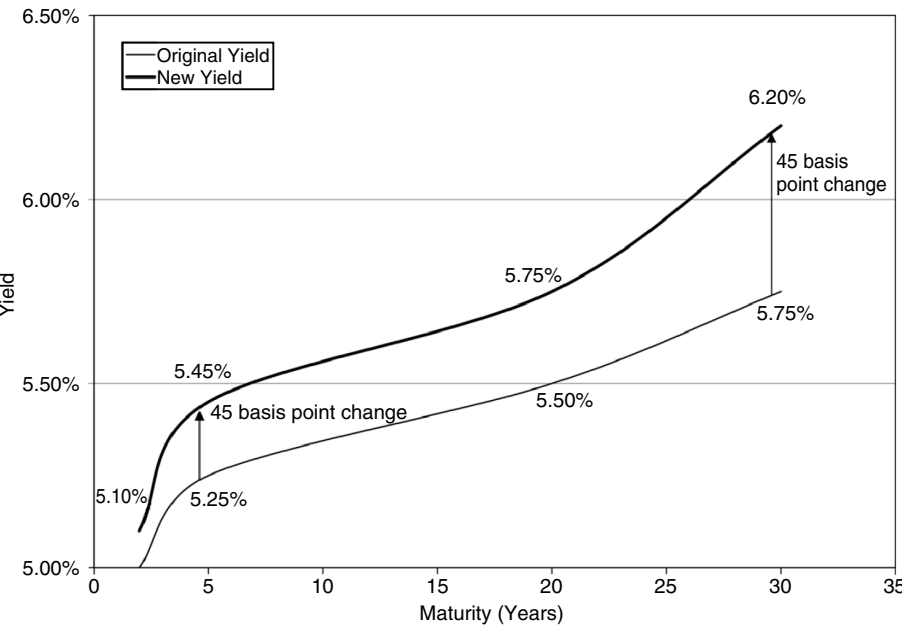
c. Nonparallel Shift of the Yield Curve

Bond	Coupon (%)	Maturity (years)	Original yield (%)	Par value (\$)	New yield (%)	New bond price	Value
A	5.00	2	5.00	5,000,000	5.05	99.9060	4,995,300
B	5.25	5	5.25	10,000,000	5.40	99.3503	9,935,033
C	5.50	20	5.50	20,000,000	5.75	97.0514	19,410,274
D	5.75	30	5.75	30,000,000	6.10	95.2082	28,562,467
Total				65,000,000			62,903,074

EXHIBIT 2 Shift in the Yield Curve

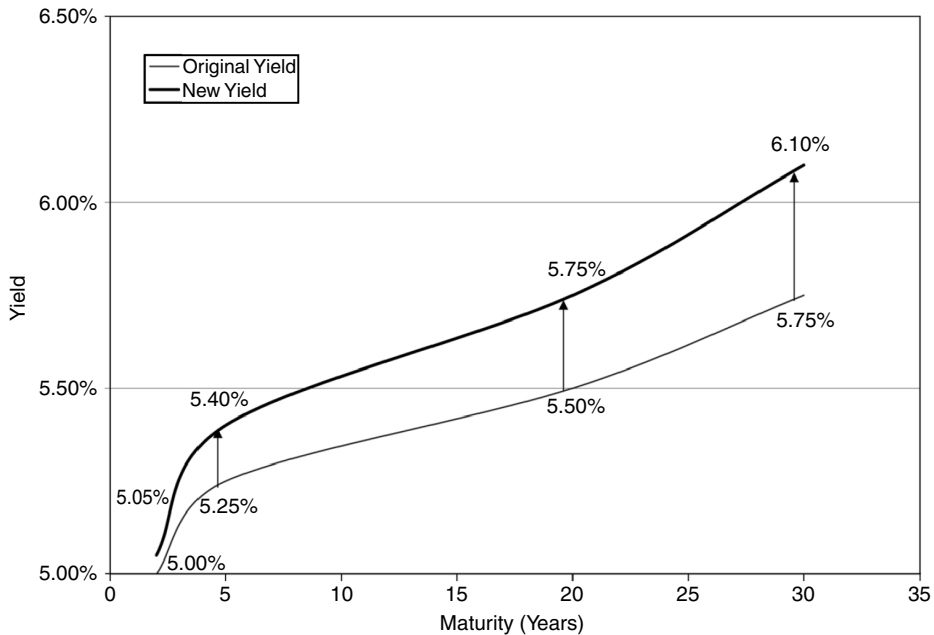


(a) Parallel Shift in the Yield Curve of +25 Basis Points



(b) Nonparallel Shift in the Yield Curve

EXHIBIT 2 (Continued)



(C) Another Nonparallel Shift in the Yield Curve

Because of the importance of yield curve risk, a good number of measures have been formulated to try to estimate the exposure of a portfolio to a non-parallel shift in the yield curve. We defer a discussion of these measures until Chapter 7. However, we introduce one basic but popular approach here. In the next chapter, we will see that the yield curve is a series of yields, one for each maturity. It is possible to determine the percentage change in the value of a portfolio if only one maturity's yield changes while the yield for all other maturities is unchanged. This is a form of duration called **rate duration**, where the word "rate" means the interest rate of a particular maturity. So, for example, suppose a portfolio consists of 40 bonds with different maturities. A "5-year rate duration" of 2 would mean that the portfolio's value will change by approximately 2% for a 100 basis point change in the 5-year yield, assuming all other rates do not change.

Consequently, in theory, there is not one rate duration but a rate duration for each maturity. In practice, a rate duration is not computed for all maturities. Instead, the rate duration is computed for several key maturities on the yield curve and this is referred to as **key rate duration**. Key rate duration is therefore simply the rate duration with respect to a change in a "key" maturity sector. Vendors of analytical systems report key rate durations for the maturities that in their view are the key maturity sectors. Key rate duration will be discussed further later.

IV. CALL AND PREPAYMENT RISK

As explained in Chapter 1, a bond may include a provision that allows the issuer to retire, or call, all or part of the issue before the maturity date. From the investor's perspective, there are three disadvantages to call provisions:

Disadvantage 1: The cash flow pattern of a callable bond is not known with certainty because it is not known when the bond will be called.

Disadvantage 2: Because the issuer is likely to call the bonds when interest rates have declined below the bond's coupon rate, the investor is exposed to reinvestment risk, i.e., the investor will have to reinvest the proceeds when the bond is called at interest rates lower than the bond's coupon rate.

Disadvantage 3: The price appreciation potential of the bond will be reduced relative to an otherwise comparable option-free bond. (This is called **price compression**.)

We explained the third disadvantage in Section II when we discussed how the price of a callable bond may not rise as much as an otherwise comparable option-free bond when interest rates decline.

Because of these three disadvantages faced by the investor, a callable bond is said to expose the investor to **call risk**. The same disadvantages apply to mortgage-backed and asset-backed securities where the borrower can prepay principal prior to scheduled principal payment dates. This risk is referred to as **prepayment risk**.

V. REINVESTMENT RISK

Reinvestment risk is the risk that the proceeds received from the payment of interest and principal (i.e., scheduled payments, called proceeds, and principal prepayments) that are available for reinvestment must be reinvested at a lower interest rate than the security that generated the proceeds. We already saw how reinvestment risk is present when an investor purchases a callable or principal prepayable bond. When the issuer calls a bond, it is typically done to lower the issuer's interest expense because interest rates have declined after the bond is issued. The investor faces the problem of having to reinvest the called bond proceeds received from the issuer in a lower interest rate environment.

Reinvestment risk also occurs when an investor purchases a bond and relies on the yield of that bond as a measure of return. We have not yet explained how to compute the "yield" for a bond. When we do, it will be demonstrated that for the yield computed at the time of purchase to be realized, the investor must be able to reinvest any coupon payments at the computed yield. So, for example, if an investor purchases a 20-year bond with a yield of 6%, to realize the yield of 6%, every time a coupon interest payment is made, it is necessary to reinvest the payment at an interest rate of at 6% until maturity. So, it is assumed that the first coupon payment can be reinvested for the next 19.5 years at 6%; the second coupon payment can be reinvested for the next 19 years at 6%, and so on. The risk that the coupon payments will be reinvested at less than 6% is also reinvestment risk.

When dealing with amortizing securities (i.e., securities that repay principal periodically), reinvestment risk is even greater. Typically, amortizing securities pay interest and principal monthly and permit the borrower to prepay principal prior to schedule payment dates. Now the investor is more concerned with reinvestment risk due to principal prepayments usually resulting from a decline in interest rates, just as in the case of a callable bond. However, since payments are monthly, the investor has to make sure that the interest and principal can be reinvested at no less than the computed yield every month as opposed to semiannually.

This reinvestment risk for an amortizing security is important to understand. Too often it is said by some market participants that securities that pay both interest and principal monthly

are advantageous because the investor has the opportunity to reinvest more frequently and to reinvest a larger amount (because principal is received) relative to a bond that pays only semiannual coupon payments. This is not the case in a declining interest rate environment, which will cause borrowers to accelerate their principal prepayments and force the investor to reinvest at lower interest rates.

With an understanding of reinvestment risk, we can now appreciate why zero-coupon bonds may be attractive to certain investors. Because there are no coupon payments to reinvest, there is no reinvestment risk. That is, zero-coupon bonds eliminate reinvestment risk. Elimination of reinvestment risk is important to some investors. That's the plus side of the risk equation. The minus side is that, as explained in Section II, the lower the coupon rate the greater the interest rate risk for two bonds with the same maturity. Thus, zero-coupon bonds of a given maturity expose investors to the greatest interest rate risk.

Once we cover our basic analytical tools in later chapters, we will see how to quantify a bond issue's reinvestment risk.

VI. CREDIT RISK

An investor who lends funds by purchasing a bond issue is exposed to **credit risk**. There are three types of credit risk:

1. default risk
2. credit spread risk
3. downgrade risk

We discuss each type below.

A. Default Risk

Default risk is defined as the risk that the issuer will fail to satisfy the terms of the obligation with respect to the timely payment of interest and principal.

Studies have examined the probability of issuers defaulting. The percentage of a population of bonds that is expected to default is called the **default rate**. If a default occurs, this does not mean the investor loses the entire amount invested. An investor can expect to recover a certain percentage of the investment. This is called the **recovery rate**. Given the default rate and the recovery rate, the estimated expected loss due to a default can be computed. We will explain the findings of studies on default rates and recovery rates in Chapter 3.

B. Credit Spread Risk

Even in the absence of default, an investor is concerned that the market value of a bond will decline and/or the price performance of a bond will be worse than that of other bonds. To understand this, recall that the price of a bond changes in the opposite direction to the change in the yield required by the market. Thus, if yields in the economy increase, the price of a bond declines, and vice versa.

As we will see in Chapter 3, the yield on a bond is made up of two components: (1) the yield on a similar default-free bond issue and (2) a premium above the yield on a default-free bond issue necessary to compensate for the risks associated with the bond. The risk premium

is referred to as a **yield spread**. In the United States, Treasury issues are the benchmark yields because they are believed to be default free, they are highly liquid, and they are not callable (with the exception of some old issues). The part of the risk premium or yield spread attributable to default risk is called the **credit spread**.

The price performance of a non-Treasury bond issue and the return over some time period will depend on how the credit spread changes. If the credit spread increases, investors say that the spread has “widened” and the market price of the bond issue will decline (assuming U.S. Treasury rates have not changed). The risk that an issuer’s debt obligation will decline due to an increase in the credit spread is called **credit spread risk**.

This risk exists for an individual issue, for issues in a particular industry or economic sector, and for all non-Treasury issues in the economy. For example, in general during economic recessions, investors are concerned that issuers will face a decline in cash flows that would be used to service their bond obligations. As a result, the credit spread tends to widen for U.S. non-Treasury issuers and the prices of all such issues throughout the economy will decline.

C. Downgrade Risk

While portfolio managers seek to allocate funds among different sectors of the bond market to capitalize on anticipated changes in credit spreads, an analyst investigating the credit quality of an individual issue is concerned with the prospects of the credit spread increasing for that particular issue. But how does the analyst assess whether he or she believes the market will change the credit spread associated with an individual issue?

One tool investors use to gauge the default risk of an issue is the credit ratings assigned to issues by rating companies, popularly referred to as **rating agencies**. There are three rating agencies in the United States: Moody’s Investors Service, Inc., Standard & Poor’s Corporation, and Fitch Ratings.

A **credit rating** is an indicator of the potential default risk associated with a particular bond issue or issuer. It represents in a simplistic way the credit rating agency’s assessment of an issuer’s ability to meet the payment of principal and interest in accordance with the terms of the indenture. Credit rating symbols or characters are uncomplicated representations of more complex ideas. In effect, they are summary opinions. Exhibit 3 identifies the ratings assigned by Moody’s, S&P, and Fitch for bonds and the meaning of each rating.

In all systems, the term **high grade** means low credit risk, or conversely, a high probability of receiving future payments is promised by the issuer. The highest-grade bonds are designated by Moody’s by the symbol Aaa, and by S&P and Fitch by the symbol AAA. The next highest grade is denoted by the symbol Aa (Moody’s) or AA (S&P and Fitch); for the third grade, all three rating companies use A. The next three grades are Baa or BBB, Ba or BB, and B, respectively. There are also C grades. Moody’s uses 1, 2, or 3 to provide a narrower credit quality breakdown within each class, and S&P and Fitch use plus and minus signs for the same purpose.

Bonds rated triple A (AAA or Aaa) are said to be **prime grade**; double A (AA or Aa) are of **high quality grade**; single A issues are called **upper medium grade**, and triple B are **lower medium grade**. Lower-rated bonds are said to have **speculative grade** elements or to be **distinctly speculative grade**.

Bond issues that are assigned a rating in the top four categories (that is, AAA, AA, A, and BBB) are referred to as **investment-grade bonds**. Issues that carry a rating below the top four categories are referred to as **non-investment-grade bonds** or **speculative bonds**, or

EXHIBIT 3 Bond Rating Symbols and Summary Description

Moody's	S&P	Fitch	Summary Description
Investment Grade—High Credit Worthiness			
Aaa	AAA	AAA	Gilt edge, prime, maximum safety
Aa1	AA+	AA+	High-grade, high-credit quality
Aa2	AA	AA	
Aa3	AA−	AA−	
A1	A+	A+	Upper-medium grade
A2	A	A	
A3	A−	A−	
Baa1	BBB+	BBB+	Lower-medium grade
Baa2	BBB	BBB	
Baa3	BBB−	BBB−	
Speculative—Lower Credit Worthiness			
Ba1	BB+	BB+	Low grade, speculative
Ba2	BB	BB	
Ba3	BB−	BB−	
B1	B	B+	Highly speculative
B2		B	
B3		B−	
Predominantly Speculative, Substantial Risk, or in Default			
Caa	CCC+	CCC+	Substantial risk, in poor standing
	CCC	CCC	
Ca	CC	CC	May be in default, very speculative
C	C	C	Extremely speculative
	CI		Income bonds—no interest being paid
	D	DDD	Default
		DD	
		D	

more popularly as **high yield bonds** or **junk bonds**. Thus, the bond market can be divided into two sectors: the investment grade and non-investment grade markets as summarized below:

Investment grade bonds	AAA, AA, A, and BBB
Non-investment grade bonds (speculative/high yield)	Below BBB

Once a credit rating is assigned to a debt obligation, a rating agency monitors the credit quality of the issuer and can reassign a different credit rating. An improvement in the credit quality of an issue or issuer is rewarded with a better credit rating, referred to as an **upgrade**; a deterioration in the credit rating of an issue or issuer is penalized by the assignment of an inferior credit rating, referred to as a **downgrade**. An unanticipated downgrading of an issue or issuer increases the credit spread and results in a decline in the price of the issue or the issuer's bonds. This risk is referred to as **downgrade risk** and is closely related to credit spread risk.

As we have explained, the credit rating is a measure of potential default risk. An analyst must be aware of how rating agencies gauge default risk for purposes of assigning ratings in order to understand the other aspects of credit risk. The agencies' assessment of potential

EXHIBIT 4 Hypothetical 1-Year Rating Transition Matrix

Rating at start of year	Rating at end of year								Total
	AAA	AA	A	BBB	BB	B	CCC	D	
AAA	93.20	6.00	0.60	0.12	0.08	0.00	0.00	0.00	100
AA	1.60	92.75	5.07	0.36	0.11	0.07	0.03	0.01	100
A	0.18	2.65	91.91	4.80	0.37	0.02	0.02	0.05	100
BBB	0.04	0.30	5.20	87.70	5.70	0.70	0.16	0.20	100
BB	0.03	0.11	0.61	6.80	81.65	7.10	2.60	1.10	100
B	0.01	0.09	0.55	0.88	7.90	75.67	8.70	6.20	100
CCC	0.00	0.01	0.31	0.84	2.30	8.10	62.54	25.90	100

default drives downgrade risk, and in turn, both default potential and credit rating changes drive credit spread risk.

A popular tool used by managers to gauge the prospects of an issue being downgraded or upgraded is a **rating transition matrix**. This is simply a table constructed by the rating agencies that shows the percentage of issues that were downgraded or upgraded in a given time period. So, the table can be used to approximate downgrade risk and default risk.

Exhibit 4 shows a hypothetical rating transition matrix for a 1-year period. The first column shows the ratings at the start of the year and the top row shows the rating at the end of the year. Let's interpret one of the numbers. Look at the cell where the rating at the beginning of the year is AA and the rating at the end of the year is AA. This cell represents the percentage of issues rated AA at the beginning of the year that did not change their rating over the year. That is, there were no downgrades or upgrades. As can be seen, 92.75% of the issues rated AA at the start of the year were rated AA at the end of the year. Now look at the cell where the rating at the beginning of the year is AA and at the end of the year is A. This shows the percentage of issues rated AA at the beginning of the year that were downgraded to A by the end of the year. In our hypothetical 1-year rating transition matrix, this percentage is 5.07%. One can view these percentages as probabilities. There is a probability that an issue rated AA will be downgraded to A by the end of the year and it is 5.07%. One can estimate total downgrade risk as well. Look at the row that shows issues rated AA at the beginning of the year. The cells in the columns A, BBB, BB, B, CCC, and D all represent downgrades from AA. Thus, if we add all of these columns in this row (5.07%, 0.36%, 0.11%, 0.07%, 0.03%, and 0.01%), we get 5.65% which is an estimate of the probability of an issue being downgraded from AA in one year. Thus, 5.65% can be viewed as an estimate of downgrade risk.

A rating transition matrix also shows the potential for upgrades. Again, using Exhibit 4 look at the row that shows issues rated AA at the beginning of the year. Looking at the cell shown in the column AAA rating at the end of the year, one finds 1.60%. This is the percentage of issues rated AA at the beginning of the year that were upgraded to AAA by the end of the year.

Finally, look at the D rating category. These are issues that go into default. We can use the information in the column with the D rating at the end of the year to estimate the probability that an issue with a particular rating will go into default at the end of the year. Hence, this would be an estimate of default risk. So, for example, the probability that an issue rated AA at the beginning of the year will go into default by the end of the year is 0.01%. In contrast, the probability of an issue rated CCC at the beginning of the year will go into default by the end of the year is 25.9%.

VII. LIQUIDITY RISK

When an investor wants to sell a bond prior to the maturity date, he or she is concerned with whether or not the bid price from broker/dealers is close to the indicated value of the issue. For example, if recent trades in the market for a particular issue have been between \$90 and \$90.5 and market conditions have not changed, an investor would expect to sell the bond somewhere in the \$90 to \$90.5 range.

Liquidity risk is the risk that the investor will have to sell a bond below its indicated value, where the indication is revealed by a recent transaction. The primary measure of liquidity is the size of the spread between the bid price (the price at which a dealer is willing to buy a security) and the ask price (the price at which a dealer is willing to sell a security). The wider the bid-ask spread, the greater the liquidity risk.

A liquid market can generally be defined by “small bid-ask spreads which do not materially increase for large transactions.”⁶ How to define the bid-ask spread in a multiple dealer market is subject to interpretation. For example, consider the bid-ask prices for four dealers. Each quote is for \$92 plus the number of 32nds shown in Exhibit 5. The bid-ask spread shown in the exhibit is measured relative to a specific dealer. The best bid-ask spread is for $\frac{2}{32}$ Dealers 2 and 3.

From the perspective of the overall market, the bid-ask spread can be computed by looking at the best bid price (high price at which a broker/dealer is willing to buy a security) and the lowest ask price (lowest offer price at which a broker/dealer is willing to sell the same security). This liquidity measure is called the **market bid-ask spread**. For the four dealers, the highest bid price is $92\frac{2}{32}$ and the lowest ask price is $92\frac{2}{32}$. Thus, the market bid-ask spread is $3\frac{1}{32}$.

A. Liquidity Risk and Marking Positions to Market

For investors who plan to hold a bond until maturity and need *not* mark the position to market, liquidity risk is not a major concern. An institutional investor who plans to hold an issue to maturity but is periodically marked-to-market is concerned with liquidity risk. By marking a position to market, the security is revalued in the portfolio based on its current market price. For example, mutual funds are required to mark to market at the end of each

EXHIBIT 5 Broker/Dealer Bid-Ask Spreads for a Specific Security

	Dealer			
	1	2	3	4
Bid price	1	1	2	2
Ask price	4	3	4	5

Bid-ask spread for each dealer (in 32nds):

	Dealer			
	1	2	3	4
Bid-ask spread	3	2	2	3

⁶Robert I. Gerber, “A User’s Guide to Buy-Side Bond Trading,” Chapter 16 in Frank J. Fabozzi (ed.), *Managing Fixed Income Portfolios* (New Hope, PA: Frank J. Fabozzi Associates, 1997, p. 278.)

day the investments in their portfolio in order to compute the mutual fund's net asset value (NAV). While other institutional investors may not mark-to-market as frequently as mutual funds, they are marked-to-market when reports are periodically sent to clients or the board of directors or trustees.

Where are the prices obtained to mark a position to market? Typically, a portfolio manager will solicit bids from several broker/dealers and then use some process to determine the bid price used to mark (i.e., value) the position. The less liquid the issue, the greater the variation there will be in the bid prices obtained from broker/dealers. With an issue that has little liquidity, the price may have to be determined from a pricing service (i.e., a service company that employs models to determine the fair value of a security) rather than from dealer bid prices.

In Chapter 1 we discussed the use of repurchase agreements as a form of borrowing funds to purchase bonds. The bonds purchased are used as collateral. The bonds purchased are marked-to-market periodically in order to determine whether or not the collateral provides adequate protection to the lender for funds borrowed (i.e., the dealer providing the financing). When liquidity in the market declines, a portfolio manager who has borrowed funds must rely solely on the bid prices determined by the dealer lending the funds.

B. Changes in Liquidity Risk

Bid-ask spreads, and therefore liquidity risk, change over time. Changing market liquidity is a concern to portfolio managers who are contemplating investing in new complex bond structures. Situations such as an unexpected change in interest rates might cause a widening of the bid-ask spread, as investors and dealers are reluctant to take new positions until they have had a chance to assess the new market level of interest rates.

Here is another example of where market liquidity may change. While there are opportunities for those who invest in a new type of bond structure, there are typically few dealers making a market when the structure is so new. If subsequently the new structure becomes popular, more dealers will enter the market and liquidity improves. In contrast, if the new bond structure turns out to be unappealing, the initial buyers face a market with less liquidity because some dealers exit the market and others offer bids that are unattractive because they do not want to hold the bonds for a potential new purchaser.

Thus, we see that the liquidity risk of an issue changes over time. An actual example of a change in market liquidity occurred during the Spring of 1994. One sector of the mortgage-backed securities market, called the derivative mortgage market, saw the collapse of an important investor (a hedge fund) and the resulting exit from the market of several dealers. As a result, liquidity in the market substantially declined and bid-ask spreads widened dramatically.

VIII. EXCHANGE RATE OR CURRENCY RISK

A bond whose payments are not in the domestic currency of the portfolio manager has unknown cash flows in his or her domestic currency. The cash flows in the manager's domestic currency are dependent on the exchange rate at the time the payments are received from the issuer. For example, suppose a portfolio manager's domestic currency is the U.S. dollar and that manager purchases a bond whose payments are in Japanese yen. If the yen depreciates

relative to the U.S. dollar at the time a payment is made, then fewer U.S. dollars can be exchanged.

As another example, consider a portfolio manager in the United Kingdom. This manager's domestic currency is the pound. If that manager purchases a U.S. dollar denominated bond, then the manager is concerned that the U.S. dollar will depreciate relative to the British pound when the issuer makes a payment. If the U.S. dollar does depreciate, then fewer British pounds will be received on the foreign exchange market.

The risk of receiving less of the domestic currency when investing in a bond issue that makes payments in a currency other than the manager's domestic currency is called **exchange rate risk** or **currency risk**.

IX. INFLATION OR PURCHASING POWER RISK

Inflation risk or **purchasing power risk** arises from the decline in the value of a security's cash flows due to inflation, which is measured in terms of purchasing power. For example, if an investor purchases a bond with a coupon rate of 5%, but the inflation rate is 3%, the purchasing power of the investor has not increased by 5%. Instead, the investor's purchasing power has increased by only about 2%.

For all but inflation protection bonds, an investor is exposed to inflation risk because the interest rate the issuer promises to make is fixed for the life of the issue.

X. VOLATILITY RISK

In our discussion of the impact of embedded options on the interest rate risk of a bond in Section II, we said that a change in the factors that affect the value of the embedded options will affect how the bond's price will change. Earlier, we looked at how a change in the level of interest rates will affect the price of a bond with an embedded option. But there are other factors that will affect the price of an embedded option.

While we discuss these other factors later, we can get an appreciation of one important factor from a general understanding of option pricing. A major factor affecting the value of an option is "expected volatility." In the case of an option on common stock, expected volatility refers to "expected price volatility." The relationship is as follows: the greater the expected price volatility, the greater the value of the option. The same relationship holds for options on bonds. However, instead of expected price volatility, for bonds it is the "expected yield volatility." The greater the expected yield volatility, the greater the value (price) of an option. The interpretation of yield volatility and how it is estimated are explained at in Chapter 8.

Now let us tie this into the pricing of a callable bond. We repeat the formula for the components of a callable bond below:

$$\text{Price of callable bond} = \text{Price of option-free bond} - \text{Price of embedded call option}$$

If expected yield volatility increases, holding all other factors constant, the price of the embedded call option will increase. As a result, the price of a callable bond will decrease (because the former is subtracted from the price of the option-free bond).

To see how a change in expected yield volatility affects the price of a puttable bond, we can write the price of a puttable bond as follows:

$$\text{Price of puttable bond} = \text{Price of option-free bond} + \text{Price of embedded put option}$$

A decrease in expected yield volatility reduces the price of the embedded put option and therefore will decrease the price of a puttable bond. Thus, the volatility risk of a puttable bond is that expected yield volatility will decrease.

This risk that the price of a bond with an embedded option will decline when expected yield volatility changes is called **volatility risk**. Below is a summary of the effect of changes in expected yield volatility on the price of callable and puttable bonds:

Type of embedded option	Volatility risk due to
Callable bonds	an increase in expected yield volatility
Puttable bonds	a decrease in expected yield volatility

XI. EVENT RISK

Occasionally the ability of an issuer to make interest and principal payments changes dramatically and unexpectedly because of factors including the following:

1. a natural disaster (such as an earthquake or hurricane) or an industrial accident that impairs an issuer's ability to meet its obligations
2. a takeover or corporate restructuring that impairs an issuer's ability to meet its obligations
3. a regulatory change

These factors are commonly referred to as **event risk**.

A. Corporate Takeover/Restructurings

The first type of event risk results in a credit rating downgrade of an issuer by rating agencies and is therefore a form of downgrade risk. However, downgrade risk is typically confined to the particular issuer whereas event risk from a natural disaster usually affects more than one issuer.

The second type of event risk also results in a downgrade and can also impact other issuers. An excellent example occurred in the fall of 1988 with the leveraged buyout (LBO) of RJR Nabisco, Inc. The entire industrial sector of the bond market suffered as bond market participants withdrew from the market, new issues were postponed, and secondary market activity came to a standstill as a result of the initial LBO bid announcement. The yield that investors wanted on Nabisco's bonds increased by about 250 basis points. Moreover, because the RJR LBO demonstrated that size was not an obstacle for an LBO, other large industrial firms that market participants previously thought were unlikely candidates for an LBO were fair game. The spillover effect to other industrial companies of the RJR LBO resulted in required yields' increasing dramatically.

B. Regulatory Risk

The third type of risk listed above is **regulatory risk**. This risk comes in a variety of forms. Regulated entities include investment companies, depository institutions, and insurance companies. Pension funds are regulated by ERISA. Regulation of these entities is in terms of the acceptable securities in which they may invest and/or the treatment of the securities for regulatory accounting purposes.

Changes in regulations may require a regulated entity to divest itself from certain types of investments. A flood of the divested securities on the market will adversely impact the price of similar securities.

XII. SOVEREIGN RISK

When an investor acquires a bond issued by a foreign entity (e.g., a French investor acquiring a Brazilian government bond), the investor faces **sovereign risk**. This is the risk that, as a result of actions of the foreign government, there may be either a default or an adverse price change even in the absence of a default. This is analogous to the forms of credit risk described in Section VI—credit risk spread and downgrade risk. That is, even if a foreign government does not default, actions by a foreign government can increase the credit risk spread sought by investors or increase the likelihood of a downgrade. Both of these will have an adverse impact on a bond's price.

Sovereign risk consists of two parts. First is the unwillingness of a foreign government to pay. A foreign government may simply repudiate its debt. The second is the inability to pay due to unfavorable economic conditions in the country. Historically, most foreign government defaults have been due to a government's inability to pay rather than unwillingness to pay.

OVERVIEW OF BOND SECTORS AND INSTRUMENTS

I. INTRODUCTION

Thus far we have covered the general features of bonds and the risks associated with investing in bonds. In this chapter, we will review the major sectors of a country's bond market and the securities issued. This includes sovereign bonds, semi-government bonds, municipal or province securities, corporate debt securities, mortgage-backed securities, asset-backed securities, and collateralized debt obligations. Our coverage in this chapter is to describe the instruments found in these sectors.

II. SECTORS OF THE BOND MARKET

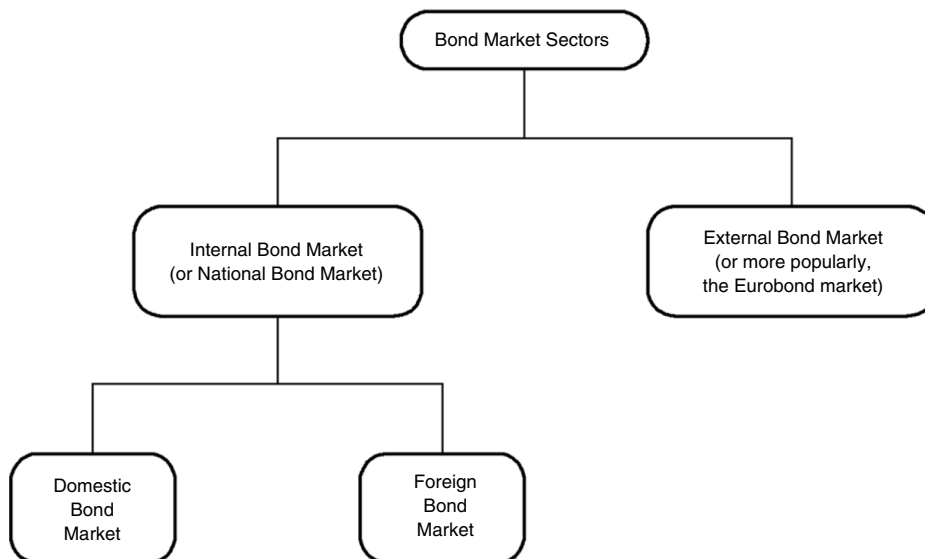
While there is no uniform system for classifying the sectors of the bond markets throughout the world, we will use the classification shown in Exhibit 1. From the perspective of a given country, the bond market can be classified into two markets: an **internal bond market** and an **external bond market**.

A. Internal Bond Market

The internal bond market of a country is also called the **national bond market**. It is divided into two parts: the **domestic bond market** and the **foreign bond market**. The domestic bond market is where issuers domiciled in the country issue bonds and where those bonds are subsequently traded.

The foreign bond market of a country is where bonds of issuers not domiciled in the country are issued and traded. For example, in the United States, the foreign bond market is the market where bonds are issued by non-U.S. entities and then subsequently traded in the United States. In the U.K., a sterling-denominated bond issued by a Japanese corporation and subsequently traded in the U.K. bond market is part of the U.K. foreign bond market. Bonds in the foreign sector of a bond market have nicknames. For example, foreign bonds in the U.S. market are nicknamed "Yankee bonds" and sterling-denominated bonds in the U.K. foreign bond market are nicknamed "Bulldog bonds." Foreign bonds can be denominated in

EXHIBIT 1 Overview of the Sectors of the Bond Market



any currency. For example, a foreign bond issued by an Australian corporation in the United States can be denominated in U.S. dollars, Australian dollars, or euros.

Issuers of foreign bonds include central governments and their subdivisions, corporations, and supranationals. A **supranational** is an entity that is formed by two or more central governments through international treaties. Supranationals promote economic development for the member countries. Two examples of supranationals are the International Bank for Reconstruction and Development, popularly referred to as the World Bank, and the Inter-American Development Bank.

B. External Bond Market

The external bond market includes bonds with the following distinguishing features:

- they are underwritten by an international syndicate
- at issuance, they are offered simultaneously to investors in a number of countries
- they are issued outside the jurisdiction of any single country
- they are in unregistered form.

The external bond market is referred to as the **international bond market**, the **offshore bond market**, or, more popularly, the **Eurobond market**.¹ Throughout this book we will use the term Eurobond market to describe this sector of the bond market.

Eurobonds are classified based on the currency in which the issue is denominated. For example, when Eurobonds are denominated in U.S. dollars, they are referred to as Eurodollar bonds. Eurobonds denominated in Japanese yen are referred to as Euroyen bonds.

¹It should be noted that the classification used here is by no means universally accepted. Some market observers refer to the external bond market as consisting of the foreign bond market and the Eurobond market.

A **global bond** is a debt obligation that is issued and traded in the foreign bond market of one or more countries and the Eurobond market.

III. SOVEREIGN BONDS

In many countries that have a bond market, the largest sector is often bonds issued by a country's central government. These bonds are referred to as **sovereign bonds**. A government can issue securities in its national bond market which are subsequently traded within that market. A government can also issue bonds in the Eurobond market or the foreign sector of another country's bond market. While the currency denomination of a government security is typically the currency of the issuing country, a government can issue bonds denominated in any currency.

A. Credit Risk

An investor in any bond is exposed to credit risk. The perception throughout the world is that the credit risk of bonds issued by the U.S. government are virtually free of credit risk. Consequently, the market views these bonds as default-free bonds. Sovereign bonds of non-U.S. central governments are rated by the credit rating agencies. These ratings are referred to as **sovereign ratings**. Standard & Poor's and Moody's rate sovereign debt. We will discuss the factors considered in rating sovereign bonds later.

The rating agencies assign two types of ratings to sovereign debt. One is a **local currency debt rating** and the other a **foreign currency debt rating**. The reason for assigning two ratings is, historically, the default frequency differs by the currency denomination of the debt. Specifically, defaults have been greater on foreign currency denominated debt. The reason for the difference in default rates for local currency debt and foreign currency debt is that if a government is willing to raise taxes and control its domestic financial system, it can generate sufficient local currency to meet its local currency debt obligation. This is not the case with foreign currency denominated debt. A central government must purchase foreign currency to meet a debt obligation in that foreign currency and therefore has less control with respect to its exchange rate. Thus, a significant depreciation of the local currency relative to a foreign currency denominated debt obligation will impair a central government's ability to satisfy that obligation.

B. Methods of Distributing New Government Securities

Four methods have been used by central governments to distribute new bonds that they issue: (1) regular auction cycle/multiple-price method, (2) regular auction cycle/single-price method, (3) ad hoc auction method, and (4) tap method.

With the **regular auction cycle/multiple-price method**, there is a regular auction cycle and winning bidders are allocated securities at the yield (price) they bid. For the **regular auction cycle/single-price method**, there is a regular auction cycle and all winning bidders are awarded securities at the highest yield accepted by the government. For example, if the highest yield for a single-price auction is 7.14% and someone bid 7.12%, that bidder would be awarded the securities at 7.14%. In contrast, with a multiple-price auction that bidder would be awarded securities at 7.12%. U.S. government bonds are currently issued using a regular auction cycle/single-price method.

In the **ad hoc auction system**, governments announce auctions when prevailing market conditions appear favorable. It is only at the time of the auction that the amount to be auctioned and the maturity of the security to be offered is announced. This is one of the methods used by the Bank of England in distributing British government bonds. In a **tap system**, additional bonds of a previously outstanding bond issue are auctioned. The government announces periodically that it is adding this new supply. The tap system has been used in the United Kingdom, the United States, and the Netherlands.

1. United States Treasury Securities U.S. Treasury securities are issued by the U.S. Department of the Treasury and are backed by the full faith and credit of the U.S. government. As noted above, market participants throughout the world view U.S. Treasury securities as having no credit risk. Because of the importance of the U.S. government securities market, we will take a close look at this market.

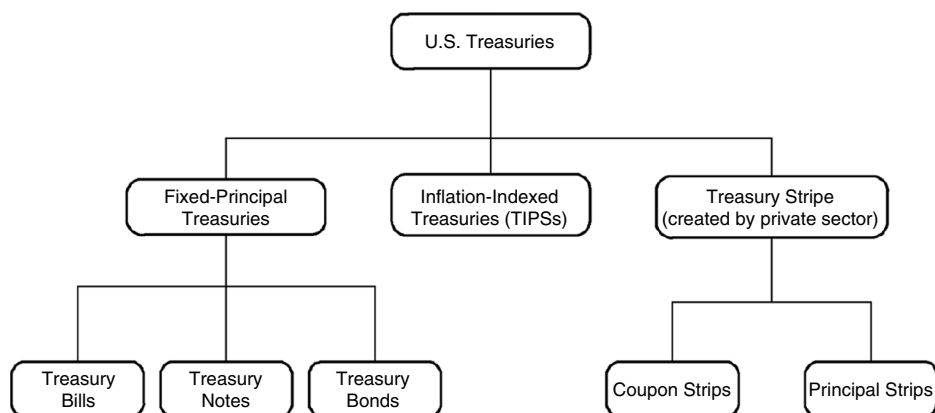
Treasury securities are sold in the primary market through sealed-bid auctions on a regular cycle using a single-price method. Each auction is announced several days in advance by means of a Treasury Department press release or press conference. The auction for Treasury securities is conducted on a competitive bid basis.

The secondary market for Treasury securities is an over-the-counter market where a group of U.S. government securities dealers offer continuous bid and ask prices on outstanding Treasuries. There is virtually 24-hour trading of Treasury securities. The most recently auctioned issue for a maturity is referred to as the **on-the-run issue** or the **current issue**. Securities that are replaced by the on-the-run issue are called **off-the-run issues**.

Exhibit 2 provides a summary of the securities issued by the U.S. Department of the Treasury. U.S. Treasury securities are categorized as **fixed-principal securities** or **inflation-indexed securities**.

a. Fixed-Principal Treasury Securities Fixed principal securities include Treasury bills, Treasury notes, and Treasury bonds. **Treasury bills** are issued at a discount to par value, have no coupon rate, mature at par value, and have a maturity date of less than 12 months. As discount securities, Treasury bills do not pay coupon interest; the return to the investor is the difference between the maturity value and the purchase price. We will explain how the price and the yield for a Treasury bill are computed in Chapter 6.

EXHIBIT 2 Overview of U.S. Treasury Debt Instruments



Treasury coupon securities issued with original maturities of more than one year and no more than 10 years are called **Treasury notes**. Coupon securities are issued at approximately par value and mature at par value. Treasury coupon securities with original maturities greater than 10 years are called **Treasury bonds**. While a few issues of the outstanding bonds are callable, the U.S. Treasury has not issued callable Treasury securities since 1984. As of this writing, the U.S. Department of the Treasury has stopped issuing Treasury bonds.

b. Inflation-Indexed Treasury Securities The U.S. Department of the Treasury issues Treasury notes and bonds that provide protection against inflation. These securities are popularly referred to as **Treasury inflation protection securities** or TIPS. (The Treasury refers to these securities as **Treasury inflation indexed securities**, TIIS.)

TIPS work as follows. The coupon rate on an issue is set at a fixed rate. That rate is determined via the auction process described later in this section. The coupon rate is called the “real rate” because it is the rate that the investor ultimately earns above the inflation rate. The inflation index that the government uses for the inflation adjustment is the non-seasonally adjusted U.S. City Average All Items Consumer Price Index for All Urban Consumers (CPI-U).

The principal that the Treasury Department will base both the dollar amount of the coupon payment and the maturity value on is adjusted semiannually. This is called the **inflation-adjusted principal**. The adjustment for inflation is as follows. Suppose that the coupon rate for a TIPS is 3.5% and the annual inflation rate is 3%. Suppose further that an investor purchases on January 1, \$100,000 of par value (principal) of this issue. The semiannual inflation rate is 1.5% (3% divided by 2). The inflation-adjusted principal at the end of the first six-month period is found by multiplying the original par value by $(1 + \text{the semiannual inflation rate})$. In our example, the inflation-adjusted principal at the end of the first six-month period is \$101,500. It is this inflation-adjusted principal that is the basis for computing the coupon interest for the first six-month period. The coupon payment is then 1.75% (one half the real rate of 3.5%) multiplied by the inflation-adjusted principal at the coupon payment date (\$101,500). The coupon payment is therefore \$1,776.25.

Let’s look at the next six months. The inflation-adjusted principal at the beginning of the period is \$101,500. Suppose that the semiannual inflation rate for the second six-month period is 1%. Then the inflation-adjusted principal at the end of the second six-month period is the inflation-adjusted principal at the beginning of the six-month period (\$101,500) increased by the semiannual inflation rate (1%). The adjustment to the principal is \$1,015 (1% times \$101,500). So, the inflation-adjusted principal at the end of the second six-month period (December 31 in our example) is \$102,515 ($\$101,500 + \$1,015$). The coupon interest that will be paid to the investor at the second coupon payment date is found by multiplying the inflation-adjusted principal on the coupon payment date (\$102,515) by one half the real rate (i.e., one half of 3.5%). That is, the coupon payment will be \$1,794.01.

As can be seen, part of the adjustment for inflation comes in the coupon payment since it is based on the inflation-adjusted principal. However, the U.S. government taxes the adjustment each year. This feature reduces the attractiveness of TIPS as investments for tax-paying entities.

Because of the possibility of disinflation (i.e., price declines), the inflation-adjusted principal at maturity may turn out to be less than the initial par value. However, the Treasury has structured TIPS so that they are redeemed at the greater of the inflation-adjusted principal and the initial par value.

An inflation-adjusted principal must be calculated for a settlement date. The inflation-adjusted principal is defined in terms of an index ratio, which is the ratio of the reference CPI

for the settlement date to the reference CPI for the issue date. The reference CPI is calculated with a 3-month lag. For example, the reference CPI for May 1 is the CPI-U reported in February. The U.S. Department of the Treasury publishes and makes available on its web site (www.publicdebt.treas.gov) a daily index ratio for an issue.

c. Treasury STRIPs The Treasury does not issue zero-coupon notes or bonds. However, because of the demand for zero-coupon instruments with no credit risk and a maturity greater than one year, the private sector has created such securities.

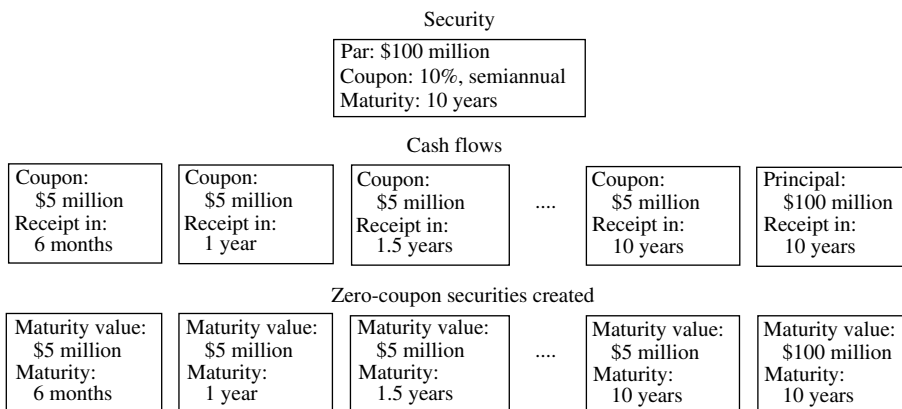
To illustrate the process, suppose \$100 million of a Treasury note with a 10-year maturity and a coupon rate of 10% is purchased to create zero-coupon Treasury securities (see Exhibit 3). The cash flows from this Treasury note are 20 semiannual payments of \$5 million each (\$100 million times 10% divided by 2) and the repayment of principal (“corpus”) of \$100 million 10 years from now. As there are 21 different payments to be made by the Treasury, a receipt representing a single payment claim on each payment is issued at a discount, creating 21 zero-coupon instruments. The amount of the maturity value for a receipt on a particular payment, whether coupon or principal, depends on the amount of the payment to be made by the Treasury on the underlying Treasury note. In our example, 20 coupon receipts each have a maturity value of \$5 million, and one receipt, the principal, has a maturity value of \$100 million. The maturity dates for the receipts coincide with the corresponding payment dates for the Treasury security.

Zero-coupon instruments are issued through the Treasury’s Separate Trading of Registered Interest and Principal Securities (STRIPS) program, a program designed to facilitate the stripping of Treasury securities. The zero-coupon Treasury securities created under the STRIPS program are direct obligations of the U.S. government.

Stripped Treasury securities are simply referred to as **Treasury strips**. Strips created from coupon payments are called **coupon strips** and those created from the principal payment are called **principal strips**. The reason why a distinction is made between coupon strips and the principal strips has to do with the tax treatment by non-U.S. entities as discussed below.

A disadvantage of a taxable entity investing in Treasury coupon strips is that accrued interest is taxed each year even though interest is not paid until maturity. Thus, these instruments have negative cash flows until the maturity date because tax payments must be made on interest earned but not received in cash must be made. One reason for distinguishing

EXHIBIT 3 Coupon Stripping: Creating Zero-Coupon Treasury Securities



between strips created from the principal and coupon is that some foreign buyers have a preference for the strips created from the principal (i.e., the principal strips). This preference is due to the tax treatment of the interest in their home country. Some country's tax laws treat the interest as a capital gain if the principal strip is purchased. The capital gain receives a preferential tax treatment (i.e., lower tax rate) compared to ordinary income.

2. Non-U.S. Sovereign Bond Issuers It is not possible to discuss the bonds/notes of all governments in the world. Instead, we will take a brief look at a few major sovereign issuers.

The German government issues bonds (called *Bunds*) with maturities from 8–30 years and notes (*Bundesobligationen*, Bobls) with a maturity of five years. Ten-year Bunds are the largest sector of the German government securities market in terms of amount outstanding and secondary market turnover. Bunds and Bobls have a fixed-rate coupons and are bullet structures.

The bonds issued by the United Kingdom are called “gilt-edged stocks” or simply *gilts*. There are more types of gilts than there are types of issues in other government bond markets. The largest sector of the gilt market is straight fixed-rate coupon bonds. The second major sector of the gilt market is index-linked issues, referred to as “linkers.” There are a few issues of outstanding gilts called “irredeemables.” These are issues with no maturity date and are therefore called “undated gilts.” Government designated gilt issues may be stripped to create gilt strips, a process that began in December 1997.

The French Treasury issues long-dated bonds, *Obligation Assimilable du Trésor* (OATS), with maturities up to 30 years and notes, *Bons du Trésor à Taux Fixe et à Intérêt Annuel* (BTANs), with maturities between 2 and 5 years. OATs are not callable. While most OAT issues have a fixed-rate coupon, there are some special issues with a floating-rate coupon. Long-dated OATs can be stripped to create OAT strips. The French government was one of the first countries after the United States to allow stripping.

The Italian government issues (1) bonds, *Buoni del Tesoro Poliennali* (BTPs), with a fixed-rate coupon that are issued with original maturities of 5, 10, and 30 years, (2) floating-rate notes, *Certificati di Credito del Tesoro* (CCTs), typically with a 7-year maturity and referenced to the Italian Treasury bill rate, (3) 2-year zero-coupon notes, *Certificati di Tesoro a Zero Coupon* (CTZs), and (4) bonds with put options, *Certificati del Tesoro con Opzione* (CTOs). The puttable bonds are issued with the same maturities as the BTPs. The investor has the right to put the bond to the Italian government halfway through its stated maturity date. The Italian government has not issued CTOs since 1992.

The Canadian government bond market has been closely related to the U.S. government bond market and has a similar structure, including types of issues. Bonds have a fixed coupon rate except for the inflation protection bonds (called “real return bonds”). All new Canadian bonds are in “bullet” form; that is, they are not callable or puttable.

About three quarters of the Australian government securities market consists of fixed-rate bonds and inflation protections bonds called “Treasury indexed bonds.” Treasury indexed bonds have either interest payments or capital linked to the Australian Consumer Price Index. The balance of the market consists of floating-rate issues, referred to as “Treasury adjustable bonds,” that have a maturity between 3 to 5 years and the reference rate is the Australian Bank Bill Index.

There are two types of Japanese government securities (referred to as JGBs) issued publicly: (1) medium-term bonds and (2) long-dated bonds. There are two types of medium-term bonds: bonds with coupons and zero-coupon bonds. Bonds with coupons have maturities of 2, 3, and 4 years. The other type of medium-term bond is the 5-year zero-coupon bond. Long-dated bonds are interest bearing.

The financial markets of Latin America, Asia with the exception of Japan, and Eastern Europe are viewed as “emerging markets.” Investing in the government bonds of emerging market countries entails considerably more credit risk than investing in the government bonds of major industrialized countries. A good amount of secondary trading of government debt of emerging markets is in **Brady bonds** which represent a restructuring of nonperforming bank loans to emerging market governments into marketable securities. There are two types of Brady bonds. The first type covers the interest due on these loans (“past-due interest bonds”). The second type covers the principal amount owed on the bank loans (“principal bonds”).

IV. SEMI-GOVERNMENT / AGENCY BONDS

A central government can establish an agency or organization that issues bonds. The bonds of such entities are not issued directly by the central government but may have either a direct or implied government guarantee. These bonds are generically referred to as **semi-government bonds** or **government agency bonds**. In some countries, semi-government bonds include bonds issued by regions of the country.

Here are a few examples of semi-government bonds. In Australia, there are the bonds issued by Telstra or a State electric power supplier such as Pacific Power. These bonds are guaranteed by the full faith and credit of the Commonwealth of Australia. Government agency bonds are issued by Germany’s Federal Railway (*Bundesbahn*) and the Post Office (*Bundespost*) with the full faith and credit of the central government.

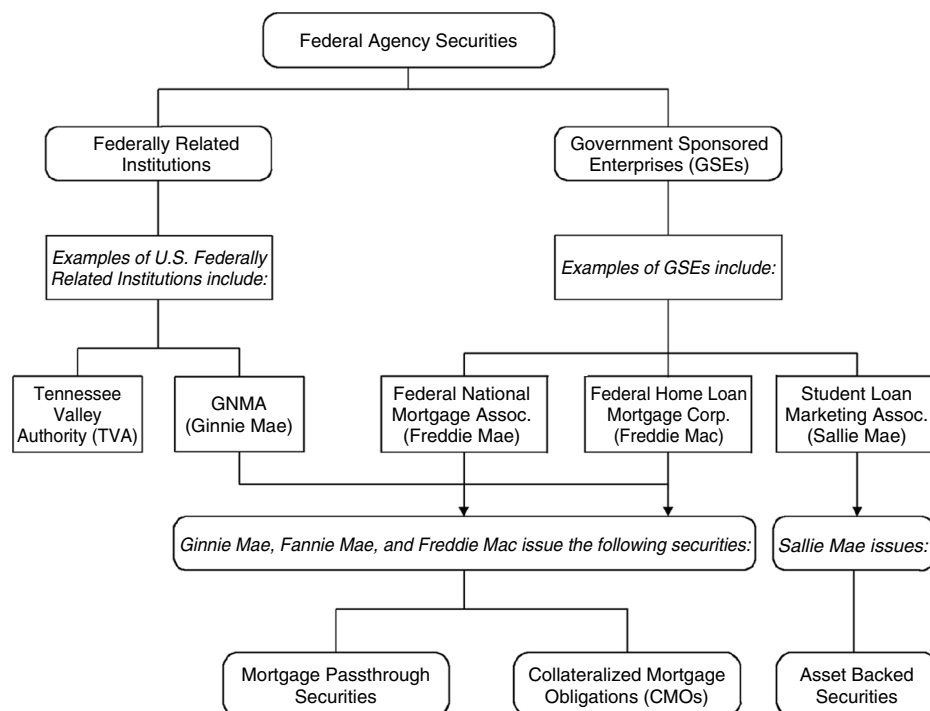
In the United States, semi-government bonds are referred to as **federal agency securities**. They are further classified by the types of issuer—those issued by **federally related institutions** and those issued by **government-sponsored enterprises**. Our focus in the remainder of this section is on U.S. federal agency securities. Exhibit 4 provides an overview of the U.S. federal agency securities market.

Federally related institutions are arms of the federal government. They include the Export-Import Bank of the United States, the Tennessee Valley Authority (TVA), the Commodity Credit Corporation, the Farmers Housing Administration, the General Services Administration, the Government National Mortgage Association (Ginnie Mae), the Maritime Administration, the Private Export Funding Corporation, the Rural Electrification Administration, the Rural Telephone Bank, the Small Business Administration, and the Washington Metropolitan Area Transit Authority. With the exception of securities of the TVA and the Private Export Funding Corporation, the securities are backed by the full faith and credit of the U.S. government. In recent years, the TVA has been the only issuer of securities directly into the marketplace.

Government-sponsored enterprises (GSEs) are privately owned, publicly chartered entities. They were created by Congress to reduce the cost of capital for certain borrowing sectors of the economy deemed to be important enough to warrant assistance. The entities in these sectors include farmers, homeowners, and students. The enabling legislation dealing with a GSE is reviewed periodically. GSEs issue securities directly in the marketplace. The market for these securities, while smaller than that of Treasury securities, has in recent years become an active and important sector of the bond market.

Today there are six GSEs that currently issue securities: Federal National Mortgage Association (Fannie Mae), Federal Home Loan Mortgage Corporation (Freddie Mac), Federal Agricultural Mortgage Corporation (Farmer Mac), Federal Farm Credit System, Federal Home Loan Bank System, and Student Loan Marketing Association (Sallie Mae). Fannie

EXHIBIT 4 Overview of U.S. Federal Agency Securities



Mae, Freddie Mac, and the Federal Home Loan Bank are responsible for providing credit to the residential housing sector. Farmer Mac provides the same function for farm properties. The Federal Farm Credit Bank System is responsible for the credit market in the agricultural sector of the economy. Sallie Mae provides funds to support higher education.

A. U.S. Agency Debentures and Discount Notes

Generally, GSEs issue two types of debt: debentures and discount notes. Debentures and discount notes do not have any specific collateral backing the debt obligation. The ability to pay debtholders depends on the ability of the issuing GSE to generate sufficient cash flows to satisfy the obligation.

Debentures can be either notes or bonds. GSE issued notes, with minor exceptions, have 1 to 20 year maturities and bonds have maturities longer than 20 years. **Discount notes** are short-term obligations, with maturities ranging from overnight to 360 days.

Several GSEs are frequent issuers and therefore have developed regular programs for the securities that they issue. For example, let's look at the debentures issued by Federal National Mortgage Association (Fannie Mae) and Freddie Mac (Federal Home Loan Mortgage Corporation). Fannie Mae issues **Benchmark Notes, Benchmark Bonds, Callable Benchmark Notes**, medium-term notes, and global bonds. The debentures issued by Freddie Mac are **Reference Notes, Reference Bonds, Callable Reference Notes**, medium-term notes, and global bonds. (We will discuss medium-term notes and global bonds in Section VI and Section VIII, respectively.) Callable Reference Notes have maturities of 2 to 10 years. Both

Benchmark Notes and Bonds and Reference Notes and Bonds are eligible for stripping to create zero-coupon bonds.

B. U.S. Agency Mortgage-Backed Securities

The two GSEs charged with providing liquidity to the mortgage market—Fannie Mae and Freddie Mac—also issue securities backed by the mortgage loans that they purchase. That is, they use the mortgage loans they underwrite or purchase as collateral for the securities they issue. These securities are called **agency mortgage-backed securities** and include mortgage passthrough securities, collateralized mortgage obligations (CMOs), and stripped mortgage-backed securities. The latter two mortgage-backed securities are referred to as derivative mortgage-backed securities because they are created from mortgage passthrough securities.

While we confine our discussion to the U.S. mortgage-backed securities market, most developed countries have similar mortgage products.

1. Mortgage Loans A **mortgage loan** is a loan secured by the collateral of some specified real estate property which obliges the borrower to make a predetermined series of payments. The mortgage gives the lender the right, if the borrower defaults, to “foreclose” on the loan and seize the property in order to ensure that the debt is paid off. The interest rate on the mortgage loan is called the **mortgage rate** or **contract rate**.

There are many types of mortgage designs available in the United States. A mortgage design is a specification of the mortgage rate, term of the mortgage, and the manner in which the borrowed funds are repaid. For now, we will use the most common mortgage design to explain the characteristics of a mortgage-backed security: a fixed-rate, level-payment, fully amortizing mortgage.

The basic idea behind this mortgage design is that each monthly mortgage payment is the same dollar amount and includes interest and principal payment. The monthly payments are such that at the end of the loan’s term, the loan has been fully amortized (i.e., there is no mortgage principal balance outstanding).

Each monthly mortgage payment for this mortgage design is due on the first of each month and consists of:

1. interest of $\frac{1}{12}$ of the fixed annual interest rate times the amount of the outstanding mortgage balance at the end of the previous month, and
2. a payment of a portion of the outstanding mortgage principal balance.

The difference between the monthly mortgage payment and the portion of the payment that represents interest equals the amount that is applied to reduce the outstanding mortgage principal balance. This amount is referred to as the **amortization**. We shall also refer to it as the **scheduled principal payment**.

To illustrate this mortgage design, consider a 30-year (360-month), \$100,000 mortgage with an 8.125% mortgage rate. The monthly mortgage payment would be \$742.50.² Exhibit

²The calculation of the monthly mortgage payment is simply an application of the present value of an annuity. The formula as applied to mortgage payments is as follows:

$$MP = B \left[\frac{r(1+r)^n}{(1+r)^n - 1} \right]$$

5 shows for selected months how each monthly mortgage payment is divided between interest and scheduled principal payment. At the beginning of month 1, the mortgage balance is \$100,000, the amount of the original loan. The mortgage payment for month 1 includes interest on the \$100,000 borrowed for the month. Since the interest rate is 8.125%, the monthly interest rate is 0.0067708 (0.08125 divided by 12). Interest for month 1 is therefore \$677.08 (\$100,000 times 0.0067708). The \$65.41 difference between the monthly mortgage payment of \$742.50 and the interest of \$677.08 is the portion of the monthly mortgage payment that represents the scheduled principal payment (i.e., amortization). This \$65.41 in month 1 reduces the mortgage balance.

The mortgage balance at the end of month 1 (beginning of month 2) is then \$99,934.59 (\$100,000 minus \$65.41). The interest for the second monthly mortgage payment is \$676.64, the monthly interest rate (0.0067708) times the mortgage balance at the beginning of month 2 (\$99,934.59). The difference between the \$742.50 monthly mortgage payment and the \$676.64 interest is \$65.86, representing the amount of the mortgage balance paid off with that monthly mortgage payment. Notice that the mortgage payment in month 360—the final payment—is sufficient to pay off the remaining mortgage principal balance.

As Exhibit 5 clearly shows, the portion of the monthly mortgage payment applied to interest declines each month and the portion applied to principal repayment increases. The reason for this is that as the mortgage balance is reduced with each monthly mortgage payment, the interest on the mortgage balance declines. Since the monthly mortgage payment is a fixed dollar amount, an increasingly larger portion of the monthly payment is applied to reduce the mortgage principal balance outstanding in each subsequent month.

To an investor in a mortgage loan (or a pool of mortgage loans), the monthly mortgage payments as described above do not equal an investor's cash flow. There are two reasons for this: (1) servicing fees and (2) prepayments.

Every mortgage loan must be serviced. Servicing of a mortgage loan involves collecting monthly payments and forwarding proceeds to owners of the loan; sending payment notices to mortgagors; reminding mortgagors when payments are overdue; maintaining records of principal balances; administering an escrow balance for real estate taxes and insurance; initiating foreclosure proceedings if necessary; and, furnishing tax information to mortgagors when applicable. The servicing fee is a portion of the mortgage rate. If the mortgage rate is 8.125% and the servicing fee is 50 basis points, then the investor receives interest of 7.625%. The interest rate that the investor receives is said to be the **net interest**.

Our illustration of the cash flow for a level-payment, fixed-rate, fully amortized mortgage assumes that the homeowner does not pay off any portion of the mortgage principal balance

where

$$\begin{aligned}
 MP &= \text{monthly mortgage payment} \\
 B &= \text{amount borrowed (i.e., original loan balance)} \\
 r &= \text{monthly mortgage rate (annual rate divided by 12)} \\
 n &= \text{number of months of the mortgage loan} \\
 \text{In our example,} \\
 B &= \$100,000 \quad r = 0.0067708 \text{ (0.08125/12)} \quad n = 360
 \end{aligned}$$

Then

$$MP = \$100,000 \left[\frac{0.0067708 (1.0067708)^{360}}{(1.0067708)^{360} - 1} \right] = \$742.50$$

EXHIBIT 5 Amortization Schedule for a Level-Payment, Fixed-Rate, Fully Amortized Mortgage (Selected Months)

Mortgage loan: \$100,000
Mortgage rate: 8.125%

Monthly payment: \$742.50
Term of loan: 30 years (360 months)

(1)	(2)	(3)	(4)	(5)	(6)
Month	Beginning of Month Mortgage Balance	Mortgage Payment	Interest	Scheduled Principal Repayment	End of Month Mortgage Balance
1	\$100,000.00	\$742.50	\$677.08	\$65.41	\$99,934.59
2	99,934.59	742.50	676.64	65.86	99,868.73
3	99,868.73	742.50	676.19	66.30	99,802.43
4	99,802.43	742.50	675.75	66.75	99,735.68
...	...	...	...	...	...
25	98,301.53	742.50	665.58	76.91	98,224.62
26	98,224.62	742.50	665.06	77.43	98,147.19
27	98,147.19	742.50	664.54	77.96	98,069.23
...	...	...	...	...	...
184	76,446.29	742.50	517.61	224.89	76,221.40
185	76,221.40	742.50	516.08	226.41	75,994.99
186	75,994.99	742.50	514.55	227.95	75,767.04
...	...	...	...	...	...
289	42,200.92	742.50	285.74	456.76	41,744.15
290	41,744.15	742.50	282.64	459.85	41,284.30
291	41,284.30	742.50	279.53	462.97	40,821.33
...	...	...	...	...	...
358	2,197.66	742.50	14.88	727.62	1,470.05
359	1,470.05	742.50	9.95	732.54	737.50
360	737.50	742.50	4.99	737.50	0.00

prior to the scheduled payment date. But homeowners do pay off all or part of their mortgage balance prior to the scheduled payment date. A payment made in excess of the monthly mortgage payment is called a **prepayment**. The prepayment may be for the entire principal outstanding principal balance or a partial additional payment of the mortgage principal balance. When a prepayment is not for the entire amount, it is called a **curtailment**. Typically, there is no penalty for prepaying a mortgage loan.

Thus, the cash flows for a mortgage loan are monthly and consist of three components: (1) net interest, (2) scheduled principal payment, and (3) prepayments. The effect of prepayments is that the amount and timing of the cash flow from a mortgage is not known with certainty. This is the risk that we referred to as **prepayment risk** in Chapter 2.³

For example, all that the investor in a \$100,000, 8.125% 30-year mortgage knows is that as long as the loan is outstanding and the borrower does not default, interest will be received and the principal will be repaid at the scheduled date each month; then at the end of the 30 years, the investor would have received \$100,000 in principal payments. What the investor does not know—the uncertainty—is for how long the loan will be outstanding, and therefore what the timing of the principal payments will be. This is true for all mortgage loans, not just the level-payment, fixed-rate, fully amortized mortgage.

³Factors affecting prepayments will be discussed in later chapters.

2. Mortgage Passthrough Securities A **mortgage passthrough security**, or simply passthrough, is a security created when one or more holders of mortgages form a collection (pool) of mortgages and sell shares or participation certificates in the pool. A pool may consist of several thousand or only a few mortgages. When a mortgage is included in a pool of mortgages that is used as collateral for a passthrough, the mortgage is said to be **securitized**.

The cash flow of a passthrough depends on the cash flow of the underlying pool of mortgages. As we just explained, the cash flow consists of monthly mortgage payments representing net interest, the scheduled principal payment, and any principal prepayments. Payments are made to security holders each month. Because of prepayments, the amount of the cash flow is uncertain in terms of the timing of the principal receipt.

To illustrate the creation of a passthrough look at Exhibits 6 and 7. Exhibit 6 shows 2,000 mortgage loans and the cash flows from these loans. For the sake of simplicity, we assume that the amount of each loan is \$100,000 so that the aggregate value of all 2,000 loans is \$200 million.

An investor who owns any one of the individual mortgage loans shown in Exhibit 6 faces prepayment risk. In the case of an individual loan, it is particularly difficult to predict prepayments. If an individual investor were to purchase all 2,000 loans, however, prepayments might become more predictable based on historical prepayment experience. However, that would call for an investment of \$200 million to buy all 2,000 loans.

Suppose, instead, that some entity purchases all 2,000 loans in Exhibit 6 and pools them. The 2,000 loans can be used as collateral to issue a security whose cash flow is based on the cash flow from the 2,000 loans, as depicted in Exhibit 7. Suppose that 200,000 certificates are issued. Thus, each certificate is initially worth \$1,000 (\$200 million divided by 200,000). Each certificate holder would be entitled to 0.0005% (1/200,000) of the cash flow. The security created is a mortgage passthrough security.

EXHIBIT 6 Mortgage Loans

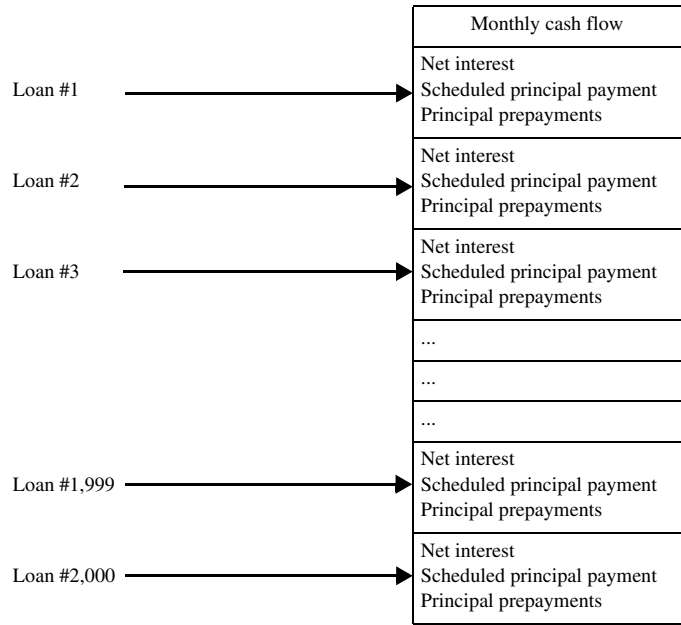
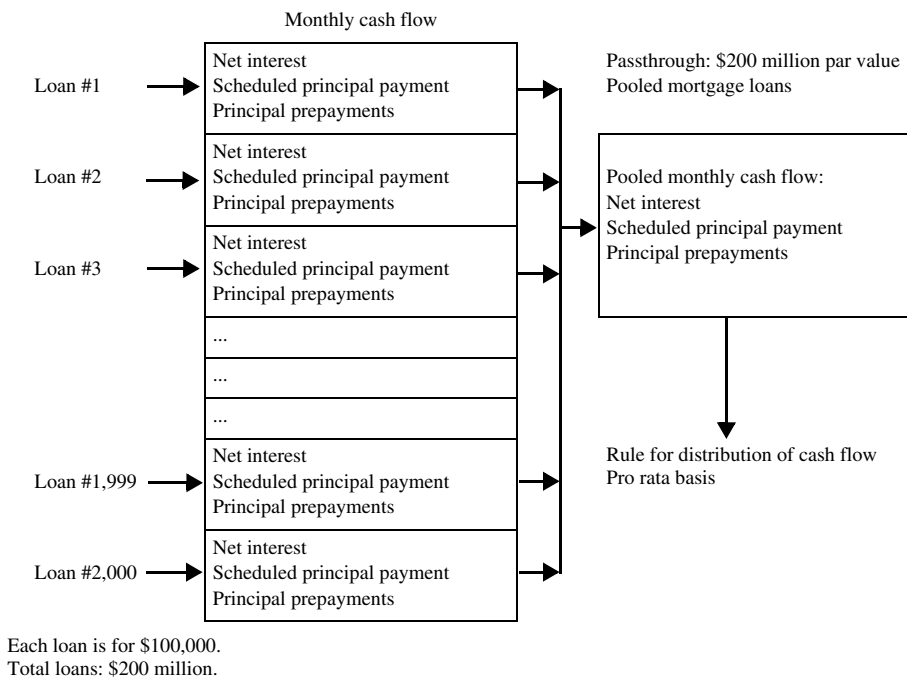


EXHIBIT 7 Creation of a Passthrough Security



Let's see what has been accomplished by creating the passthrough. The total amount of prepayment risk has not changed. Yet, the investor is now exposed to the prepayment risk spread over 2,000 loans rather than one individual mortgage loan and for an investment of less than \$200 million.

Let's compare the cash flow for a mortgage passthrough security (an amortizing security) to that of a noncallable coupon bond (a nonamortizing security). For a standard coupon bond, there are no principal payments prior to maturity while for a mortgage passthrough security the principal is paid over time. Unlike a standard coupon bond that pays interest semiannually, a mortgage passthrough makes monthly interest and principal payments. Mortgage passthrough securities are similar to coupon bonds that are callable in that there is uncertainty about the cash flows due to uncertainty about when the entire principal will be paid.

Passthrough securities are issued by Ginnie Mae, Fannie Mae, and Freddie Mac. They are guaranteed with respect to the timely payment of interest and principal.⁴ The loans that are permitted to be included in the pool of mortgage loans issued by Ginnie Mae, Fannie Mae, and Freddie Mac must meet the underwriting standards that have been established by these entities. Loans that satisfy the underwriting requirements are referred to as **conforming loans**. Mortgage-backed securities not issued by agencies are backed by pools of nonconforming loans.

⁴Freddie Mac previously issued passthrough securities that guaranteed the timely payment of interest but guaranteed only the eventual payment of principal (when it is collected or within one year).

3. **Collateralized Mortgage Obligations** Now we will show how one type of agency mortgage derivative security is created—a **collateralized mortgage obligation** (CMO). The motivation for creation of a CMO is to distribute prepayment risk among different classes of bonds.

The investor in our passthrough in Exhibit 7 remains exposed to the total prepayment risk associated with the underlying pool of mortgage loans, regardless of how many loans there are. Securities can be created, however, where investors do not share prepayment risk equally. Suppose that instead of distributing the monthly cash flow on a pro rata basis, as in the case of a passthrough, the distribution of the principal (both scheduled principal and prepayments) is carried out on some prioritized basis. How this is done is illustrated in Exhibit 8.

The exhibit shows the cash flow of our original 2,000 mortgage loans and the passthrough. Also shown are three classes of bonds, commonly referred to as **tranches**,⁵ the par value of each tranche, and a set of payment rules indicating how the principal from the passthrough is to be distributed to each tranche. Note that the sum of the par value of the three tranches is equal to \$200 million. Although it is not shown in the exhibit, for each of the three tranches, there will be certificates representing a proportionate interest in a tranche. For example, suppose that for Tranche A, which has a par value of \$80 million, there are 80,000 certificates issued. Each certificate would receive a proportionate share (0.00125%) of payments received by Tranche A.

The rule for the distribution of principal shown in Exhibit 8 is that Tranche A will receive all principal (both scheduled and prepayments) until that tranche's remaining principal balance is zero. Then, Tranche B receives all principal payments until its remaining principal balance is zero. After Tranche B is completely paid, Tranche C receives principal payments. The rule for the distribution of the cash flows in Exhibit 8 indicates that each of the three tranches receives interest on the basis of the amount of the par value outstanding.

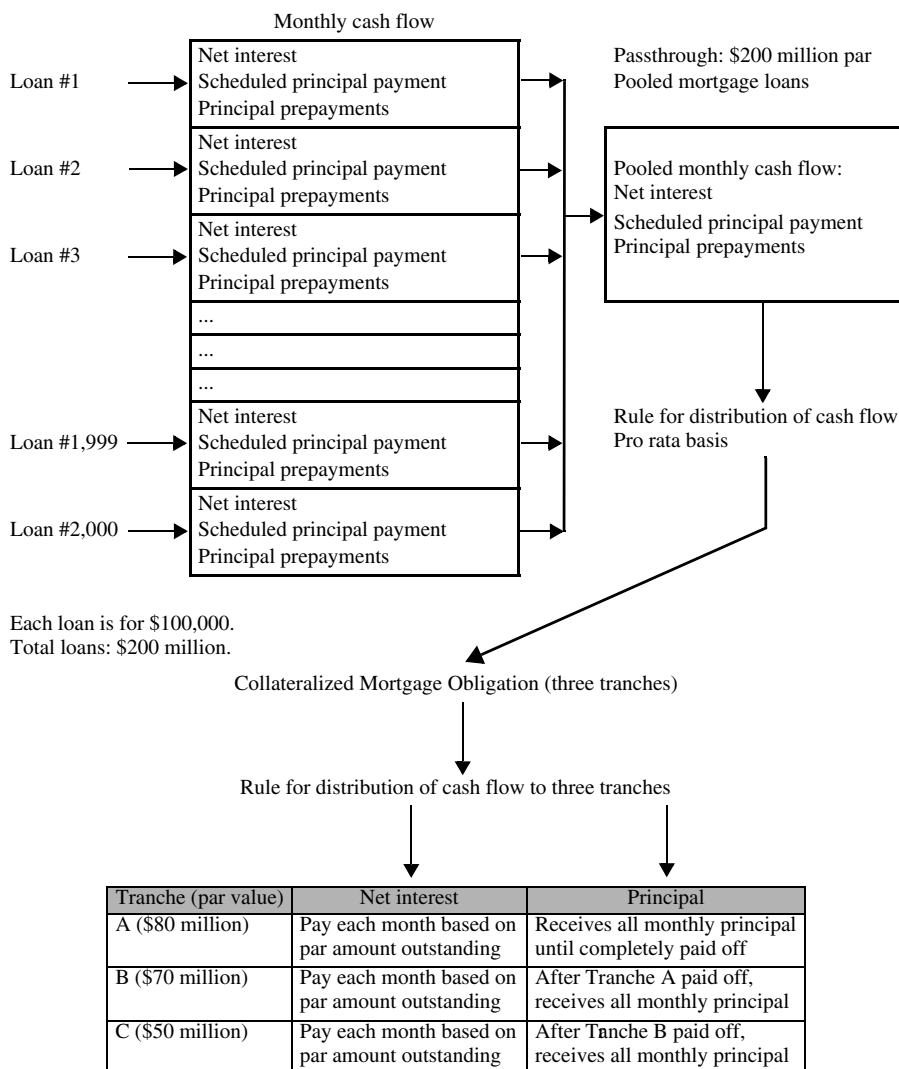
The mortgage-backed security that has been created is called a CMO. The collateral for a CMO issued by the agencies is a pool of passthrough securities which is placed in a trust. The ultimate source for the CMO's cash flow is the pool of mortgage loans.

Let's look now at what has been accomplished. Once again, the total prepayment risk for the CMO is the same as the total prepayment risk for the 2,000 mortgage loans. However, the prepayment risk has been distributed differently across the three tranches of the CMO. Tranche A absorbs prepayments first, then Tranche B, and then Tranche C. The result of this is that Tranche A effectively is a shorter term security than the other two tranches; Tranche C will have the longest maturity. Different institutional investors will be attracted to the different tranches, depending on the nature of their liabilities and the effective maturity of the CMO tranche. Moreover, there is less uncertainty about the maturity of each tranche of the CMO than there is about the maturity of the pool of passthroughs from which the CMO is created. Thus, redirection of the cash flow from the underlying mortgage pool creates tranches that satisfy the asset/liability objectives of certain institutional investors better than a passthrough. Stated differently, the rule for distributing principal repayments redistributes prepayment risk among the tranches.

The CMO we describe in Exhibit 8 has a simple set of rules for the distribution of the cash flow. Today, much more complicated CMO structures exist. The basic objective is to provide certain CMO tranches with less uncertainty about prepayment risk. Note, of course, that this can occur only if the reduction in prepayment risk for some tranches is absorbed by other

⁵"Tranche" is from an old French word meaning "slice." (The pronunciation of tranche rhymes with the English word "launch," as in launch a ship or a rocket.)

EXHIBIT 8 Creation of a Collateralized Mortgage Obligation



tranches in the CMO structure. A good example is one type of CMO tranche called a **planned amortization class tranche** or PAC tranche. This is a tranche that has a schedule for the repayment of principal (hence the name “planned amortization”) if prepayments are realized at a certain prepayment rate.⁶ As a result, the prepayment risk is reduced (not eliminated) for this type of CMO tranche. The tranche that realizes greater prepayment risk in order for the PAC tranche to have greater prepayment protection is called the **support tranche**.

We will describe in much more detail PAC tranches and support tranches, as well as other types of CMO tranches in Chapter 10.

⁶We will explain what is meant by “prepayment rate” later.

V. STATE AND LOCAL GOVERNMENTS

Non-central government entities also issue bonds. In the United States, this includes state and local governments and entities that they create. These securities are referred to as **municipal securities** or **municipal bonds**. Because the U.S. bond market has the largest and most developed market for non-central government bonds, we will focus on municipal securities in this market.

In the United States, there are both tax-exempt and taxable municipal securities. “Tax-exempt” means that interest on a municipal security is exempt from federal income taxation. The tax-exemption of municipal securities applies to interest income, not capital gains. The exemption may or may not extend to taxation at the state and local levels. Each state has its own rules as to how interest on municipal securities is taxed. Most municipal securities that have been issued are tax-exempt. Municipal securities are commonly referred to as **tax-exempt securities** despite the fact that there are taxable municipal securities that have been issued and are traded in the market. Municipal bonds are traded in the over-the-counter market supported by municipal bond dealers across the country.

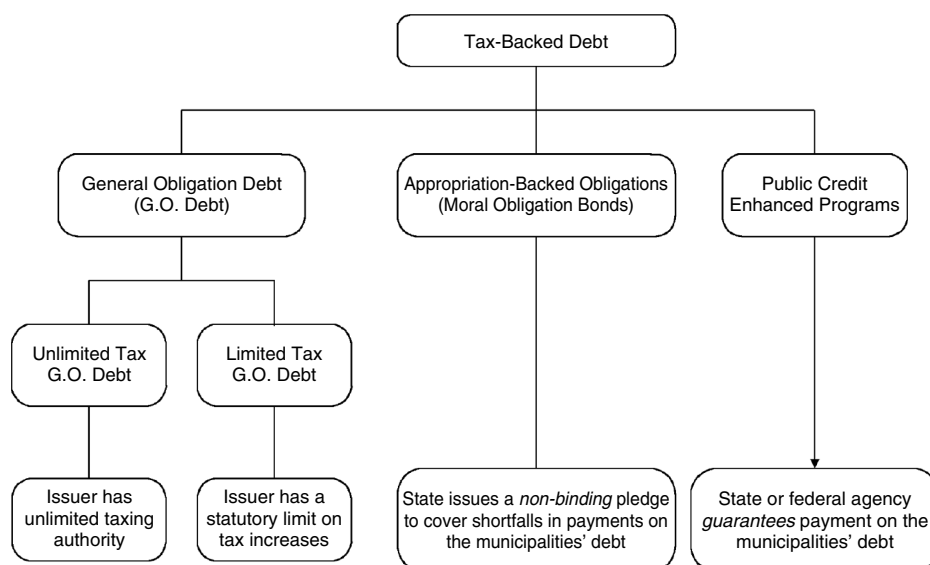
Like other non-Treasury fixed income securities, municipal securities expose investors to credit risk. The nationally recognized rating organizations rate municipal securities according to their credit risk. In later chapters, we look at the factors rating agencies consider in assessing credit risk.

There are basically two types of municipal security structures: **tax-backed debt** and **revenue bonds**. We describe each below, as well as some variants.

A. Tax-Backed Debt

Tax-backed debt obligations are instruments issued by states, counties, special districts, cities, towns, and school districts that are secured by some form of tax revenue. Exhibit 9 provides

EXHIBIT 9 Tax-Backed Debt Issues in the U.S. Municipal Securities Market



an overview of the types of tax-backed debt issued in the U.S. municipal securities market. Tax-backed debt includes **general obligation debt**, **appropriation-backed obligations**, and **debt obligations supported by public credit enhancement programs**. We discuss each below.

1. General Obligation Debt The broadest type of tax-backed debt is general obligation debt. There are two types of general obligation pledges: unlimited and limited. An **unlimited tax general obligation debt** is the stronger form of general obligation pledge because it is secured by the issuer's unlimited taxing power. The tax revenue sources include corporate and individual income taxes, sales taxes, and property taxes. Unlimited tax general obligation debt is said to be secured by the full faith and credit of the issuer. A **limited tax general obligation debt** is a limited tax pledge because, for such debt, there is a statutory limit on tax rates that the issuer may levy to service the debt.

Certain general obligation bonds are secured not only by the issuer's general taxing powers to create revenues accumulated in a general fund, but also by certain identified fees, grants, and special charges, which provide additional revenues from outside the general fund. Such bonds are known as **double-barreled in security** because of the dual nature of the revenue sources. For example, the debt obligations issued by special purpose service systems may be secured by a pledge of property taxes, a pledge of special fees/operating revenue from the service provided, or a pledge of both property taxes and special fees/operating revenues. In the last case, they are double-barreled.

2. Appropriation-Backed Obligations Agencies or authorities of several states have issued bonds that carry a potential state liability for making up shortfalls in the issuing entity's obligation. The appropriation of funds from the state's general tax revenue must be approved by the state legislature. However, the state's pledge is not binding. Debt obligations with this nonbinding pledge of tax revenue are called **moral obligation bonds**. Because a moral obligation bond requires legislative approval to appropriate the funds, it is classified as an appropriation-backed obligation. The purpose of the moral obligation pledge is to enhance the credit worthiness of the issuing entity. However, the investor must rely on the best-efforts of the state to approve the appropriation.

3. Debt Obligations Supported by Public Credit Enhancement Programs While a moral obligation is a form of credit enhancement provided by a state, it is not a legally enforceable or legally binding obligation of the state. There are entities that have issued debt that carries some form of public credit enhancement that is legally enforceable. This occurs when there is a guarantee by the state or a federal agency or when there is an obligation to automatically withhold and deploy state aid to pay any defaulted debt service by the issuing entity. Typically, the latter form of public credit enhancement is used for debt obligations of a state's school systems.

Some examples of state credit enhancement programs include Virginia's bond guarantee program that authorizes the governor to withhold state aid payments to a municipality and divert those funds to pay principal and interest to a municipality's general obligation holders in the event of a default. South Carolina's constitution requires mandatory withholding of state aid by the state treasurer if a school district is not capable of meeting its general obligation debt. Texas created the Permanent School Fund to guarantee the timely payment of principal and interest of the debt obligations of qualified school districts. The fund's income is obtained from land and mineral rights owned by the state of Texas.

More recently, states and local governments have issued increasing amounts of bonds where the debt service is to be paid from so-called “dedicated” revenues such as sales taxes, tobacco settlement payments, fees, and penalty payments. Many are structured to mimic the asset-backed bonds that are discussed later in this chapter (Section VII).

B. Revenue Bonds

The second basic type of security structure is found in a revenue bond. Revenue bonds are issued for enterprise financings that are secured by the revenues generated by the completed projects themselves, or for general public-purpose financings in which the issuers pledge to the bondholders the tax and revenue resources that were previously part of the general fund. This latter type of revenue bond is usually created to allow issuers to raise debt outside general obligation debt limits and without voter approval.

Revenue bonds can be classified by the type of financing. These include utility revenue bonds, transportation revenue bonds, housing revenue bonds, higher education revenue bonds, health care revenue bonds, sports complex and convention center revenue bonds, seaport revenue bonds, and industrial revenue bonds.

C. Special Bond Structures

Some municipal securities have special security structures. These include **insured bonds** and **prerefunded bonds**.

1. Insured Bonds Insured bonds, in addition to being secured by the issuer’s revenue, are also backed by insurance policies written by commercial insurance companies. Insurance on a municipal bond is an agreement by an insurance company to pay the bondholder principal and/or coupon interest that is due on a stated maturity date but that has not been paid by the bond issuer. Once issued, this municipal bond insurance usually extends for the term of the bond issue and cannot be canceled by the insurance company.

2. Prerefunded Bonds Although originally issued as either revenue or general obligation bonds, municipals are sometimes prerefunded and thus called **prerefunded municipal bonds**. A prerefunding usually occurs when the original bonds are escrowed or collateralized by direct obligations guaranteed by the U.S. government. By this, it is meant that a portfolio of securities guaranteed by the U.S. government is placed in a trust. The portfolio of securities is assembled such that the cash flows from the securities match the obligations that the issuer must pay. For example, suppose that a municipality has a 7% \$100 million issue with 12 years remaining to maturity. The municipality’s obligation is to make payments of \$3.5 million every six months for the next 12 years and \$100 million 12 years from now. If the issuer wants to prerefund this issue, a portfolio of U.S. government obligations can be purchased that has a cash flow of \$3.5 million every six months for the next 12 years and \$100 million 12 years from now.

Once this portfolio of securities whose cash flows match those of the municipality’s obligation is in place, the prerefunded bonds are no longer secured as either general obligation or revenue bonds. The bonds are now supported by cash flows from the portfolio of securities held in an escrow fund. Such bonds, if escrowed with securities guaranteed by the U.S. government, have little, if any, credit risk. They are the safest municipal bonds available.

The escrow fund for a prerefunded municipal bond can be structured so that the bonds to be refunded are to be called at the first possible call date or a subsequent call date established

in the original bond indenture. While prerefunded bonds are usually retired at their first or subsequent call date, some are structured to match the debt obligation to the maturity date. Such bonds are known as **escrowed-to-maturity bonds**.

VI. CORPORATE DEBT SECURITIES

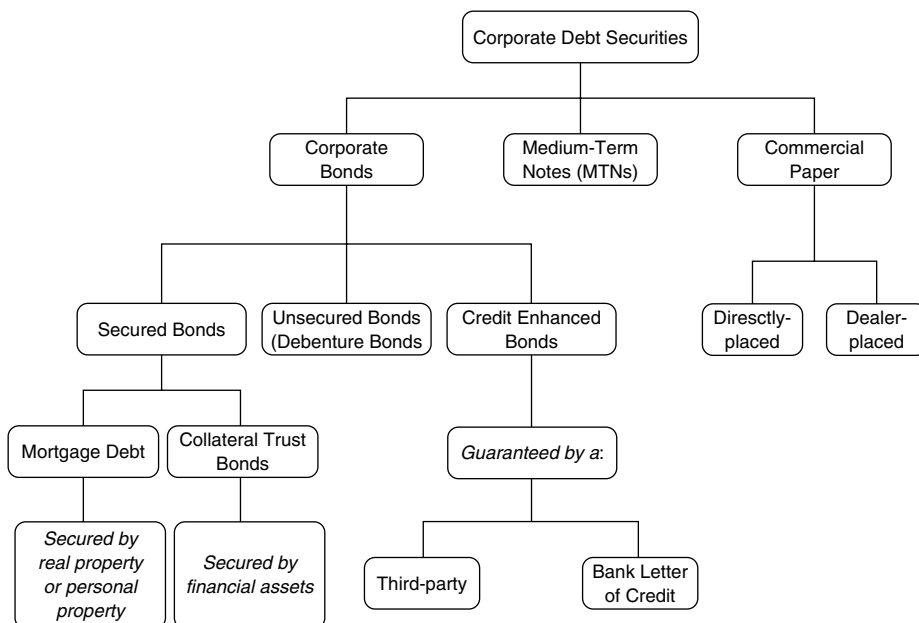
Corporations throughout the world that seek to borrow funds can do so through either bank borrowing or the issuance of debt securities. The securities issued include bonds (called corporate bonds), medium term notes, asset-backed securities, and commercial paper. Exhibit 10 provides an overview of the structures found in the corporate debt market. In many countries throughout the world, the principal form of borrowing is via bank borrowing and, as a result, a well-developed market for non-bank borrowing has not developed or is still in its infancy stage. However, even in countries where the market for corporate debt securities is small, large corporations can borrow outside of their country's domestic market.

Because in the United States there is a well developed market for corporations to borrow via the public issuance of debt obligations, we will look at this market. Before we describe the features of corporate bonds in the United States, we will discuss the rights of bondholders in a bankruptcy and the factors considered by rating agencies in assigning a credit rating.

A. Bankruptcy and Bondholder Rights in the United States

Every country has securities laws and contract laws that govern the rights of bondholders and a bankruptcy code that covers the treatment of bondholders in the case of a bankruptcy. There

EXHIBIT 10 Overview of Corporate Debt Securities



are principles that are common in the legal arrangements throughout the world. Below we discuss the features of the U.S. system.

The holder of a U.S. corporate debt instrument has priority over the equity owners in a bankruptcy proceeding. Moreover, there are creditors who have priority over other creditors. The law governing bankruptcy in the United States is the Bankruptcy Reform Act of 1978 as amended from time to time. One purpose of the act is to set forth the rules for a corporation to be either liquidated or reorganized when filing bankruptcy.

The **liquidation** of a corporation means that all the assets will be distributed to the claim holders of the corporation and no corporate entity will survive. In a **reorganization**, a new corporate entity will emerge at the end of the bankruptcy proceedings. Some security holders of the bankrupt corporation will receive cash in exchange for their claims, others may receive new securities in the corporation that results from the reorganization, and others may receive a combination of both cash and new securities in the resulting corporation.

Another purpose of the bankruptcy act is to give a corporation time to decide whether to reorganize or liquidate and then the necessary time to formulate a plan to accomplish either a reorganization or liquidation. This is achieved because when a corporation files for bankruptcy, the act grants the corporation protection from creditors who seek to collect their claims. The petition for bankruptcy can be filed either by the company itself, in which case it is called a **voluntary bankruptcy**, or be filed by its creditors, in which case it is called an **involuntary bankruptcy**. A company that files for protection under the bankruptcy act generally becomes a “debtor-in-possession” and continues to operate its business under the supervision of the court.

The bankruptcy act is comprised of 15 chapters, each chapter covering a particular type of bankruptcy. Chapter 7 deals with the liquidation of a company; Chapter 11 deals with the reorganization of a company.

When a company is liquidated, creditors receive distributions based on the **absolute priority rule** to the extent assets are available. The absolute priority rule is the principle that senior creditors are paid in full before junior creditors are paid anything. For secured and unsecured creditors, the absolute priority rule guarantees their seniority to equity holders. In liquidations, the absolute priority rule generally holds. In contrast, there is a good body of literature that argues that strict absolute priority typically has not been upheld by the courts or the SEC in reorganizations.

B. Factors Considered in Assigning a Credit Rating

In the previous chapter, we explained that there are companies that assign credit ratings to corporate issues based on the prospects of default. These companies are called rating agencies. In conducting a credit examination, each rating agency, as well as credit analysts employed by investment management companies, consider the four C's of credit—**character, capacity, collateral**, and **covenants**.

It is important to understand that a credit analysis can be for an entire company or a particular debt obligation of that company. Consequently, a rating agency may assign a different rating to the various issues of the same corporation depending on the level of seniority of the bondholders of each issue in the case of bankruptcy. For example, we will explain below that there is senior debt and subordinated debt. Senior debtholders have a better position relative to subordinated debtholders in the case of a bankruptcy for a given issuer. So, a rating agency, for example, may assign a rating of “A” to the senior debt of a corporation and a lower rating, “BBB,” to the subordinated debt of the same corporation.

Character analysis involves the analysis of the quality of management. In discussing the factors it considers in assigning a credit rating, Moody's Investors Service notes the following regarding the quality of management:

Although difficult to quantify, management quality is one of the most important factors supporting an issuer's credit strength. When the unexpected occurs, it is a management's ability to react appropriately that will sustain the company's performance.⁷

In assessing management quality, the analysts at Moody's, for example, try to understand the business strategies and policies formulated by management. Moody's considers the following factors: (1) strategic direction, (2) financial philosophy, (3) conservatism, (4) track record, (5) succession planning, and (6) control systems.⁸

In assessing the ability of an issuer to pay (i.e., capacity), the analysts conduct financial statement analysis. In addition to financial statement analysis, the factors examined by analysts at Moody's are (1) industry trends, (2) the regulatory environment, (3) basic operating and competitive position, (4) financial position and sources of liquidity, (5) company structure (including structural subordination and priority of claim), (6) parent company support agreements, and (7) special event risk.⁹

The third C, collateral, is looked at not only in the traditional sense of assets pledged to secure the debt, but also to the quality and value of those unpledged assets controlled by the issuer. Unpledged collateral is capable of supplying additional sources of funds to support payment of debt. Assets form the basis for generating cash flow which services the debt in good times as well as bad. We discuss later the various types of collateral used for a corporate debt issue and features that analysts should be cognizant of when evaluating an investor's secured position.

Covenants deal with limitations and restrictions on the borrower's activities. **Affirmative covenants** call upon the debtor to make promises to do certain things. **Negative covenants** are those which require the borrower not to take certain actions. Negative covenants are usually negotiated between the borrower and the lender or their agents. Borrowers want the least restrictive loan agreement available, while lenders should want the most restrictive, consistent with sound business practices. But lenders should not try to restrain borrowers from accepted business activities and conduct. A borrower might be willing to include additional restrictions (up to a point) if it can get a lower interest rate on the debt obligation. When borrowers seek to weaken restrictions in their favor, they are often willing to pay more interest or give other consideration. We will see examples of positive and negative covenants later in this chapter.

C. Corporate Bonds

In Chapter 1, we discussed the features of bonds including the wide range of coupon types, the provisions for principal payments, provisions for early retirement, and other embedded options. Also, in Chapter 2, we reviewed the various forms of credit risk and the ratings assigned by rating agencies. In our discussion of corporate bonds here, we will discuss secured and unsecured debt and information about default and recovery rates.

⁷"Industrial Company Rating Methodology," *Moody's Investors Service: Global Credit Research* (July 1998), p. 6.

⁸"Industrial Company Rating Methodology," p. 7.

⁹"Industrial Company Rating Methodology," p. 3.

1. **Secured Debt, Unsecured Debt, and Credit Enhancements** A corporate debt obligation may be secured or unsecured. **Secured debt** means that there is some form of collateral pledged to ensure payment of the debt. Remove the pledged collateral and we have **unsecured debt**.

It is important to recognize that while a superior legal status will strengthen a bondholder's chance of recovery in case of default, it will not absolutely prevent bondholders from suffering financial loss when the issuer's ability to generate sufficient cash flow to pay its obligations is seriously eroded. Claims against a weak borrower are often satisfied for less than par value.

a. Secured Debt Either **real property** or **personal property** may be pledged as security for secured debt. With **mortgage debt**, the issuer grants the bondholders a lien against pledged assets. A lien is a legal right to sell mortgaged property to satisfy unpaid obligations to bondholders. In practice, foreclosure and sale of mortgaged property is unusual. If a default occurs, there is usually a financial reorganization of the issuer in which provision is made for settlement of the debt to bondholders. The mortgage lien is important, though, because it gives the mortgage bondholders a strong bargaining position relative to other creditors in determining the terms of a reorganization.

Some companies do not own fixed assets or other real property and so have nothing on which they can give a mortgage lien to secure bondholders. Instead, they own securities of other companies; they are holding companies and the other companies are subsidiaries. To satisfy the desire of bondholders for security, the issuer grants investors a lien on stocks, notes, bonds or other kind of financial asset they own. Bonds secured by such assets are called **collateral trust bonds**. The eligible collateral is periodically marked to market by the trustee to ensure that the market value has a liquidation value in excess of the amount needed to repay the entire outstanding bonds and accrued interest. If the collateral is insufficient, the issuer must, within a certain period, bring the value of the collateral up to the required amount. If the issuer is unable to do so, the trustee would then sell collateral and redeem bonds.

Mortgage bonds have many different names. The following names have been used: **first mortgage bonds** (most common name), **first and general mortgage bonds**, **first refunding mortgage bonds**, and **first mortgage and collateral trusts**. There are instances (excluding prior lien bonds as mentioned above) when a company might have two or more layers of mortgage debt outstanding with different priorities. This situation usually occurs because companies cannot issue additional first mortgage debt (or the equivalent) under the existing indentures. Often this secondary debt level is called **general and refunding mortgage bonds** (G&R). In reality, this is mostly second mortgage debt. Some issuers may have third mortgage bonds.

Although an indenture may not limit the total amount of bonds that may be issued with the same lien, there are certain **issuance tests** that usually have to be satisfied before the company may sell more bonds. Typically there is an **earnings test** that must be satisfied before additional bonds may be issued with the same lien.

b. Unsecured Debt Unsecured debt is commonly referred to as **debenture bonds**. Although a debenture bond is not secured by a specific pledge of property, that does not mean that bondholders have no claim on property of issuers or on their earnings. Debenture bondholders have the claim of general creditors on all assets of the issuer not pledged specifically to secure other debt. And they even have a claim on pledged assets to the extent that these assets generate proceeds in liquidation that are greater than necessary to satisfy secured creditors. **Subordinated debenture bonds** are issues that rank after secured debt, after debenture bonds, and often after some general creditors in their claim on assets and earnings.

One of the important protective provisions for unsecured debt holders is the **negative pledge clause**. This provision, found in most senior unsecured debt issues and a few subordinated issues, prohibits a company from creating or assuming any lien to secure a debt issue without equally securing the subject debt issue(s) (with certain exceptions).

c. Credit Enhancements Some debt issuers have other companies guarantee their loans. This is normally done when a subsidiary issues debt and the investors want the added protection of a **third-party guarantee**. The use of guarantees makes it easier and more convenient to finance special projects and affiliates, although guarantees are also extended to operating company debt.

An example of a third-party (but related) guarantee was U.S. West Capital Funding, Inc. 8% Guaranteed Notes that were due October 15, 1996 (guaranteed by U.S. West, Inc.). The principal purpose of Capital Funding was to provide financing to U.S. West and its affiliates through the issuance of debt guaranteed by U.S. West. PepsiCo, Inc. has guaranteed the debt of its financing affiliate, PepsiCo Capital Resources, Inc., and The Standard Oil Company (an Ohio Corporation) has unconditionally guaranteed the debt of Sohio Pipe Line Company.

Another credit enhancing feature is the **letter of credit (LOC)** issued by a bank. A LOC requires the bank make payments to the trustee when requested so that monies will be available for the bond issuer to meet its interest and principal payments when due. Thus, the credit of the bank under the LOC is substituted for that of the debt issuer. Specialized insurance companies also lend their credit standing to corporate debt, both new issues and outstanding secondary market issues. In such cases, the credit rating of the bond is usually no better than the credit rating of the guarantor.

While a guarantee or other type of credit enhancement may add some measure of protection to a debtholder, caution should not be thrown to the wind. In effect, one's job may even become more complex as an analysis of both the issuer and the guarantor should be performed. In many cases, only the latter is needed if the issuer is merely a financing conduit without any operations of its own. However, if both concerns are operating companies, it may very well be necessary to analyze both, as the timely payment of principal and interest ultimately will depend on the stronger party. Generally, a downgrade of the credit enhancer's claims-paying ability reduces the value of the credit-enhanced bonds.

2. Default Rates and Recovery Rates Now we turn our attention to the various aspects of the historical performance of corporate issuers with respect to fulfilling their obligations to bondholders. Specifically, we will review two aspects of this performance. First, we will review the default rate of corporate borrowers. Second, we will review the default loss rate of corporate borrowers. From an investment perspective, default rates by themselves are not of paramount significance: it is perfectly possible for a portfolio of bonds to suffer defaults and to outperform Treasuries at the same time, provided the yield spread of the portfolio is sufficiently high to offset the losses from default. Furthermore, because holders of defaulted bonds typically recover some percentage of the face amount of their investment, the **default loss rate** is substantially lower than the default rate. Therefore, it is important to look at default loss rates or, equivalently, **recovery rates**.

a. Default Rates A default rate can be measured in different ways. A simple way to define a default rate is to use the issuer as the unit of study. A default rate is then measured as the number of issuers that default divided by the total number of issuers at the beginning of

the year. This measure—referred to as the **issuer default rate**—gives no recognition to the amount defaulted nor the total amount of issuance. Moody's, for example, uses this default rate statistic in its study of default rates. The rationale for ignoring dollar amounts is that the credit decision of an investor does not increase with the size of the issuer. The second measure—called the **dollar default rate**—defines the default rate as the par value of all bonds that defaulted in a given calendar year, divided by the total par value of all bonds outstanding during the year. With either default rate statistic, one can measure the default for a given year or an average annual default rate over a certain number of years.

There have been several excellent studies of corporate bond default rates. All of the studies found that the lower the credit rating, the greater the probability of a corporate issuer defaulting.

There have been extensive studies focusing on default rates for non-investment grade corporate bonds (i.e., speculative-grade issuer or high yield bonds). Studies by Edward Altman suggest that the annual default rate for speculative-grade corporate debt has been between 2.15% and 2.4% per year.¹⁰ Asquith, Mullins, and Wolff, however, found that nearly one out of every three speculative-grade bonds defaults.¹¹ The large discrepancy arises because researchers use three different definitions of “default rate”; even if applied to the same universe of bonds (which they are not), the results of these studies could be valid simultaneously.¹²

Altman defines the default rate as the dollar default rate. His estimates (2.15% and 2.40%) are simple averages of the annual dollar default rates over a number of years. Asquith, Mullins, and Wolff use a cumulative dollar default rate statistic. While both measures are useful indicators of bond default propensity, they are not directly comparable. Even when restated on an annualized basis, they do not all measure the same quantity. The default statistics reported in both studies, however, are surprisingly similar once cumulative rates have been annualized. A majority of studies place the annual dollar default rates for all original issue high-yield bonds between 3% and 4%.

b. Recovery Rates There have been several studies that have focused on recovery rates or default loss rates for corporate debt. Measuring the amount recovered is not a simple task. The final distribution to claimants when a default occurs may consist of cash and securities. Often it is difficult to track what was received and then determine the present value of any non-cash payments received.

Here we review recovery information as reported in a study by Moody's which uses the trading price at the time of default as a proxy for the amount recovered.¹³ The recovery rate is the trading price at that time divided by the par value. Moody's found that the recovery rate was 38% for all bonds. Moreover, the study found that the higher the level of seniority, the greater the recovery rate.

¹⁰Edward I. Altman and Scott A. Nammacher, *Investing in Junk Bonds* (New York: John Wiley, 1987) and Edward I. Altman, “Research Update: Mortality Rates and Losses, Bond Rating Drift,” unpublished study prepared for a workshop sponsored by Merrill Lynch Merchant Banking Group, High Yield Sales and Trading, 1989.

¹¹Paul Asquith, David W. Mullins, Jr., and Eric D. Wolff, “Original Issue High Yield Bonds: Aging Analysis of Defaults, Exchanges, and Calls,” *Journal of Finance* (September 1989), pp. 923–952.

¹²As a parallel, we know that the mortality rate in the United States is currently less than 1% per year, but we also know that 100% of all humans (eventually) die.

¹³Moody's Investors Service, *Corporate Bond Defaults and Default Rates: 1970–1994*, Moody's Special Report, January 1995, p. 13.

D. Medium-Term Notes

A **medium-term note** (MTN) is a debt instrument, with the unique characteristic that notes are offered continuously to investors by an agent of the issuer. Investors can select from several maturity ranges: 9 months to 1 year, more than 1 year to 18 months, more than 18 months to 2 years, and so on up to 30 years. Medium-term notes are registered with the Securities and Exchange Commission under Rule 415 (the shelf registration rule) which gives a borrower (corporation, agency, sovereign, or supranational) the maximum flexibility for issuing securities on a continuous basis. As with corporate bonds, MTNs are rated by the nationally recognized statistical rating organizations.

The term “medium-term note” used to describe this debt instrument is misleading. Traditionally, the term “note” or “medium-term” was used to refer to debt issues with a maturity greater than one year but less than 15 years. Certainly this is not a characteristic of MTNs since they have been sold with maturities from nine months to 30 years, and even longer. For example, in July 1993, Walt Disney Corporation issued a security with a 100-year maturity off its medium-term note shelf registration. From the perspective of the borrower, the initial purpose of the MTN was to fill the funding gap between commercial paper and long-term bonds. It is for this reason that they are referred to as “medium term.”

Borrowers have flexibility in designing MTNs to satisfy their own needs. They can issue fixed- or floating-rate debt. The coupon payments can be denominated in U.S. dollars or in a foreign currency. MTNs have been designed with the same features as corporate bonds.

1. The Primary Market Medium-term notes differ from bonds in the manner in which they are distributed to investors when they are initially sold. Although some corporate bond issues are sold on a “best-efforts basis” (i.e., the underwriter does not purchase the securities from the issuer but only agrees to sell them),¹⁴ typically corporate bonds are underwritten by investment bankers. When “underwritten,” the investment banker purchases the bonds from the issuer at an agreed upon price and yield and then attempts to sell them to investors. This is discussed further in Section IX. MTNs have been traditionally distributed on a best-efforts basis by either an investment banking firm or other broker/dealers acting as agents. Another difference between bonds and MTNs is that when offered, MTNs are usually sold in relatively small amounts on either a continuous or an intermittent basis, while bonds are sold in large, discrete offerings.

An entity that wants to initiate a MTN program will file a shelf registration¹⁵ with the SEC for the offering of securities. While the SEC registration for MTN offerings are between \$100 million and \$1 billion, once completely sold, the issuer can file another shelf registration for a new MTN offering. The registration will include a list of the investment banking firms, usually two to four, that the borrower has arranged to act as agents to distribute the MTNs.

¹⁴The primary market for bonds is described in Section IX A.

¹⁵SEC Rule 415 permits certain issuers to file a single registration document indicating that it intends to sell a certain amount of a certain class of securities at one or more times within the next two years. Rule 415 is popularly referred to as the “shelf registration rule” because the securities can be viewed as sitting on the issuer’s “shelf” and can be taken off that shelf and sold to the public without obtaining additional SEC approval. In essence, the filing of a single registration document allows the issuer to come to market quickly because the sale of the security has been preapproved by the SEC. Prior to establishment of Rule 415, there was a lengthy period required before a security could be sold to the public. As a result, in a fast-moving market, issuers could not come to market quickly with an offering to take advantage of what it perceived to be attractive financing opportunities.

The issuer then posts rates over a range of maturities: for example, nine months to one year, one year to 18 months, 18 months to two years, and annually thereafter. In an offering rate schedule, an issuer will post rates as a spread over a Treasury security of comparable maturity. Rates will not be posted for maturity ranges that the issuer does not desire to sell.

The agents will then make the offering rate schedule available to their investor base interested in MTNs. An investor who is interested in the offering will contact the agent. In turn, the agent contacts the issuer to confirm the terms of the transaction. Since the maturity range in an offering rate schedule does not specify a specific maturity date, the investor can choose the final maturity subject to approval by the issuer.

The rate offering schedule can be changed at any time by the issuer either in response to changing market conditions or because the issuer has raised the desired amount of funds at a given maturity. In the latter case, the issuer can either not post a rate for that maturity range or lower the rate.

2. Structured MTNs At one time, the typical MTN was a fixed-rate debenture that was noncallable. It is common today for issuers of MTNs to couple their offerings with transactions in the derivative markets (options, futures/forwards, swaps, caps, and floors) so they may create debt obligations with more complex risk/return features than are available in the corporate bond market. Specifically, an issue can have a floating-rate over all or part of the life of the security and the coupon formula can be based on a benchmark interest rate, equity index, individual stock price, foreign exchange rate, or commodity index. There are MTNs with inverse floating coupon rates and can include various embedded options.

MTNs created when the issuer simultaneously transacts in the derivative markets are called **structured notes**. The most common derivative instrument used in creating structured notes is a swap, an instrument described in Chapter 13. By using the derivative markets in combination with an offering, issuers are able to create investment vehicles that are more customized for institutional investors to satisfy their investment objectives, but who are forbidden from using swaps for hedging or speculating. Moreover, it allows institutional investors who are restricted to investing in investment grade debt issues the opportunity to participate in other asset classes such as the equity market. Hence, structured notes are sometimes referred to as “rule busters.” For example, an investor who buys an MTN whose coupon rate is tied to the performance of the S&P 500 (the reference rate) is participating in the equity market without owning common stock. If the coupon rate is tied to a foreign stock index, the investor is participating in the equity market of a foreign country without owning foreign common stock. In exchange for creating a structured note product, issuers can reduce their funding costs.

Common structured notes include: step-up notes, inverse floaters, deleveraged floaters, dual-indexed floaters, range notes, and index amortizing notes.

a. Deleveraged Floaters A **deleveraged floater** is a floater that has a coupon formula where the coupon rate is computed as a fraction of the reference rate plus a quoted margin. The general formula for a deleveraged floater is:

$$\text{coupon rate} = b \times (\text{reference rate}) + \text{quoted margin}$$

where b is a value between zero and one.

b. Dual-Indexed Floaters The coupon rate for a **dual-indexed floater** is typically a fixed percentage plus the difference between two reference rates. For example, the Federal Home Loan Bank System issued a floater whose coupon rate (reset quarterly) as follows:

$$(10\text{-year Constant Maturity Treasury rate}) - (3\text{-month LIBOR}) + 160 \text{ basis points}$$

c. Range Notes A **range note** is a type of floater whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range at the reset date. If the reference rate is outside of the range, the coupon rate is zero for that period. For example, a 3-year range note might specify that the reference rate is the 1-year Treasury rate and that the coupon rate resets every year. The coupon rate for the year is the Treasury rate as long as the Treasury rate at the coupon reset date falls within the range as specified below:

	Year 1	Year 2	Year 3
Lower limit of range	4.5%	5.25%	6.00%
Upper limit of range	6.5%	7.25%	8.00%

If the 1-year Treasury rate is outside of the range, the coupon rate is zero. For example, if in Year 1 the 1-year Treasury rate is 5% at the coupon reset date, the coupon rate for the year is 5%. However, if the 1-year Treasury rate is 7%, the coupon rate for the year is zero since the 1-year Treasury rate is greater than the upper limit for Year 1 of 6.5%.

d. Index Amortizing Notes An **index amortizing note (IAN)** is a structured note with a fixed coupon rate but whose principal payments are made prior to the stated maturity date based on the prevailing value for some reference interest rate. The principal payments are structured so that the time to maturity of an IAN increases when the reference interest rate increases and the maturity decreases when the reference interest rate decreases.

From our understanding of reinvestment risks, we can see the risks associated with investing in an IAN. Since the coupon rate is fixed, when interest rates rise, an investor would prefer to receive principal back faster in order to reinvest the proceeds received at the prevailing higher rate. However, with an IAN, the rate of principal repayment is decreased. In contrast, when interest rates decline, an investor does not want principal repaid quickly because the investor would then be forced to reinvest the proceeds received at the prevailing lower interest rate. With an IAN, when interest rates decline, the investor will, in fact, receive principal back faster.

E. Commercial Paper

Commercial paper is a short-term unsecured promissory note that is issued in the open market and represents the obligation of the issuing corporation. Typically, commercial paper is issued as a zero-coupon instrument. In the United States, the maturity of commercial paper is typically less than 270 days and the most common maturity is 50 days or less.

To pay off holders of maturing paper, issuers generally use the proceeds obtained from selling new commercial paper. This process is often described as “rolling over” short-term paper. The risk that the investor in commercial paper faces is that the issuer will be unable to issue new paper at maturity. As a safeguard against this “roll-over risk,” commercial paper is typically backed by unused bank credit lines.

There is very little secondary trading of commercial paper. Typically, an investor in commercial paper is an entity that plans to hold it until maturity. This is understandable since an investor can purchase commercial paper in a direct transaction with the issuer which will issue paper with the specific maturity the investor desires.

Corporate issuers of commercial paper can be divided into financial companies and nonfinancial companies. There has been significantly greater use of commercial paper by financial companies compared to nonfinancial companies. There are three types of financial

EXHIBIT 11 Commercial Paper Ratings

Category	Commercial rating company		
	Fitch	Moody's	S&P
Investment grade	F-1+		A-1+
	F-1	P-1	A-1
	F-2	P-2	A-2
	F-3	P-3	A-3
Noninvestment grade	F-S	NP (Not Prime)	B
			C
			D
In default	D		

companies: captive finance companies, bank-related finance companies, and independent finance companies. Captive finance companies are subsidiaries of manufacturing companies. Their primary purpose is to secure financing for the customers of the parent company. For example, U.S. automobile manufacturers have captive finance companies. Furthermore, a bank holding company may have a subsidiary that is a finance company, providing loans to enable individuals and businesses to acquire a wide range of products. Independent finance companies are those that are not subsidiaries of equipment manufacturing firms or bank holding companies.

Commercial paper is classified as either directly placed paper or dealer-placed paper. **Directly placed paper** is sold by the issuing firm to investors without the help of an agent or an intermediary. A large majority of the issuers of directly placed paper are financial companies. These entities require continuous funds in order to provide loans to customers. As a result, they find it cost effective to establish a sales force to sell their commercial paper directly to investors. General Electric Capital Corporation (GE Capital)—the principal financial services arm of General Electric Company—is the largest and most active direct issuer of commercial paper in the United States. **Dealer-placed commercial paper** requires the services of an agent to sell an issuer's paper.

The three nationally recognized statistical rating organizations that rate corporate bonds and medium-term notes also rate commercial paper. The ratings are shown in Exhibit 11. Commercial paper ratings, as with the ratings on other securities, are categorized as either investment grade or noninvestment grade.

F. Bank Obligations

Commercial banks are special types of corporations. Larger banks will raise funds using the various debt obligations described earlier. In this section, we describe two other debt obligations of banks—negotiable certificates of deposit and bankers acceptances—that are used by banks to raise funds.

1. Negotiable CDs A **certificate of deposit** (CD) is a financial asset issued by a bank (or other deposit-accepting entity) that indicates a specified sum of money has been deposited at the issuing depository institution. A CD bears a maturity date and a specified interest rate; it can be issued in any denomination. In the United States, CDs issued by most banks are insured by the Federal Deposit Insurance Corporation (FDIC), but only for amounts up to \$100,000. There is no limit on the maximum maturity. A CD may be nonnegotiable or negotiable. In the former case, the initial depositor must wait until the maturity date of the CD to obtain the funds. If the depositor chooses to withdraw funds prior to the maturity

date, an early withdrawal penalty is imposed. In contrast, a **negotiable CD** allows the initial depositor (or any subsequent owner of the CD) to sell the CD in the open market prior to the maturity date. Negotiable CDs are usually issued in denominations of \$1 million or more. Hence, an investor in a negotiable CD issued by an FDIC insured bank is exposed to the credit risk for any amount in excess of \$100,000.

An important type of negotiable CD is the **Eurodollar CD**, which is a U.S. dollar-denominated CD issued primarily in London by U.S., European, Canadian, and Japanese banks. The interest rates paid on Eurodollar CDs play an important role in the world financial markets because they are viewed globally as the cost of bank borrowing. This is due to the fact that these interest rates represent the rates at which major international banks offer to pay each other to borrow money by issuing a Eurodollar CD with given maturities. The interest rate paid is called the **London interbank offered rate** (LIBOR). The maturities for the Eurodollar CD range from overnight to five years. So, references to “3-month LIBOR” indicate the interest rate that major international banks are offering to pay to other such banks on a Eurodollar CD that matures in three months. During the 1990s, LIBOR has increasingly become the reference rate of choice for borrowing arrangements—loans and floating-rate securities.

2. Bankers Acceptances Simply put, a **bankers acceptance** is a vehicle created to facilitate commercial trade transactions. The instrument is called a bankers acceptance because a bank accepts the ultimate responsibility to repay a loan to its holder. The use of bankers acceptances to finance a commercial transaction is referred to as “acceptance financing.” In the United States, the transactions in which bankers acceptances are created include (1) the importing of goods; (2) the exporting of goods to foreign entities; (3) the storing and shipping of goods between two foreign countries where neither the importer nor the exporter is a U.S. firm; and (4) the storing and shipping of goods between two U.S. entities in the United States. Bankers acceptances are sold on a discounted basis just as Treasury bills and commercial paper.

The best way to explain the creation of a bankers acceptance is by an illustration. Several entities are involved in our hypothetical transaction:

- Luxury Cars USA (Luxury Cars), a firm in Pennsylvania that sells automobiles
- Italian Fast Autos Inc. (IFA), a manufacturer of automobiles in Italy
- First Doylestown Bank (Doylestown Bank), a commercial bank in Doylestown, Pennsylvania
- *Banco di Francesco*, a bank in Naples, Italy
- The Izzabof Money Market Fund, a U.S. mutual fund

Luxury Cars and IFA are considering a commercial transaction. Luxury Cars wants to import 45 cars manufactured by IFA. IFA is concerned with the ability of Luxury Cars to make payment on the 45 cars when they are received.

Acceptance financing is suggested as a means for facilitating the transaction. Luxury Cars offers \$900,000 for the 45 cars. The terms of the sale stipulate payment to be made to IFA 60 days after it ships the 45 cars to Luxury Cars. IFA determines whether it is willing to accept the \$900,000. In considering the offering price, IFA must calculate the present value of the \$900,000, because it will not be receiving payment until 60 days after shipment. Suppose that IFA agrees to these terms.

Luxury Cars arranges with its bank, Doylestown Bank, to issue a letter of credit. The letter of credit indicates that Doylestown Bank will make good on the payment of \$900,000

that Luxury Cars must make to IFA 60 days after shipment. The letter of credit, or time draft, will be sent by Doylestown Bank to IFA's bank, *Banco di Francesco*. Upon receipt of the letter of credit, *Banco di Francesco* will notify IFA, which will then ship the 45 cars. After the cars are shipped, IFA presents the shipping documents to *Banco di Francesco* and receives the present value of \$900,000. IFA is now out of the picture.

Banco di Francesco presents the time draft and the shipping documents to Doylestown Bank. The latter will then stamp "accepted" on the time draft. By doing so, Doylestown Bank has created a bankers acceptance. This means that Doylestown Bank agrees to pay the holder of the bankers acceptance \$900,000 at the maturity date. Luxury Cars will receive the shipping documents so that it can procure the 45 cars once it signs a note or some other type of financing arrangement with Doylestown Bank.

At this point, the holder of the bankers acceptance is *Banco di Francesco*. It has two choices. It can continue to hold the bankers acceptance as an investment in its loan portfolio, or it can request that Doylestown Bank make a payment of the present value of \$900,000. Let's assume that *Banco di Francesco* requests payment of the present value of \$900,000. Now the holder of the bankers acceptance is Doylestown Bank. It has two choices: retain the bankers acceptance as an investment as part of its loan portfolio or sell it to an investor. Suppose that Doylestown Bank chooses the latter, and that The Izzabof Money Market Fund is seeking a high-quality investment with the same maturity as that of the bankers acceptance. Doylestown Bank sells the bankers acceptance to the money market fund at the present value of \$900,000. Rather than sell the instrument directly to an investor, Doylestown Bank could sell it to a dealer, who would then resell it to an investor such as a money market fund. In either case, at the maturity date, the money market fund presents the bankers acceptance to Doylestown Bank, receiving \$900,000, which the bank in turn recovers from Luxury Cars.

Investing in bankers acceptances exposes the investor to credit risk and liquidity risk. Credit risk arises because neither the borrower nor the accepting bank may be able to pay the principal due at the maturity date. When the bankers acceptance market was growing in the early 1980s, there were over 25 dealers. By 1989, the decline in the amount of bankers acceptances issued drove many one-time major dealers out of the business. Today, there are only a few major dealers and therefore bankers acceptances are considered illiquid. Nevertheless, since bankers acceptances are typically purchased by investors who plan to hold them to maturity, liquidity risk is not a concern to such investors.

VII. ASSET-BACKED SECURITIES

In Section IVB we described how residential mortgage loans have been securitized. While residential mortgage loans is by far the largest type of asset that has been securitized, the major types of assets that have been securitized in many countries have included the following:

- auto loans and leases
- consumer loans
- commercial assets (e.g., including aircraft, equipment leases, trade receivables)
- credit cards
- home equity loans
- manufactured housing loans

Asset-backed securities are securities backed by a pool of loans or receivables. Our objective in this section is to provide a brief introduction to asset-backed securities.

A. The Role of the Special Purpose Vehicle

The key question for investors first introduced to the asset-backed securities market is why doesn't a corporation simply issue a corporate bond or medium-term note rather than an asset-backed security? To understand why, consider a triple B rated corporation that manufactures construction equipment. We will refer to this corporation as XYZ Corp. Some of its sales are for cash and others are on an installment sales basis. The installment sales are assets on the balance sheet of XYZ Corp., shown as "installment sales receivables."

Suppose XYZ Corp. wants to raise \$75 million. If it issues a corporate bond, for example, XYZ Corp.'s funding cost would be whatever the benchmark Treasury yield is plus a yield spread for BBB issuers. Suppose, instead, that XYZ Corp. has installment sales receivables that are more than \$75 million. XYZ Corp. can use the installment sales receivables as collateral for a bond issue. What will its funding cost be? It will probably be the same as if it issued a corporate bond. The reason is if XYZ Corp. defaults on any of its obligations, the creditors will have claim on all of its assets, including the installment sales receivables to satisfy payment of their bonds.

However, suppose that XYZ Corp. can create another corporation or legal entity and sell the installment sales receivables to that entity. We'll refer to this entity as SPV Corp. If the transaction is done properly, SPV Corp. owns the installment sales receivables, not XYZ Corp. It is important to understand that SPV Corp. is *not* a subsidiary of XYZ Corp.; therefore, the assets in SPV Corp. (i.e., the installment sales receivables) are not owned by XYZ Corp. This means that if XYZ Corp. is forced into bankruptcy, its creditors cannot claim the installment sales receivables because they are owned by SPV Corp. What are the implications?

Suppose that SPV Corp. sells securities backed by the installment sales receivables. Now creditors will evaluate the credit risk associated with collecting the receivables independent of the credit rating of XYZ Corp. What credit rating will be received for the securities issued by SPV Corp.? Whatever SPV Corp. wants the rating to be! It may seem strange that the issuer (SPV Corp.) can get any rating it wants, but that is the case. The reason is that SPV Corp. will show the characteristics of the collateral for the security (i.e., the installment sales receivables) to a rating agency. In turn, the rating agency will evaluate the credit quality of the collateral and inform the issuer what must be done to obtain specific ratings.

More specifically, the issuer will be asked to "credit enhance" the securities. There are various forms of credit enhancement. Basically, the rating agencies will look at the potential losses from the pool of installment sales receivables and make a determination of how much credit enhancement is needed for it to issue a specific rating. The higher the credit rating sought by the issuer, the greater the credit enhancement. Thus, XYZ Corp. which is BBB rated can obtain funding using its installment sales receivables as collateral to obtain a better credit rating for the securities issued. In fact, with enough credit enhancement, it can issue a AAA-rated security.

The key to a corporation issuing a security with a higher credit rating than the corporation's own credit rating is using SPV Corp. as the issuer. Actually, this legal entity that a corporation sells the assets to is called a **special purpose vehicle** or **special purpose corporation**. It plays a critical role in the ability to create a security—an asset-backed security—that separates the assets used as collateral from the corporation that is seeking financing.¹⁶

¹⁶There are other advantages to the corporation having to do with financial accounting for the assets sold. We will not discuss this aspect of financing via asset securitization here since it is not significant for the investor.

Why doesn't a corporation always seek the highest credit rating (AAA) for its securities backed by collateral? The answer is that credit enhancement does not come without a cost. Credit enhancement mechanisms increase the costs associated with a securitized borrowing via an asset-backed security. So, the corporation must monitor the trade-off when seeking a higher rating between the additional cost of credit enhancing the security versus the reduction in funding cost by issuing a security with a higher credit rating.

Additionally, if bankruptcy occurs, there is the risk that a bankruptcy judge may decide that the assets of the special purpose vehicle are assets that the creditors of the corporation seeking financing (XYZ Corp. in our example) may claim after all. This is an important but unresolved legal issue in the United States. Legal experts have argued that this is unlikely. In the prospectus of an asset-backed security, there will be a legal opinion addressing this issue. This is the reason why special purpose vehicles in the United States are referred to as "bankruptcy remote" entities.

B. Credit Enhancement Mechanisms

Later, we will review how rating agencies analyze collateral in order to assign ratings. What is important to understand is that the amount of credit enhancement will be determined relative to a particular rating. There are two general types of credit enhancement structures: external and internal.

External credit enhancements come in the form of third-party guarantees. The most common forms of external credit enhancements are (1) a corporate guarantee, (2) a letter of credit, and (3) bond insurance. A corporate guarantee could be from the issuing entity seeking the funding (XYZ Corp. in our illustration above) or its parent company. Bond insurance provides the same function as in municipal bond structures and is referred to as an insurance "wrap."

A disadvantage of an external credit enhancement is that it is subject to the credit risk of the third-party guarantor. Should the third-party guarantor be downgraded, the issue itself could be subject to downgrade even if the collateral is performing as expected. This is based on the "weak link" test followed by rating agencies. According to this test, when evaluating a proposed structure, the credit quality of the issue is only as good as the weakest link in credit enhancement regardless of the quality of the underlying loans. Basically, an external credit enhancement exposes the investor to event risk since the downgrading of one entity (the third-party guarantor) can result in a downgrade of the asset-backed security.

Internal credit enhancements come in more complicated forms than external credit enhancements. The most common forms of internal credit enhancements are reserve funds, over collateralization, and senior/subordinate structures.

VIII. COLLATERALIZED DEBT OBLIGATIONS

A fixed income product that is also classified as part of the asset-backed securities market is the **collateralized debt obligation** (CDO). CDOs deserve special attention because of their growth since 2000. Moreover, while a CDO is backed by various assets, it is managed in a way that is not typical in other asset-backed security transactions. CDOs have been issued in both developed and developing countries.

A CDO is a product backed by a diversified pool of one or more of the following types of debt obligations:

- U.S. domestic investment-grade and high-yield corporate bonds
- U.S. domestic bank loans
- emerging market bonds
- special situation loans and distressed debt
- foreign bank loans
- asset-backed securities
- residential and commercial mortgage-backed securities
- other CDOs

When the underlying pool of debt obligations consists of bond-type instruments (corporate and emerging market bonds), a CDO is referred to as a **collateralized bond obligation** (CBO). When the underlying pool of debt obligations are bank loans, a CDO is referred to as a **collateralized loan obligation** (CLO).

In a CDO structure, an asset manager is responsible for managing the portfolio of assets (i.e., the debt obligations in which it invests). The funds to purchase the underlying assets (i.e., the bonds and loans) are obtained from the issuance of a CDO. The CDO is structured into notes or *tranches* similar to a CMO issue. The tranches are assigned ratings by a rating agency. There are restrictions as to how the manager manages the CDO portfolio, usually in the form of specific tests that must be satisfied. If any of the restrictions are violated by the asset manager, the notes can be downgraded and it is possible that the trustee begin paying principal to the senior noteholders in the CDO structure.

CDOs are categorized based on the motivation of the sponsor of the transaction. If the motivation of the sponsor is to earn the spread between the yield offered on the fixed income products held in the portfolio of the underlying pool (i.e., the collateral) and the payments made to the noteholders in the structure, then the transaction is referred to as an **arbitrage transaction**. (Moreover, a CDO is a vehicle for a sponsor that is an investment management firm to gather additional assets to manage and thereby generate additional management fees.) If the motivation of the sponsor is to remove debt instruments (primarily loans) from its balance sheet, then the transaction is referred to as a **balance sheet transaction**. Sponsors of balance sheet transactions are typically financial institutions such as banks and insurance companies seeking to reduce their capital requirements by removing loans due to their higher risk-based capital requirements.

IX. PRIMARY MARKET AND SECONDARY MARKET FOR BONDS

Financial markets can be categorized as those dealing with financial claims that are newly issued, called the primary market, and those for exchanging financial claims previously issued, called the secondary market.

A. Primary Market

The primary market for bonds involves the distribution to investors of newly issued securities by central governments, its agencies, municipal governments, and corporations. Investment bankers work with issuers to distribute newly issued securities. The traditional process for issuing new securities involves investment bankers performing one or more of the following three functions: (1) advising the issuer on the terms and the timing of the offering, (2) buying

the securities from the issuer, and (3) distributing the issue to the public. The advisor role may require investment bankers to design a security structure that is more palatable to investors than a particular traditional instrument.

In the sale of new securities, investment bankers need not undertake the second function—buying the securities from the issuer. An investment banker may merely act as an advisor and/or distributor of the new security. The function of buying the securities from the issuer is called underwriting. When an investment banking firm buys the securities from the issuer and accepts the risk of selling the securities to investors at a lower price, it is referred to as an underwriter. When the investment banking firm agrees to buy the securities from the issuer at a set price, the underwriting arrangement is referred to as a firm commitment. In contrast, in a best efforts arrangement, the investment banking firm only agrees to use its expertise to sell the securities—it does not buy the entire issue from the issuer. The fee earned from the initial offering of a security is the difference between the price paid to the issuer and the price at which the investment bank reoffers the security to the public (called the reoffering price).

1. Bought Deal and Auction Process Not all bond issues are underwritten using the traditional firm commitment or best effort process we just described. Variations in the United States, the Euromarkets, and foreign markets for bonds include the **bought deal** and the **auction process**. The mechanics of a bought deal are as follows. The underwriting firm or group of underwriting firms offers a potential issuer of debt securities a firm bid to purchase a specified amount of securities with a certain coupon rate and maturity. The issuer is given a day or so (maybe even a few hours) to accept or reject the bid. If the bid is accepted, the underwriting firm has “bought the deal.” It can, in turn, sell the securities to other investment banking firms for distribution to their clients and/or distribute the securities to its clients. Typically, the underwriting firm that buys the deal will have presold most of the issue to its institutional clients. Thus, the risk of capital loss for the underwriting firm in a bought deal may not be as great as it first appears. There are some deals that are so straightforward that a large underwriting firm may have enough institutional investor interest to keep the risks of distributing the issue at the reoffering price quite small. Moreover, hedging strategies using interest rate risk control tools can reduce or eliminate the risk of realizing a loss of selling the bonds at a price below the reoffering price.

In the auction process, the issuer announces the terms of the issue and interested parties submit bids for the entire issue. This process is more commonly referred to as a competitive bidding underwriting. For example, suppose that a public utility wishes to issue \$400 million of bonds. Various underwriters will form syndicates and bid on the issue. The syndicate that bids the lowest yield (i.e., the lowest cost to the issuer) wins the entire \$400 million bond issue and then reoffers it to the public.

2. Private Placement of Securities Public and private offerings of securities differ in terms of the regulatory requirements that must be satisfied by the issuer. For example, in the United States, the Securities Act of 1933 and the Securities Exchange Act of 1934 require that all securities offered to the general public must be registered with the SEC, unless there is a specific exemption. The Securities Acts allow certain exemptions from federal registration. Section 4(2) of the 1933 Act exempts from registration “transactions by an issuer not involving any public offering.”

The exemption of an offering does not mean that the issuer need not disclose information to potential investors. The issuer must still furnish the same information deemed material by

the SEC. This is provided in a private placement memorandum, as opposed to a prospectus for a public offering. The distinction between the private placement memorandum and the prospectus is that the former does not include information deemed by the SEC as “non-material,” whereas such information is required in a prospectus. Moreover, unlike a prospectus, the private placement memorandum is not subject to SEC review.

In the United States, one restriction that was imposed on buyers of privately placed securities is that they may not be sold for two years after acquisition. Thus, there was no liquidity in the market for that time period. Buyers of privately placed securities must be compensated for the lack of liquidity which raises the cost to the issuer of the securities. SEC Rule 144A, which became effective in 1990, eliminates the two-year holding period by permitting large institutions to trade securities acquired in a private placement among themselves without having to register these securities with the SEC. Private placements are therefore now classified as Rule 144A offerings or non-Rule 144A offerings. The latter are more commonly referred to as traditional private placements. Rule 144A offerings are underwritten by investment bankers.

B. Secondary Market

In the secondary market, an issuer of a bond—whether it is a corporation or a governmental unit—may obtain regular information about the bond’s value. The periodic trading of a bond reveals to the issuer the consensus price that the bond commands in an open market. Thus, issuers can discover what value investors attach to their bonds and the implied interest rates investors expect and demand from them. Bond investors receive several benefits from a secondary market. The market obviously offers them liquidity for their bond holdings as well as information about fair or consensus values. Furthermore, secondary markets bring together many interested parties and thereby reduces the costs of searching for likely buyers and sellers of bonds.

A bond can trade on an exchange or in an over-the-counter market. Traditionally, bond trading has taken place predominately in the over-the-counter market where broker-dealer trading desks take principal positions to fill customer buy and sell orders. In recent years, however, there has been an evolution away from this form of traditional bond trading and toward electronic bond trading. This evolution toward electronic bond trading is likely to continue.

There are several related reasons for the transition to the electronic trading of bonds. First, because the bond business has been a principal business (where broker-dealer firms risk their own capital) rather than an agency business (where broker-dealer firms act merely as an agent or broker), the capital of the market makers is critical. The amount of capital available to institutional investors to invest throughout the world has placed significant demands on the capital of broker-dealer firms. As a result, making markets in bonds has become more risky for broker-dealer firms. Second, the increase in bond market volatility has increased the capital required of broker-dealer firms in the bond business. Finally, the profitability of bond market trading has declined since many of the products have become more commodity-like and their bid-offer spreads have decreased.

The combination of the increased risk and the decreased profitability of bond market trading has induced the major broker-dealer firms to deemphasize this business in the allocation of capital. Broker-dealer firms have determined that it is more efficient to employ their capital in other activities such as underwriting and asset management, rather than in principal-type market-making businesses. As a result, the liquidity of the traditionally principal-oriented bond

markets has declined, and this decline in liquidity has opened the way for other market-making mechanisms. This retreat by traditional market-making firms opened the door for electronic trading. In fact, the major broker-dealer firms in bonds have supported electronic trading in bonds.

Electronic trading in bonds has helped fill this developing vacuum and provided liquidity to the bond markets. In addition to the overall advantages of electronic trading in providing liquidity to the markets and price discovery (particularly for less liquid markets) is the resulting trading and portfolio management efficiencies that have been realized. For example, portfolio managers can load their buy/sell orders into a web site, trade from these orders, and then clear these orders.

There are a variety of types of electronic trading systems for bonds. The two major types of electronic trading systems are **dealer-to-customer systems** and **exchange systems**. Dealer-to-customer systems can be a single-dealer system or multiple-dealer system. Single-dealer systems are based on a customer dealing with a single, identified dealer over the computer. The single-dealer system simply computerizes the traditional customer-dealer market-making mechanism. Multi-dealer systems provide some advancement over the single-dealer method. A customer can select from any of several identified dealers whose bids and offers are provided on a computer screen. The customer knows the identity of the dealer.

In an exchange system, dealer and customer bids and offers are entered into the system on an anonymous basis, and the clearing of the executed trades is done through a common process. Two different major types of exchange systems are those based on continuous trading and call auctions. Continuous trading permits trading at continuously changing market-determined prices throughout the day and is appropriate for liquid bonds, such as Treasury and agency securities. Call auctions provide for fixed price auctions (that is, all the transactions or exchanges occur at the same “fixed” price) at specific times during the day and are appropriate for less liquid bonds such as corporate bonds and municipal bonds.

CHAPTER 4

UNDERSTANDING YIELD SPREADS

I. INTRODUCTION

The interest rate offered on a particular bond issue depends on the interest rate that can be earned on (1) risk-free instruments and (2) the perceived risks associated with the issue. We refer to the interest rates on risk-free instruments as the “level of interest rates.” The actions of a country’s central bank influence the level of interest rates as does the state of the country’s economy. In the United States, the level of interest rates depends on the state of the economy, the interest rate policies implemented by the Board of Governors of the Federal Reserve Board, and the government’s fiscal policies.

A casual examination of the financial press and dealer quote sheets shows a wide range of interest rates reported at any given point in time. Why are there differences in interest rates among debt instruments? We provided information on this topic in Chapters 1 and 2. In Chapter 1, we explained the various features of a bond while in Chapter 2 we explained how those features affect the risk characteristics of a bond relative to bonds without that feature.

In this chapter, we look more closely at the differences in yields offered by bonds in different sectors of the bond market and within a sector of the bond market. This information is used by investors in assessing the “relative value” of individual securities within a bond sector, or among sectors of the bond market. Relative value analysis is a process of ranking individual securities or sectors with respect to expected return potential. We will continue to use the terms “interest rate” and “yield” interchangeably.

II. INTEREST RATE DETERMINATION

Our focus in this chapter is on (1) the relationship between interest rates offered on different bond issues at a point in time and (2) the relationships among interest rates offered in different sectors of the economy at a given point in time. We will provide a brief discussion of the role of the U.S. Federal Reserve (the Fed), the policy making body whose interest rate policy tools directly influence short-term interest rates and indirectly influence long-term interest rates.

Once the Fed makes a policy decision it immediately announces the policy in a statement issued at the close of its meeting. The Fed also communicates its future intentions via public speeches or its Chairman’s testimony before Congress. Managers who pursue an active strategy of positioning a portfolio to take advantage of expected changes in interest rates watch closely the same key economic indicators that the Fed watches in order to anticipate a change in

the Fed's monetary policy and to assess the expected impact on short-term interest rates. The indicators that are closely watched by the Fed include non-farm payrolls, industrial production, housing starts, motor vehicle sales, durable good orders, National Association of Purchasing Management supplier deliveries, and commodity prices.

In implementing monetary policy, the Fed uses the following interest rate policy tools:

1. open market operations
2. the discount rate
3. bank reserve requirements
4. verbal persuasion to influence how bankers supply credit to businesses and consumers

Engaging in open market operations and changing the discount rate are the tools most often employed. Together, these tools can raise or lower the cost of funds in the economy. Open market operations do this through the Fed's buying and selling of U.S. Treasury securities. This action either adds funds to the market (when Treasury securities are purchased) or withdraws funds from the market (when Treasury securities are sold). Fed open market operations influence the federal funds rate, the rate at which banks borrow and lend funds from each other. The discount rate is the interest rate at which banks can borrow on a collateralized basis at the Fed's discount window. Increasing the discount rate makes the cost of funds more expensive for banks; the cost of funds is reduced when the discount rate is lowered. Changing bank reserve requirements is a less frequently used policy, as is the use of verbal persuasion to influence the supply of credit.

III. U.S. TREASURY RATES

The securities issued by the U.S. Department of the Treasury are backed by the full faith and credit of the U.S. government. Consequently, market participants throughout the world view these securities as being "default risk-free" securities. However, there are risks associated with owning U.S. Treasury securities.

The Treasury issues the following securities:

Treasury bills: Zero-coupon securities with a maturity at issuance of one year or less. The Treasury currently issues 1-month, 3-month, and 6-month bills.

Treasury notes: Coupon securities with maturity at issuance greater than 1 year but not greater than 10 years. The Treasury currently issues 2-year, 5-year, and 10-year notes.

Treasury bonds: Coupon securities with maturity at issuance greater than 10 years. Although Treasury bonds have traditionally been issued with maturities up to 30 years, the Treasury suspended issuance of the 30-year bond in October 2001.

Inflation-protection securities: Coupon securities whose principal's reference rate is the Consumer Price Index.

The on-the-run issue or current issue is the most recently auctioned issue of Treasury notes and bonds of each maturity. The off-the-run issues are securities that were previously issued and are replaced by the on-the-run issue. Issues that have been replaced by several more recent issues are said to be "well off-the-run issues."

The secondary market for Treasury securities is an over-the-counter market where a group of U.S. government securities dealers provides continuous bids and offers on specific

outstanding Treasuries. This secondary market is the most liquid financial market in the world. Off-the-run issues are less liquid than on-the-run issues.

A. Risks of Treasury Securities

With this brief review of Treasury securities, let's look at their risks. We listed the general risks in Chapter 2 and repeat them here: (1) interest rate risk, (2) call and prepayment risk, (3) yield curve risk, (4) reinvestment risk, (5) credit risk, (6) liquidity risk, (7) exchange-rate risk, (8) volatility risk, (9) inflation or purchasing power risk, and (10) event risk.

All fixed income securities, including Treasury securities, expose investors to interest rate risk.¹ However, the degree of interest rate risk is not the same for all securities. The reason is that maturity and coupon rate affect how much the price changes when interest rates change. One measure of a security's interest rate risk is its *duration*.² Since Treasury securities, like other fixed income securities, have different durations, they have different exposures to interest rate risk as measured by duration.

Technically, yield curve risk and volatility risk are risks associated with Treasury securities. However, at this early stage of our understanding of fixed income analysis, we will not attempt to explain these risks. It is not necessary to understand these risks at this point in order to appreciate the material that follows in this section.

Because Treasury securities are noncallable, there is no reinvestment risk due to an issue being called.³ Treasury coupon securities carry reinvestment risk because in order to realize the yield offered on the security, the investor must reinvest the coupon payments received at an interest rate equal to the computed yield. So, all Treasury coupon securities are exposed to reinvestment risk. Treasury bills are not exposed to reinvestment risk because they are zero-coupon instruments.

As for credit risk, the perception in the global financial community is that Treasury securities have no credit risk. In fact, when market participants and the popular press state that Treasury securities are "risk free," they are referring to credit risk.

Treasury securities are highly liquid. However, on-the-run and off-the-run Treasury securities trade with different degrees of liquidity. Consequently, the yields offered by on-the-run and off-the-run issues reflect different degrees of liquidity.

Since U.S. Treasury securities are dollar denominated, there is no exchange-rate risk for an investor whose domestic currency is the U.S. dollar. However, non-U.S. investors whose domestic currency is not the U.S. dollar are exposed to exchange-rate risk.

Fixed-rate Treasury securities are exposed to inflation risk. Treasury inflation protection securities (TIPS) have a coupon rate that is effectively adjusted for the rate of inflation and therefore have protection against inflation risk.

¹Interest rate risk is the risk of an adverse movement in the price of a bond due to changes in interest rates.

²Duration is a measure of a bond's price sensitivity to a change in interest rates.

³The Treasury no longer issues callable bonds. The Treasury issued callable bonds in the early 1980s and all of these issues will mature no later than November 2014 (assuming that they are not called before then). Moreover, as of 2004, the longest maturity of these issues is 10 years. Consequently, while outstanding callable issues of the Treasury are referred to as "bonds," based on their current maturity these issues would not be compared to long-term bonds in any type of relative value analysis. Therefore, because the Treasury no longer issues callable bonds and the outstanding issues do not have the maturity characteristics of a long-term bond, we will ignore these callable issues and simply treat Treasury bonds as noncallable.

EXHIBIT 1 Relationship Between Yield and Maturity
for On-the-Run Treasury Issues on February 8, 2002

Issue (maturity)	Yield (%)
1 month	1.68
3 months	1.71
6 months	1.81
1 year ¹	2.09
2 years	2.91
5 years	4.18
10 years	4.88
30 years ²	5.38

¹The 1-year issue is based on the 2-year issue closest to maturing in one year.

²The 30-year issue shown is based on the last 30-year issue before the Treasury suspended issuance of Treasury bonds in October 2001.

Source: *Global Relative Value*, Lehman Brothers, Fixed Income Research, February 11, 2002, p. 128.

Finally, the yield on Treasury securities is impacted by a myriad of events that can be classified as political risk, a form of event risk. The actions of monetary and fiscal policy in the United States, as well as the actions of other central banks and governments, can have an adverse or favorable impact on U.S. Treasury yields.

B. The Treasury Yield Curve

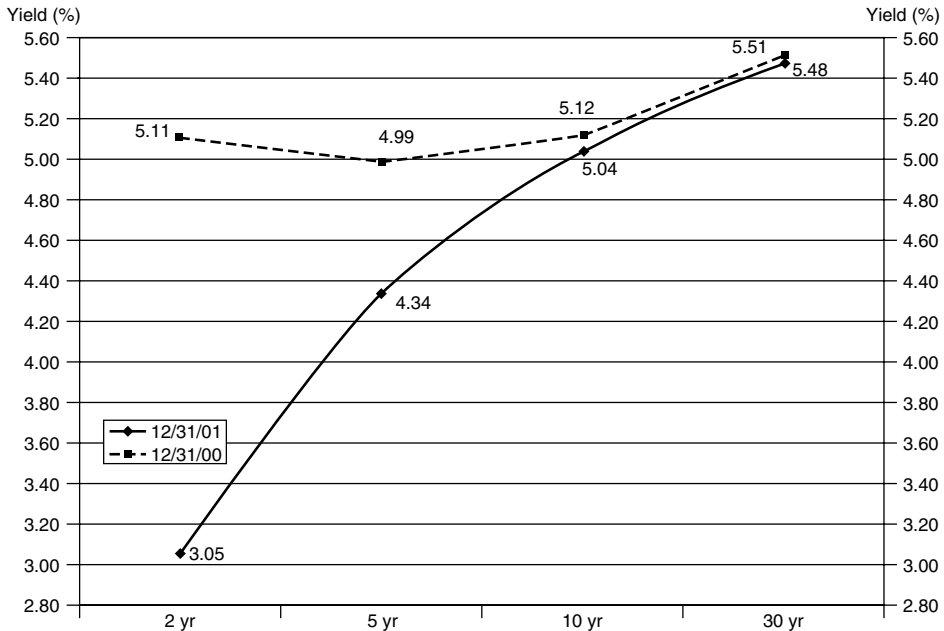
Given that Treasury securities do not expose investors to credit risk, market participants look at the yield offered on an on-the-run Treasury security as the minimum interest rate required on a non-Treasury security with the same maturity. The relationship between yield and maturity of on-the-run Treasury securities on February 8, 2002 is displayed in Exhibit 1 in tabular form. The relationship shown in Exhibit 1 is called the **Treasury yield curve**—even though the “curve” shown in the exhibit is presented in tabular form.

The information presented in Exhibit 1 indicates that the longer the maturity the higher the yield and is referred to as an **upward sloping yield curve**. Since this is the most typical shape for the Treasury yield curve, it is also referred to as a **normal yield curve**. Other relationships have been observed. An **inverted yield curve** indicates that the longer the maturity, the lower the yield. For a **flat yield curve** the yield is approximately the same regardless of maturity.

Exhibit 2 provides a graphic example of the variants of these shapes and also shows how a yield curve can change over time. In the exhibit, the yield curve at the beginning of 2001 was inverted up to the 5-year maturity but was upward sloping beyond the 5-year maturity. By December 2001, all interest rates had declined. As seen in the exhibit, interest rates less than the 10-year maturity dropped substantially more than longer-term rates resulting in an upward sloping yield curve.

The number of on-the-run securities available in constructing the yield curve has decreased over the last two decades. While the 1-year and 30-year yields are shown in the February 8, 2002 yield curve, as of this writing there is no 1-year Treasury bill and the maturity of the 30-year Treasury bond (the last one issued before suspension of the issuance of 30-year

EXHIBIT 2 U.S. Treasury Yield Curve: December 2000 and December 2001



Source: Lehman Brothers Fixed Income Research, *Global Fixed Income Strategy "Playbook,"* January 2002.

Treasury bonds) will decline over time. To get a yield for maturities where no on-the-run Treasury issue exists, it is necessary to interpolate from the yield of two on-the-run issues. Several methodologies are used in practice. (The simplest is just a linear interpolation.) Thus, when market participants talk about a yield on the Treasury yield curve that is not one of the available on-the-run maturities—for example, the 8-year yield—it is only an approximation.

It is critical to understand that any non-Treasury issue must offer a premium above the yield offered for the same maturity on-the-run Treasury issue. For example, if a corporation wanted to offer a 10-year noncallable issue on February 8, 2002, the issuer must offer a yield greater than 4.88% (the yield for the 10-year on-the-run Treasury issue). How much greater depends on the additional risks associated with investing in the 10-year corporate issue compared to investors in the 10-year on-the-run Treasury issue. Even off-the-run Treasury issues must offer a premium to reflect differences in liquidity.

Two factors complicate the relationship between maturity and yield as portrayed by the yield curve. The first is that the yield for on-the-run issues may be distorted by the fact that purchase of these securities can be financed at lower rates and as a result these issues offer artificially low yields. To clarify, some investors purchase securities with borrowed funds and use the securities purchased as collateral for the loan. This type of collateralized borrowing is called a repurchase agreement. Since dealers want to obtain use of these securities for their own trading activities, they are willing to lend funds to investors at a lower interest rate than is otherwise available for borrowing in the market. Consequently, incorporated into the price of an on-the-run Treasury security is the cheaper financing available, resulting in a lower yield for an on-the-run issue than would prevail in the absence of this financing advantage.

The second factor complicating the comparison of on-the-run and off-the-run Treasury issues (in addition to liquidity differences) is that they have different interest rate risks and different reinvestment risks. So, for example, if the coupon rate for the 5-year on-the-run Treasury issue in February 2002 is 4.18% and an off-the-run Treasury issue with just less than 5 years to maturity has a 5.25% coupon rate, the two bonds have different degrees of interest rate risk. Specifically, the on-the-run issue has greater interest rate risk (duration) because of the lower coupon rate. However, it has less reinvestment risk because the coupon rate is lower.

Because of this, when market participants talk about interest rates in the Treasury market and use these interest rates to value securities they look at another relationship in the Treasury market: the relationship between yield and maturity for zero-coupon Treasury securities. But wait, we said that the Treasury only issues three zero-coupon securities—1-month, 3-month, and 6-month Treasury bills. Where do we obtain the relationship between yield and maturity for zero-coupon Treasury securities? We discuss this next.

1. Theories of the Term Structure of Interest Rates What information does the yield curve reveal? How can we explain and interpret changes in the yield curve? These questions are of great interest to anyone concerned with such tasks as the valuation of multiperiod securities, economic forecasting, and risk management. Theories of the term structure of interest rates⁴ address these questions. Here we introduce the three main theories or explanations of the term structure. We shall present these theories intuitively.⁵

The three main term structure theories are:

- the pure expectations theory (unbiased expectations theory)
- the liquidity preference theory (or liquidity premium theory)
- the market segmentation theory

Each theory is explained below.

a. Pure Expectations Theory The pure expectations theory makes the simplest and most direct link between the yield curve and investors' expectations about future interest rates, and, because long-term interest rates are plausibly linked to investor expectations about future inflation, it also opens the door to some interesting economic interpretations.

The **pure expectations theory** explains the term structure in terms of expected future short-term interest rates. According to the pure expectations theory, the market sets the yield on a two-year bond so that the return on the two-year bond is approximately equal to the return on a one-year bond plus the expected return on a one-year bond purchased one year from today.

Under this theory, a rising term structure indicates that the market expects short-term rates to rise in the future. For example, if the yield on the two-year bond is higher than the yield on the one-year bond, according to this theory, investors expect the one-year rate a year from now to be sufficiently higher than the one-year rate available now so that the two ways of investing for two years have the same expected return. Similarly, a flat term structure reflects

⁴Term structure means the same as maturity structure—a description of how a bond's yield changes as the bond's maturity changes. In other words, term structure asks the question: Why do long-term bonds have a different yield than short-term bonds?

⁵Later, we provide a more mathematical treatment of these theories in terms of forward rates that we will discuss in Chapter 6.

an expectation that future short-term rates will be unchanged from today's short-term rates, while a falling term structure reflects an expectation that future short-term rates will decline. This is summarized below:

Shape of term structure	Implication according to pure expectations theory
upward sloping (normal)	rates expected to rise
downward sloping (inverted)	rates expected to decline
flat	rates not expected to change

The implications above are the broadest interpretation of the theory.

How does the pure expectations theory explain a humped yield curve? According to the theory, this can result when investors expect the returns on one-year securities to rise for a number of years, then fall for a number of years.

The relationships that the table above illustrates suggest that the shape of the yield curve contains information regarding investors' expectations about future inflation. A pioneer of the theory of interest rates (the economist Irving Fisher) asserted that interest rates reflect the sum of a relatively stable real rate of interest plus a premium for expected inflation. Under this hypothesis, if short-term rates are expected to rise, investors expect inflation to rise as well. An upward (downward) sloping term structure would mean that investors expected rising (declining) future inflation. Much economic discussion in the financial press and elsewhere is based on this interpretation of the yield curve.

The shortcoming of the pure expectations theory is that it assumes investors are indifferent to interest rate risk and any other risk factors associated with investing in bonds with different maturities.

b. Liquidity Preference Theory The **liquidity preference theory** asserts that market participants want to be compensated for the interest rate risk associated with holding longer-term bonds. The longer the maturity, the greater the price volatility when interest rates change and investors want to be compensated for this risk. According to the liquidity preference theory, the term structure of interest rates is determined by (1) expectations about future interest rates and (2) a yield premium for interest rate risk.⁶ Because interest rate risk increases with maturity, the liquidity preference theory asserts that the yield premium increases with maturity.

Consequently, based on this theory, an upward-sloping yield curve may reflect expectations that future interest rates either (1) will rise, or (2) will be unchanged or even fall, but with a yield premium increasing with maturity fast enough to produce an upward sloping yield curve. Thus, for an upward sloping yield curve (the most frequently observed type), the liquidity preference theory by itself has nothing to say about expected future short-term interest rates. For flat or downward sloping yield curves, the liquidity preference theory is consistent with a forecast of declining future short-term interest rates, given the theory's prediction that the yield premium for interest rate risk increases with maturity.

Because the liquidity preference theory argues that the term structure is determined by both expectations regarding future interest rates and a yield premium for interest rate risk, it is referred to as **biased expectations theory**.

⁶In the liquidity preference theory, "liquidity" is measured in terms of interest rate risk. Specifically, the more interest rate risk, the less the liquidity.

c. Market Segmentation Theory Proponents of the **market segmentation theory** argue that within the different maturity sectors of the yield curve the supply and demand for funds determine the interest rate for that sector. That is, each maturity sector is an independent or segmented market for purposes of determining the interest rate in that maturity sector. Thus, positive sloping, inverted, and humped yield curves are all possible. In fact, the market segmentation theory can be used to explain any shape that one might observe for the yield curve.

Let's understand why proponents of this theory view each maturity sector as independent or segmented. In the bond market, investors can be divided into two groups based on their return needs: investors that manage funds versus a broad-based bond market index and those that manage funds versus their liabilities. The easiest case is for those that manage funds against liabilities. Investors managing funds where liabilities represent the benchmark will restrict their activities to the maturity sector that provides the best match with the maturity of their liabilities.⁷ This is the basic principle of asset-liability management. If these investors invest funds outside of the maturity sector that provides the best match against liabilities, they are exposing themselves to the risks associated with an asset-liability mismatch. For example, consider the manager of a defined benefit pension fund. Since the liabilities of a defined benefit pension fund are long-term, the manager will invest in the long-term maturity sector of the bond market. Similarly, commercial banks whose liabilities are typically short-term focus on short-term fixed-income investments. Even if the rate on long-term bonds were considerably more attractive than that on short-term investments, according to the market segmentation theory commercial banks will restrict their activities to investments at the short end of the yield curve. Reinforcing this notion of a segmented market are restrictions imposed on financial institutions that prevent them from mismatching the maturity of assets and liabilities.

A variant of the market segmentation theory is the **preferred habitat theory**. This theory argues that investors prefer to invest in particular maturity sectors as dedicated by the nature of their liabilities. However, proponents of this theory do not assert that investors would be unwilling to shift out of their preferred maturity sector; instead, it is argued that if investors are given an inducement to do so in the form of a yield premium they will shift out of their preferred habitat. The implication of the preferred habitat theory for the shape of the yield curve is that any shape is possible.

C. Treasury Strips

Although the U.S. Department of the Treasury does not issue zero-coupon Treasury securities with maturity greater than one year, government dealers can synthetically create zero-coupon securities, which are effectively guaranteed by the full faith and credit of the U.S. government, with longer maturities. They create these securities by separating the coupon payments and the principal payment of a coupon-bearing Treasury security and selling them off separately. The process, referred to as **stripping a Treasury security**, results in securities called **Treasury strips**. The Treasury strips created from coupon payments are called Treasury coupon strips and those created from the principal payment are called Treasury principal strips. We explained the process of creating Treasury strips in Chapter 3.

⁷One of the principles of finance is the "matching principle:" short-term assets should be financed with (or matched with) short-term liabilities; long-term assets should be financed with (or matched with) long-term sources of financing.

Because zero-coupon instruments have no reinvestment risk, Treasury strips for different maturities provide a superior relationship between yield and maturity than do securities on the on-the-run Treasury yield curve. The lack of reinvestment risk eliminates the bias resulting from the difference in reinvestment risk for the securities being compared. Another advantage is that the duration of a zero-coupon security is approximately equal to its maturity. Consequently, when comparing bond issues against Treasury strips, we can compare them on the basis of duration.

The yield on a zero-coupon security has a special name: the **spot rate**. In the case of a Treasury security, the yield is called a **Treasury spot rate**. The relationship between maturity and Treasury spot rates is called the **term structure of interest rates**. Sometimes discussions of the term structure of interest rates in the Treasury market get confusing. The Treasury yield curve and the Treasury term structure of interest rates are often used interchangeably. While there is a technical difference between the two, the context in which these terms are used should be understood.

IV. YIELDS ON NON-TREASURY SECURITIES

Despite the imperfections of the Treasury yield curve as a benchmark for the minimum interest rate that an investor requires for investing in a non-Treasury security, it is commonplace to refer to the additional yield over the benchmark Treasury issue of the same maturity as the **yield spread**. In fact, because non-Treasury sectors of the fixed income market offer a yield spread to Treasury securities, non-Treasury sectors are commonly referred to as **spread sectors** and non-Treasury securities in these sectors are referred to as **spread products**.

A. Measuring Yield Spreads

While it is common to talk about spreads relative to a Treasury security of the same maturity, a yield spread between any two bond issues can be easily computed. In general, the yield spread between any two bond issues, bond X and bond Y, is computed as follows:

$$\text{yield spread} = \text{yield on bond X} - \text{yield on bond Y}$$

where bond Y is considered the reference bond (or benchmark) against which bond X is measured.

When a yield spread is computed in this manner it is referred to as an **absolute yield spread** and it is measured in basis points. For example, on February 8, 2002, the yield on the 10-year on-the-run Treasury issue was 4.88% and the yield on a single A rated 10-year industrial bond was 6.24%. If bond X is the 10-year industrial bond and bond Y is the 10-year on-the-run Treasury issue, the absolute yield spread was:

$$\text{yield spread} = 6.24\% - 4.88\% = 1.36\% \text{ or } 136 \text{ basis points}$$

Unless otherwise specified, yield spreads are typically measured in this way. Yield spreads can also be measured on a relative basis by taking the ratio of the yield spread to the yield of the reference bond. This is called a **relative yield spread** and is computed as shown below, assuming that the reference bond is bond Y:

$$\text{relative yield spread} = \frac{\text{yield on bond X} - \text{yield on bond Y}}{\text{yield on bond Y}}$$

Sometimes bonds are compared in terms of a **yield ratio**, the quotient of two bond yields, as shown below:

$$\text{yield ratio} = \frac{\text{yield on bond X}}{\text{yield on bond Y}}$$

Typically, in the U.S. bond market when these measures are computed, bond Y (the reference bond) is a Treasury issue. In that case, the equations for the yield spread measures are as follows:

$$\text{absolute yield spread} = \text{yield on bond X} - \text{yield of on-the-run Treasury}$$

$$\text{relative yield spread} = \frac{\text{yield on bond X} - \text{yield of on-the-run Treasury}}{\text{yield of on-the-run Treasury}}$$

$$\text{yield ratio} = \frac{\text{yield on bond X}}{\text{yield of on-the-run Treasury}}$$

For the above example comparing the yields on the 10-year single A rated industrial bond and the 10-year on-the-run Treasury, the relative yield spread and yield ratio are computed below:

$$\text{absolute yield spread} = 6.24\% - 4.88\% = 1.36\% = 136 \text{ basis points}$$

$$\text{relative yield spread} = \frac{6.24\% - 4.88\%}{4.88\%} = 0.279 = 27.9\%$$

$$\text{yield ratio} = \frac{6.24\%}{4.88\%} = 1.279$$

The reason for computing yield spreads in terms of a relative yield spread or a yield ratio is that the magnitude of the yield spread is affected by the level of interest rates. For example, in 1957 the yield on Treasuries was about 3%. At that time, the absolute yield spread between triple B rated utility bonds and Treasuries was 40 basis points. This was a relative yield spread of 13% (0.40% divided by 3%). However, when the yield on Treasuries exceeded 10% in 1985, an absolute yield spread of 40 basis points would have meant a relative yield spread of only 4% (0.40% divided by 10%). Consequently, in 1985 an absolute yield spread greater than 40 basis points would have been required in order to produce a similar relative yield spread.

In this chapter, we will focus on the yield spread as most commonly measured, the absolute yield spread. So, when we refer to yield spread, we mean absolute yield spread.

Whether we measure the yield spread as an absolute yield spread, a relative yield spread, or a yield ratio, the question to answer is what causes the yield spread between two bond issues. Basically, active bond portfolio strategies involve assessing the factors that cause the yield spread, forecasting how that yield spread may change over an investment horizon, and taking a position to capitalize on that forecast.

B. Intermarket Sector Spreads and Intramarket Spreads

The bond market is classified into sectors based on the type of issuer. In the United States, these sectors include the U.S. government sector, the U.S. government agencies sector, the municipal sector, the corporate sector, the mortgage-backed securities sector, the asset-backed

securities sector, and the foreign (sovereign, supranational, and corporate) sector. Different sectors are generally perceived as offering different risks and rewards.

The major market sectors are further divided into sub-sectors reflecting common economic characteristics. For example, within the corporate sector, the subsectors are: (1) industrial companies, (2) utility companies, (3) finance companies, and (4) banks. In the market for asset-backed securities, the sub-sectors are based on the type of collateral backing the security. The major types are securities backed by pools of (1) credit card receivables, (2) home equity loans, (3) automobile loans, (4) manufactured housing loans, and (5) student loans. Excluding the Treasury market sector, the other market sectors have a wide range of issuers, each with different abilities to satisfy their contractual obligations. Therefore, a key feature of a debt obligation is the nature of the issuer.

The yield spread between the yields offered in two sectors of the bond market with the same maturity is referred to as an **intermarket sector spread**. The most common intermarket sector spread calculated by market participants is the yield spread between a non-Treasury sector and Treasury securities with the same maturity.

The yield spread between two issues within a market sector is called an **intramarket sector spread**. As with Treasury securities, a yield curve can be estimated for a given issuer. The yield spread typically increases with maturity. The yield spreads for a given issuer can be added to the yield for the corresponding maturity of the on-the-run Treasury issue. The resulting yield curve is then an **issuer's on-the-run yield curve**.

The factors other than maturity that affect the intermarket and intramarket yield spreads are (1) the relative credit risk of the two issues, (2) the presence of embedded options, (3) the liquidity of the two issues, and (4) the taxability of interest received by investors.

C. Credit Spreads

The yield spread between non-Treasury securities and Treasury securities that are identical in all respects except for credit rating is referred to as a **credit spread** or **quality spread**. "Identical in all respects except credit rating" means that the maturities are the same and that there are no embedded options.

For example, Exhibit 3 shows information on the yield spread within the corporate sector by credit rating and maturity, for the 90-day period ending February 8, 2002. The high, low, and average spreads for the 90-day period are reported. Note that the lower the credit rating, the higher the credit spread. Also note that, for a given sector of the corporate market and a given credit rating, the credit spread increases with maturity.

It is argued that credit spreads between corporates and Treasuries change systematically with changes in the economy. Credit spreads widen (i.e., become larger) in a declining or contracting economy and narrow (i.e., become smaller) during economic expansion. The economic rationale is that, in a declining or contracting economy, corporations experience declines in revenue and cash flow, making it more difficult for corporate issuers to service their contractual debt obligations. To induce investors to hold spread products as credit quality deteriorates, the credit spread widens. The widening occurs as investors sell off corporates and invest the proceeds in Treasury securities (popularly referred to as a "flight to quality"). The converse is that, during economic expansion and brisk economic activity, revenue and cash flow increase, increasing the likelihood that corporate issuers will have the capacity to service their contractual debt obligations.

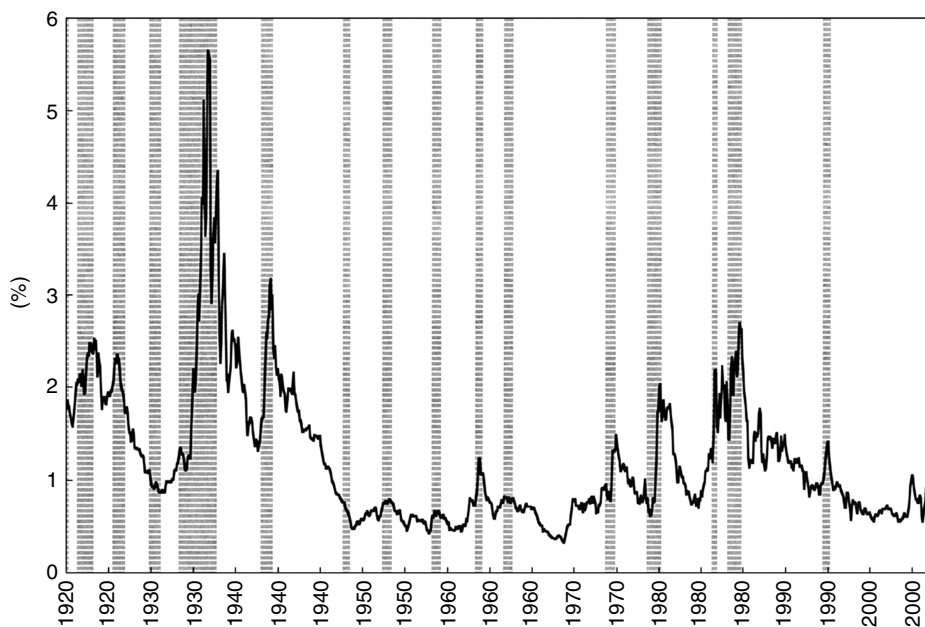
Exhibit 4 provides evidence of the impact of the business cycle on credit spreads since 1919. The credit spread in the exhibit is the difference between Baa rated and Aaa rated

EXHIBIT 3 Credit Spreads (in Basis Points) in the Corporate Sector on February 8, 2002

Maturity (years)	AA—90-day			A—90-day			BBB—90-day		
	High	Low	Avg	High	Low	Avg	High	Low	Avg
Industrials									
5	87	58	72	135	85	112	162	117	140
10	102	73	90	158	109	134	180	133	156
30	114	93	106	170	132	152	199	154	175
Utilities									
5	140	0	103	153	112	134	200	163	184
10	160	0	121	168	132	153	220	182	204
30	175	0	132	188	151	171	240	200	222
Finance									
5	103	55	86	233	177	198			
10	125	78	103	253	170	209			
30	148	100	130	253	207	228			
Banks									
5	97	60	81	113	83	100			
10	120	78	95	127	92	110			
30	138	105	121	170	127	145			

Source: Abstracted from *Global Relative Value*, Lehman Brothers, Fixed Income Research, February 11, 2002, p. 133.

EXHIBIT 4 Credit Spreads Between Baa and Aaa Corporate Bonds Over the Business Cycle Since 1919



Shaded areas = economic recession as defined by the NBER.

Source: Exhibit 1 in Leland E. Crabbe and Frank J. Fabozzi, *Managing a Corporate Portfolio* (Hoboken, NJ: John Wiley & Sons, 2002), p. 154.

corporate bonds; the shaded areas in the exhibit represent periods of economic recession as defined by the National Bureau of Economic Research (NBER). In general, corporate credit spreads tightened during the early stages of economic expansion, and spreads widened sharply during economic recessions. In fact, spreads typically begin to widen before the official beginning of an economic recession.⁸

Some market observers use the yield spread between issuers in cyclical and non-cyclical industry sectors as a proxy for yield spreads due to expected economic conditions. The rationale is as follows. While companies in both cyclical and non-cyclical industries are adversely affected by expectations of a recession, the impact is greater for cyclical industries. As a result, the yield spread between issuers in cyclical and non-cyclical industry sectors will widen with expectations of a contracting economy.

D. Including Embedded Options

It is not uncommon for a bond issue to include a provision that gives either the issuer and/or the bondholder an option to take some action against the other party. The most common type of option in a bond issue is the call provision that grants the issuer the right to retire the debt, fully or partially, before the scheduled maturity date.

The presence of an embedded option has an effect on both the yield spread of an issue relative to a Treasury security and the yield spread relative to otherwise comparable issues that do not have an embedded option. In general, investors require a larger yield spread to a comparable Treasury security for an issue with an embedded option that is favorable to the issuer (e.g. a call option) than for an issue without such an option. In contrast, market participants require a smaller yield spread to a comparable Treasury security for an issue with an embedded option that is favorable to the investor (e.g., put option or conversion option). In fact, for a bond with an option favorable to an investor, the interest rate may be less than that on a comparable Treasury security.

Even for callable bonds, the yield spread depends on the type of call feature. For a callable bond with a deferred call, the longer the deferred call period, the greater the call protection provided to the investor. Thus, all other factors equal, the longer the deferred call period, the lower the yield spread attributable to the call feature.

A major part of the bond market is the mortgage-backed securities sector.⁹ These securities expose an investor to prepayment risk and the yield spread between a mortgage-backed security and a comparable Treasury security reflects this prepayment risk. To see this, consider a basic mortgage-backed security called a Ginnie Mae passthrough security. This security is backed by the full faith and credit of the U.S. government. Consequently, the yield spread between a Ginnie Mae passthrough security and a comparable Treasury security is not due to credit risk. Rather, it is primarily due to prepayment risk. For example, Exhibit 5 reports the yield on 30-year Ginnie Mae passthrough securities with different coupon rates. The first issue to be addressed is the maturity of the comparable Treasury issue against which the Ginnie Mae should be benchmarked in order to calculate a yield spread. This is an issue because a mortgage passthrough security is an amortizing security that repays principal over time rather than just at the stated maturity date (30 years in our illustration). Consequently, while the

⁸For a further discussion and evidence regarding business cycles and credit spreads, see Chapter 10 in Leland E. Crabbe and Frank J. Fabozzi, *Managing a Corporate Portfolio* (Hoboken, NJ: John Wiley & Sons, 2002).

⁹The mortgage-backed securities sector is often referred to as simply the “mortgage sector.”

EXHIBIT 5 Yield Spreads and Option-Adjusted Spread (OAS) for Ginnie Mae 30-Year Passthrough Securities (February 8, 2002)

Coupon rate (%)	Yield spread (bps)	Benchmark Treasury	OAS on 2/8/02 (bps)	90-Day OAS (bps)		
				High	Low	Avg
6.5	203	5 year	52	75	46	59
7.0	212	5 year	57	83	54	65
7.5	155	3 year	63	94	62	74
8.0	105	3 year	73	108	73	88
9.0	244	2 year	131	160	124	139

Source: Abstracted from *Global Relative Value*, Lehman Brothers, Fixed Income Research, February 11, 2002, p. 132.

stated maturity of a Ginnie Mae passthrough is 30 years, its yield should not be compared to the yield on a 30-year Treasury issue. For now, you can see that the Treasury benchmark in Exhibit 5 depends on the coupon rate. The yield spread, shown in the second column, depends on the coupon rate.

In general, when a yield spread is cited for an issue that is callable, part of the spread reflects the risk associated with the embedded option. Reported yield spreads do not adjust for embedded options. The raw yield spreads are sometimes referred to as **nominal spreads**—nominal in the sense that the value of embedded options has not been removed in computing an adjusted yield spread. The yield spread that adjusts for the embedded option is OAS.

The last four columns in Exhibit 5 show Lehman Brothers' estimate of the option-adjusted spread for the 30-year Ginnie Mae passthroughs shown in the exhibit—the option-adjusted spread on February 8, 2002 and for the prior 90-day period (high, low, and average). The nominal spread is the yield spread shown in the second column. Notice that the option-adjusted spread is considerably less than the nominal spread. For example, for the 7.5% coupon issue the nominal spread is 155 basis points. After adjusting for the prepayment risk (i.e., the embedded option), the spread as measured by the option-adjusted spread is considerably less, 63 basis points.

E. Liquidity

Even within the Treasury market, a yield spread exists between off-the-run Treasury issues and on-the-run Treasury issues of similar maturity due to differences in liquidity and the effects of the repo market. Similarly, in the spread sectors, generic on-the-run yield curves can be estimated and the liquidity spread due to an off-the-run issue can be computed.

A Lehman Brother's study found that one factor that affects liquidity (and therefore the yield spread) is the size of an issue—the larger the issue, the greater the liquidity relative to a smaller issue, and the greater the liquidity, the lower the yield spread.¹⁰

F. Taxability of Interest Income

In the United States, unless exempted under the federal income tax code, interest income is taxable at the federal income tax level. In addition to federal income taxes, state and local taxes may apply to interest income.

¹⁰*Global Relative Value*, Lehman Brothers, Fixed Income Research, June 28, 1999, COR-2 AND 3.

EXHIBIT 6 Yield Ratio for AAA General Obligation Municipal Bonds to U.S. Treasuries of the Same Maturity (February 12, 2002)

Maturity	Yield on AAA General obligation (%)	Yield on U.S. Treasury (%)	Yield ratio
3 months	1.29	1.72	0.75
6 months	1.41	1.84	0.77
1 year	1.69	2.16	0.78
2 years	2.20	3.02	0.73
3 years	2.68	3.68	0.73
4 years	3.09	4.13	0.75
5 years	3.42	4.42	0.77
7 years	3.86	4.84	0.80
10 years	4.25	4.95	0.86
15 years	4.73	5.78	0.82
20 years	4.90	5.85	0.84
30 years	4.95	5.50	0.90

Source: Bloomberg Financial Markets.

The federal tax code specifically exempts interest income from *qualified* municipal bond issues from taxation.¹¹ Because of the tax-exempt feature of these municipal bonds, the yield on municipal bonds is less than that on Treasuries with the same maturity. Exhibit 6 shows this relationship on February 12, 2002, as reported by Bloomberg Financial Markets. The yield ratio shown for municipal bonds is the ratio of AAA general obligation bond yields to yields for the same maturity on-the-run Treasury issue.¹²

The difference in yield between tax-exempt securities and Treasury securities is typically measured not in terms of the absolute yield spread but as a yield ratio. More specifically, it is measured as the quotient of the yield on a tax-exempt security relative to the yield on a comparable Treasury security. This is reported in Exhibit 6. The yield ratio has changed over time due to changes in tax rates, as well as other factors. The higher the tax rate, the more attractive the tax-exempt feature and the lower the yield ratio.

The U.S. municipal bond market is divided into two bond sectors: general obligation bonds and revenue bonds. For the tax-exempt bond market, the benchmark for calculating yield spreads is not Treasury securities, but rather a generic AAA general obligation yield curve constructed by dealer firms active in the municipal bond market and by data/analytics vendors.

1. After-Tax Yield and Taxable-Equivalent Yield The yield on a taxable bond issue after federal income taxes are paid is called the **after-tax yield** and is computed as follows:

$$\text{after-tax yield} = \text{pre-tax yield} \times (1 - \text{marginal tax rate})$$

Of course, the marginal tax rate¹³ varies among investors. For example, suppose a taxable bond issue offers a yield of 5% and is acquired by an investor facing a marginal tax rate of 31%. The after-tax yield would then be:

$$\text{after-tax yield} = 0.05 \times (1 - 0.31) = 0.0345 = 3.45\%$$

¹¹As explained in Chapter 3, some municipal bonds are taxable.

¹²Some maturities for Treasury securities shown in the exhibit are not on-the-run issues. These are estimates for the market yields.

¹³The marginal tax rate is the tax rate at which an additional dollar is taxed.

Alternatively, we can determine the yield that must be offered on a taxable bond issue to give the same after-tax yield as a tax-exempt issue. This yield is called the **taxable-equivalent yield** or **tax-equivalent yield** and is computed as follows:

$$\text{taxable-equivalent yield} = \frac{\text{tax-exempt yield}}{(1 - \text{marginal tax rate})}$$

For example, consider an investor facing a 31% marginal tax rate who purchases a tax-exempt issue with a yield of 4%. The taxable-equivalent yield is then:

$$\text{taxable-equivalent yield} = \frac{0.04}{(1 - 0.31)} = 0.058 = 5.80\%$$

Notice that the higher the marginal tax rate, the higher the taxable equivalent yield. For instance, in our last example if the marginal tax rate is 40% rather than 31%, the taxable-equivalent yield would be 6.67% rather than 5.80%, as shown below:

$$\text{taxable-equivalent yield} = \frac{0.04}{(1 - 0.40)} = 0.0667 = 6.67\%$$

Some state and local governments tax interest income from bond issues that are exempt from federal income taxes. Some municipalities exempt interest income from all municipal issues from taxation, while others do not. Some states exempt interest income from bonds issued by municipalities within the state but tax the interest income from bonds issued by municipalities outside of the state. The implication is that two municipal securities with the same credit rating and the same maturity may trade at different yield spreads because of the relative demand for bonds of municipalities in different states. For example, in a high income tax state such as New York, the demand for bonds of New York municipalities drives down their yields relative to bonds issued by municipalities in a zero income tax state such as Texas.

G. Technical Factors

At times, deviations from typical yield spreads are caused by temporary imbalances between supply and demand. For example, in the second quarter of 1999, issuers became concerned that the Fed would pursue a policy to increase interest rates. In response, a record issuance of corporate securities resulted in an increase in the yield spread between corporates and Treasuries.

In the municipal market, yield spreads are affected by the temporary oversupply of issues within a market sector. For example, a substantial new issue volume of high-grade state general obligation bonds may tend to decrease the yield spread between high-grade and low-grade revenue bonds. In a weak market environment, it is easier for high-grade municipal bonds to come to market than for weaker credits. So at times high grades flood weak markets even when there is a relative scarcity of medium- and low-grade municipal bond issues.

Since technical factors cause temporary misalignments of the yield spread relationship, some investors look at the forward calendar of planned offerings to project the impact on future yield spreads. Some corporate analysts identify the risk of yield spread changes due to the supply of new issues when evaluating issuers or sectors.

V. NON-U.S. INTEREST RATES

The same factors that affect yield spreads in the United States are responsible for yield spreads in other countries and between countries. Major non-U.S. bond markets have a government benchmark yield curve similar to that of the U.S. Treasury yield curve. Exhibit 7 shows the government yield curve as of the beginning and end of 2001 for Germany, Japan, the U.K., and France. These yield curves are presented to illustrate the different shapes

EXHIBIT 7 Yield Curves in Germany, Japan, the U.K., and France: 2001

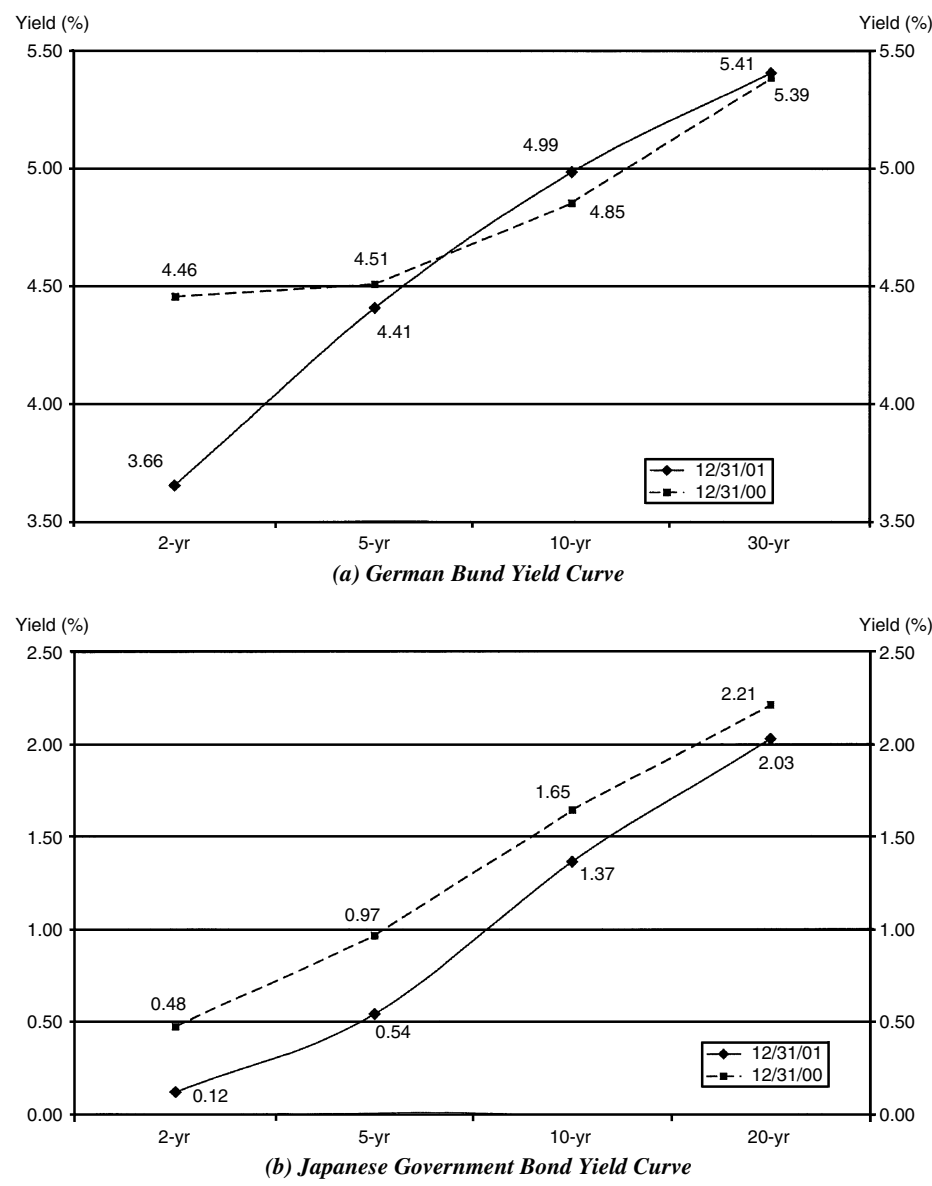
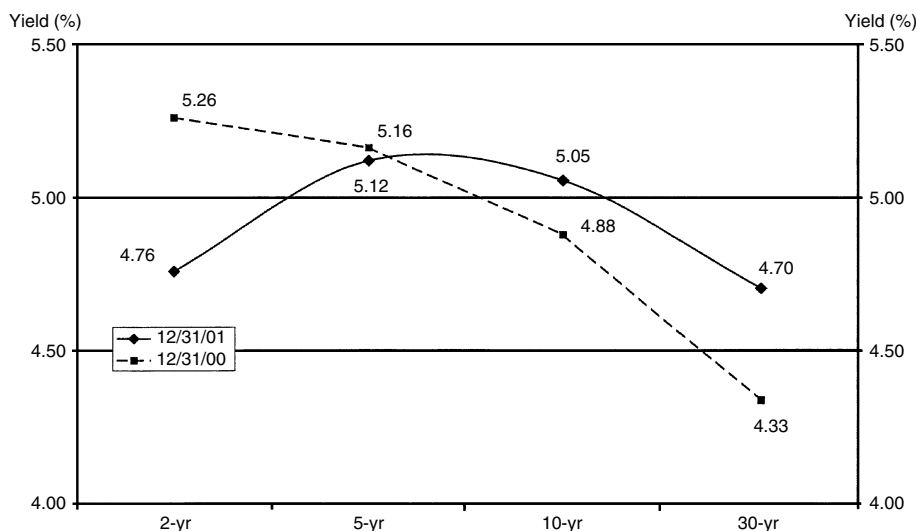
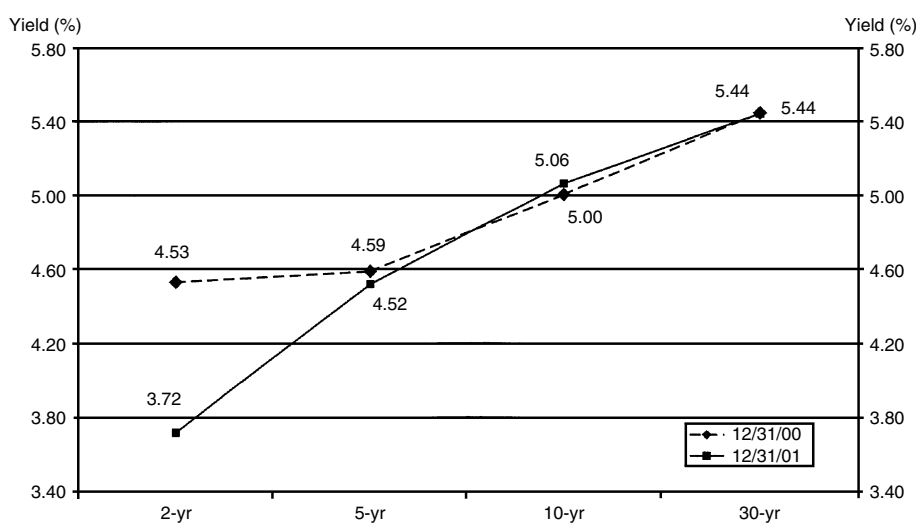


EXHIBIT 7 (Continued)



(c) U.K. Gilt Yield Curve



(d) French OAT Yield Curve

Source: Lehman Brothers Fixed Income Research, *Global Fixed Income Strategy "Playbook,"* January 2002.

and the way in which they can change. Notice that only the Japanese yield curve shifted in an almost parallel fashion (i.e., the rate for all maturities changed by approximately the same number of basis points).

The German bond market is the largest market for publicly issued bonds in Europe. The yields on German government bonds are viewed as benchmark interest rates in Europe. Because of the important role of the German bond market, nominal spreads are typically computed relative to German government bonds (German bunds).

Institutional investors who borrow funds on a short-term basis to invest (referred to as “funded investors”) obviously desire to earn an amount in excess of their borrowing cost. The most popular borrowing cost reference rate is the **London interbank offered rate (LIBOR)**. LIBOR is the interest rate at which banks pay to borrow funds from other banks in the London interbank market. The borrowing occurs via a cash deposit of one bank (the lender) into a certificate of deposit (CD) in another bank (the borrower). The maturity of the CD can be from overnight to five years. So, 3-month LIBOR represents the interest rate paid on a CD that matures in three months. The CD can be denominated in one of several currencies. The currencies for which LIBOR is reported are the U.S. dollar, the British pound, the Euro, the Canadian dollar, the Australian dollar, the Japanese yen, and Swiss francs. When it is denominated in U.S. dollars, it is referred to as a Eurodollar CD. LIBOR is determined for every London business day by the British Bank Association (BBA) by maturity and for each currency and is reported by various services.

Entities seeking to borrow funds pays a spread over LIBOR and seek to earn a spread over that funding cost when they invest the borrowed funds. So, for example, if the 3-month borrowing cost for a funded investor is 3-month LIBOR plus 25 basis points and the investor can earn 3-month LIBOR plus 125 basis points for three months, then the investor earns a spread of 100 basis points for three months (125 basis points-25 basis points).

VI. SWAP SPREADS

Another important spread measure is the **swap spread**.

A. Interest Rate Swap and the Swap Spread

In an interest rate swap, two parties (called **counterparties**) agree to exchange periodic interest payments. The dollar amount of the interest payments exchanged is based on a predetermined dollar principal, which is called the **notional principal** or **notional amount**. The dollar amount each counterparty pays to the other is the agreed-upon periodic interest rate times the notional principal. The only dollars exchanged between the parties are the interest payments, not the notional principal. In the most common type of swap, one party agrees to pay the other party fixed interest payments at designated dates for the life of the swap. This party is referred to as the **fixed-rate payer**. The fixed rate that the fixed-rate payer pays is called the **swap rate**. The other party, who agrees to make interest rate payments that float with some reference rate, is referred to as the **fixed-rate receiver**.

The reference rates used for the floating rate in an interest rate swap is one of various money market instruments: LIBOR (the most common reference rate used in swaps), Treasury bill rate, commercial paper rate, bankers' acceptance rate, federal funds rate, and prime rate.

The convention that has evolved for quoting a swap rate is that a dealer sets the floating rate equal to the reference rate and then quotes the fixed rate that will apply. The fixed rate has a specified “spread” above the yield for a Treasury with the same term to maturity as the swap. This specified spread is called the **swap spread**. The **swap rate** is the sum of the yield for a Treasury with the same maturity as the swap plus the swap spread.

To illustrate an interest rate swap in which one party pays fixed and receives floating, assume the following:

term of swap: 5 years
 swap spread: 50 basis points
 reference rate: 3-month LIBOR
 notional amount: \$50 million
 frequency of payments: every three months

Suppose also that the 5-year Treasury rate is 5.5% at the time the swap is entered into. Then the swap rate will be 6%, found by adding the swap spread of 50 basis points to the 5-year Treasury yield of 5.5%.

This means that the fixed-rate payer agrees to pay a 6% annual rate for the next five years with payments made quarterly and receive from the fixed-rate receiver 3-month LIBOR with the payments made quarterly. Since the notional amount is \$50 million, this means that every three months, the fixed-rate payer pays \$750,000 (6% times \$50 million divided by 4). The fixed-rate receiver pays 3-month LIBOR times \$50 million divided by 4. The table below shows the payment made by the fixed-rate receiver to the fixed-rate payer for different values of 3-month LIBOR:¹⁴

If 3-month LIBOR is	Annual dollar amount	Quarterly payment
4%	\$2,000,000	\$500,000
5%	2,500,000	625,000
6%	3,000,000	750,000
7%	3,500,000	875,000
8%	4,000,000	1,000,000

In practice, the payments are netted out. For example, if 3-month LIBOR is 4%, the fixed-rate receiver would receive \$750,000 and pay to the fixed-rate payer \$500,000. Netting the two payments, the fixed-rate payer pays the fixed-rate receiver \$250,000 (\$750,000 – \$500,000).

B. Role of Interest Rate Swaps

Interest rate swaps have many important applications in fixed income portfolio management and risk management. They tie together the fixed-rate and floating-rate sectors of the bond market. As a result, investors can convert a fixed-rate asset into a floating-rate asset with an interest rate swap.

Suppose a financial institution has invested in 5-year bonds with a \$50 million par value and a coupon rate of 9% and that this bond is selling at par value. Moreover, this institution borrows \$50 million on a quarterly basis (to fund the purchase of the bonds) and its cost of funds is 3-month LIBOR plus 50 basis points. The “income spread” between its assets (i.e., 5-year bonds) and its liabilities (its funding cost) for any 3-month period depends on 3-month LIBOR. The following table shows how the annual spread varies with 3-month LIBOR:

¹⁴The amount of the payment is found by dividing the annual dollar amount by four because payments are made quarterly. In a real world application, both the fixed-rate and floating-rate payments are adjusted for the number of days in a quarter, but it is unnecessary for us to deal with this adjustment here.

Asset yield	3-month LIBOR	Funding cost	Annual income spread
9.00%	4.00%	4.50%	4.50%
9.00%	5.00%	5.50%	3.50%
9.00%	6.00%	6.50%	2.50%
9.00%	7.00%	7.50%	1.50%
9.00%	8.00%	8.50%	0.50%
9.00%	8.50%	9.00%	0.00%
9.00%	9.00%	9.50%	−0.50%
9.00%	10.00%	10.50%	−1.50%
9.00%	11.00%	11.50%	−2.50%

As 3-month LIBOR increases, the income spread decreases. If 3-month LIBOR exceeds 8.5%, the income spread is negative (i.e., it costs more to borrow than is earned on the bonds in which the borrowed funds are invested).

This financial institution has a mismatch between its assets and its liabilities. An interest rate swap can be used to hedge this mismatch. For example, suppose the manager of this financial institution enters into a 5-year swap with a \$50 million notional amount in which it agrees to pay a fixed rate (i.e., to be the fixed-rate payer) in exchange for 3-month LIBOR. Suppose further that the swap rate is 6%. Then the annual income spread taking into account the swap payments is as follows for different values of 3-month LIBOR:

Asset yield	3-month LIBOR	Funding cost	Fixed rate paid in swap	3-month LIBOR rec. in swap	Annual income spread
9.00%	4.00%	4.50%	6.00%	4.00%	2.50%
9.00%	5.00%	5.50%	6.00%	5.00%	2.50%
9.00%	6.00%	6.50%	6.00%	6.00%	2.50%
9.00%	7.00%	7.50%	6.00%	7.00%	2.50%
9.00%	8.00%	8.50%	6.00%	8.00%	2.50%
9.00%	8.50%	9.00%	6.00%	8.50%	2.50%
9.00%	9.00%	9.50%	6.00%	9.00%	2.50%
9.00%	10.00%	10.50%	6.00%	10.00%	2.50%
9.00%	11.00%	11.50%	6.00%	11.00%	2.50%

Assuming the bond does not default and is not called, the financial institution has locked in a spread of 250 basis points.

Effectively, the financial institution using this interest rate swap converted a fixed-rate asset into a floating-rate asset. The reference rate for the synthetic floating-rate asset is 3-month LIBOR and the liabilities are in terms of 3-month LIBOR. Alternatively, the financial institution could have converted its liabilities to a fixed-rate by entering into a 5-year \$50 million notional amount swap by being the fixed-rate payer and the results would have been the same.

This simple illustration shows the critical importance of an interest rate swap. Investors and issuers with a mismatch of assets and liabilities can use an interest rate swap to better match assets and liabilities, thereby reducing their risk.

C. Determinants of the Swap Spread

Market participants throughout the world view the swap spread as the appropriate spread measure for valuation and relative value analysis. Here we discuss the determinants of the swap spread.

We know that

$$\text{swap rate} = \text{Treasury rate} + \text{swap spread}$$

where Treasury rate is equal to the yield on a Treasury with the same maturity as the swap. Since the parties are swapping the future reference rate for the swap rate, then:

$$\text{reference rate} = \text{Treasury rate} + \text{swap spread}$$

Solving for the swap spread we have:

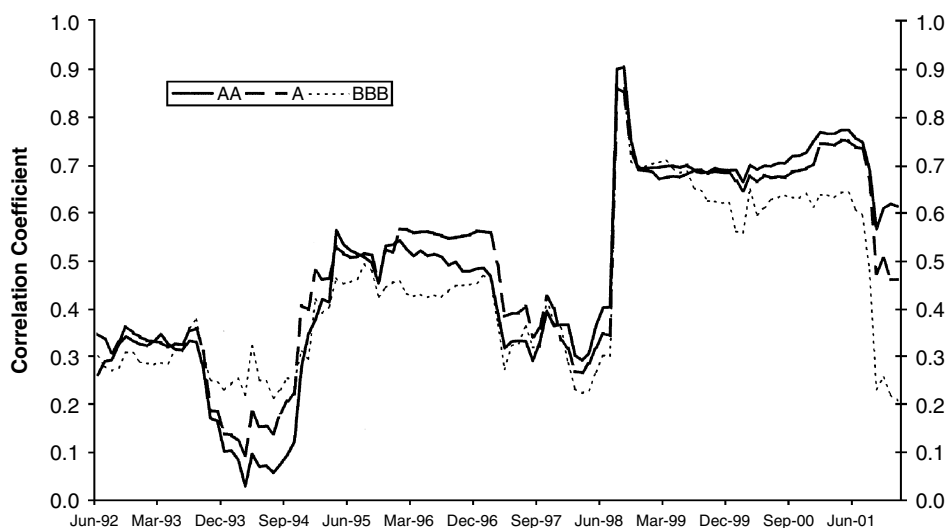
$$\text{swap spread} = \text{reference rate} - \text{Treasury rate}$$

Since the most common reference rate is LIBOR, we can substitute this into the above formula getting:

$$\text{swap spread} = \text{LIBOR} - \text{Treasury rate}$$

Thus, the swap spread is a spread of the global cost of short-term borrowing over the Treasury rate.

EXHIBIT 8 Three-Year Trailing Correlation Between Swap Spreads and Credit Spreads (AA, A, and BB): June 1992 to December 2001



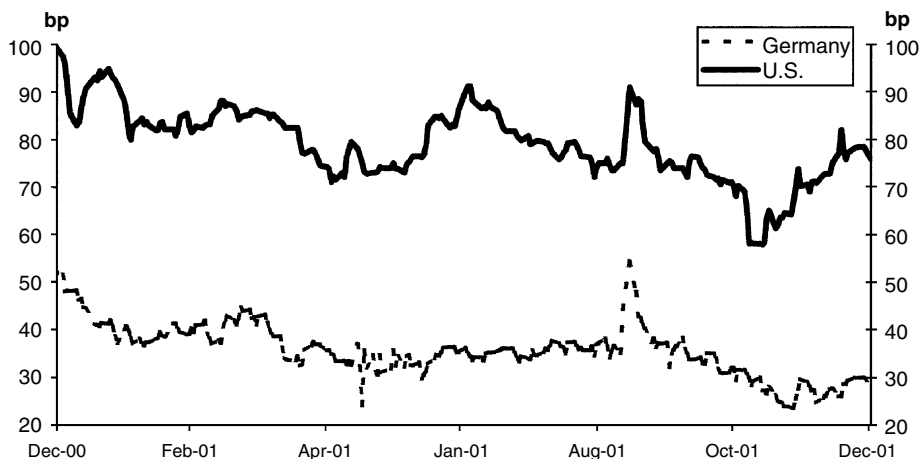
Source: Lehman Brothers Fixed Income Research, *Global Fixed Income Strategy "Playbook,"* January 2002.

EXHIBIT 9 January and December 2001 Swap Spread Curves for Germany, Japan, U.K., and U.S.

	Germany				Japan				U.K.				U.S.			
	2-Year	5-Year	10-Year	30-Year	2-Year	5-Year	10-Year	30-Year	2-Year	5-Year	10-Year	30-Year	2-Year	5-Year	10-Year	30-Year
Jan-01	23	40	54	45	8	10	14	29	40	64	83	91	63	82	81	73
Dec-01	22	28	28	14	3	(2)	(1)	8	36	45	52	42	46	76	77	72

Source: Lehman Brothers Fixed Income Research, *Global Fixed Income Strategy "Playbook,"* January 2002.

EXHIBIT 10 Daily 5-Year Swap Spreads in Germany and the United States: 2001



Source: Lehman Brothers Fixed Income Research, *Global Fixed Income Strategy "Playbook,"* January 2002.

The swap spread primarily reflects the credit spreads in the corporate bond market.¹⁵ Studies have found a high correlation between swap spreads and credit spreads in various sectors of the fixed income market. This can be seen in Exhibit 8 (on the previous page) which shows the 3-year trailing correlation from June 1992 to December 2001 between swap spreads and AA, A, and BBB credit spreads. Note from the exhibit that the highest correlation is with AA credit spreads.

D. Swap Spread Curve

A **swap spread curve** shows the relationship between the swap rate and swap maturity. A swap spread curve is available by country. The swap spread is the amount added to the yield of the respective country's government bond with the same maturity as the maturity of the swap. Exhibit 9 shows the swap spread curves for Germany, Japan, the U.K., and the U.S. for January 2001 and December 2001. The swap spreads move together. For example, Exhibit 10 shows the daily 5-year swap spreads from December 2000 to December 2001 for the U.S. and Germany.

¹⁵We say primarily because there are also technical factors that affect the swap spread. For a discussion of these factors, see Richard Gordon, "The Truth about Swap Spreads," in Frank J. Fabozzi (ed.), *Professional Perspectives on Fixed Income Portfolio Management: Volume 1* (New Hope, PA: Frank J. Fabozzi Associates, 2000), pp. 97–104.

INTRODUCTION TO THE VALUATION OF DEBT SECURITIES

I. INTRODUCTION

Valuation is the process of determining the fair value of a financial asset. The process is also referred to as “valuing” or “pricing” a financial asset. In this chapter, we will explain the general principles of fixed income security valuation. In this chapter, we will limit our discussion to the valuation of option-free bonds.

II. GENERAL PRINCIPLES OF VALUATION

The fundamental principle of financial asset valuation is that its value is equal to the present value of its expected cash flows. This principle applies regardless of the financial asset. Thus, the valuation of a financial asset involves the following three steps:

Step 1: Estimate the expected cash flows.

Step 2: Determine the appropriate interest rate or interest rates that should be used to discount the cash flows.

Step 3: Calculate the present value of the expected cash flows found in step 1 using the interest rate or interest rates determined in step 2.

A. Estimating Cash Flows

Cash flow is simply the cash that is expected to be received in the future from an investment. In the case of a fixed income security, it does not make any difference whether the cash flow is interest income or payment of principal. The **cash flows** of a security are the collection of each period's cash flow. Holding aside the risk of default, the cash flows for few fixed income securities are simple to project. Noncallable U.S. Treasury securities have known cash flows. For Treasury coupon securities, the cash flows are the coupon interest payments every six months up to and including the maturity date and the principal payment at the maturity date.

At times, investors will find it difficult to estimate the cash flows when they purchase a fixed income security. For example, if

1. the issuer or the investor has the option to change the contractual due date for the payment of the principal, or
2. the coupon payment is reset periodically by a formula based on some value or values of reference rates, prices, or exchange rates, or
3. the investor has the choice to convert or exchange the security into common stock.

Callable bonds, putable bonds, mortgage-backed securities, and asset-backed securities are examples of (1). Floating-rate securities are an example of (2). Convertible bonds and exchangeable bonds are examples of (3).

For securities that fall into the first category, future interest rate movements are the key factor to determine if the option will be exercised. Specifically, if interest rates fall far enough, the issuer can sell a new issue of bonds at the lower interest rate and use the proceeds to pay off (call) the older bonds that have the higher coupon rate. (This assumes that the interest savings are larger than the costs involved in refunding.) Similarly, for a loan, if rates fall enough that the interest savings outweigh the refinancing costs, the borrower has an incentive to refinance. For a putable bond, the investor will put the issue if interest rates rise enough to drive the market price below the put price (i.e., the price at which it must be repurchased by the issuer).

What this means is that to properly estimate the cash flows of a fixed income security, it is necessary to incorporate into the analysis how, in the future, changes in interest rates and other factors affecting the embedded option may affect cash flows.

B. Determining the Appropriate Rate or Rates

Once the cash flows for a fixed income security are estimated, the next step is to determine the appropriate interest rate to be used to discount the cash flows. As we did in the previous chapter, we will use the terms *interest rate* and *yield* interchangeably. The minimum interest rate that an investor should require is the yield available in the marketplace on a default-free cash flow. In the United States, this is the yield on a U.S. Treasury security. This is *one* of the reasons that the Treasury market is closely watched. What is the *minimum* interest rate U.S. investors demand? At this point, we can assume that it is the yield on the on-the-run Treasury security with the same as the security being valued.¹ We will qualify this shortly.

For a security that is not issued by the U.S. government, investors will require a yield premium over the yield available on an on-the-run Treasury issue. This yield premium reflects the additional risks that the investor accepts.

For each cash flow estimated, the same interest rate can be used to calculate the present value. However, since each cash flow is unique, it is more appropriate to value each cash flow using an interest rate specific to that cash flow's maturity. In the traditional approach to valuation a single interest rate is used. In Section IV, we will see that the proper approach to valuation uses multiple interest rates each specific to a particular cash flow. In that section, we will also demonstrate why this must be the case.

C. Discounting the Expected Cash Flows

Given expected (estimated) cash flows and the appropriate interest rate or interest rates to be used to discount the cash flows, the final step in the valuation process is to value the cash flows.

¹As explained in Chapter 3, the on-the-run Treasury issues are the most recently auctioned Treasury issues.

What is the value of a single cash flow to be received in the future? It is the amount of money that must be invested today to generate that future value. The resulting value is called the **present value** of a cash flow. (It is also called the **discounted value**.) The present value of a cash flow will depend on (1) when a cash flow will be received (i.e., the **timing** of a cash flow) and (2) the interest rate used to calculate the present value. The interest rate used is called the **discount rate**.

First, we calculate the present value for each expected cash flow. Then, to determine the value of the security, we calculate the sum of the present values (i.e., for all of the security's expected cash flows).

If a discount rate i can be earned on any sum invested today, the present value of the expected cash flow to be received t years from now is:

$$\text{present value}_t = \frac{\text{expected cash flow in period } t}{(1 + i)^t}$$

The value of a financial asset is then the sum of the present value of all the expected cash flows. That is, assuming that there are N expected cash flows:

$$\text{value} = \text{present value}_1 + \text{present value}_2 + \cdots + \text{present value}_N$$

To illustrate the present value formula, consider a simple bond that matures in four years, has a coupon rate of 10%, and has a maturity value of \$100. For simplicity, let's assume the bond pays interest annually and a discount rate of 8% should be used to calculate the present value of each cash flow. The cash flow for this bond is:

Year	Cash flow
1	\$10
2	10
3	10
4	110

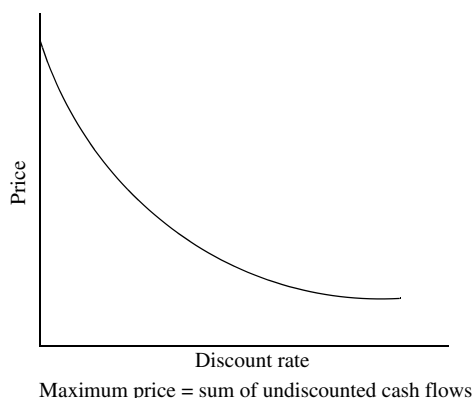
The present value of each cash flow is:

$$\begin{aligned} \text{Year 1: } \text{present value}_1 &= \frac{\$10}{(1.08)^1} = \$9.2593 \\ \text{Year 2: } \text{present value}_2 &= \frac{\$10}{(1.08)^2} = \$8.5734 \\ \text{Year 3: } \text{present value}_3 &= \frac{\$10}{(1.08)^3} = \$7.9383 \\ \text{Year 4: } \text{present value}_4 &= \frac{\$110}{(1.08)^4} = \$80.8533 \end{aligned}$$

The value of this security is then the sum of the present values of the four cash flows. That is, the present value is \$106.6243 (\$9.2593 + \$8.5734 + \$7.9383 + \$80.8533).

1. Present Value Properties An important property about the present value can be seen from the above illustration. For the first three years, the cash flow is the same (\$10) and the discount rate is the same (8%). The present value decreases as we go further into the future. *This is an important property of the present value: for a given discount rate, the further into the*

EXHIBIT 1 Price/Discount Rate Relationship
for an Option-Free Bond



future a cash flow is received, the lower its present value. This can be seen in the present value formula. As t increases, present value _{t} decreases.

Suppose that instead of a discount rate of 8%, a 12% discount rate is used for each cash flow. Then, the present value of each cash flow is:

$$\begin{aligned} \text{Year 1: present value}_1 &= \frac{\$10}{(1.12)^1} = \$8.9286 \\ \text{Year 2: present value}_2 &= \frac{\$10}{(1.12)^2} = \$7.9719 \\ \text{Year 3: present value}_3 &= \frac{\$10}{(1.12)^3} = \$7.1178 \\ \text{Year 4: present value}_4 &= \frac{\$110}{(1.12)^4} = \$69.9070 \end{aligned}$$

The value of this security is then \$93.9253 (\$8.9286 + \$7.9719 + \$7.1178 + \$69.9070). The security's value is lower if a 12% discount rate is used compared to an 8% discount rate (\$93.9253 versus \$106.6243). This is another general property of present value: *the higher the discount rate, the lower the present value.* Since the value of a security is the present value of the expected cash flows, this property carries over to the value of a security: *the higher the discount rate, the lower a security's value.* The reverse is also true: *the lower the discount rate, the higher a security's value.*

Exhibit 1 shows, for an option-free bond, this inverse relationship between a security's value and the discount rate. The shape of the curve in Exhibit 1 is referred to as **convex**. By convex, it is meant the curve is bowed in from the origin. As we will see in Chapter 7, this convexity or bowed shape has implications for the price volatility of a bond when interest rates change. What is important to understand is that the relationship is not linear.

2. Relationship between Coupon Rate, Discount Rate, and Price Relative to Par Value In Chapter 2, we described the relationship between a bond's coupon rate, required market yield, and price relative to its par value (i.e., premium, discount, or equal to par). The required yield is equivalent to the discount rate discussed above. We stated the following relationship:

coupon rate = yield required by market, *therefore* price = par value
 coupon rate < yield required by market, *therefore* price < par value (discount)
 coupon rate > yield required by market, *therefore* price > par value (premium)

Now that we know how to value a bond, we can demonstrate the relationship. The coupon rate on our hypothetical bond is 10%. When an 8% discount rate is used, the bond's value is \$106.6243. That is, the price is greater than par value (premium). This is because the coupon rate (10%) is greater than the required yield (the 8% discount rate). We also showed that when the discount rate is 12% (i.e., greater than the coupon rate of 10%), the price of the bond is \$93.9253. That is, the bond's value is less than par value when the coupon rate is less than the required yield (discount). When the discount rate is the same as the coupon rate, 10%, the bond's value is equal to par value as shown below:

Year	Cash flow	Present value at 10%
1	\$10	\$9.0909
2	10	8.2645
3	10	7.5131
4	110	75.1315
Total		\$100.0000

3. Change in a Bond's Value as it Moves Toward Maturity As a bond moves closer to its maturity date, its value changes. More specifically, assuming that the discount rate does not change, a bond's value:

1. decreases over time if the bond is selling at a premium
2. increases over time if the bond is selling at a discount
3. is unchanged if the bond is selling at par value

At the maturity date, the bond's value is equal to its par value. So, over time as the bond moves toward its maturity date, its price will move to its par value—a characteristic sometimes referred to as a “pull to par value.”

To illustrate what happens to a bond selling at a premium, consider once again the 4-year 10% coupon bond. When the discount rate is 8%, the bond's price is 106.6243. Suppose that one year later, the discount rate is still 8%. There are only three cash flows remaining since the bond is now a 3-year security. The cash flow and the present value of the cash flows are given below:

Year	Cash flow	Present value at 8%
1	\$10	\$9.2593
2	10	8.5734
3	110	87.3215
Total		\$105.1542

The price has declined from \$106.6243 to \$105.1542.

Now suppose that the bond's price is initially below par value. For example, as stated earlier, if the discount rate is 12%, the 4-year 10% coupon bond's value is \$93.9253. Assuming the discount rate remains at 12%, one year later the cash flow and the present value of the cash flow would be as shown:

Year	Cash flow	Present value at 12%
1	\$10	\$8.9286
2	10	7.9719
3	110	78.2958
	Total	\$95.1963

The bond's price increases from \$93.9253 to \$95.1963.

To understand how the price of a bond changes as it moves towards maturity, consider the following three 20-year bonds for which the yield required by the market is 8%: a premium bond (10% coupon), a discount bond (6% coupon), and a par bond (8% coupon). To simplify the example, it is assumed that each bond pays interest annually. Exhibit 2 shows the price of each bond as it moves toward maturity, assuming that the 8% yield required by the market does not change. The premium bond with an initial price of 119.6363 decreases in price until it reaches par value at the maturity date. The discount bond with an initial price of 80.3637 increases in price until it reaches par value at the maturity date.

In practice, over time the discount rate will change. So, the bond's value will change due to both the change in the discount rate and the change in the cash flow as the bond moves toward maturity. For example, again suppose that the discount rate for the 4-year 10% coupon is 8% so that the bond is selling for \$106.6243. One year later, suppose that the discount rate appropriate for a 3-year 10% coupon bond increases from 8% to 9%. Then the cash flow and present value of the cash flows are shown below:

Year	Cash flow	Present value at 9%
1	\$10	\$9.1743
2	10	8.4168
3	110	84.9402
	Total	\$102.5313

The bond's price will decline from \$106.6243 to \$102.5313. As shown earlier, if the discount rate did not increase, the price would have declined to only \$105.1542. The price decline of \$4.0930 ($\$106.6243 - \$102.5313$) can be decomposed as follows:

Price change attributable to moving to maturity (no change in discount rate)	\$1.4701 ($\$106.6243 - \$105.1542$)
Price change attribute to an increase in the discount rate from 8% to 9%	<u>\$2.6229</u> ($\$105.1542 - \$102.5313$)
Total price change	\$4.0930

D. Valuation Using Multiple Discount Rates

Thus far, we have used one discount rate to compute the present value of each cash flow. As we will see shortly, the proper way to value the cash flows of a bond is to use a different discount rate that is unique to the time period in which a cash flow will be received. So, let's look at how we would value a security using a different discount rate for each cash flow.

Suppose that the appropriate discount rates are as follows:

year 1	6.8%
year 2	7.2%
year 3	7.6%
year 4	8.0%

EXHIBIT 2 Movement of a Premium, Discount, and Par Bond as a Bond Moves Towards Maturity

Information about the three bonds:

All bonds mature in 20 years and have a yield required by the market of 8%

Coupon payments are annual

Premium bond = 10% coupon selling for 119.6363

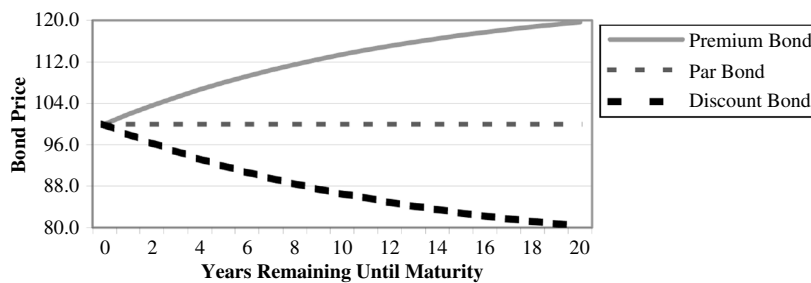
Discount bond = 6% coupon selling for 80.3637

Par bond = 8% coupon selling at par value

Assumption: The yield required by the market is unchanged over the life of the bond at 8%.

Time to maturity in years	Premium bond	Discount bond	Par bond
20	119.6363	80.3637	100.0000
19	119.2072	80.7928	100.0000
18	118.7438	81.2562	100.0000
17	118.2433	81.7567	100.0000
16	117.7027	82.2973	100.0000
15	117.1190	82.8810	100.0000
14	116.4885	83.5115	100.0000
13	115.8076	84.1924	100.0000
12	115.0722	84.9278	100.0000
11	114.2779	85.7221	100.0000
10	113.4202	86.5798	100.0000
9	112.4938	87.5062	100.0000
8	111.4933	88.5067	100.0000
7	110.4127	89.5873	100.0000
6	109.2458	90.7542	100.0000
5	107.9854	92.0146	100.0000
4	106.6243	93.3757	100.0000
3	105.1542	94.8458	100.0000
2	103.5665	96.4335	100.0000
1	101.8519	98.1481	100.0000
0	100.0000	100.0000	100.0000

The Effect of Time on a Bond's Price



Then, for the 4-year 10% coupon bond, the present value of each cash flow is:

$$\text{Year 1: present value}_1 = \frac{\$10}{(1.068)^1} = \$9.3633$$

$$\text{Year 2: present value}_2 = \frac{\$10}{(1.072)^2} = \$8.7018$$

$$\begin{aligned} \text{Year 3: present value}_3 &= \frac{\$10}{(1.076)^3} = \$8.0272 \\ \text{Year 4: present value}_4 &= \frac{\$110}{(1.080)^4} = \$80.8533 \end{aligned}$$

The present value of this security, assuming the above set of discount rates, is \$106.9456.

E. Valuing Semiannual Cash Flows

In our illustrations, we assumed coupon payments are paid once per year. For most bonds, the coupon payments are semiannual. This does not introduce any complexities into the calculation. The procedure is to simply adjust the coupon payments by dividing the annual coupon payment by 2 and adjust the discount rate by dividing the annual discount rate by 2. The time period t in the present value formula is treated in terms of 6-month periods rather than years.

For example, consider once again the 4-year 10% coupon bond with a maturity value of \$100. The cash flow for the first 3.5 years is equal to \$5 (\$10/2). The last cash flow is equal to the final coupon payment (\$5) plus the maturity value (\$100). So the last cash flow is \$105.

Now the tricky part. If an annual discount rate of 8% is used, how do we obtain the semiannual discount rate? We will simply use one-half the annual rate, 4% (or 8%/2). The reader should have a problem with this: a 4% semiannual rate is not an 8% effective annual rate. That is correct. However, as we will see in the next chapter, the *convention* in the bond market is to quote annual interest rates that are just double semiannual rates. This will be explained more fully in the next chapter. Don't let this throw you off here. For now, just accept the fact that one-half an annual discount rate is used to obtain a semiannual discount rate in the balance of the chapter.

Given the cash flows and the semiannual discount rate of 4%, the present value of each cash flow is shown below:

$$\begin{aligned} \text{Period 1: present value}_1 &= \frac{\$5}{(1.04)^1} = \$4.8077 \\ \text{Period 2: present value}_2 &= \frac{\$5}{(1.04)^2} = \$4.6228 \\ \text{Period 3: present value}_3 &= \frac{\$5}{(1.04)^3} = \$4.4450 \\ \text{Period 4: present value}_4 &= \frac{\$5}{(1.04)^4} = \$4.2740 \\ \text{Period 5: present value}_5 &= \frac{\$5}{(1.04)^5} = \$4.1096 \\ \text{Period 6: present value}_6 &= \frac{\$5}{(1.04)^6} = \$3.9516 \\ \text{Period 7: present value}_7 &= \frac{\$5}{(1.04)^7} = \$3.7996 \\ \text{Period 8: present value}_8 &= \frac{\$105}{(1.04)^8} = \$76.7225 \end{aligned}$$

The security's value is equal to the sum of the present value of the eight cash flows, \$106.7327. Notice that this price is greater than the price when coupon payments are annual (\$106.6243). This is because one-half the annual coupon payment is received six months sooner than when payments are annual. This produces a higher present value for the semiannual coupon payments relative to the annual coupon payments.

The value of a non-amortizing bond can be divided into two components: (1) the present value of the coupon payments and (2) the present value of the maturity value. For a fixed-rate coupon bond, the coupon payments represent an annuity. A short-cut formula can be used to compute the value of a bond when using a single discount rate: compute the present value of the annuity and then add the present value of the maturity value.²

The present value of an annuity is equal to:

$$\text{annuity payment} \times \left[\frac{1 - \frac{1}{(1+i)^{\text{no. of periods}}}}{i} \right]$$

For a bond with annual interest payments, i is the annual discount rate and the “no. of periods” is equal to the number of years.

Applying this formula to a semiannual-pay bond, the annuity payment is one half the annual coupon payment and the number of periods is double the number of years to maturity. So, the present value of the coupon payments can be expressed as:

$$\text{semiannual coupon payment} \times \left[\frac{1 - \frac{1}{(1+i)^{\text{no. of years} \times 2}}}{i} \right]$$

where i is the semiannual discount rate (annual rate/2). Notice that in the formula, we use the number of years multiplied by 2 since a period in our illustration is six months.

The present value of the maturity value is equal to

$$\text{present value of maturity value} = \frac{\$100}{(1+i)^{\text{no. of years} \times 2}}$$

To illustrate this computation, consider once again the 4-year 10% coupon bond with an annual discount rate of 8% and a semiannual discount rate of one half this rate (4%) for the reason cited earlier. Then:

$$\begin{aligned} \text{semiannual coupon payment} &= \$5 \\ \text{semiannual discount rate}(i) &= 4\% \\ \text{number of years} &= 4 \end{aligned}$$

then the present value of the coupon payments is

$$\$5 \times \left[\frac{1 - \frac{1}{(1.04)^{4 \times 2}}}{0.04} \right] = \$33.6637$$

²Note that in our earlier illustration, we computed the present value of the semiannual coupon payments before the maturity date and then added the present value of the last cash flow (last semiannual coupon payment plus the maturity value). In the presentation of how to use the short-cut formula, we are computing the present value of all the semiannual coupon payments and then adding the present value of the maturity value. Both approaches will give the same answer for the value of a bond.

To determine the price, the present value of the maturity value must be added to the present value of the coupon payments. The present value of the maturity value is

$$\text{present value of maturity value} = \frac{\$100}{(1.04)^{4 \times 2}} = \$73.0690$$

The price is then \$106.7327 (\$33.6637 + \$73.0690). This agrees with our previous calculation for the price of this bond.

F. Valuing a Zero-Coupon Bond

For a zero-coupon bond, there is only one cash flow—the maturity value. The value of a zero-coupon bond that matures N years from now is

$$\frac{\text{maturity value}}{(1 + i)^{\text{no. of years} \times 2}}$$

where i is the semiannual discount rate.

It may seem surprising that the number of periods is double the number of years to maturity. In computing the value of a zero-coupon bond, the number of 6-month periods (i.e., “no. of years $\times 2$ ”) is used in the denominator of the formula. The rationale is that the pricing of a zero-coupon bond should be consistent with the pricing of a semiannual coupon bond. Therefore, the use of 6-month periods is required in order to have uniformity between the present value calculations.

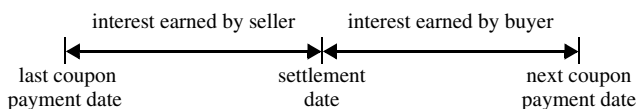
To illustrate the application of the formula, the value of a 5-year zero-coupon bond with a maturity value of \$100 discounted at an 8% interest rate is \$67.5564, as shown below:

$$\begin{aligned} i &= 0.04 (= 0.08/2) \\ N &= 5 \\ \frac{\$100}{(1.04)^{5 \times 2}} &= \$67.5564 \end{aligned}$$

G. Valuing a Bond Between Coupon Payments

For coupon-paying bonds, a complication arises when we try to price a bond between coupon payments. The amount that the buyer pays the seller in such cases is the present value of the cash flow. But one of the cash flows, the very next cash flow, encompasses two components as shown below:

1. interest earned by the seller
2. interest earned by the buyer



The interest earned by the seller is the interest that has accrued³ between the last coupon payment date and the settlement date.⁴ This interest is called **accrued interest**. At the time of purchase, the buyer must compensate the seller for the accrued interest. The buyer recovers the accrued interest when the next coupon payment is received.

When the price of a bond is computed using the present value calculations described earlier, it is computed with accrued interest embodied in the price. This price is referred to as the **full price**. (Some market participants refer to it as the **dirty price**.) It is the full price that the buyer pays the seller. From the full price, the accrued interest must be deducted to determine the **price** of the bond, sometimes referred to as the **clean price**.

Below, we show how the present value formula is modified to compute the full price when a bond is purchased between coupon periods.

1. Computing the Full Price To compute the full price, it is first necessary to determine the fractional periods between the settlement date and the next coupon payment date. This is determined as follows:

$$w \text{ periods} = \frac{\text{days between settlement date and next coupon payment date}}{\text{days in coupon period}}$$

Then the present value of the expected cash flow to be received t periods from now using a discount rate i assuming the first coupon payment is w periods from now is:

$$\text{present value}_t = \frac{\text{expected cash flow}}{(1 + i)^{t-1+w}}$$

This procedure for calculating the present value when a security is purchased between coupon payments is called the “Street method.”

To illustrate the calculation, suppose that there are five semiannual coupon payments remaining for a 10% coupon bond. Also assume the following:

1. 78 days between the settlement date and the next coupon payment date
2. 182 days in the coupon period

Then w is 0.4286 periods ($= 78/182$). The present value of each cash flow assuming that each is discounted at 8% annual discount rate is

$$\text{Period 1: present value}_1 = \frac{\$5}{(1.04)^{0.4286}} = \$4.9167$$

$$\text{Period 2: present value}_2 = \frac{\$5}{(1.04)^{1.4286}} = \$4.7276$$

$$\text{Period 3: present value}_3 = \frac{\$5}{(1.04)^{2.4286}} = \$4.5457$$

$$\text{Period 4: present value}_4 = \frac{\$5}{(1.04)^{3.4286}} = \$4.3709$$

$$\text{Period 5: present value}_5 = \frac{\$105}{(1.04)^{4.4286}} = \$88.2583$$

³“Accrued” means that the interest is earned but not distributed to the bondholder.

⁴The settlement date is the date a transaction is completed.

The full price is the sum of the present value of the cash flows, which is \$106.8192. Remember that the full price includes the accrued interest that the buyer is paying the seller.

2. Computing the Accrued Interest and the Clean Price To find the price without accrued interest, called the **clean price** or simply **price**, the accrued interest must be computed. To determine the accrued interest, it is first necessary to determine the number of days in the accrued interest period. The number of days in the accrued interest period is determined as follows:

$$\begin{aligned} &\text{days in accrued interest period} = \\ &\quad \text{days in coupon period} - \text{days between settlement and next coupon payment} \end{aligned}$$

The percentage of the next semiannual coupon payment that the seller has earned as accrued interest is found as follows:

$$\frac{\text{days in accrued interest period}}{\text{days in coupon period}}$$

So, for example, returning to our illustration where the full price was computed, since there are 182 days in the coupon period and there are 78 days from the settlement date to the next coupon payment, the days in the accrued interest period is 182 minus 78, or 104 days. Therefore, the percentage of the coupon payment that is accrued interest is:

$$\frac{104}{182} = 0.5714 = 57.14\%$$

This is the same percentage found by simply subtracting w from 1. In our illustration, w was 0.4286. Then $1 - 0.4286 = 0.5714$.

Given the value of w , the amount of accrued interest (AI) is equal to:

$$\text{AI} = \text{semiannual coupon payment} \times (1 - w)$$

So, for the 10% coupon bond whose full price we computed, since the semiannual coupon payment per \$100 of par value is \$5 and w is 0.4286, the accrued interest is:

$$\$5 \times (1 - 0.4286) = \$2.8570$$

The clean price is then:

$$\text{full price} - \text{accrued interest}$$

In our illustration, the clean price is⁵

$$\$106.8192 - \$2.8570 = \$103.9622$$

3. Day Count Conventions The practice for calculating the number of days between two dates depends on *day count conventions* used in the bond market. The convention differs by the type of security. Day count conventions are also used to calculate the number of days in the numerator and denominator of the ratio w .

⁵Notice that in computing the full price the present value of the next coupon payment is computed. However, the buyer pays the seller the accrued interest now despite the fact that it will be recovered at the next coupon payment date.

The accrued interest (AI) assuming semiannual payments is calculated as follows:

$$AI = \frac{\text{annual coupon}}{2} \times \frac{\text{days in AI period}}{\text{days in coupon period}}$$

In calculating the number of days between two dates, the actual number of days is not always the same as the number of days that should be used in the accrued interest formula. The number of days used depends on the day count convention for the particular security. Specifically, day count conventions differ for Treasury securities and government agency securities, municipal bonds, and corporate bonds.

For coupon-bearing Treasury securities, the day count convention used is to determine the actual number of days between two dates. This is referred to as the “actual/actual” day count convention. For example, consider a coupon-bearing Treasury security whose previous coupon payment was March 1. The next coupon payment would be on September 1. Suppose this Treasury security is purchased with a settlement date of July 17th. The actual number of days between July 17 (the settlement date) and September 1 (the date of the next coupon payment) is 46 days, as shown below:

July 17 to July 31	14 days
August	31 days
September 1	1 day
	46 days

Note that the settlement date (July 17) is not counted. The number of days in the coupon period is the actual number of days between March 1 and September 1, which is 184 days. The number of days between the last coupon payment (March 1) through July 17 is therefore 138 days (184 days – 46 days).

For coupon-bearing agency, municipal, and corporate bonds, a different day count convention is used. It is assumed that every month has 30 days, that any 6-month period has 180 days, and that there are 360 days in a year. This day count convention is referred to as “30/360.” For example, consider once again the Treasury security purchased with a settlement date of July 17, the previous coupon payment on March 1, and the next coupon payment on September 1. If the security is an agency, municipal, or corporate bond, the number of days until the next coupon payment is 44 days as shown below:

July 17 to July 31	13 days
August	30 days
September 1	1 day
	44 days

Note that the settlement date, July 17, is not counted. Since July is treated as having 30 days, there are 13 days (30 days minus the first 17 days in July). The number of days from March 1 to July 17 is 136, which is the number of days in the accrued interest period.

III. TRADITIONAL APPROACH TO VALUATION

The traditional approach to valuation has been to discount every cash flow of a fixed income security by the same interest rate (or discount rate). For example, consider the three

EXHIBIT 3 Cash Flows for Three 10-Year
Hypothetical Treasury Securities Per \$100 of
Par Value

Each period is six months

Period	Coupon rate		
	12%	8%	0%
1–19	\$6	\$4	\$0
20	106	104	100

hypothetical 10-year Treasury securities shown in Exhibit 3: a 12% coupon bond, an 8% coupon bond, and a zero-coupon bond. The cash flows for each bond are shown in the exhibit. Since the cash flows of all three bonds are viewed as default free, the traditional practice is to use the same discount rate to calculate the present value of all three bonds and use the same discount rate for the cash flow for each period. The discount rate used is the yield for the on-the-run issue obtained from the Treasury yield curve. For example, suppose that the yield for the 10-year on-the-run Treasury issue is 10%. Then, the practice is to discount each cash flow for each bond using a 10% discount rate.

For a non-Treasury security, a yield premium or yield spread is added to the on-the-run Treasury yield. The yield spread is the same regardless of when a cash flow is to be received in the traditional approach. For a 10-year non-Treasury security, suppose that 90 basis points is the appropriate yield spread. Then all cash flows would be discounted at the yield for the on-the-run 10-year Treasury issue of 10% plus 90 basis points.

IV. THE ARBITRAGE-FREE VALUATION APPROACH

The fundamental flaw of the traditional approach is that it views each security as the same package of cash flows. For example, consider a 10-year U.S. Treasury issue with an 8% coupon rate. The cash flows per \$100 of par value would be 19 payments of \$4 every six months and \$104 twenty 6-month periods from now. The traditional practice would discount each cash flow using the same discount rate.

The proper way to view the 10-year 8% coupon Treasury issue is as a package of zero-coupon bonds whose maturity value is equal to the amount of the cash flow and whose maturity date is equal to each cash flow's payment date. Thus, the 10-year 8% coupon Treasury issue should be viewed as 20 zero-coupon bonds. The reason this is the proper way to value a security is that it does not allow arbitrage profit by taking apart or "stripping" a security and selling off the stripped securities at a higher aggregate value than it would cost to purchase the security in the market. We'll illustrate this later. We refer to this approach to valuation as the **arbitrage-free valuation approach**.⁶

⁶In its simple form, arbitrage is the simultaneous buying and selling of an asset at two different prices in two different markets. The arbitrageur profits without risk by buying cheap in one market and simultaneously selling at the higher price in the other market. Such opportunities for arbitrage are rare. Less obvious arbitrage opportunities exist in situations where a package of assets can produce a payoff (expected return) identical to an asset that is priced differently. This arbitrage relies on a fundamental principle of finance called the "law of one price" which states that a given asset must have the same price

EXHIBIT 4 Comparison of Traditional Approach and Arbitrage-Free Approach in Valuing a Treasury Security

Each period is six months

Period	Discount (base interest) rate		Cash flows for*		
	Traditional approach	Arbitrage-free approach	12%	8%	0%
1	10-year Treasury rate	1-period Treasury spot rate	\$6	\$4	\$0
2	10-year Treasury rate	2-period Treasury spot rate	6	4	0
3	10-year Treasury rate	3-period Treasury spot rate	6	4	0
4	10-year Treasury rate	4-period Treasury spot rate	6	4	0
5	10-year Treasury rate	5-period Treasury spot rate	6	4	0
6	10-year Treasury rate	6-period Treasury spot rate	6	4	0
7	10-year Treasury rate	7-period Treasury spot rate	6	4	0
8	10-year Treasury rate	8-period Treasury spot rate	6	4	0
9	10-year Treasury rate	9-period Treasury spot rate	6	4	0
10	10-year Treasury rate	10-period Treasury spot rate	6	4	0
11	10-year Treasury rate	11-period Treasury spot rate	6	4	0
12	10-year Treasury rate	12-period Treasury spot rate	6	4	0
13	10-year Treasury rate	13-period Treasury spot rate	6	4	0
14	10-year Treasury rate	14-period Treasury spot rate	6	4	0
15	10-year Treasury rate	15-period Treasury spot rate	6	4	0
16	10-year Treasury rate	16-period Treasury spot rate	6	4	0
17	10-year Treasury rate	17-period Treasury spot rate	6	4	0
18	10-year Treasury rate	18-period Treasury spot rate	6	4	0
19	10-year Treasury rate	19-period Treasury spot rate	6	4	0
20	10-year Treasury rate	20-period Treasury spot rate	106	104	100

*Per \$100 of par value.

By viewing any financial asset as a package of zero-coupon bonds, a consistent valuation framework can be developed. Viewing a financial asset as a package of zero-coupon bonds means that any two bonds would be viewed as different packages of zero-coupon bonds and valued accordingly.

The difference between the traditional valuation approach and the arbitrage-free approach is illustrated in Exhibit 4, which shows how the three bonds whose cash flows are depicted in Exhibit 3 should be valued. With the traditional approach, the discount rate for all three bonds is the yield on a 10-year U.S. Treasury security. With the arbitrage-free approach, the discount rate for a cash flow is the theoretical rate that the U.S. Treasury would have to pay if it issued a zero-coupon bond with a maturity date equal to the maturity date of the cash flow.

Therefore, to implement the arbitrage-free approach, it is necessary to determine the theoretical rate that the U.S. Treasury would have to pay on a zero-coupon Treasury security for each maturity. As explained in the previous chapter, the name given to the zero-coupon Treasury rate is the **Treasury spot rate**. In Chapter 6, we will explain how the Treasury spot rate can be calculated. The spot rate for a Treasury security is the interest rate that should be used to discount a default-free cash flow with the same maturity. We call the value of a bond based on spot rates the **arbitrage-free value**.

regardless of the means by which one goes about creating that asset. The law of one price implies that if the payoff of an asset can be synthetically created by a package of assets, the price of the package and the price of the asset whose payoff it replicates must be equal.

EXHIBIT 5 Determination of the Arbitrage-Free Value of an 8% 10-year Treasury

Period	Years	Cash flow (\$)	Spot rate (%)*	Present value (\$) **
1	0.5	4	3.0000	3.9409
2	1.0	4	3.3000	3.8712
3	1.5	4	3.5053	3.7968
4	2.0	4	3.9164	3.7014
5	2.5	4	4.4376	3.5843
6	3.0	4	4.7520	3.4743
7	3.5	4	4.9622	3.3694
8	4.0	4	5.0650	3.2747
9	4.5	4	5.1701	3.1791
10	5.0	4	5.2772	3.0829
11	5.5	4	5.3864	2.9861
12	6.0	4	5.4976	2.8889
13	6.5	4	5.6108	2.7916
14	7.0	4	5.6643	2.7055
15	7.5	4	5.7193	2.6205
16	8.0	4	5.7755	2.5365
17	8.5	4	5.8331	2.4536
18	9.0	4	5.9584	2.3581
19	9.5	4	6.0863	2.2631
20	10.0	104	6.2169	56.3830
			<i>Total</i>	\$115.2621

*The spot rate is an annual discount rate. The convention to obtain a semiannual discount rate is to take one-half the annual discount rate. So, for period 6 (i.e., 3 years), the spot rate is 4.7520%. The semiannual discount rate is 2.376%.

** The present value for the cash flow is equal to:

$$\frac{\text{Cash flow}}{(1 + \text{Spot rate}/2)^{\text{period}}}$$

A. Valuation Using Treasury Spot Rates

For the purposes of our discussion, we will take the Treasury spot rate for each maturity as given. To illustrate how Treasury spot rates are used to compute the arbitrage-free value of a Treasury security, we will use the hypothetical Treasury spot rates shown in the fourth column of Exhibit 5 to value an 8% 10-year Treasury security. The present value of each period's cash flow is shown in the last column. The sum of the present values is the arbitrage-free value for the Treasury security. For the 8% 10-year Treasury, it is \$115.2619.

As a second illustration, suppose that a 4.8% coupon 10-year Treasury bond is being valued based on the Treasury spot rates shown in Exhibit 5. The arbitrage-free value of this bond is \$90.8428 as shown in Exhibit 6.

In the next chapter, we discuss yield measures. The yield to maturity is a measure that would be computed for this bond. We won't show how it is computed in this chapter, but simply state the result. The yield for the 4.8% coupon 10-year Treasury bond is 6.033%. Notice that the spot rates are used to obtain the price and the price is then used to compute a conventional yield measure. *It is important to understand that there are an infinite number of spot rate curves that can generate the same price of \$90.8428 and therefore the same yield.* (We return to this point in the next chapter.)

EXHIBIT 6 Determination of the Arbitrage-Free Value of a 4.8% 10-year Treasury

Period	Years	Cash flow (\$)	Spot rate (%)*	Present value (\$) **
1	0.5	2.4	3.0000	2.3645
2	1.0	2.4	3.3000	2.3227
3	1.5	2.4	3.5053	2.2781
4	2.0	2.4	3.9164	2.2209
5	2.5	2.4	4.4376	2.1506
6	3.0	2.4	4.7520	2.0846
7	3.5	2.4	4.9622	2.0216
8	4.0	2.4	5.0650	1.9648
9	4.5	2.4	5.1701	1.9075
10	5.0	2.4	5.2772	1.8497
11	5.5	2.4	5.3864	1.7916
12	6.0	2.4	5.4976	1.7334
13	6.5	2.4	5.6108	1.6750
14	7.0	2.4	5.6643	1.6233
15	7.5	2.4	5.7193	1.5723
16	8.0	2.4	5.7755	1.5219
17	8.5	2.4	5.8331	1.4722
18	9.0	2.4	5.9584	1.4149
19	9.5	2.4	6.0863	1.3578
20	10.0	102.4	6.2169	55.5156
			<i>Total</i>	90.8430

*The spot rate is an annual discount rate. The convention to obtain a semiannual discount rate is to take one-half the annual discount rate. So, for period 6 (i.e., 3 years), the spot rate is 4.7520%. The semiannual discount rate is 2.376%.

** The present value for the cash flow is equal to:

$$\frac{\text{Cash flow}}{(1 + \text{Spot rate}/2)^{\text{period}}}$$

B. Reason for Using Treasury Spot Rates

Thus far, we simply asserted that the value of a Treasury security should be based on discounting each cash flow using the corresponding Treasury spot rate. But what if market participants value a security using the yield for the on-the-run Treasury with a maturity equal to the maturity of the Treasury security being valued? (In other words, what if participants use the yield on coupon-bearing securities rather than the yield on zero-coupon securities?) Let's see why a Treasury security will have to trade close to its arbitrage-free value.

1. Stripping and the Arbitrage-Free Valuation The key in the process is the existence of the Treasury strips market. As explained in Chapter 3, a dealer has the ability to take apart the cash flows of a Treasury coupon security (i.e., strip the security) and create zero-coupon securities. These zero-coupon securities, which we called Treasury strips, can be sold to investors. At what interest rate or yield can these Treasury strips be sold to investors? They can be sold at the Treasury spot rates. If the market price of a Treasury security is less than its value using the arbitrage-free valuation approach, then a dealer can buy the Treasury security, strip it, and sell off the Treasury strips so as to generate greater proceeds than the cost of purchasing the Treasury security. The resulting profit is an arbitrage profit. Since, as we will see, the value

determined by using the Treasury spot rates does not allow for the generation of an arbitrage profit, this is the reason why the approach is referred to as an “arbitrage-free” approach.

To illustrate this, suppose that the yield for the on-the-run 10-year Treasury issue is 6%. (We will see in Chapter 6 that the Treasury spot rate curve in Exhibit 5 was generated from a yield curve where the on-the-run 10-year Treasury issue was 6%.) Suppose that the 8% coupon 10-year Treasury issue is valued using the traditional approach based on 6%. Exhibit 7 shows the value based on discounting all the cash flows at 6% is \$114.8775.

Consider what would happen if the market priced the security at \$114.8775. The value based on the Treasury spot rates (Exhibit 5) is \$115.2621. What can the dealer do? The dealer can buy the 8% 10-year issue for \$114.8775, strip it, and sell the Treasury strips at the spot rates shown in Exhibit 5. By doing so, the proceeds that will be received by the dealer are \$115.2621. This results in an arbitrage profit of \$0.3846 ($= \$115.2621 - \$114.8775$).⁷ Dealers recognizing this arbitrage opportunity will bid up the price of the 8% 10-year Treasury issue in order to acquire it and strip it. At what point will the arbitrage profit disappear? When the security is priced at \$115.2621, the value that we said is the arbitrage-free value.

To understand in more detail where this arbitrage profit is coming from, look at Exhibit 8. The third column shows how much each cash flow can be sold for by the dealer if it is stripped. The values in the third column are simply the present values in Exhibit 5 based on discounting the cash flows at the Treasury spot rates. The fourth column shows how much the dealer is effectively purchasing the cash flow if each cash flow is discounted at 6%. This is the last column in Exhibit 7. The sum of the arbitrage profit from each cash flow stripped is the total arbitrage profit.

2. Reconstitution and Arbitrage-Free Valuation We have just demonstrated how coupon stripping of a Treasury issue will force its market value to be close to the value determined by arbitrage-free valuation when the market price is less than the arbitrage-free value. What happens when a Treasury issue’s market price is greater than the arbitrage-free value? Obviously, a dealer will not want to strip the Treasury issue since the proceeds generated from stripping will be less than the cost of purchasing the issue.

When such situations occur, the dealer will follow a procedure called **reconstitution**.⁸ Basically, the dealer can purchase a package of Treasury strips so as to create a synthetic (i.e., artificial) Treasury coupon security that is worth more than the same maturity and same coupon Treasury issue.

To illustrate this, consider the 4.8% 10-year Treasury issue whose arbitrage-free value was computed in Exhibit 6. The arbitrage-free value is \$90.8430. Exhibit 9 shows the price assuming the traditional approach where all the cash flows are discounted at a 6% interest rate. The price is \$91.0735. What the dealer can do is purchase the Treasury strip for each 6-month period at the prices shown in Exhibit 6 and sell short the 4.8% 10-year Treasury coupon issue whose cash flows are being replicated. By doing so, the dealer has the cash flow of a 4.8% coupon 10-year Treasury security at a cost of \$90.8430, thereby generating an arbitrage profit of \$0.2305 ($\$91.0735 - \$90.8430$). The cash flows from the package of Treasury strips

⁷This may seem like a small amount, but remember that this is for a single \$100 par value bond. Multiply this by thousands of bonds and you can see a dealer’s profit potential.

⁸The definition of *reconstitute* is to provide with a new structure, often by assembling various parts into a whole. *Reconstitution* then, as used here, means to assemble the parts (the Treasury strips) in such a way that a new whole (a Treasury coupon bond) is created. That is, it is the opposite of *stripping* a coupon bond.

EXHIBIT 7 Price of an 8% 10-year Treasury Valued at a 6% Discount Rate

Period	Years	Cash flow (\$)	Spot rate (%)*	Present value (\$) **
1	0.5	4	6.0000	3.8835
2	1.0	4	6.0000	3.7704
3	1.5	4	6.0000	3.6606
4	2.0	4	6.0000	3.5539
5	2.5	4	6.0000	3.4504
6	3.0	4	6.0000	3.3499
7	3.5	4	6.0000	3.2524
8	4.0	4	6.0000	3.1576
9	4.5	4	6.0000	3.0657
10	5.0	4	6.0000	2.9764
11	5.5	4	6.0000	2.8897
12	6.0	4	6.0000	2.8055
13	6.5	4	6.0000	2.7238
14	7.0	4	6.0000	2.6445
15	7.5	4	6.0000	2.5674
16	8.0	4	6.0000	2.4927
17	8.5	4	6.0000	2.4201
18	9.0	4	6.0000	2.3496
19	9.5	4	6.0000	2.2811
20	10.0	104	6.0000	57.5823
			Total	114.8775

*The discount rate is an annual discount rate. The convention to obtain a semiannual discount rate is to take one-half the annual discount rate. So, since the discount rate for each period is 6%, the semiannual discount rate is 3%.

** The present value for the cash flow is equal to:

$$\frac{\text{Cash flow}}{(1.03)^{\text{period}}}$$

purchased is used to make the payments for the Treasury coupon security shorted. Actually, in practice, this can be done in a more efficient manner using a procedure for reconstitution provided for by the Department of the Treasury.

What forces the market price to the arbitrage-free value of \$90.8430? As dealers sell short the Treasury coupon issue (4.8% 10-year issue), the price of the issue decreases. When the price is driven down to \$90.8430, the arbitrage profit no longer exists.

This process of stripping and reconstitution assures that the price of a Treasury issue will not depart materially from its arbitrage-free value. In other countries, as governments permit the stripping and reconstitution of their issues, the value of non-U.S. government issues have also moved toward their arbitrage-free value.

C. Credit Spreads and the Valuation of Non-Treasury Securities

The Treasury spot rates can be used to value any default-free security. For a non-Treasury security, the theoretical value is not as easy to determine. The value of a non-Treasury security is found by discounting the cash flows by the Treasury spot rates plus a yield spread to reflect the additional risks.

The spot rate used to discount the cash flow of a non-Treasury security can be the Treasury spot rate plus a constant credit spread. For example, suppose the 6-month Treasury

EXHIBIT 8 Arbitrage Profit from Stripping the 8% 10-Year Treasury

Period	Years	Sell for	Buy for	Arbitrage profit
1	0.5	3.9409	3.8835	0.0574
2	1.0	3.8712	3.7704	0.1008
3	1.5	3.7968	3.6606	0.1363
4	2.0	3.7014	3.5539	0.1475
5	2.5	3.5843	3.4504	0.1339
6	3.0	3.4743	3.3499	0.1244
7	3.5	3.3694	3.2524	0.1170
8	4.0	3.2747	3.1576	0.1170
9	4.5	3.1791	3.0657	0.1134
10	5.0	3.0829	2.9764	0.1065
11	5.5	2.9861	2.8897	0.0964
12	6.0	2.8889	2.8055	0.0834
13	6.5	2.7916	2.7238	0.0678
14	7.0	2.7055	2.6445	0.0611
15	7.5	2.6205	2.5674	0.0531
16	8.0	2.5365	2.4927	0.0439
17	8.5	2.4536	2.4201	0.0336
18	9.0	2.3581	2.3496	0.0086
19	9.5	2.2631	2.2811	−0.0181
20	10.0	56.3830	57.5823	−1.1993
		115.2621	114.8775	0.3846

EXHIBIT 9 Price of a 4.8% 10-Year Treasury Valued at a 6% Discount Rate

Period	Years	Cash flow (\$)	Spot rate (%)	Present value (\$)
1	0.5	2.4	6.0000	2.3301
2	1.0	2.4	6.0000	2.2622
3	1.5	2.4	6.0000	2.1963
4	2.0	2.4	6.0000	2.1324
5	2.5	2.4	6.0000	2.0703
6	3.0	2.4	6.0000	2.0100
7	3.5	2.4	6.0000	1.9514
8	4.0	2.4	6.0000	1.8946
9	4.5	2.4	6.0000	1.8394
10	5.0	2.4	6.0000	1.7858
11	5.5	2.4	6.0000	1.7338
12	6.0	2.4	6.0000	1.6833
13	6.5	2.4	6.0000	1.6343
14	7.0	2.4	6.0000	1.5867
15	7.5	2.4	6.0000	1.5405
16	8.0	2.4	6.0000	1.4956
17	8.5	2.4	6.0000	1.4520
18	9.0	2.4	6.0000	1.4097
19	9.5	2.4	6.0000	1.3687
20	10.0	102.4	6.0000	56.6964
			<i>Total</i>	91.0735

spot rate is 3% and the 10-year Treasury spot rate is 6%. Also suppose that a suitable credit spread is 90 basis points. Then a 3.9% spot rate is used to discount a 6-month cash flow of a non-Treasury bond and a 6.9% discount rate to discount a 10-year cash flow. (Remember that when each semiannual cash flow is discounted, the discount rate used is one-half the spot rate — 1.95% for the 6-month spot rate and 3.45% for the 10-year spot rate.)

The drawback of this approach is that there is no reason to expect the credit spread to be the same regardless of when the cash flow is received. We actually observed this in the previous chapter when we saw how credit spreads increase with maturity. Consequently, it might be expected that credit spreads increase with the maturity of the bond. That is, there is a **term structure of credit spreads**.

Dealer firms typically estimate a term structure for credit spreads for each credit rating and market sector. Generally, the credit spread increases with maturity. This is a typical shape for the term structure of credit spreads. In addition, the shape of the term structure is not the same for all credit ratings. Typically, the lower the credit rating, the steeper the term structure of credit spreads.

When the credit spreads for a given credit rating and market sector are added to the Treasury spot rates, the resulting term structure is used to value bonds with that credit rating in that market sector. This term structure is referred to as the **benchmark spot rate curve** or **benchmark zero-coupon rate curve**.

For example, Exhibit 10 reproduces the Treasury spot rate curve in Exhibit 5. Also shown in the exhibit is a hypothetical credit spread for a non-Treasury security. The resulting benchmark spot rate curve is in the next-to-the-last column. It is this spot rate curve that is used to value the securities that have the same credit rating and are in the same market sector. This is done in Exhibit 10 for a hypothetical 8% 10-year issue. The arbitrage-free value is \$108,4616. Notice that the theoretical value is less than that for an otherwise comparable Treasury security. The arbitrage-free value for an 8% 10-year Treasury is \$115,2621 (see Exhibit 5).

V. VALUATION MODELS

A **valuation model** provides the fair value of a security. Thus far, the two valuation approaches we have presented have dealt with valuing simple securities. By simple we mean that it assumes the securities do not have an embedded option. A Treasury security and an option-free non-Treasury security can be valued using the arbitrage-free valuation approach.

More general valuation models handle securities with embedded options. In the fixed income area, two common models used are the **binomial model** and the **Monte Carlo simulation model**. The former model is used to value callable bonds, puttable bonds, floating-rate notes, and structured notes in which the coupon formula is based on an interest rate. The Monte Carlo simulation model is used to value mortgage-backed securities and certain types of asset-backed securities.⁹

In very general terms, the following five features are common to the binomial and Monte Carlo simulation valuation models:

1. Each model begins with the yields on the on-the-run Treasury securities and generates Treasury spot rates.

⁹A short summary reason is: mortgage-backed securities and certain asset-backed securities are interest rate path dependent securities and the binomial model cannot value such securities.

EXHIBIT 10 Calculation of Arbitrage-Free Value of a Hypothetical 8% 10-Year Non-Treasury Security Using Benchmark Spot Rate Curve

Period	Years	Cash flow	Treasury spot rate (%)	Credit spread (%)	Benchmark spot (%)	Present value (\$)
1	0.5	4	3.0000	0.20	3.2000	3.9370
2	1.0	4	3.3000	0.20	3.5000	3.8636
3	1.5	4	3.5053	0.25	3.7553	3.7829
4	2.0	4	3.9164	0.30	4.2164	3.6797
5	2.5	4	4.4376	0.35	4.7876	3.5538
6	3.0	4	4.7520	0.35	5.1020	3.4389
7	3.5	4	4.9622	0.40	5.3622	3.3237
8	4.0	4	5.0650	0.45	5.5150	3.2177
9	4.5	4	5.1701	0.45	5.6201	3.1170
10	5.0	4	5.2772	0.50	5.7772	3.0088
11	5.5	4	5.3864	0.55	5.9364	2.8995
12	6.0	4	5.4976	0.60	6.0976	2.7896
13	6.5	4	5.6108	0.65	6.2608	2.6794
14	7.0	4	5.6643	0.70	6.3643	2.5799
15	7.5	4	5.7193	0.75	6.4693	2.4813
16	8.0	4	5.7755	0.80	6.5755	2.3838
17	8.5	4	5.8331	0.85	6.6831	2.2876
18	9.0	4	5.9584	0.90	6.8584	2.1801
19	9.5	4	6.0863	0.95	7.0363	2.0737
20	10.0	104	6.2169	1.00	7.2169	51.1835
					Total	\$108.4616

- Each model makes an assumption about the expected volatility of short-term interest rates. This is a critical assumption in both models since it can significantly affect the security's fair value.
- Based on the volatility assumption, different "branches" of an interest rate tree (in the case of the binomial model) and interest rate "paths" (in the case of the Monte Carlo model) are generated.
- The model is calibrated to the Treasury market. This means that if an "on-the-run" Treasury issue is valued using the model, the model will produce the observed market price.
- Rules are developed to determine when an issuer/borrower will exercise embedded options—a call/put rule for callable/puttable bonds and a prepayment model for mortgage-backed and certain asset-backed securities.

The user of any valuation model is exposed to **modeling risk**. This is the risk that the output of the model is incorrect because the assumptions upon which it is based are incorrect. Consequently, it is imperative the results of a valuation model be stress-tested for modeling risk by altering assumptions.

YIELD MEASURES, SPOT RATES, AND FORWARD RATES

I. INTRODUCTION

Frequently, investors assess the relative value of a security by some yield or yield spread measure quoted in the market. These measures are based on assumptions that limit their use to gauge relative value. This chapter explains the various yield and yield spread measures and their limitations.

In this chapter, we will see a basic approach to computing the spot rates from the on-the-run Treasury issues. We will see the limitations of the nominal spread measure and explain two measures that overcome these limitations—zero-volatility spread and option-adjusted spread.

II. SOURCES OF RETURN

When an investor purchases a fixed income security, he or she can expect to receive a dollar return from one or more of the following sources:

1. the coupon interest payments made by the issuer
2. any capital gain (or capital loss—a negative dollar return) when the security matures, is called, or is sold
3. income from reinvestment of interim cash flows (interest and/or principal payments prior to stated maturity)

Any yield measure that purports to measure the potential return from a fixed income security should consider all three sources of return described above.

A. Coupon Interest Payments

The most obvious source of return on a bond is the periodic coupon interest payments. For zero-coupon instruments, the return from this source is zero. By purchasing a security below its par value and receiving the full par value at maturity, the investor in a zero-coupon instrument is effectively receiving interest in a lump sum.

B. Capital Gain or Loss

An investor receives cash when a bond matures, is called, or is sold. If these proceeds are greater than the purchase price, a capital gain results. For a bond held to maturity, there will be a capital gain if the bond is purchased below its par value. For example, a bond purchased for \$94.17 with a par value of \$100 will generate a capital gain of \$5.83 ($\$100 - \$94.17$) if held to maturity. For a callable bond, a capital gain results if the price at which the bond is called (i.e., the call price) is greater than the purchase price. For example, if the bond in our previous example is callable and subsequently called at \$100.5, a capital gain of \$6.33 ($\$100.50 - \$94.17$) will be realized. If the same bond is sold prior to its maturity or before it is called, a capital gain will result if the proceeds exceed the purchase price. So, if our hypothetical bond is sold prior to the maturity date for \$103, the capital gain would be \$8.83 ($\$103 - \$94.17$).

Similarly, for all three outcomes, a capital loss is generated when the proceeds received are less than the purchase price. For a bond held to maturity, there will be a capital loss if the bond is purchased for more than its par value (i.e., purchased at a premium). For example, a bond purchased for \$102.50 with a par value of \$100 will generate a capital loss of \$2.50 ($\$102.50 - \$100$) if held to maturity. For a callable bond, a capital loss results if the price at which the bond is called is less than the purchase price. For example, if the bond in our example is callable and subsequently called at \$100.50, a capital loss of \$2 ($\$102.50 - \$100.50$) will be realized. If the same bond is sold prior to its maturity or before it is called, a capital loss will result if the sale price is less than the purchase price. So, if our hypothetical bond is sold prior to the maturity date for \$98.50, the capital loss would be \$4 ($\$102.50 - \$98.50$).

C. Reinvestment Income

Prior to maturity, with the exception of zero-coupon instruments, fixed income securities make periodic interest payments that can be reinvested. Amortizing securities (such as mortgage-backed securities and asset-backed securities) make periodic principal payments that can be reinvested prior to final maturity. The interest earned from reinvesting the interim cash flows (interest and/or principal payments) prior to final or stated maturity is called **reinvestment income**.

III. TRADITIONAL YIELD MEASURES

Yield measures cited in the bond market include current yield, yield to maturity, yield to call, yield to put, yield to worst, and cash flow yield. These yield measures are expressed as a percent return rather than a dollar return. Below we explain how each measure is calculated and its limitations.

A. Current Yield

The **current yield** relates the annual dollar coupon interest to a bond's market price. The formula for the current yield is:

$$\text{current yield} = \frac{\text{annual dollar coupon interest}}{\text{price}}$$

For example, the current yield for a 7% 8-year bond whose price is \$94.17 is 7.43% as shown below:

$$\text{annual dollar coupon interest} = 0.07 \times \$100 = \$7$$

$$\text{price} = \$94.17$$

$$\text{current yield} = \frac{\$7}{\$94.17} = 0.0743 \text{ or } 7.43\%$$

The current yield will be greater than the coupon rate when the bond sells at a discount; the reverse is true for a bond selling at a premium. For a bond selling at par, the current yield will be equal to the coupon rate.

The drawback of the current yield is that it considers only the coupon interest and no other source for an investor's return. No consideration is given to the capital gain an investor will realize when a bond purchased at a discount is held to maturity; nor is there any recognition of the capital loss an investor will realize if a bond purchased at a premium is held to maturity. No consideration is given to reinvestment income.

B. Yield to Maturity

The most popular measure of yield in the bond market is the **yield to maturity**. The yield to maturity is the interest rate that will make the present value of a bond's cash flows equal to its market price plus accrued interest. To find the yield to maturity, we first determine the expected cash flows and then search, by trial and error, for the interest rate that will make the present value of cash flows equal to the market price plus accrued interest. (This is simply a special case of an **internal rate of return** (IRR) calculation where the cash flows are those received if the bond is held to the maturity date.) In the illustrations presented in this chapter, we assume that the next coupon payment will be six months from now so that there is no accrued interest.

To illustrate, consider a 7% 8-year bond selling for \$94.17. The cash flows for this bond are (1) 16 payments every 6-months of \$3.50 and (2) a payment sixteen 6-month periods from now of \$100. The present value using various *semiannual* discount (interest) rates is:

Semiannual interest rate	3.5%	3.6%	3.7%	3.8%	3.9%	4.0%
Present value	100.00	98.80	97.62	96.45	95.30	94.17

When a 4.0% interest rate is used, the present value of the cash flows is equal to \$94.17, which is the price of the bond. Hence, 4.0% is the *semiannual* yield to maturity.

The market convention adopted to annualize the semiannual yield to maturity is to double it and call that the yield to maturity. Thus, the yield to maturity for the above bond is 8% (2 times 4.0%). The yield to maturity computed using this convention—doubling the semiannual yield—is called a **bond-equivalent yield**.

The following relationships between the price of a bond, coupon rate, current yield, and yield to maturity hold:

Bond selling at	Relationship
par	coupon rate = current yield = yield to maturity
discount	coupon rate < current yield < yield to maturity
premium	coupon rate > current yield > yield to maturity

1. The Bond-Equivalent Yield Convention The *convention* developed in the bond market to move from a semiannual yield to an annual yield is to simply double the semiannual yield. As just noted, this is called the bond-equivalent yield. In general, when one doubles a semiannual yield (or a semiannual return) to obtain an annual measure, one is said to be computing the measure on a **bond-equivalent basis**.

Students of the bond market are troubled by this convention. The two questions most commonly asked are: First, why is the practice of simply doubling a semiannual yield followed? Second, wouldn't it be more appropriate to compute the effective annual yield by compounding the semiannual yield?¹

The answer to the first question is that it is simply a convention. There is no danger with a convention unless you use it improperly. The fact is that market participants recognize that a yield (or return) is computed on a semiannual basis by convention and adjust accordingly when using the number. So, if the bond-equivalent yield on a security purchased by an investor is 6%, the investor knows the semiannual yield is 3%. Given that, the investor can use that semiannual yield to compute an effective annual yield or any other annualized measure desired. For a manager comparing the yield on a security as an asset purchased to a yield required on a liability to satisfy, the yield figure will be measured in a manner consistent with that of the yield required on the liability.

The answer to the second question is that it is true that computing an effective annual yield would be better. But so what? Once we discover the limitations of yield measures in general, we will question whether or not an investor should use a bond-equivalent yield measure or an effective annual yield measure in making investment decisions. That is, when we identify the major problems with yield measures, the doubling of a semiannual yield is the least of our problems.

So, don't lose any sleep over this convention. Just make sure that you use a bond-equivalent yield measure properly.

2. Limitations of Yield-to-Maturity Measure The yield to maturity considers not only the coupon income but any capital gain or loss that the investor will realize by holding the bond to maturity. The yield to maturity also considers the timing of the cash flows. *It does consider reinvestment income; however, it assumes that the coupon payments can be reinvested at an interest rate equal to the yield to maturity.* So, if the yield to maturity for a bond is 8%, for example, to earn that yield the coupon payments must be reinvested at an interest rate equal to 8%.

The illustrations below clearly demonstrate this. In the illustrations, the analysis will be in terms of dollars. Be sure you keep in mind the difference between the **total future dollars**, which is equal to all the dollars an investor expects to receive (including the recovery of the principal), and the **total dollar return**, which is equal to the dollars an investor expects to realize from the three sources of return (coupon payments, capital gain/loss, and reinvestment income).

Suppose an investor has \$94.17 and places the funds in a certificate of deposit (CD) that matures in 8 years. Let's suppose that the bank agrees to pay 4% interest every six months. This means that the bank is agreeing to pay 8% on a bond equivalent basis (i.e., doubling the semiannual yield). We can translate all of this into the total future dollars that will be generated by this investment at the end of 8 years. From the standard formula for the future

¹By compounding the semiannual yield it is meant that the annual yield is computed as follows:

$$\text{effective annual yield} = (1 + \text{semiannual yield})^2 - 1$$

value of an investment today, we can determine the total future dollars as:

$$\$94.17 \times (1.04)^{16} = \$176.38$$

So, to an investor who invests \$94.17 for 8 years at an 8% yield on a bond equivalent basis and interest is paid semiannually, the investment will generate \$176.38. Decomposing the total future dollars we see that:

$$\text{Total future dollars} = \$176.38$$

$$\text{Return of principal} = \$94.17$$

$$\text{Total interest from CD} = \$82.21$$

Thus, any investment that promises a yield of 8% on a bond equivalent basis for 8 years on an investment of \$94.17 must generate total future dollars of \$176.38 or equivalently a return from all sources of \$82.21. That is, if we look at the three sources of a bond return that offered an 8% yield with semiannual coupon payments and sold at a price of \$94.17, the following would have to hold:

$$\begin{aligned} &\text{Coupon interest} \\ &+ \text{Capital gain} \\ &+ \text{Reinvestment income} \\ &= \overline{\text{Total dollar return} = \text{Total interest from CD} = \$82.21} \end{aligned}$$

Now, instead of a certificate of deposit, suppose that an investor purchases a bond with a coupon rate of 7% that matures in 8 years. We know that the three sources of return are coupon income, capital gain/loss, and reinvestment income. Suppose that the price of this bond is \$94.17. The yield to maturity for this bond (on a bond equivalent basis) is 8%. Notice that this is the same type of investment as the certificate of deposit—the bank offered an 8% yield on a bond equivalent basis for 8 years and made payments semiannually. So, what should the investor in this bond expect in terms of *total future dollars*? As we just demonstrated, an investment of \$94.17 must generate \$176.38 in order to say that it provided a yield of 8%. Or equivalently, the total dollar return that must be generated is \$82.21. Let's look at what in fact is generated in terms of dollar return.

The coupon is \$3.50 every six months. So the dollar return from the coupon interest is \$3.50 for 16 six-month periods, or \$56. When the bond matures, there is a capital gain of \$5.83 (\$100 — \$94.17). Therefore, based on these two sources of return we have:

$$\text{Coupon interest} = \$56.00$$

$$\text{Capital gain} = \$5.83$$

$$\text{Dollar return without reinvestment income} = \overline{\$61.83}$$

Something's wrong here. Only \$61.83 is generated from the bond whereas \$82.21 is needed in order to say that this bond provided an 8% yield. That is, there is a dollar return shortfall of \$20.38 (\$82.21 — \$61.83). How is this dollar return shortfall generated?

Recall that in the case of the certificate of deposit, the bank does the reinvesting of the principal and interest, and pays 4% every six months or 8% on a bond equivalent basis. In

contrast, for the bond, the investor has to reinvest any coupon interest until the bond matures. It is the reinvestment income that must generate the dollar return shortfall of \$20.38. But at what yield will the investor have to reinvest the coupon payments in order to generate the \$20.38? The answer is: the yield to maturity.² That is, the reinvestment income will be \$20.38 if each semiannual coupon payment of \$3.50 can be reinvested at a semiannual yield of 4% (one half the yield to maturity). The reinvestment income earned on a given coupon payment of \$3.50, if it is invested from the time of receipt in period t to the maturity date (16 periods in our example) at a 4% semiannual rate, is:

$$\$3.50 (1.04)^{16-t} - \$3.50$$

The first coupon payment ($t = 1$) can be reinvested for 15 periods. Applying the formula above we find the reinvestment income earned on the first coupon payment is:

$$\$3.50 (1.04)^{16-1} - \$3.50 = \$2.80$$

Similarly, the reinvestment income for all coupon payments is shown below:

Period	Periods reinvested	Coupon payment	Reinvestment income
1	15	\$3.5	\$2.80
2	14	3.5	2.56
3	13	3.5	2.33
4	12	3.5	2.10
5	11	3.5	1.89
6	10	3.5	1.68
7	9	3.5	1.48
8	8	3.5	1.29
9	7	3.5	1.11
10	6	3.5	0.93
11	5	3.5	0.76
12	4	3.5	0.59
13	3	3.5	0.44
14	2	3.5	0.29
15	1	3.5	0.14
16	0	3.5	0.00
		Total	\$20.39

²This can be verified by using the future value of an annuity. The future of an annuity is given by the following formula:

$$\text{Annuity payment} = \left[\frac{(1+i)^n - 1}{i} \right]$$

where i is the interest rate and n is the number of periods.

In our example, i is 4%, n is 16, and the amount of the annuity is the semiannual coupon of \$3.50. Therefore, the future value of the coupon payment is

$$\$3.50 \left[\frac{(1.04)^{16} - 1}{0.04} \right] = \$76.38$$

Since the coupon payments are \$56, the reinvestment income is \$20.38 (\$76.38 - \$56). This is the amount that is necessary to produce the dollar return shortfall in our example.

The total reinvestment income is \$20.39 (differing from \$20.38 due to rounding).

So, with the reinvestment income of \$20.38 at 4% semiannually (i.e., one half the yield to maturity on a bond-equivalent basis), the total dollar return is

$$\begin{aligned}\text{Coupon interest} &= \$56.00 \\ \text{Capital gain} &= \$5.83 \\ \text{Reinvestment income} &= \$20.38 \\ \text{Total dollar return} &= \underline{\$82.21}\end{aligned}$$

In our illustration, we used an investment in a certificate of deposit to show what the total future dollars will have to be in order to obtain a yield of 8% on an investment of \$94.17 for 8 years when interest payments are semiannual. However, this holds for any type of investment, not just a certificate of deposit. For example, if an investor is told that he or she can purchase a debt instrument for \$94.17 that offers an 8% yield (on a bond-equivalent basis) for 8 years and makes interest payments semiannually, then the investor should translate this yield into the following:

I should be receiving total future dollars of \$176.38
I should be receiving a total dollar return of \$82.21

It is always important to think in terms of dollars (or pound sterling, yen, or other currency) because “yield measures” are misleading.

We can also see that the reinvestment income can be a significant portion of the total dollar return. In our example, the total dollar return is \$82.21 and the total dollar return from reinvestment income to make up the shortfall is \$20.38. This means that reinvestment income is about 25% of the total dollar return.

This is such an important point that we should go through this one more time for another bond. Suppose an investor purchases a 15-year 8% coupon bond at par value (\$100). The yield for this bond is simple to determine since the bond is trading at par. The yield is equal to the coupon rate, 8%. Let’s translate this into dollars. We know that if an investor makes an investment of \$100 for 15 years that offers an 8% yield and the interest payments are semiannual, the total future dollars will be:

$$\$100 \times (1.04)^{30} = \$324.34$$

Decomposing the total future dollars we see that:

$$\begin{aligned}\text{Total future dollars} &= \$324.34 \\ \text{Return of principal} &= \$100.00 \\ \text{Total dollar return} &= \$224.34\end{aligned}$$

Without reinvestment income, the dollar return is:

$$\begin{aligned}\text{Coupon interest} &= \$120 \\ \text{Capital gain} &= \$0 \\ \text{Dollar return without reinvestment income} &= \underline{\$120}\end{aligned}$$

Note that the capital gain is \$0 because the bond is purchased at par value.

The dollar return shortfall is therefore \$104.34 (\$224.34 – \$120). This shortfall is made up if the coupon payments can be reinvested at a yield of 8% (the yield on the bond at the time of purchase). For this bond, the reinvestment income is 46.5% of the total dollar return needed to produce a yield of 8% (\$104.34/\$224.34).³

Clearly, the investor will only realize the yield to maturity stated at the time of purchase if the following two assumptions hold:

Assumption 1: the coupon payments can be reinvested at the yield to maturity

Assumption 2: the bond is held to maturity

With respect to the first assumption, the risk that an investor faces is that future interest rates will be less than the yield to maturity at the time the bond is purchased, known as **reinvestment risk**. If the bond is not held to maturity, the investor faces the risk that he may have to sell for less than the purchase price, resulting in a return that is less than the yield to maturity, known as **interest rate risk**.

3. Factors Affecting Reinvestment Risk There are two characteristics of a bond that affect the degree of reinvestment risk:

Characteristic 1. For a given yield to maturity and a given non-zero coupon rate, the longer the maturity, the more the bond's total dollar return depends on reinvestment income to realize the yield to maturity at the time of purchase. That is, the greater the reinvestment risk.

The implication is the yield to maturity measure for long-term maturity coupon bonds tells little about the potential return that an investor may realize if the bond is held to maturity. For long-term bonds, in high interest rate environments, the reinvestment income component may be as high as 70% of the bond's total dollar return.

Characteristic 2. For a coupon paying bond, for a given maturity and a given yield to maturity, the higher the coupon rate, the more dependent the bond's total dollar return will be on the reinvestment of the coupon payments in order to produce the yield to maturity at the time of purchase.

This means that holding maturity and yield to maturity constant, bonds selling at a premium will be more dependent on reinvestment income than bonds selling at par. This is because the reinvestment income has to make up the capital loss due to amortizing the price premium when holding the bond to maturity. In contrast, a bond selling at a discount will be less dependent on reinvestment income than a bond selling at par because a portion of the return

³The future value of the coupon payments of \$4 for 30 six-month periods is:

$$\$4.00 \left[\frac{(1.04)^{30} - 1}{0.04} \right] = \$224.34$$

Since the coupon payments are \$120 and the capital gain is \$0, the reinvestment income is \$104.34. This is the amount that is necessary to produce the dollar return shortfall in our example.

EXHIBIT 1 Percentage of Total Dollar Return from Reinvestment Income for a Bond to Generate an 8% Yield (BEY)

	Years to maturity				
	2	3	5	8	15
Bond with a 7% coupon					
Price	98.19	97.38	95.94	94.17	91.35
% of total	5.2%	8.6%	15.2%	24.8%	44.5%
Bond with an 8% coupon					
Price	100.00	100.00	100.00	100.00	100.00
% of total	5.8%	9.5%	16.7%	26.7%	46.5%
Bond with a 12% coupon					
Price	107.26	110.48	116.22	122.30	134.58
% of total	8.1%	12.9%	21.6%	31.0%	51.8%

is coming from the capital gain due to accrediting the price discount when holding the bond to maturity. For zero-coupon bonds, none of the bond's total dollar return is dependent on reinvestment income. So, a zero-coupon bond has no reinvestment risk if held to maturity.

The dependence of the total dollar return on reinvestment income for bonds with different coupon rates and maturities is shown in Exhibit 1.

4. Comparing Semiannual-Pay and Annual-Pay Bonds In our yield calculations, we have been dealing with bonds that pay interest semiannually. A non-U.S. bond may pay interest annually rather than semiannually. This is the case for many government bonds in Europe and Eurobonds. In such instances, an adjustment is required to make a direct comparison between the yield to maturity on a U.S. fixed-rate bond and that on an annual-pay non-U.S. fixed-rate bond.

Given the yield to maturity on an annual-pay bond, its bond-equivalent yield is computed as follows:

$$\text{bond-equivalent yield of an annual-pay bond} = 2[(1 + \text{yield on annual-pay bond})^{0.5} - 1]$$

The term in the square brackets involves determining what semiannual yield, when compounded, produces the yield on an annual-pay bond. Doubling this semiannual yield (i.e., multiplying the term in the square brackets by 2), gives the bond-equivalent yield.

For example, suppose that the yield to maturity on an annual-pay bond is 6%. Then the bond-equivalent yield is:

$$2[(1.06)^{0.5} - 1] = 5.91\%$$

Notice that the bond-equivalent yield will always be less than the annual-pay bond's yield to maturity.

To convert the bond-equivalent yield of a U.S. bond issue to an annual-pay basis so that it can be compared to the yield on an annual-pay bond, the following formula can be used:

$$\text{yield on an annual-pay basis} = \left[\left(1 + \frac{\text{yield on a bond-equivalent basis}}{2} \right)^2 - 1 \right]$$

By dividing the yield on a bond-equivalent basis by 2 in the above expression, the semiannual yield is computed. The semiannual yield is then compounded to get the yield on an annual-pay basis.

For example, suppose that the yield of a U.S. bond issue quoted on a bond-equivalent basis is 6%. The yield to maturity on an annual-pay basis would be:

$$[(1.03)^2 - 1] = 6.09\%$$

The yield on an annual-pay basis is always greater than the yield on a bond-equivalent basis because of compounding.

C. Yield to Call

When a bond is callable, the practice has been to calculate a yield to call as well as a yield to maturity. A callable bond may have a call schedule.⁴ The yield to call assumes the issuer will call a bond on some assumed call date and that the call price is the price specified in the call schedule. Typically, investors calculate a yield to first call or yield to next call, a yield to first par call, and a yield to refunding. The **yield to first call** is computed for an issue that is not currently callable, while the **yield to next call** is computed for an issue that is currently callable.

Yield to refunding is used when bonds are currently callable but have some restrictions on the source of funds used to buy back the debt when a call is exercised. Namely, if a debt issue contains some refunding protection, bonds cannot be called for a certain period of time with the proceeds of other debt issues sold at a lower cost of money. As a result, the bondholder is afforded some protection if interest rates decline and the issuer can obtain lower-cost funds to pay off the debt. It should be stressed that the bonds can be called with funds derived from other sources (e.g., cash on hand) during the refunded-protected period. The refunding date is the first date the bond can be called using lower-cost debt.

The procedure for calculating any yield to call measure is the same as for any yield to maturity calculation: determine the interest rate that will make the present value of the expected cash flows equal to the price plus accrued interest. In the case of yield to first call, the expected cash flows are the coupon payments to the first call date and the call price. For the **yield to first par call**, the expected cash flows are the coupon payments to the first date at which the issuer can call the bond at par and the par value. For the yield to refunding, the expected cash flows are the coupon payments to the first refunding date and the call price at the first refunding date.

To illustrate the computation, consider a 7% 8-year bond with a maturity value of \$100 selling for \$106.36. Suppose that the first call date is three years from now and the call price is \$103. The cash flows for this bond if it is called in three years are (1) 6 coupon payments of \$3.50 every six months and (2) \$103 in six 6-month periods from now.

The present value for several semiannual interest rates is shown in Exhibit 2. Since a semiannual interest rate of 2.8% makes the present value of the cash flows equal to the price, 2.8% is the yield to first call. Therefore, the yield to first call on a bond-equivalent basis is 5.6%.

⁴A call schedule shows the call price that the issuer must pay based on the date when the issue is called. An example of a call schedule is provided in Chapter 1.

EXHIBIT 2 Yield to Call for an 8-year 7% Coupon Bond with a Maturity Value of \$100. First Call Date Is the End of Year 3, and Call Price of \$103.

Annual interest rate (%)	Semiannual interest rate (%)	Present value of 6 payments of \$3.5	Present value of \$103 6 periods from now	Present value of cash flows
5.0	2.5	\$19.28	\$88.82	\$108.10
5.2	2.6	19.21	88.30	107.51
5.4	2.7	19.15	87.78	106.93
5.6	2.8	19.09	87.27	106.36

For our 7% 8-year callable bond, suppose that the first par call date is 5 years from now. The cash flows for computing the first par call are then: (1) a total 10 coupon payments of \$3.50 each paid every six months and (2) \$100 in ten 6-month periods. The yield to par call is 5.53%. Let's verify that this is the case. The semiannual yield is 2.765% (one half of 5.53%). The present value of the 10 coupon payments of \$3.50 every six months when discounted at 2.765% is \$30.22. The present value of \$100 (the call price of par) at the end of five years (10 semiannual periods) is \$76.13. The present value of the cash flow is then \$106.35 ($= \$30.22 + \$76.13$). Since the price of the bond is \$106.36 and since using a yield of 5.53% produces a value for this callable bond that differs from \$106.36 by only 1 penny, 5.53% is the yield to first par call.

Let's take a closer look at the yield to call as a measure of the potential return of a security. The yield to call considers all three sources of potential return from owning a bond. However, as in the case of the yield to maturity, it assumes that all cash flows can be reinvested at the yield to call until the assumed call date. As we just demonstrated, this assumption may be inappropriate. Moreover, the yield to call assumes that

Assumption 1: the investor will hold the bond to the assumed call date

Assumption 2: the issuer will call the bond on that date

These assumptions underlying the yield to call are unrealistic. Moreover, comparison of different yields to call with the yield to maturity are meaningless because the cash flows stop at the assumed call date. For example, consider two bonds, M and N. Suppose that the yield to maturity for bond M, a 5-year noncallable bond, is 7.5% while for bond N the yield to call, assuming the bond will be called in three years, is 7.8%. Which bond is better for an investor with a 5-year investment horizon? It's not possible to tell from the yields cited. If the investor intends to hold the bond for five years and the issuer calls bond N after three years, the total dollar return that will be available at the end of five years will depend on the interest rate that can be earned from investing funds from the call date to the end of the investment horizon.

D. Yield to Put

When a bond is puttable, the yield to the first put date is calculated. The yield to put is the interest rate that will make the present value of the cash flows to the first put date equal to the price plus accrued interest. As with all yield measures (except the current yield), yield to put assumes that any interim coupon payments can be reinvested at the yield calculated. Moreover, the yield to put assumes that the bond will be put on the first put date.

For example, suppose that a 6.2% coupon bond maturing in 8 years is puttable at par in 3 years. The price of this bond is \$102.19. The cash flows for this bond if it is put in three

years are: (1) a total of 6 coupon payments of \$3.10 each paid every six months and (2) the \$100 put price in six 6-month periods from now. The semiannual interest rate that will make the present value of the cash flows equal to the price of \$102.19 is 2.7%. Therefore, 2.7% is the semiannual yield to put and 5.4% is the yield to put on a bond equivalent basis.

E. Yield to Worst

A yield can be calculated for every possible call date and put date. In addition, a yield to maturity can be calculated. The lowest of all these possible yields is called the **yield to worst**. For example, suppose that there are only four possible call dates for a callable bond, that the yield to call assuming each possible call date is 6%, 6.2%, 5.8%, and 5.7%, and that the yield to maturity is 7.5%. Then the yield to worst is the minimum of these yields, 5.7% in our example.

The yield to worst measure holds little meaning as a measure of potential return. It supposedly states that this is the worst possible yield that the investor will realize. However, as we have noted about any yield measure, it does not identify the potential return over some investment horizon. Moreover, the yield to worst does not recognize that each yield calculation used in determining the yield to worst has different exposures to reinvestment risk.

F. Cash Flow Yield

Mortgage-backed securities and asset-backed securities are backed by a pool of loans or receivables. The cash flows for these securities include principal payment as well as interest. The complication that arises is that the individual borrowers whose loans make up the pool typically can prepay their loan in whole or in part prior to the scheduled principal payment dates. Because of principal prepayments, in order to project cash flows it is necessary to make an assumption about the rate at which principal prepayments will occur. This rate is called the **prepayment rate** or **prepayment speed**.

Given cash flows based on an assumed prepayment rate, a yield can be calculated. The yield is the interest rate that will make the present value of the projected cash flows equal to the price plus accrued interest. The yield calculated is commonly referred to as a **cash flow yield**.⁵

1. Bond-Equivalent Yield Typically, the cash flows for mortgage-backed and asset-backed securities are monthly. Therefore the interest rate that will make the present value of projected principal and interest payments equal to the market price plus accrued interest is a monthly rate. The monthly yield is then annualized as follows.

First, the semiannual effective yield is computed from the monthly yield by compounding it for six months as follows:

$$\text{effective semiannual yield} = (1 + \text{monthly yield})^6 - 1$$

⁵Some firms such as Prudential Securities refer to this yield as yield to maturity rather than cash flow yield.

Next, the effective semiannual yield is doubled to get the annual cash flow yield on a bond-equivalent basis. That is,

$$\begin{aligned}\text{cash flow yield} &= 2 \times \text{effective semiannual yield} \\ &= 2[(1 + \text{monthly yield})^6 - 1]\end{aligned}$$

For example, if the monthly yield is 0.5%, then:

$$\text{cash flow yield on a bond-equivalent basis} = 2[(1.005)^6 - 1] = 6.08\%$$

The calculation of the cash flow yield may seem strange because it first requires the computing of an effective semiannual yield given the monthly yield and then doubling. This is simply a market convention. Of course, the student of the bond market can always ask the same two questions as with the yield to maturity: Why it is done? Isn't it better to just compound the monthly yield to get an effective annual yield? The answers are the same as given earlier for the yield to maturity. Moreover, as we will see next, this is the least of our problems in using a cash flow yield measure for an asset-backed and mortgage-backed security.

2. Limitations of Cash Flow Yield As we have noted, the yield to maturity has two shortcomings as a measure of a bond's potential return: (1) it is assumed that the coupon payments can be reinvested at a rate equal to the yield to maturity and (2) it is assumed that the bond is held to maturity. These shortcomings are equally present in application of the cash flow yield measure: (1) the projected cash flows are assumed to be reinvested at the cash flow yield and (2) the mortgage-backed or asset-backed security is assumed to be held until the final payoff of all the loans, based on some prepayment assumption. The significance of reinvestment risk, the risk that the cash flows will be reinvested at a rate less than the cash flow yield, is particularly important for mortgage-backed and asset-backed securities since payments are typically monthly and include principal payments (scheduled and prepaid), and interest. Moreover, the cash flow yield is dependent on realizing of the projected cash flows according to some prepayment rate. If actual prepayments differ significantly from the prepayment rate assumed, the cash flow yield will not be realized.

G. Spread/Margin Measures for Floating-Rate Securities

The coupon rate for a floating-rate security (or floater) changes periodically according to a reference rate (such as LIBOR or a Treasury rate). Since the future value for the reference rate is unknown, it is not possible to determine the cash flows. This means that a yield to maturity cannot be calculated. Instead, "margin" measures are computed. Margin is simply some spread above the floater's reference rate.

Several spread or margin measures are routinely used to evaluate floaters. Two margin measures commonly used are spread for life and discount margin.⁶

⁶For a discussion of other traditional measures, see Chapter 3 in Frank J. Fabozzi and Steven V. Mann, *Floating Rate Securities* (New Hope, PA; Frank J. Fabozzi Associates, 2000).

1. **Spread for Life** When a floater is selling at a premium/discount to par, investors consider the premium or discount as an additional source of dollar return. **Spread for life** (also called **simple margin**) is a measure of potential return that accounts for the accretion (amortization) of the discount (premium) as well as the constant quoted margin over the security's remaining life. Spread for life (in basis points) is calculated using the following formula:

$$\text{Spread for life} = \left[\frac{100(100 - \text{Price})}{\text{Maturity}} + \text{Quoted margin} \right] \times \left(\frac{100}{\text{Price}} \right)$$

where

Price = market price per \$100 of par value

Maturity = number of years to maturity

Quoted margin = quoted margin in the coupon reset formula measured in basis points

For example, suppose that a floater with a quoted margin of 80 basis points is selling for 99.3098 and matures in 6 years. Then,

$$\text{Price} = 99.3098$$

$$\text{Maturity} = 6$$

$$\text{Quoted margin} = 80$$

$$\begin{aligned} \text{Spread for life} &= \left[\frac{100(100 - 99.3098)}{6} + 80 \right] \times \left(\frac{100}{99.3098} \right) \\ &= 92.14 \text{ basis points} \end{aligned}$$

The limitations of the spread for life are that it considers only the accretion/amortization of the discount/premium over the floater's remaining term to maturity and does not consider the level of the coupon rate or the time value of money.

2. **Discount Margin** **Discount margin** estimates the average margin over the reference rate that the investor can expect to earn over the life of the security. The procedure for calculating the discount margin is as follows:

Step 1. Determine the cash flows assuming that the reference rate does *not* change over the life of the security.

Step 2. Select a margin.

Step 3. Discount the cash flows found in Step 1 by the current value of the reference rate plus the margin selected in Step 2.

Step 4. Compare the present value of the cash flows as calculated in Step 3 to the price plus accrued interest. If the present value is equal to the security's price plus accrued interest, the discount margin is the margin assumed in Step 2. If the present value is not equal to the security's price plus accrued interest, go back to Step 2 and try a different margin.

For a security selling at par, the discount margin is simply the quoted margin in the coupon reset formula.

To illustrate the calculation, suppose that the coupon reset formula for a 6-year floating-rate security selling for \$99.3098 is 6-month LIBOR plus 80 basis points. The coupon rate is reset every 6 months. Assume that the current value for the reference rate is 10%.

Exhibit 3 shows the calculation of the discount margin for this security. The second column shows the current value for 6-month LIBOR. The third column sets forth the cash flows for the security. The cash flow for the first 11 periods is equal to one-half the current 6-month LIBOR (5%) plus the semiannual quoted margin of 40 basis points multiplied by \$100. At the maturity date (i.e., period 12), the cash flow is \$5.4 plus the maturity value of \$100. The column headings of the last five columns show the assumed margin. The rows below the assumed margin show the present value of each cash flow. The last row gives the total present value of the cash flows.

For the five assumed margins, the present value is equal to the price of the floating-rate security (\$99.3098) when the assumed margin is 96 basis points. Therefore, the discount margin is 96 basis points. Notice that the discount margin is 80 basis points, the same as the quoted margin, when this security is selling at par.

There are two drawbacks of the discount margin as a measure of the potential return from investing in a floating-rate security. First, the measure assumes that the reference rate will not change over the life of the security. Second, if the floating-rate security has a cap or floor, this is not taken into consideration.

H. Yield on Treasury Bills

Treasury bills are zero-coupon instruments with a maturity of one year or less. The convention in the Treasury bill market is to calculate a bill's **yield on a discount basis**. This yield is determined by two variables:

1. the settlement price per \$1 of maturity value (denoted by p)
2. the number of days to maturity which is calculated as the number of days between the settlement date and the maturity date (denoted by N_{SM})

The yield on a discount basis (denoted by d) is calculated as follows:

$$d = (1 - p) \left(\frac{360}{N_{SM}} \right)$$

We will use two actual Treasury bills to illustrate the calculation of the yield on a discount basis assuming a settlement date in both cases of 8/6/97. The first bill has a maturity date of 1/8/98 and a price of 0.97769722. For this bill, the number of days from the settlement date to the maturity date, N_{SM} , is 155. Therefore, the yield on a discount basis is

$$d = (1 - 0.97769722) \left(\frac{360}{155} \right) = 5.18\%$$

For our second bill, the maturity date is 7/23/98 and the price is 0.9490075. Assuming a settlement date of 8/6/97, the number of days from the settlement date to the maturity date is 351. The yield on a discount basis for this bill is

$$d = (1 - 0.9490075) \left(\frac{360}{351} \right) = 5.23\%$$

EXHIBIT 3 Calculation of the Discount Margin for a Floating-Rate Security

Floating rate security:

Maturity = 6 years

Price = 99.3098

Coupon formula = LIBOR + 80 basis points

Reset every six months

Period	LIBOR (%)	Cash flow (\$)*	Present value (\$) at assumed margin of **				
			80 bp	84 bp	88 bp	96 bp	100 bp
1	10	5.4	5.1233	5.1224	5.1214	5.1195	5.1185
2	10	5.4	4.8609	4.8590	4.8572	4.8535	4.8516
3	10	5.4	4.6118	4.6092	4.6066	4.6013	4.5987
4	10	5.4	4.3755	4.3722	4.3689	4.3623	4.3590
5	10	5.4	4.1514	4.1474	4.1435	4.1356	4.1317
6	10	5.4	3.9387	3.9342	3.9297	3.9208	3.9163
7	10	5.4	3.7369	3.7319	3.7270	3.7171	3.7122
8	10	5.4	3.5454	3.5401	3.5347	3.5240	3.5186
9	10	5.4	3.3638	3.3580	3.3523	3.3409	3.3352
10	10	5.4	3.1914	3.1854	3.1794	3.1673	3.1613
11	10	5.4	3.0279	3.0216	3.0153	3.0028	2.9965
12	10	105.4	56.0729	55.9454	55.8182	55.5647	55.4385
Present value			100.0000	99.8269	99.6541	99.3098	99.1381

*For periods 1–11: cash flow = \$100 (0.5) (LIBOR + assumed margin)

For period 12: cash flow = \$100 (0.5) (LIBOR + assumed margin) + 100

**The discount rate is found as follows. To LIBOR of 10%, the assumed margin is added. Thus, for an 80 basis point assumed margin, the discount rate is 10.80%. This is an annual discount rate on a bond-equivalent basis. The semiannual discount rate is then half this amount, 5.4%. It is this discount rate that is used to compute the present value of the cash flows for an assumed margin of 80 basis points.

Given the yield on a discount basis, the price of a bill (per \$1 of maturity value) is computed as follows:

$$p = 1 - d(N_{SM}/360)$$

For the 155-day bill selling for a yield on a discount basis of 5.18%, the price per \$1 of maturity value is

$$p = 1 - 0.0518 (155/360) = 0.97769722$$

For the 351-day bill selling for a yield on a discount basis of 5.23%, the price per \$1 of maturity value is

$$p = 1 - 0.0523 (351/360) = 0.9490075$$

The quoted yield on a discount basis is not a meaningful measure of the return from holding a Treasury bill for two reasons. First, the measure is based on a maturity value investment rather than on the actual dollar amount invested. Second, the yield is annualized according to a 360-day year rather than a 365-day year, making it difficult to compare yields on Treasury bills with Treasury notes and bonds which pay interest based on the actual number of days in a year. The use of 360 days for a year is a convention for money market instruments. Despite its shortcomings as a measure of return, this is the method dealers have adopted to quote Treasury bills.

Market participants recognize this limitation of yield on a discount basis and consequently make adjustments to make the yield quoted on a Treasury bill comparable to that on a Treasury coupon security. For investors who want to compare the yield on Treasury bills to that of other money market instruments (i.e., debt obligations with a maturity that does not exceed one year), there is a formula to convert the yield on a discount basis to that of a money market yield. The key point is that while the convention is to quote the yield on a Treasury bill in terms of a yield on a discount basis, no one uses that yield measure other than to compute the price given the quoted yield.

IV. THEORETICAL SPOT RATES

The theoretical spot rates for Treasury securities represent the appropriate set of interest rates that should be used to value default-free cash flows. A default-free theoretical spot rate curve can be constructed from the observed Treasury yield curve. There are several approaches that are used in practice. The approach that we describe below for creating a theoretical spot rate curve is called **bootstrapping**. (The bootstrapping method described here is also used in constructing a theoretical spot rate curve for LIBOR.)

A. Bootstrapping

Bootstrapping begins with the yield for the on-the-run Treasury issues because there is no credit risk and no liquidity risk. In practice, however, there is a problem of obtaining a sufficient number of data points for constructing the U.S. Treasury yield curve. In the United States, the U.S. Department of the Treasury currently issues 3-month and 6-month Treasury bills and

2-year, 5-year, and 10-year Treasury notes. Treasury bills are zero-coupon instruments and Treasury notes are coupon-paying instruments. Hence, there are not many data points from which to construct a Treasury yield curve, particularly after two years. At one time, the U.S. Treasury issued 30-year Treasury bonds. Since the Treasury no longer issues 30-year bonds, market participants currently use the last issued Treasury bond (which has a maturity less than 30 years) to estimate the 30-year yield. The 2-year, 5-year, and 10-year Treasury notes and an estimate of the 30-year Treasury bond are used to construct the Treasury yield curve.

On September 5, 2003, Lehman Brothers reported the following values for these four yields:

2 year	1.71%
5 year	3.25%
10 year	4.35%
30 year	5.21%

To fill in the yield for the 25 missing whole year maturities (3 year, 4 year, 6 year, 7 year, 8 year, 9 year, 11 year, and so on to the 29-year maturity), the yield for the 25 whole year maturities are interpolated from the yield on the surrounding maturities. The simplest interpolation, and the one most commonly used in practice, is simple linear interpolation.

For example, suppose that we want to fill in the gap for each one year of maturity. To determine the amount to add to the on-the-run Treasury yield as we go from the lower maturity to the higher maturity, the following formula is used:

$$\frac{\text{Yield at higher maturity} - \text{Yield at lower maturity}}{\text{Number of years between two observed maturity points}}$$

The estimated on-the-run yield for all intermediate whole-year maturities is found by adding the amount computed from the above formula to the yield at the lower maturity.

For example, using the September 5, 2003 yields, the 5-year yield of 3.25% and the 10-year yield of 4.35% are used to obtain the interpolated 6-year, 7-year, 8-year, and 9-year yields by first calculating:

$$\frac{4.35\% - 3.25\%}{5} = 0.22\%$$

Then,

$$\text{interpolated 6-year yield} = 3.25\% + 0.22\% = 3.47\%$$

$$\text{interpolated 7-year yield} = 3.47\% + 0.22\% = 3.69\%$$

$$\text{interpolated 8-year yield} = 3.69\% + 0.22\% = 3.91\%$$

$$\text{interpolated 9-year yield} = 3.91\% + 0.22\% = 4.13\%$$

Thus, when market participants talk about a yield on the Treasury yield curve that is not one of the on-the-run maturities—for example, the 8-year yield—it is only an approximation. Notice that there is a large gap between maturity points. This may result in misleading yields

for the interim maturity points when estimated using the linear interpolation method, a point that we return to later in this chapter.

To illustrate bootstrapping, we will use the Treasury yields shown in Exhibit 4 for maturities up to 10 years using 6-month periods.⁷ Thus, there are 20 Treasury yields shown. The yields shown are assumed to have been interpolated from the on-the-run Treasury issues. Exhibit 5 shows the Treasury yield curve based on the yields shown in Exhibit 4. Our objective is to show how the values in the last column of Exhibit 4 (labeled “Spot rate”) are obtained.

Throughout the analysis and illustrations to come, it is important to remember that the basic principle is the value of the Treasury coupon security should be equal to the value of the package of zero-coupon Treasury securities that duplicates the coupon bond’s cash flows. We saw this in Chapter 5 when we discussed arbitrage-free valuation.

Consider the 6-month and 1-year Treasury securities in Exhibit 4. As we explained in Chapter 5, these two securities are called Treasury bills and they are issued as zero-coupon instruments. Therefore, the annualized yield (not the discount yield) of 3.00% for the 6-month

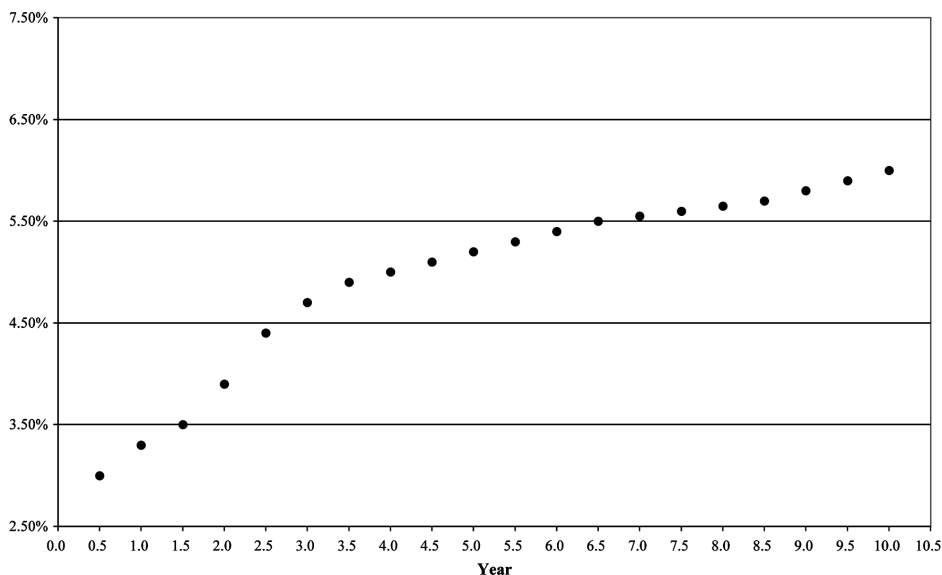
EXHIBIT 4 Hypothetical Treasury Yields (Interpolated)

Period	Years	Annual par yield to maturity (BEY) (%)*	Price	Spot rate (BEY) (%)*
1	0.5	3.00	—	3.0000
2	1.0	3.30	—	3.3000
3	1.5	3.50	100.00	3.5053
4	2.0	3.90	100.00	3.9164
5	2.5	4.40	100.00	4.4376
6	3.0	4.70	100.00	4.7520
7	3.5	4.90	100.00	4.9622
8	4.0	5.00	100.00	5.0650
9	4.5	5.10	100.00	5.1701
10	5.0	5.20	100.00	5.2772
11	5.5	5.30	100.00	5.3864
12	6.0	5.40	100.00	5.4976
13	6.5	5.50	100.00	5.6108
14	7.0	5.55	100.00	5.6643
15	7.5	5.60	100.00	5.7193
16	8.0	5.65	100.00	5.7755
17	8.5	5.70	100.00	5.8331
18	9.0	5.80	100.00	5.9584
19	9.5	5.90	100.00	6.0863
20	10.0	6.00	100.00	6.2169

*The yield to maturity and the spot rate are annual rates. They are reported as bond-equivalent yields. To obtain the semiannual yield or rate, one half the annual yield or annual rate is used.

⁷Two points should be noted about the yields reported in Exhibit 4. First, the yields are unrelated to our earlier Treasury yields on September 5, 2003 that we used to show how to calculate the yield on interim maturities using linear interpolation. Second, the Treasury yields in our illustration after the first year are all shown at par value. Hence the Treasury yield curve in Exhibit 4 is called a *par yield curve*.

EXHIBIT 5 Treasury Par Yield Curve



Treasury security is equal to the 6-month spot rate.⁸ Similarly, for the 1-year Treasury security, the cited yield of 3.30% is the 1-year spot rate. Given these two spot rates, we can compute the spot rate for a theoretical 1.5-year zero-coupon Treasury. The value of a theoretical 1.5-year Treasury should equal the present value of the three cash flows from the 1.5-year coupon Treasury, where the yield used for discounting is the spot rate corresponding to the time of receipt of each six-month cash flow. Since all the coupon bonds are selling at par, as explained in the previous section, the yield to maturity for each bond is the coupon rate. Using \$100 par, the cash flows for the 1.5-year coupon Treasury are:

$$\begin{array}{llll}
 0.5 \text{ year} & 0.035 \times \$100 \times 0.5 & = & \$1.75 \\
 1.0 \text{ year} & 0.035 \times \$100 \times 0.5 & = & \$1.75 \\
 1.5 \text{ years} & 0.035 \times \$100 \times 0.5 + 100 & = & \$101.75
 \end{array}$$

The present value of the cash flows is then:

$$\frac{1.75}{(1 + z_1)^1} + \frac{1.75}{(1 + z_2)^2} + \frac{101.75}{(1 + z_3)^3}$$

where

z_1 = one-half the annualized 6-month theoretical spot rate

z_2 = one-half the 1-year theoretical spot rate

z_3 = one-half the 1.5-year theoretical spot rate

⁸We will assume that the annualized yield for the Treasury bill is computed on a bond-equivalent basis. Earlier in this chapter, we saw how the yield on a Treasury bill is quoted. The quoted yield can be converted into a bond-equivalent yield; we assume this has already been done in Exhibit 4.

Since the 6-month spot rate is 3% and the 1-year spot rate is 3.30%, we know that:

$$z_1 = 0.0150 \text{ and } z_2 = 0.0165$$

We can compute the present value of the 1.5-year coupon Treasury security as:

$$\frac{1.75}{(1+z_1)^1} + \frac{1.75}{(1+z_2)^2} + \frac{101.75}{(1+z_3)^3} = \frac{1.75}{(1.015)^1} + \frac{1.75}{(1.0165)^2} + \frac{101.75}{(1+z_3)^3}$$

Since the price of the 1.5-year coupon Treasury security is par value (see Exhibit 4), the following relationship must hold:⁹

$$\frac{1.75}{(1.015)^1} + \frac{1.75}{(1.0165)^2} + \frac{101.75}{(1+z_3)^3} = 100$$

We can solve for the theoretical 1.5-year spot rate as follows:

$$\begin{aligned} 1.7241 + 1.6936 + \frac{101.75}{(1+z_3)^3} &= 100 \\ \frac{101.75}{(1+z_3)^3} &= 96.5822 \\ (1+z_3)^3 &= \frac{101.75}{96.5822} \\ z_3 &= 0.0175265 = 1.7527\% \end{aligned}$$

Doubling this yield, we obtain the bond-equivalent yield of 3.5053%, which is the theoretical 1.5-year spot rate. That rate is the rate that the market would apply to a 1.5-year zero-coupon Treasury security if, in fact, such a security existed. In other words, all Treasury cash flows to be received 1.5 years from now should be valued (i.e., discounted) at 3.5053%.

Given the theoretical 1.5-year spot rate, we can obtain the theoretical 2-year spot rate. The cash flows for the 2-year coupon Treasury in Exhibit 3 are:

0.5 year	$0.039 \times \$100 \times 0.5$	=	\$1.95
1.0 year	$0.039 \times \$100 \times 0.5$	=	\$1.95
1.5 years	$0.039 \times \$100 \times 0.5$	=	\$1.95
2.0 years	$0.039 \times \$100 \times 0.5 + 100$	=	\$101.95

The present value of the cash flows is then:

$$\frac{1.95}{(1+z_1)^1} + \frac{1.95}{(1+z_2)^2} + \frac{1.95}{(1+z_3)^3} + \frac{101.95}{(1+z_4)^4}$$

where z_4 = one-half the 2-year theoretical spot rate.

⁹If we had not been working with a par yield curve, the equation would have been set equal to whatever the market price for the 1.5-year issue is.

Since the 6-month spot rate, 1-year spot rate, and 1.5-year spot rate are 3.00%, 3.30%, and 3.5053%, respectively, then:

$$z_1 = 0.0150 \quad z_2 = 0.0165 \quad z_3 = 0.017527$$

Therefore, the present value of the 2-year coupon Treasury security is:

$$\frac{1.95}{(1.0150)^1} + \frac{1.95}{(1.0165)^2} + \frac{1.95}{(1.07527)^3} + \frac{101.95}{(1 + z_4)^4}$$

Since the price of the 2-year coupon Treasury security is par, the following relationship must hold:

$$\frac{1.95}{(1.0150)^1} + \frac{1.95}{(1.0165)^2} + \frac{1.95}{(1.017527)^3} + \frac{101.95}{(1 + z_4)^4} = 100$$

We can solve for the theoretical 2-year spot rate as follows:

$$\begin{aligned} \frac{101.95}{(1 + z_4)^4} &= 94.3407 \\ (1 + z_4)^4 &= \frac{101.95}{94.3407} \\ z_4 &= 0.019582 = 1.9582\% \end{aligned}$$

Doubling this yield, we obtain the theoretical 2-year spot rate bond-equivalent yield of 3.9164%.

One can follow this approach sequentially to derive the theoretical 2.5-year spot rate from the calculated values of z_1 , z_2 , z_3 , and z_4 (the 6-month-, 1-year-, 1.5-year-, and 2-year rates), and the price and coupon of the 2.5-year bond in Exhibit 4. Further, one could derive theoretical spot rates for the remaining 15 half-yearly rates.

The spot rates thus obtained are shown in the last column of Exhibit 4. They represent the term structure of default-free spot rate for maturities up to 10 years at the particular time to which the bond price quotations refer. In fact, it is the default-free spot rates shown in Exhibit 4 that were used in our illustrations in the previous chapter.

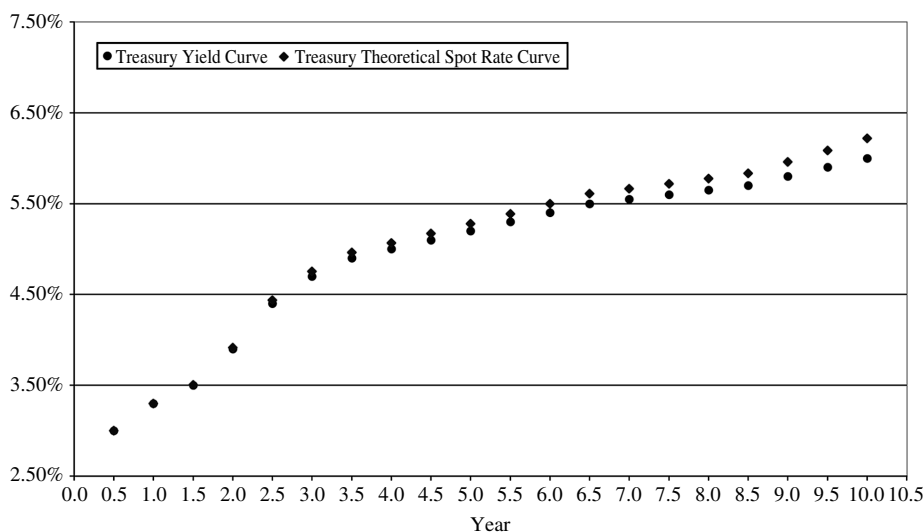
Exhibit 6 shows a plot of the spot rates. The graph is called the **theoretical spot rate curve**. Also shown on Exhibit 6 is a plot of the par yield curve from Exhibit 5. Notice that the theoretical spot rate curve lies above the par yield curve. This will always be the case when the par yield curve is upward sloping. When the par yield curve is downward sloping, the theoretical spot rate curve will lie below the par yield curve.

B. Yield Spread Measures Relative to a Spot Rate Curve

Traditional analysis of the yield spread for a non-Treasury bond involves calculating the difference between the bond's yield and the yield to maturity of a benchmark Treasury coupon security. The latter is obtained from the Treasury yield curve. For example, consider the following 10-year bonds:

Issue	Coupon	Price	Yield to maturity
Treasury	6%	100.00	6.00%
Non-Treasury	8%	104.19	7.40%

EXHIBIT 6 Theoretical Spot Rate Curve and Treasury Yield Curve



The yield spread for these two bonds as traditionally computed is 140 basis points (7.4% minus 6%). We have referred to this traditional yield spread as the **nominal spread**.

Exhibit 7 shows the Treasury yield curve from Exhibit 5. The nominal spread of 140 basis points is the difference between the 7.4% yield to maturity for the 10-year non-Treasury security and the yield on the 10-year Treasury, 6%.

What is the nominal spread measuring? It is measuring the compensation for the additional credit risk, option risk (i.e., the risk associated with embedded options),¹⁰ and liquidity risk an investor is exposed to by investing in a non-Treasury security rather than a Treasury security with the same maturity.

The drawbacks of the nominal spread measure are

1. for both bonds, the yield fails to take into consideration the term structure of spot rates and
2. in the case of callable and/or putable bonds, expected interest rate volatility may alter the cash flows of the non-Treasury bond.

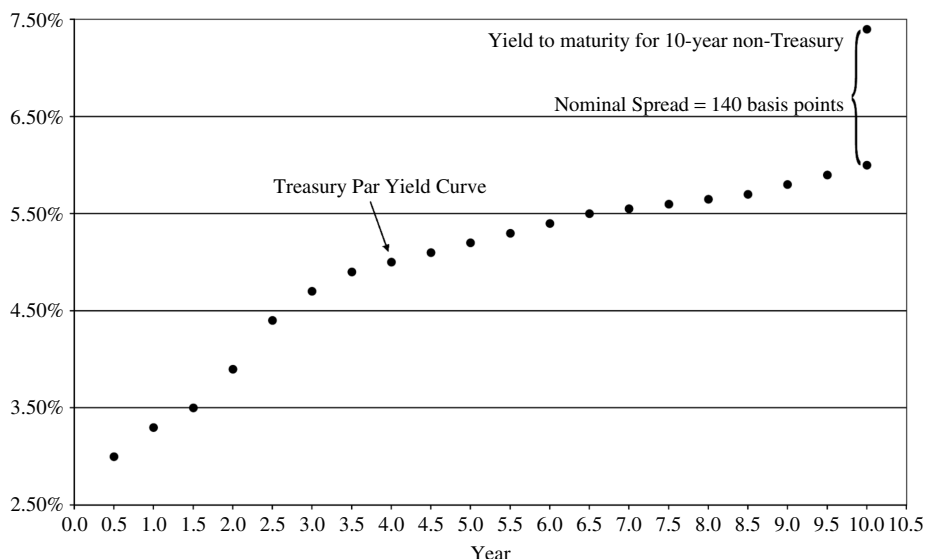
Let's examine each of the drawbacks and alternative spread measures for handling them.

1. Zero-Volatility Spread The **zero-volatility spread** or **Z-spread** is a measure of the spread that the investor would realize over the entire Treasury spot rate curve if the bond is held to maturity. It is not a spread off one point on the Treasury yield curve, as is the nominal spread. The Z-spread, also called the **static spread**, is calculated as the spread that will make the present value of the cash flows from the non-Treasury bond, when discounted at the Treasury spot rate plus the spread, equal to the non-Treasury bond's price. A trial-and-error procedure is required to determine the Z-spread.

To illustrate how this is done, let's use the non-Treasury bond in our previous illustration and the Treasury spot rates in Exhibit 4. These spot rates are repeated in Exhibit 8. The

¹⁰Option risk includes prepayment and call risk.

EXHIBIT 7 Illustration of the Nominal Spread



third column in Exhibit 8 shows the cash flows for the 8% 10-year non-Treasury issue. The goal is to determine the spread that, when added to all the Treasury spot rates, will produce a present value for the cash flows of the non-Treasury bond equal to its market price of \$104.19.

Suppose we select a spread of 100 basis points. To each Treasury spot rate shown in the fourth column of Exhibit 8, 100 basis points is added. So, for example, the 5-year (period 10) spot rate is 6.2772% (5.2772% plus 1%). The spot rate plus 100 basis points is then used to calculate the present values as shown in the fifth column. The total present value of the fifth column is \$107.5414. Because the present value is not equal to the non-Treasury issue's price (\$104.19), the Z-spread is not 100 basis points. If a spread of 125 basis points is tried, it can be seen from the next-to-the-last column of Exhibit 8 that the present value is \$105.7165; again, because this is not equal to the non-Treasury issue's price, 125 basis points is not the Z-spread. The last column of Exhibit 8 shows the present value when a 146 basis point spread is tried. The present value is equal to the non-Treasury issue's price. Therefore 146 basis points is the Z-spread, compared to the nominal spread of 140 basis points.

A graphical presentation of the Z-spread is shown in Exhibit 9. Since the benchmark for computing the Z-spread is the theoretical spot rate curve, that curve is shown in the exhibit. Above each yield at each maturity on the theoretical spot rate curve is a yield that is 146 basis points higher. This is the Z-spread. It is a spread over the entire spot rate curve.

What should be clear is that the difference between the nominal spread and the Z-spread is the benchmark that is being used: the nominal spread is a spread off of one point on the Treasury yield curve (see Exhibit 7) while the Z-spread is a spread over the entire theoretical Treasury spot rate curve.

What does the Z-spread represent for this non-Treasury security? Since the Z-spread is measured relative to the Treasury spot rate curve, it represents a spread to compensate for the non-Treasury security's credit risk, liquidity risk, and any option risk (i.e., the risks associated with any embedded options).

EXHIBIT 8 Determining Z-Spread for an 8% Coupon, 10-Year Non-Treasury Issue Selling at \$104.19 to Yield 7.4%

Period	Years	Cash flow (\$)	Spot rate (%) [*]	Present value (\$) assuming a spread of ^{**}		
				100 bp	125 bp	146 bp
1	0.5	4.00	3.0000	3.9216	3.9168	3.9127
2	1.0	4.00	3.3000	3.8334	3.8240	3.8162
3	1.5	4.00	3.5053	3.7414	3.7277	3.7163
4	2.0	4.00	3.9164	3.6297	3.6121	3.5973
5	2.5	4.00	4.4376	3.4979	3.4767	3.4590
6	3.0	4.00	4.7520	3.3742	3.3497	3.3293
7	3.5	4.00	4.9622	3.2565	3.2290	3.2061
8	4.0	4.00	5.0650	3.1497	3.1193	3.0940
9	4.5	4.00	5.1701	3.0430	3.0100	2.9825
10	5.0	4.00	5.2772	2.9366	2.9013	2.8719
11	5.5	4.00	5.3864	2.8307	2.7933	2.7622
12	6.0	4.00	5.4976	2.7255	2.6862	2.6536
13	6.5	4.00	5.6108	2.6210	2.5801	2.5463
14	7.0	4.00	5.6643	2.5279	2.4855	2.4504
15	7.5	4.00	5.7193	2.4367	2.3929	2.3568
16	8.0	4.00	5.7755	2.3472	2.3023	2.2652
17	8.5	4.00	5.8331	2.2596	2.2137	2.1758
18	9.0	4.00	5.9584	2.1612	2.1148	2.0766
19	9.5	4.00	6.0863	2.0642	2.0174	1.9790
20	10.0	104.00	6.2169	51.1835	49.9638	48.9632
Total				107.5416	105.7165	104.2146

* The spot rate is an annual rate.

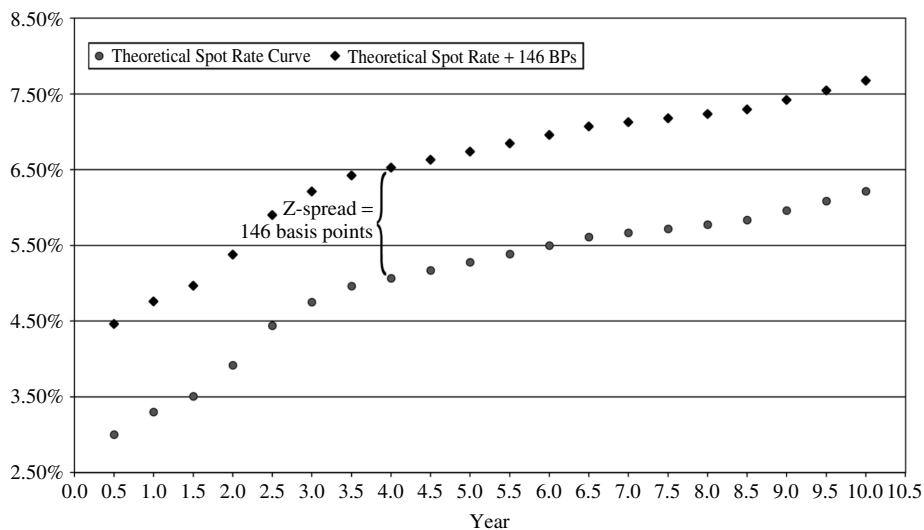
** The discount rate used to compute the present value of each cash flow in the third column is found by adding the assumed spread to the spot rate and then dividing by 2. For example, for period 4 the spot rate is 3.9164%. If the assumed spread is 100 basis points, then 100 basis points is added to 3.9164% to give 4.9164%. Dividing this rate by 2 gives the semiannual rate of 2.4582%. The present value is then:

$$\frac{\text{cash flow in period } t}{(1.024582)^t}$$

a. Divergence Between Z-Spread and Nominal Spread Typically, for standard coupon-paying bonds with a bullet maturity (i.e., a single payment of principal) the Z-spread and the nominal spread will not differ significantly. In our example, it is only 6 basis points. In general terms, the divergence (i.e., amount of difference) is a function of (1) the shape of the term structure of interest rates and (2) the characteristics of the security (i.e., coupon rate, time to maturity, and type of principal payment provision—non-amortizing versus amortizing).

For short-term issues, there is little divergence. The main factor causing any difference is the shape of the Treasury spot rate curve. The steeper the spot rate curve, the greater the difference. To illustrate this, consider the two spot rate curves shown in Exhibit 10. The yield for the longest maturity of both spot rate curves is 6%. The first curve is steeper than the one used in Exhibit 8; the second curve is flat, with the yield for all maturities equal to 6%. For our 8% 10-year non-Treasury issue, it can be shown that for the first spot rate curve in Exhibit 10 the Z-spread is 192 basis points. Thus, with this steeper spot rate curve, the difference between the Z-spread and the nominal spread is 52 basis points. For the flat curve the Z-spread is 140 basis points, the same as the nominal spread. This will always be the case because the nominal

EXHIBIT 9 Illustration of the Z Spread



spread assumes that the same yield is used to discount each cash flow and, with a flat yield curve, the same yield is being used to discount each flow. Thus, the nominal yield spread and the Z-spread will produce the same value for this security.

The difference between the Z-spread and the nominal spread is greater for issues in which the principal is repaid over time rather than only at maturity. Thus, the difference between the nominal spread and the Z-spread will be considerably greater for mortgage-backed and

EXHIBIT 10 Two Hypothetical Spot Rate Curves

Period	Years	Steep curve (%)	Flat curve (%)
1	0.5	2.00	6.00
2	1.0	2.40	6.00
3	1.5	2.80	6.00
4	2.0	2.90	6.00
5	2.5	3.00	6.00
6	3.0	3.10	6.00
7	3.5	3.30	6.00
8	4.0	3.80	6.00
9	4.5	3.90	6.00
10	5.0	4.20	6.00
11	5.5	4.40	6.00
12	6.0	4.50	6.00
13	6.5	4.60	6.00
14	7.0	4.70	6.00
15	7.5	4.90	6.00
16	8.0	5.00	6.00
17	8.5	5.30	6.00
18	9.0	5.70	6.00
19	9.5	5.80	6.00
20	10.0	6.00	6.00

asset-backed securities in a steep yield curve environment. We can see this intuitively if we think in terms of a 10-year zero-coupon bond and a 10-year amortizing security with equal semiannual cash flows (that includes interest and principal payment). The Z-spread for the zero-coupon bond will not be affected by the shape of the term structure but the amortizing security will be.

b. Z-Spread Relative to Any Benchmark In the same way that a Z-spread relative to a Treasury spot rate curve can be calculated, a Z-spread to any benchmark spot rate curve can be calculated. To illustrate, suppose that a hypothetical non-Treasury security with a coupon rate of 8% and a 10-year maturity is trading at \$105.5423. Assume that the *benchmark spot rate curve for this issuer* is the one given in Exhibit 10 of the previous chapter. The Z-spread relative to that issuer's benchmark spot rate curve is the spread that must be added to the spot rates shown in the next-to-last column of that exhibit that will make the present value of the cash flows equal to the market price. In our illustration, the Z-spread relative to this benchmark is 40 basis points.

What does the Z-spread mean when the benchmark is not the Treasury spot rate curve (i.e., default-free spot rate curve)? When the Treasury spot rate curve is the benchmark, we said that the Z-spread for a non-Treasury issue embodies credit risk, liquidity risk, and any option risk. When the benchmark is the spot rate curve for the issuer, the Z-spread is measuring the spread attributable to the liquidity risk of the issue and any option risk.

Thus, when a Z-spread is cited, it must be cited relative to some benchmark spot rate curve. This is necessary because it indicates the credit and sector risks that are being considered when the Z-spread was calculated. While Z-spreads are typically calculated using Treasury securities as the benchmark interest rates, this need not be the case. Vendors of analytical systems commonly allow the user to select a benchmark spot rate curve. Moreover, in non-U.S. markets, Treasury securities are typically not the benchmark. The key point is that an investor should always ask what benchmark was used to compute the Z-spread.

2. Option-Adjusted Spread The Z-spread seeks to measure the spread over a spot rate curve thus overcoming the first problem of the nominal spread that we cited earlier. Now let's look at the second shortcoming—failure to take future interest rate volatility into account which could change the cash flows for bonds with embedded options.

a. Valuation Models What investors seek to do is to buy undervalued securities (securities whose value is greater than their price). Before they can do this though, they need to know what the security is worth (i.e., a fair price to pay). A valuation model is designed to provide precisely this. If a model determines the fair price of a share of common stock is \$36 and the market price is currently \$24, then the stock is considered to be undervalued. If a bond is selling for less than its fair value, then it too is considered undervalued.

A valuation model need not stop here, however. Market participants find it more convenient to think about yield spread than about price differences. A valuation model can take this difference between the fair price and the market price and convert it into a yield spread measure. Instead of asking, "How much is this security undervalued?", the model can ask, "How much return will I earn in exchange for taking on these risks?"

The **option-adjusted spread** (OAS) was developed as a way of doing just this: taking the dollar difference between the fair price and market price and converting it into a yield spread measure. Thus, the OAS is used to reconcile the fair price (or value) to the market price by finding a return (spread) that will equate the two (using a trial and error procedure). This is

somewhat similar to what we did earlier when calculating yield to maturity, yield to call, etc., only in this case, we are calculating a spread (measured in basis points) rather than a percentage rate of return as we did then.

The OAS is model dependent. That is, the OAS computed depends on the valuation model used. In particular, OAS models differ considerably in how they forecast interest rate changes, leading to variations in the level of OAS. What are two of these key modeling differences?

- Interest rate volatility is a critical assumption. Specifically, the higher the interest rate volatility assumed, the lower the OAS. In comparing the OAS of dealer firms, it is important to check on the volatility assumption made.
- The OAS is a spread, but what is it a “spread” over? The OAS is a spread over the Treasury spot rate curve or the issuer’s benchmark used in the analysis. In the model, the spot rate curve is actually the result of a series of assumptions that allow for changes in interest rates. Again, different models yield different results.

Why is the spread referred to as “option adjusted”? Because the security’s embedded option can change the cash flows; the value of the security should take this change of cash flow into account. Note that the Z-spread doesn’t do this—it ignores the fact that interest rate changes can affect the cash flows. In essence, it assumes that interest rate volatility is zero. This is why the Z-spread is also referred to as the **zero-volatility OAS**.

b. Option Cost The implied cost of the option embedded in any security can be obtained by calculating the difference between the OAS at the assumed interest rate or yield volatility and the Z-spread. That is, since the Z-spread is just the sum of the OAS and option cost, i.e.,

$$\text{Z-spread} = \text{OAS} + \text{option cost}$$

it follows that:

$$\text{option cost} = \text{Z-spread} - \text{OAS}$$

The reason that the option cost is measured in this way is as follows. In an environment in which interest rates are assumed not to change, the investor would earn the Z-spread. When future interest rates are uncertain, the spread is different because of the embedded option(s); the OAS reflects the spread after adjusting for this option. Therefore, the option cost is the difference between the spread that would be earned in a static interest rate environment (the Z-spread, or equivalently, the zero-volatility OAS) and the spread after adjusting for the option (the OAS).

For callable bonds and most mortgage-backed and asset-backed securities, the option cost is positive. This is because the issuer’s ability to alter the cash flows will result in an OAS that is less than the Z-spread. In the case of a putable bond, the OAS is greater than the Z-spread so that the option cost is negative. This occurs because of the investor’s ability to alter the cash flows.

In general, when the option cost is positive, this means that the investor has sold an option to the issuer or borrower. This is true for callable bonds and most mortgage-backed and asset-backed securities. A negative value for the option cost means that the investor has purchased an option from the issuer or borrower. A putable bond is an example of this negative option cost. There are certain securities in the mortgage-backed securities market that also have an option cost that is negative.

c. Highlighting the Pitfalls of the Nominal Spread We can use the concepts presented in this chapter to highlight the pitfalls of the nominal spread. First, we can recast the relationship between the option cost, Z-spread, and OAS as follows:

$$\text{Z-spread} = \text{OAS} + \text{option cost}$$

Next, recall that the nominal spread and the Z-spread may not diverge significantly. Suppose that the nominal spread is approximately equal to the Z-spread. Then, we can substitute nominal spread for Z-spread in the previous relationship giving:

$$\text{nominal spread} \approx \text{OAS} + \text{option cost}$$

This relationship tells us that a high nominal spread could be hiding a high option cost. The option cost represents the portion of the spread that the investor has given to the issuer or borrower. Thus, while the nominal spread for a security that can be called or prepaid might be, say 200 basis points, the option cost may be 190 and the OAS only 10 basis points. But, an investor is only compensated for the OAS. An investor that relies on the nominal spread may not be adequately compensated for taking on the option risk associated with a security with an embedded option.

3. Summary of Spread Measures We have just described three spread measures:

- nominal spread
- zero-volatility spread
- option-adjusted spread

To understand different spread measures we ask two questions:

1. What is the benchmark for computing the spread? That is, what is the spread measured relative to?
2. What is the spread measuring?

The table below provides a summary showing for each of the three spread measures the benchmark and the risks for which the spread is compensating.

Spread measure	Benchmark	Reflects compensation for:
Nominal	Treasury yield curve	Credit risk, option risk, liquidity risk
Zero-volatility	Treasury spot rate curve	Credit risk, option risk, liquidity risk
Option-adjusted	Treasury spot rate curve	Credit risk, liquidity risk

V. FORWARD RATES

We have seen how a default-free theoretical spot rate curve can be extrapolated from the Treasury yield curve. Additional information useful to market participants can be extrapolated from the default-free theoretical spot rate curve: **forward rates**. Under certain assumptions described later, these rates can be viewed as the market's consensus of future interest rates.

Examples of forward rates that can be calculated from the default-free theoretical spot rate curve are the:

- 6-month forward rate six months from now
- 6-month forward rate three years from now
- 1-year forward rate one year from now
- 3-year forward rate two years from now
- 5-year forward rates three years from now

Since the forward rates are implicitly extrapolated from the default-free theoretical spot rate curve, these rates are sometimes referred to as **implied forward rates**. We begin by showing how to compute the 6-month forward rates. Then we explain how to compute any forward rate.

While we continue to use the Treasury yield curve in our illustrations, as noted earlier, a LIBOR spot rate curve can also be constructed using the bootstrapping methodology and forward rates for LIBOR can be obtained in the same manner as described below.

A. Deriving 6-Month Forward Rates

To illustrate the process of extrapolating 6-month forward rates, we will use the yield curve and corresponding spot rate curve from Exhibit 4. We will use a very simple arbitrage principle as we did earlier in this chapter to derive the spot rates. Specifically, if two investments have the same cash flows and have the same risk, they should have the same value.

Consider an investor who has a 1-year investment horizon and is faced with the following two alternatives:

- buy a 1-year Treasury bill, or
- buy a 6-month Treasury bill and, when it matures in six months, buy another 6-month Treasury bill.

The investor will be indifferent toward the two alternatives if they produce the same return over the 1-year investment horizon. The investor knows the spot rate on the 6-month Treasury bill and the 1-year Treasury bill. However, he does not know what yield will be on a 6-month Treasury bill purchased six months from now. That is, he does not know the 6-month forward rate six months from now. Given the spot rates for the 6-month Treasury bill and the 1-year Treasury bill, the forward rate on a 6-month Treasury bill is the rate that equalizes the dollar return between the two alternatives.

To see how that rate can be determined, suppose that an investor purchased a 6-month Treasury bill for $\$X$. At the end of six months, the value of this investment would be:

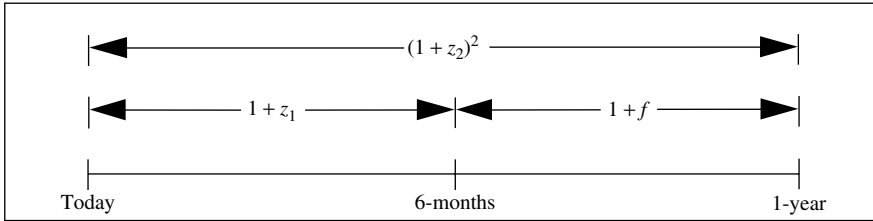
$$X(1 + z_1)$$

where z_1 is one-half the bond-equivalent yield (BEY) of the theoretical 6-month spot rate.

Let f represent one-half the forward rate (expressed as a BEY) on a 6-month Treasury bill available six months from now. If the investor were to rollover his investment by purchasing that bill at that time, then the future dollars available at the end of one year from the X investment would be:

$$X(1 + z_1)(1 + f)$$

EXHIBIT 11 Graphical Depiction of the Six-Month Forward Rate Six Months from Now



Now consider the alternative of investing in a 1-year Treasury bill. If we let z_2 represent one-half the BEY of the theoretical 1-year spot rate, then the future dollars available at the end of one year from the X investment would be:

$$X(1 + z_2)^2$$

The reason that the squared term appears is that the amount invested is being compounded for two periods. (Recall that each period is six months.)

The two choices are depicted in Exhibit 11. Now we are prepared to analyze the investor's choices and what this says about forward rates. The investor will be indifferent toward the two alternatives confronting him if he makes the same dollar investment ($\$X$) and receives the same future dollars from both alternatives at the end of one year. That is, the investor will be indifferent if:

$$X(1 + z_1)(1 + f) = X(1 + z_2)^2$$

Solving for f , we get:

$$f = \frac{(1 + z_2)^2}{(1 + z_1)} - 1$$

Doubling f gives the BEY for the 6-month forward rate six months from now.

We can illustrate the use of this formula with the theoretical spot rates shown in Exhibit 4. From that exhibit, we know that:

6-month bill spot rate = 0.030, therefore $z_1 = 0.0150$

1-year bill spot rate = 0.033, therefore $z_2 = 0.0165$

Substituting into the formula, we have:

$$f = \frac{(1.0165)^2}{(1.0150)} - 1 = 0.0180 = 1.8\%$$

Therefore, the 6-month forward rate six months from now is 3.6% ($1.8\% \times 2$) BEY.

Let's confirm our results. If X is invested in the 6-month Treasury bill at 1.5% and the proceeds then reinvested for six months at the 6-month forward rate of 1.8%, the total proceeds from this alternative would be:

$$X(1.015)(1.018) = 1.03327 X$$

Investment of X in the 1-year Treasury bill at one-half the 1-year rate, 1.0165%, would produce the following proceeds at the end of one year:

$$X(1.0165)^2 = 1.03327 X$$

Both alternatives have the same payoff if the 6-month Treasury bill yield six months from now is 1.8% (3.6% on a BEY). This means that, if an investor is guaranteed a 1.8% yield (3.6% BEY) on a 6-month Treasury bill six months from now, he will be indifferent toward the two alternatives.

The same line of reasoning can be used to obtain the 6-month forward rate beginning at any time period in the future. For example, the following can be determined:

- the 6-month forward rate three years from now
- the 6-month forward rate five years from now

The notation that we use to indicate 6-month forward rates is ${}_1f_m$ where the subscript 1 indicates a 1-period (6-month) rate and the subscript m indicates the period beginning m periods from now. When m is equal to zero, this means the current rate. Thus, the first 6-month forward rate is simply the current 6-month spot rate. That is, ${}_1f_0 = z_1$.

The general formula for determining a 6-month forward rate is:

$${}_1f_m = \frac{(1 + z_{m+1})^{m+1}}{(1 + z_m)^m} - 1$$

For example, suppose that the 6-month forward rate four years (eight 6-month periods) from now is sought. In terms of our notation, m is 8 and we seek ${}_1f_8$. The formula is then:

$${}_1f_8 = \frac{(1 + z_9)^9}{(1 + z_8)^8} - 1$$

From Exhibit 4, since the 4-year spot rate is 5.065% and the 4.5-year spot rate is 5.1701%, z_8 is 2.5325% and z_9 is 2.58505%. Then,

$${}_1f_8 = \frac{(1.0258505)^9}{(1.025325)^8} - 1 = 3.0064\%$$

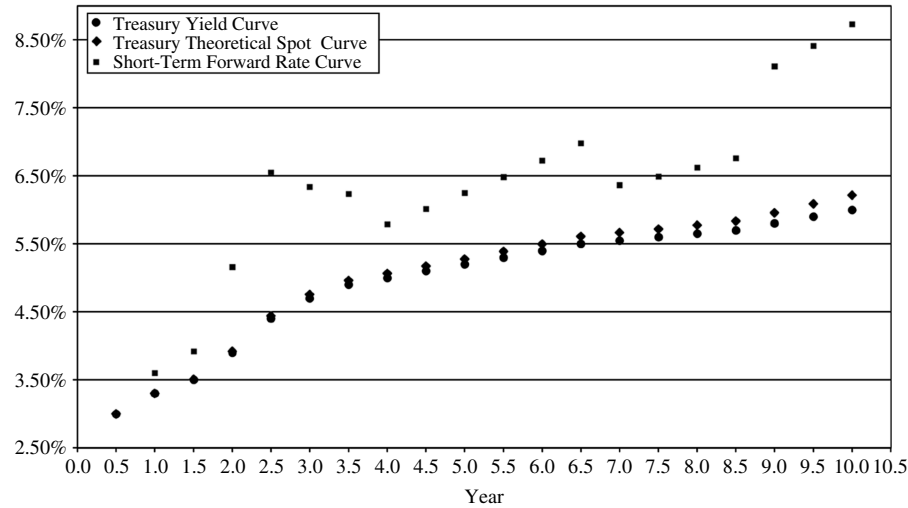
Doubling this rate gives a 6-month forward rate four years from now of 6.01%

Exhibit 12 shows all of the 6-month forward rates for the Treasury yield curve shown in Exhibit 4. The forward rates reported in Exhibit 12 are the annualized rates on a bond-equivalent basis. In Exhibit 13, the short-term forward rates are plotted along with the Treasury par yield curve and theoretical spot rate curve. The graph of the short-term forward rates is called the **short-term forward-rate curve**. Notice that the short-term forward rate curve lies above the other two curves. This will always be the case if the par yield curve is upward sloping. If the par yield curve is downward sloping, the short-term forward rate curve will be the lowest curve. Notice the unusual shape for the short-term forward rate curve. There is a mathematical reason for this shape. In practice, analysts will use statistical techniques to create a smooth short-term forward rate curve.

EXHIBIT 12 Six-Month Forward Rates (Annualized Rates on a Bond-Equivalent Basis)

Notation	Forward rate
${}_1f_0$	3.00
${}_1f_1$	3.60
${}_1f_2$	3.92
${}_1f_3$	5.15
${}_1f_4$	6.54
${}_1f_5$	6.33
${}_1f_6$	6.23
${}_1f_7$	5.79
${}_1f_8$	6.01
${}_1f_9$	6.24
${}_1f_{10}$	6.48
${}_1f_{11}$	6.72
${}_1f_{12}$	6.97
${}_1f_{13}$	6.36
${}_1f_{14}$	6.49
${}_1f_{15}$	6.62
${}_1f_{16}$	6.76
${}_1f_{17}$	8.10
${}_1f_{18}$	8.40
${}_1f_{19}$	8.71

EXHIBIT 13 Graph of Short-Term Forward Rate Curve



B. Relationship between Spot Rates and Short-Term Forward Rates

Suppose an investor invests \$X in a 3-year zero-coupon Treasury security. The total proceeds three years (six periods) from now would be:

$$X(1 + z_6)^6$$

The investor could instead buy a 6-month Treasury bill and reinvest the proceeds every six months for three years. The future dollars or dollar return will depend on the 6-month forward rates. Suppose that the investor can actually reinvest the proceeds maturing every six months at the calculated 6-month forward rates shown in Exhibit 12. At the end of three years, an investment of X would generate the following proceeds:

$$X(1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2)(1 + {}_1f_3)(1 + {}_1f_4)(1 + {}_1f_5)$$

Since the two investments must generate the same proceeds at the end of three years, the two previous equations can be equated:

$$X(1 + z_6)^6 = X(1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2)(1 + {}_1f_3)(1 + {}_1f_4)(1 + {}_1f_5)$$

Solving for the 3-year (6-period) spot rate, we have:

$$z_6 = [(1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2)(1 + {}_1f_3)(1 + {}_1f_4)(1 + {}_1f_5)]^{1/6} - 1$$

This equation tells us that the 3-year spot rate depends on the current 6-month spot rate and the five 6-month forward rates. In fact, the right-hand side of this equation is a geometric average of the current 6-month spot rate and the five 6-month forward rates.

Let's use the values in Exhibits 4 and 12 to confirm this result. Since the 6-month spot rate in Exhibit 4 is 3%, z_1 is 1.5% and therefore¹¹

$$\begin{aligned} z_6 &= [(1.015)(1.018)(1.0196)(1.0257)(1.0327)(1.03165)]^{1/6} - 1 \\ &= 0.023761 = 2.3761\% \end{aligned}$$

Doubling this rate gives 4.7522%. This agrees with the spot rate shown in Exhibit 4.

In general, the relationship between a T -period spot rate, the current 6-month spot rate, and the 6-month forward rates is as follows:

$$z_T = [(1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2) \dots (1 + {}_1f_{T-1})]^{1/T} - 1$$

Therefore, discounting at the forward rates will give the same present value as discounting at spot rates.

C. Valuation Using Forward Rates

Since a spot rate is simply a package of short-term forward rates, it will not make any difference whether we discount cash flows using spot rates or forward rates. That is, suppose that the cash flow in period T is \$1. Then the present value of the cash flow can be found using the spot rate for period T as follows:

$$\text{PV of \$1 in } T \text{ periods} = \frac{1}{(1 + z_T)^T}$$

¹¹Actually, the semiannual forward rates are based on annual rates calculated to more decimal places. For example, $f_{1,3}$ is 5.15% in Exhibit 12 but based on the more precise value, the semiannual rate is 2.577%.

Alternatively, since we know that

$$z_T = [(1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2) \cdots (1 + {}_1f_{T-1})]^{1/T} - 1$$

then, adding 1 to both sides of the equation,

$$(1 + z_T) = [(1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2) \cdots (1 + {}_1f_{T-1})]^{1/T}$$

Raising both sides of the equation to the T -th power we get:

$$(1 + z_T)^T = (1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2) \cdots (1 + {}_1f_{T-1})$$

Substituting the right-hand side of the above equation into the present value formula we get:

$$\text{PV of \$1 in } T \text{ periods} = \frac{1}{(1 + z_1)(1 + {}_1f_1)(1 + {}_1f_2) \cdots (1 + {}_1f_{T-1})}$$

In practice, the present value of \$1 in T periods is called the **forward discount factor for period T** .

For example, consider the forward rates shown in Exhibit 12. The forward discount rate for period 4 is found as follows:

$$\begin{aligned} z_1 &= 3\%/2 = 1.5\% & {}_1f_1 &= 3.6\%/2 = 1.8\% \\ {}_1f_2 &= 3.92\%/2 = 1.958\% & {}_1f_3 &= 5.15\%/2 = 2.577\% \end{aligned}$$

$$\begin{aligned} \text{forward discount factor of \$1 in 4 periods} &= \frac{\$1}{(1.015)(1.018)(1.01958)(1.02577)} \\ &= 0.925369 \end{aligned}$$

To see that this is the same present value that would be obtained using the spot rates, note from Exhibit 4 that the 2-year spot rate is 3.9164%. Using that spot rate, we find:

$$\begin{aligned} z_4 &= 3.9164\%/2 = 1.9582\% \\ \text{PV of \$1 in 4 periods} &= \frac{\$1}{(1.019582)^4} = 0.925361 \end{aligned}$$

The answer is the same as the forward discount factor (the slight difference is due to rounding).

Exhibit 14 shows the computation of the forward discount factor for each period based on the forward rates in Exhibit 12. Let's show how both the forward rates and the spot rates can be used to value a 2-year 6% coupon Treasury bond. The present value for each cash flow is found as follows using spot rates:

$$\frac{\text{cash flow for period } t}{(1 + z_t)^t}$$

EXHIBIT 14 Calculation of the Forward Discount Factor for Each Period

Period	Years	Notation	Forward rate*	$0.5 \times$ Forward rate**	$1 +$ Forward rate	Forward discount factor
1	0.5	${}_1f_0$	3.00%	1.5000%	1.01500	0.985222
2	1.0	${}_1f_1$	3.60%	1.8002%	1.01800	0.967799
3	1.5	${}_1f_2$	3.92%	1.9583%	1.01958	0.949211
4	2.0	${}_1f_3$	5.15%	2.5773%	1.02577	0.925362
5	2.5	${}_1f_4$	6.54%	3.2679%	1.03268	0.896079
6	3.0	${}_1f_5$	6.33%	3.1656%	1.03166	0.868582
7	3.5	${}_1f_6$	6.23%	3.1139%	1.03114	0.842352
8	4.0	${}_1f_7$	5.79%	2.8930%	1.02893	0.818668
9	4.5	${}_1f_8$	6.01%	3.0063%	1.03006	0.794775
10	5.0	${}_1f_9$	6.24%	3.1221%	1.03122	0.770712
11	5.5	${}_1f_{10}$	6.48%	3.2407%	1.03241	0.746520
12	6.0	${}_1f_{11}$	6.72%	3.3622%	1.03362	0.722237
13	6.5	${}_1f_{12}$	6.97%	3.4870%	1.03487	0.697901
14	7.0	${}_1f_{13}$	6.36%	3.1810%	1.03181	0.676385
15	7.5	${}_1f_{14}$	6.49%	3.2450%	1.03245	0.655126
16	8.0	${}_1f_{15}$	6.62%	3.3106%	1.03310	0.634132
17	8.5	${}_1f_{16}$	6.76%	3.3778%	1.03378	0.613412
18	9.0	${}_1f_{17}$	8.10%	4.0504%	1.04050	0.589534
19	9.5	${}_1f_{18}$	8.40%	4.2009%	1.04201	0.565767
20	10.0	${}_1f_{19}$	8.72%	4.3576%	1.04357	0.542142

*The rates in this column are rounded to two decimal places.

**The rates in this column used the forward rates in the previous column carried to four decimal places.

The following table uses the spot rates in Exhibit 4 to value this bond:

Period	Spot rate BEY (%)	Semiannual spot rate (%)	PV of \$1	Cash flow	PV of cash flow
1	3.0000	1.50000	0.9852217	3	2.955665
2	3.3000	1.65000	0.9677991	3	2.903397
3	3.5053	1.75266	0.9492109	3	2.847633
4	3.9164	1.95818	0.9253619	103	95.312278
				Total	104.018973

Based on the spot rates, the value of this bond is \$104.0190.

Using forward rates and the forward discount factors, the present value of the cash flow in period t is found as follows:

$$\text{cash flow in period } t \times \text{discount factor for period } t$$

The following table uses the forward rates and the forward discount factors in Exhibit 14 to value this bond:

Period	Semiann. forward rate	Forward discount factor	Cash flow	PV of cash flow
1	1.5000%	0.985222	3	2.955665
2	1.8002%	0.967799	3	2.903397
3	1.9583%	0.949211	3	2.847633
4	2.5773%	0.925362	103	95.312278
			Total	104.018973

The present value of this bond using forward rates is \$104.0190.

So, it does not matter whether one discounts cash flows by spot rates or forward rates, the value is the same.

D. Computing Any Forward Rate

Using spot rates, we can compute any forward rate. Using the same arbitrage arguments as used above to derive the 6-month forward rates, any forward rate can be obtained.

There are two elements to the forward rate. The first is when in the future the rate begins. The second is the length of time for the rate. For example, the 2-year forward rate 3 years from now means a rate three years from now for a length of two years. The notation used for a forward rate, f , will have two subscripts—one before f and one after f as shown below:

$${}_t f_m$$

The subscript before f is t and is the length of time that the rate applies. The subscript after f is m and is when the forward rate begins. That is,

$$\text{the length of time of the forward rate } {}_t f_m \text{ when the forward rate begins}$$

Remember our time periods are still 6-month periods. Given the above notation, here is what the following mean:

Notation	Interpretation for the forward rate
${}_1 f_{12}$	6-month (1-period) forward rate beginning 6 years (12 periods) from now
${}_2 f_8$	1-year (2-period) forward rate beginning 4 years (8 periods) from now
${}_6 f_4$	3-year (6-period) forward rate beginning 2 years (4 periods) from now
${}_8 f_{10}$	4-year (8-period) forward rate beginning 5 years (10 periods) from now

To see how the formula for the forward rate is derived, consider the following two alternatives for an investor who wants to invest for $m + t$ periods:

- buy a zero-coupon Treasury bond that matures in $m + t$ periods, or
- buy a zero-coupon Treasury bond that matures in m periods and invest the proceeds at the maturity date in a zero-coupon Treasury bond that matures in t periods.

The investor will be indifferent between the two alternatives if they produce the same return over the $m + t$ investment horizon.

For \$100 invested in the first alternative, the proceeds for this investment at the horizon date assuming that the semiannual rate is z_{m+t} is

$$\$100 (1 + z_{m+t})^{m+t}$$

For the second alternative, the proceeds for this investment at the end of m periods assuming that the semiannual rate is z_m is

$$\$100 (1 + z_m)^m$$

When the proceeds are received in m periods, they are reinvested at the forward rate, ${}_t f_m$, producing a value for the investment at the end of $m + t$ periods of

$$\$100 (1 + z_m)^m (1 + {}_t f_m)^t$$

For the investor to be indifferent to the two alternatives, the following relationship must hold:

$$\$100 (1 + z_{m+t})^{m+t} = \$100 (1 + z_m)^m (1 + {}_t f_m)^t$$

Solving for ${}_t f_m$ we get:

$${}_t f_m = \left[\frac{(1 + z_{m+t})^{m+t}}{(1 + z_m)^m} \right]^{1/t} - 1$$

Notice that if t is equal to 1, the formula reduces to the 1-period (6-month) forward rate.

To illustrate, for the spot rates shown in Exhibit 4, suppose that an investor wants to know the 2-year forward rate three years from now. In terms of the notation, t is equal to 4 and m is equal to 6. Substituting for t and m into the equation for the forward rate we have:

$${}_4 f_6 = \left[\frac{(1 + z_{10})^{10}}{(1 + z_6)^6} \right]^{1/4} - 1$$

This means that the following two spot rates are needed: z_6 (the 3-year spot rate) and z_{10} (the 5-year spot rate). From Exhibit 4 we know

$$z_6 (\text{the 3-year spot rate}) = 4.752\%/2 = 0.02376$$

$$z_{10} (\text{the 5-year spot rate}) = 5.2772\%/2 = 0.026386$$

then

$${}_4 f_6 = \left[\frac{(1.026386)^{10}}{(1.02376)^6} \right]^{1/4} - 1 = 0.030338$$

Therefore, ${}_4 f_6$ is equal to 3.0338% and doubling this rate gives 6.0675% the forward rate on a bond-equivalent basis.

We can verify this result. Investing \$100 for 10 periods at the spot rate of 2.6386% will produce the following value:

$$\$100 (1.026386)^{10} = \$129.7499$$

Investing \$100 for 6 periods at 2.376% and reinvesting the proceeds for 4 periods at the forward rate of 3.030338% gives the same value:

$$\$100 (1.02376)^6 (1.030338)^4 = \$129.75012$$

INTRODUCTION TO THE MEASUREMENT OF INTEREST RATE RISK

I. INTRODUCTION

In Chapter 2, we discussed the interest rate risk associated with investing in bonds. We know that the value of a bond moves in the opposite direction to a change in interest rates. If interest rates increase, the price of a bond will decrease. For a short bond position, a loss is generated if interest rates fall. However, a manager wants to know more than simply when a position generates a loss. To control interest rate risk, a manager must be able to quantify that result.

What is the key to measuring the interest rate risk? It is the accuracy in estimating the value of the position after an adverse interest rate change. A valuation model determines the value of a position after an adverse interest rate move. Consequently, if a reliable valuation model is not used, there is no way to properly measure interest rate risk exposure.

There are two approaches to measuring interest rate risk—the full valuation approach and the duration/convexity approach.

II. THE FULL VALUATION APPROACH

The most obvious way to measure the interest rate risk exposure of a bond position or a portfolio is to re-value it when interest rates change. The analysis is performed for different scenarios with respect to interest rate changes. For example, a manager may want to measure the interest rate exposure to a 50 basis point, 100 basis point, and 200 basis point instantaneous change in interest rates. This approach requires the re-valuation of a bond or bond portfolio for a given interest rate change scenario and is referred to as the **full valuation approach**. It is sometimes referred to as **scenario analysis** because it involves assessing the exposure to interest rate change scenarios.

To illustrate this approach, suppose that a manager has a \$10 million par value position in a 9% coupon 20-year bond. The bond is option-free. The current price is 134.6722 for a yield (i.e., yield to maturity) of 6%. The market value of the position is \$13,467,220 ($134.6722\% \times \$10$ million). Since the manager owns the bond, she is concerned with a rise in yield since this will decrease the market value of the position. To assess the exposure to a rise in market yields, the manager decides to look at how the value of the bond will change if yields change instantaneously for the following three scenarios: (1) 50 basis point increase,

EXHIBIT 1 Illustration of Full Valuation Approach to Assess the Interest Rate Risk of a Bond Position for Three Scenarios

Current bond position: 9% coupon 20-year bond (option-free)

Price: 134.6722

Yield to maturity: 6%

Par value owned: \$10 million

Market value of position: \$13,467,220.00

Scenario	Yield change (bp)	New yield	New price	New market value (\$)	Percentage change in market value (%)
1	50	6.5%	127.7606	12,776,050	−5.13%
2	100	7.0%	121.3551	12,135,510	−9.89%
3	200	8.0%	109.8964	10,989,640	−18.40%

(2) 100 basis point increase, and (3) 200 basis point increase. This means that the manager wants to assess what will happen to the bond position if the yield on the bond increases from 6% to (1) 6.5%, (2) 7%, and (3) 8%. Because this is an option-free bond, valuation is straightforward. In the examples that follow, we will use one yield to discount each of the cash flows. In other words, to simplify the calculations, we will assume a flat yield curve (even though that assumption doesn't fit the examples perfectly). The price of this bond per \$100 par value and the market value of the \$10 million par position is shown in Exhibit 1. Also shown is the new market value and the percentage change in market value.

In the case of a portfolio, each bond is valued for a given scenario and then the total value of the portfolio is computed for a given scenario. For example, suppose that a manager has a portfolio with the following two option-free bonds: (1) 6% coupon 5-year bond and (2) 9% coupon 20-year bond. For the shorter term bond, \$5 million of par value is owned and the price is 104.3760 for a yield of 5%. For the longer term bond, \$10 million of par value is owned and the price is 134.6722 for a yield of 6%. Suppose that the manager wants to assess the interest rate risk of this portfolio for a 50, 100, and 200 basis point increase in interest rates assuming both the 5-year yield and 20-year yield change by the same number of basis points. Exhibit 2 shows the interest rate risk exposure. Panel a of the exhibit shows the market value of the 5-year bond for the three scenarios. Panel b does the same for the 20-year bond. Panel c shows the total market value of the two-bond portfolio and the percentage change in the market value for the three scenarios.

In the illustration in Exhibit 2, it is assumed that both the 5-year and the 20-year yields changed by the same number of basis points. The full valuation approach can also handle scenarios where the yield curve does not change in a parallel fashion. Exhibit 3 illustrates this for our portfolio that includes the 5-year and 20-year bonds. The scenario analyzed is a yield curve shift combined with shifts in the level of yields. In the illustration in Exhibit 3, the following yield changes for the 5-year and 20-year yields are assumed:

Scenario	Change in 5-year rate (bp)	Change in 20-year rate (bp)
1	50	10
2	100	50
3	200	100

The last panel in Exhibit 3 shows how the market value of the portfolio changes for each scenario.

EXHIBIT 2 Illustration of Full Valuation Approach to Assess the Interest Rate Risk of a Two Bond Portfolio (Option-Free) for Three Scenarios Assuming a Parallel Shift in the Yield Curve

Panel a

Bond 1: 6% coupon 5-year bond Par value: \$5,000,000
 Initial price: 104.3760 Initial market value: \$5,218,800
 Yield: 5%

Scenario	Yield change (bp)	New yield	New price	New market value (\$)
1	50	5.5%	102.1600	5,108,000
2	100	6.0%	100.0000	5,000,000
3	200	7.0%	95.8417	4,792,085

Panel b

Bond 2: 9% coupon 20-year bond Par value: \$10,000,000
 Initial price: 134.6722 Initial market value: \$13,467,220
 Yield: 6%

Scenario	Yield change (bp)	New yield	New price	New market value (\$)
1	50	6.5%	127.7605	12,776,050
2	100	7.0%	121.3551	12,135,510
3	200	8.0%	109.8964	10,989,640

Panel c

Initial Portfolio Market value: \$18,686,020.00

Scenario	Yield change (bp)	Market value of			Percentage change in market value (%)
		Bond 1 (\$)	Bond 2 (\$)	Portfolio (\$)	
1	50	5,108,000	12,776,050	17,884,020	-4.29%
2	100	5,000,000	12,135,510	17,135,510	-8.30%
3	200	4,792,085	10,989,640	15,781,725	-15.54%

EXHIBIT 3 Illustration of Full Valuation Approach to Assess the Interest Rate Risk of a Two Bond Portfolio (Option-Free) for Three Scenarios Assuming a Nonparallel Shift in the Yield Curve

Panel a

Bond 1: 6% coupon 5-year bond Par value: \$5,000,000
 Initial price: 104.3760 Initial market value: \$5,218,800
 Yield: 5%

Scenario	Yield change (bp)	New yield	New price	New market value (\$)
1	50	5.5%	102.1600	5,108,000
2	100	6.0%	100.0000	5,000,000
3	200	7.0%	95.8417	4,792,085

Panel b

Bond 2: 9% coupon 20-year bond Par value: \$10,000,000
 Initial price: 134.6722 Initial market value: \$13,467,220
 Yield: 6%

Scenario	Yield change (bp)	New yield	New price	New market value (\$)
1	10	6.1%	133.2472	13,324,720
2	50	6.5%	127.7605	12,776,050
3	100	7.0%	121.3551	12,135,510

Panel c

Initial Portfolio Market value: \$18,686,020.00

Scenario	Market value of			Percentage change in market value (%)
	Bond 1 (\$)	Bond 2 (\$)	Portfolio (\$)	
1	5,108,000	13,324,720	18,432,720	-1.36%
2	5,000,000	12,776,050	17,776,050	-4.87%
3	4,792,085	12,135,510	16,927,595	-9.41%

The full valuation approach seems straightforward. If one has a good valuation model, assessing how the value of a portfolio or individual bond will change for different scenarios for parallel and nonparallel yield curve shifts measures the interest rate risk of a portfolio.

A common question that often arises when using the full valuation approach is which scenarios should be evaluated to assess interest rate risk exposure. For some regulated entities, there are specified scenarios established by regulators. For example, it is common for regulators of depository institutions to require entities to determine the impact on the value of their bond portfolio for a 100, 200, and 300 basis point instantaneous change in interest rates (up and down). (Regulators tend to refer to this as “simulating” interest rate scenarios rather than scenario analysis.) Risk managers and highly leveraged investors such as hedge funds tend to look at extreme scenarios to assess exposure to interest rate changes. This practice is referred to as **stress testing**.

Of course, in assessing how changes in the yield curve can affect the exposure of a portfolio, there are an infinite number of scenarios that can be evaluated. The state-of-the-art technology involves using a complex statistical procedure¹ to determine a likely set of yield curve shift scenarios from historical data.

It seems like the chapter should end right here. We can use the full valuation approach to assess the exposure of a bond or portfolio to interest rate changes to evaluate any scenario, assuming—and this must be repeated continuously—that *the manager has a good valuation model to estimate what the price of the bonds will be in each interest rate scenario*. However, we are not stopping here. In fact, the balance of this chapter is considerably longer than this section. Why? The reason is that the full valuation process can be very time consuming. This is particularly true if the portfolio has a large number of bonds, even if a minority of those bonds are complex (i.e., have embedded options). While the full valuation approach is the recommended method, managers want one simple measure that they can use to get an idea of how bond prices will change if rates change in a parallel fashion, rather than having to revalue an entire portfolio. In Chapter 2, such a measure was introduced—duration. We will discuss this measure as well as a supplementary measure (convexity) in Sections IV and V, respectively. To build a foundation to understand the limitations of these measures, we describe the basic price volatility characteristics of bonds in Section III. The fact that there are limitations of using one or two measures to describe the interest rate exposure of a position or portfolio should not be surprising. These measures provide a starting point for assessing interest rate risk.

III. PRICE VOLATILITY CHARACTERISTICS OF BONDS

In Chapter 2, we described the characteristics of a bond that affect its price volatility: (1) maturity, (2) coupon rate, and (3) presence of embedded options. We also explained how the level of yields affects price volatility. In this section, we will take a closer look at the price volatility of bonds.

A. Price Volatility Characteristics of Option-Free Bonds

Let's begin by focusing on option-free bonds (i.e., bonds that do not have embedded options). A fundamental characteristic of an option-free bond is that the price of the bond changes in

¹The procedure used is principal component analysis.

EXHIBIT 4 Price/Yield Relationship for Four Hypothetical Option-Free Bonds

Yield (%)	Price (\$)			
	6%/5 year	6%/20 year	9%/5 year	9%/20 year
4.00	108.9826	127.3555	122.4565	168.3887
5.00	104.3760	112.5514	117.5041	150.2056
5.50	102.1600	106.0195	115.1201	142.1367
5.90	100.4276	101.1651	113.2556	136.1193
5.99	100.0427	100.1157	112.8412	134.8159
6.00	100.0000	100.0000	112.7953	134.6722
6.01	99.9574	99.8845	112.7494	134.5287
6.10	99.5746	98.8535	112.3373	133.2472
6.50	97.8944	94.4479	110.5280	127.7605
7.00	95.8417	89.3225	108.3166	121.3551
8.00	91.8891	80.2072	104.0554	109.8964

the opposite direction to a change in the bond's yield. Exhibit 4 illustrates this property for four hypothetical bonds assuming a par value of \$100.

When the price/yield relationship for any option-free bond is graphed, it exhibits the shape shown in Exhibit 5. Notice that as the yield increases, the price of an option-free bond declines. However, this relationship is not linear (i.e., not a straight line relationship). The shape of the price/yield relationship for any option-free bond is referred to as **convex**. This price/yield relationship reflects an instantaneous change in the required yield.

The price sensitivity of a bond to changes in the yield can be measured in terms of the dollar price change or the percentage price change. Exhibit 6 uses the four hypothetical bonds in Exhibit 4 to show the percentage change in each bond's price for various changes in yield, assuming that the initial yield for all four bonds is 6%. An examination of Exhibit 6 reveals the following properties concerning the price volatility of an option-free bond:

Property 1: Although the price moves in the opposite direction from the change in yield, the percentage price change is not the same for all bonds.

EXHIBIT 5 Price/Yield Relationship for a Hypothetical Option-Free Bond

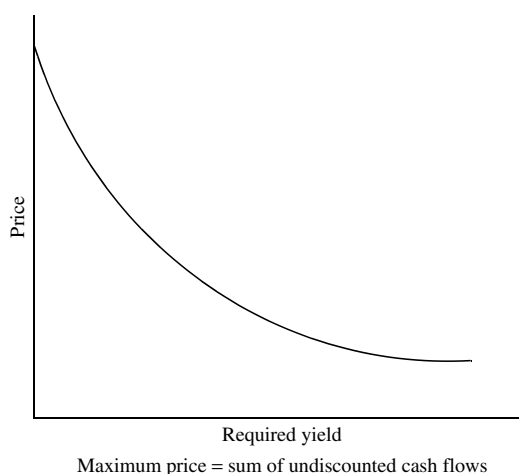


EXHIBIT 6 Instantaneous Percentage Price Change for Four Hypothetical Bonds (Initial yield for all four bonds is 6%)

New Yield (%)	Percentage Price Change			
	6%/5 year	6%/20 year	9%/5 year	9%/20 year
4.00	8.98	27.36	8.57	25.04
5.00	4.38	12.55	4.17	11.53
5.50	2.16	6.02	2.06	5.54
5.90	0.43	1.17	0.41	1.07
5.99	0.04	0.12	0.04	0.11
6.01	-0.04	-0.12	-0.04	-0.11
6.10	-0.43	-1.15	-0.41	-1.06
6.50	-2.11	-5.55	-2.01	-5.13
7.00	-4.16	-10.68	-3.97	-9.89
8.00	-8.11	-19.79	-7.75	-18.40

Property 2: For small changes in the yield, the percentage price change for a given bond is roughly the same, whether the yield increases or decreases.

Property 3: For large changes in yield, the percentage price change is not the same for an increase in yield as it is for a decrease in yield.

Property 4: For a given large change in yield, the percentage price increase is greater than the percentage price decrease.

While the properties are expressed in terms of percentage price change, they also hold for dollar price changes.

An explanation for these last two properties of bond price volatility lies in the convex shape of the price/yield relationship. Exhibit 7 illustrates this. The following notation is used in the exhibit

Y = initial yield

Y_1 = lower yield

Y_2 = higher yield

P = initial price

P_1 = price at lower yield Y_1

P_2 = price at higher yield Y_2

What was done in the exhibit was to change the initial yield (Y) up and down by the same number of basis points. That is, in Exhibit 7, the yield is decreased from Y to Y_1 and increased from Y to Y_2 such that the change is the same:

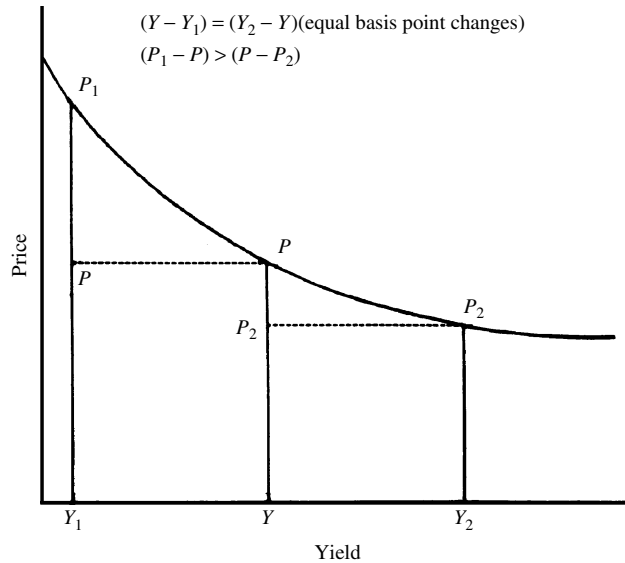
$$Y - Y_1 = Y_2 - Y$$

Also, the change in yield is a large number of basis points.

The vertical distance from the horizontal axis (the yield) to the intercept on the graph shows the price. The change in the initial price (P) when the yield declines from Y to Y_1 is equal to the difference between the new price (P_1) and the initial price (P). That is,

$$\text{change in price when yield decreases} = P_1 - P$$

EXHIBIT 7 Graphical Illustration of Properties 3 and 4 for an Option-Free Bond



The change in the initial price (P) when the yield increases from Y to Y_2 is equal to the difference between the new price (P_2) and the initial price (P). That is,

$$\text{change in price when yield increases} = P_2 - P$$

As can be seen in the exhibit, the change in price when yield decreases is not equal to the change in price when yield increases by the same number of basis points. That is,

$$P_1 - P \neq P_2 - P$$

This is what Property 3 states.

A comparison of the price change shows that the change in price when yield decreases is greater than the change in price when yield increases. That is,

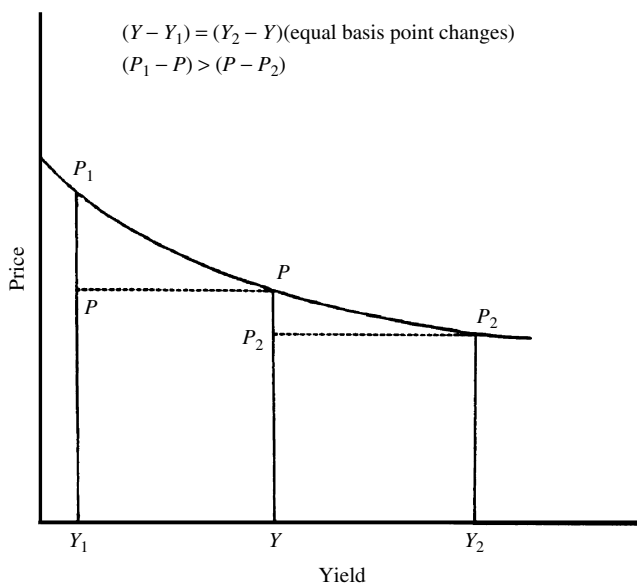
$$P_1 - P > P_2 - P$$

This is Property 4.

The implication of Property 4 is that if an investor owns a bond, the capital gain that will be realized if the yield decreases is greater than the capital loss that will be realized if the yield increases by the same number of basis points. For an investor who is short a bond (i.e., sold a bond not owned), the reverse is true: the potential capital loss is greater than the potential capital gain if the yield changes by a given number of basis points.

The convexity of the price/yield relationship impacts Property 4. Exhibit 8 shows a less convex price/yield relationship than Exhibit 7. That is, the price/yield relationship in Exhibit 8 is less bowed than the price/yield relationship in Exhibit 7. Because of the difference in the convexities, look at what happens when the yield increases and decreases by the same number of basis points and the yield change is a large number of basis points. We use the same notation

EXHIBIT 8 Impact of Convexity on Property 4: Less Convex Bond



in Exhibits 8 and 9 as in Exhibit 7. Notice that while the price gain when the yield decreases is greater than the price decline when the yield increases, the gain is not much greater than the loss. In contrast, Exhibit 9 has much greater convexity than the bonds in Exhibits 7 and 8 and the price gain is significantly greater than the loss for the bonds depicted in Exhibits 7 and 8.

B. Price Volatility of Bonds with Embedded Options

Now let's turn to the price volatility of bonds with embedded options. As explained in previous chapters, the price of a bond with an embedded option is comprised of two components. The first is the value of the same bond if it had no embedded option (that is, the price if the bond is option free). The second component is the value of the embedded option. In other words, the value of a bond with embedded options is equal to the value of an option-free bond plus or minus the value of embedded options.

The two most common types of embedded options are call (or prepay) options and put options. As interest rates in the market decline, the issuer may call or prepay the debt obligation prior to the scheduled principal payment date. The other type of option is a put option. This option gives the investor the right to require the issuer to purchase the bond at a specified price. Below we will examine the price/yield relationship for bonds with both types of embedded options (calls and puts) and implications for price volatility.

1. Bonds with Call and Prepay Options In the discussion below, we will refer to a bond that may be called or is prepayable as a callable bond. Exhibit 10 shows the price/yield relationship for an option-free bond and a callable bond. The convex curve given by $a-a'$ is the price/yield relationship for an option-free bond. The unusual shaped curve denoted by $a-b$ in the exhibit is the price/yield relationship for the callable bond.

The reason for the price/yield relationship for a callable bond is as follows. When the prevailing market yield for comparable bonds is higher than the coupon rate on the callable

EXHIBIT 9 Impact of Convexity on Property 4: Highly Convex Bond

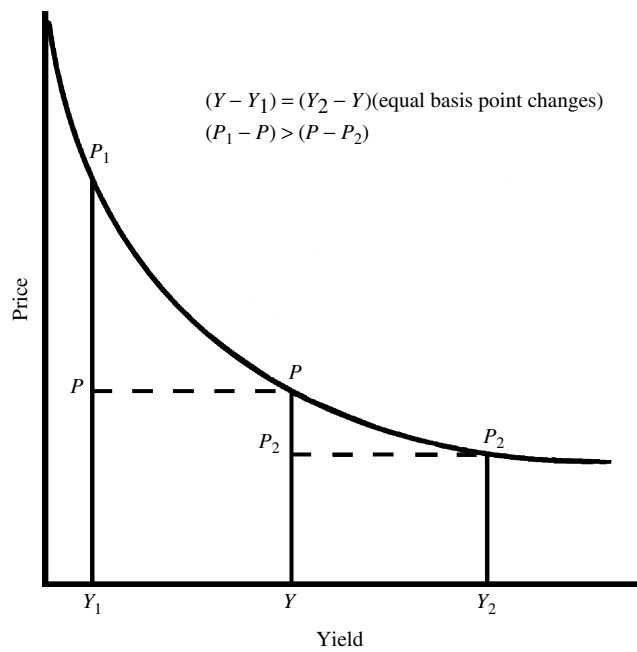
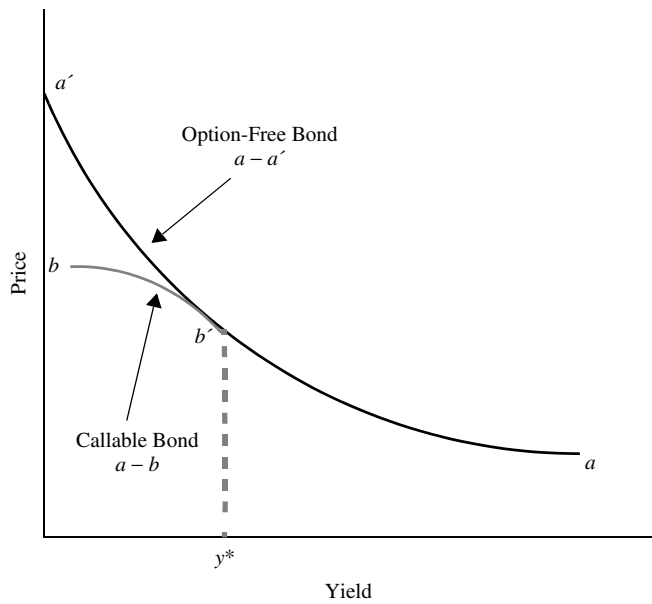


EXHIBIT 10 Price/Yield Relationship for a Callable Bond and an Option-Free Bond



bond, it is unlikely that the issuer will call the issue. For example, if the coupon rate on a bond is 7% and the prevailing market yield on comparable bonds is 12%, it is highly unlikely that the issuer will call a 7% coupon bond so that it can issue a 12% coupon bond. Since the bond is unlikely to be called, the callable bond will have a similar price/yield relationship to an otherwise comparable option-free bond. Consequently, the callable bond will be valued as if it is an option-free bond. However, since there is still some value to the call option,² the bond won't trade exactly like an option-free bond.

As yields in the market decline, the concern is that the issuer will call the bond. The issuer won't necessarily exercise the call option as soon as the market yield drops below the coupon rate. Yet, the value of the embedded call option increases as yields approach the coupon rate from higher yield levels. For example, if the coupon rate on a bond is 7% and the market yield declines to 7.5%, the issuer will most likely not call the issue. However, market yields are now at a level at which the investor is concerned that the issue may eventually be called if market yields decline further. Cast in terms of the value of the embedded call option, that option becomes more valuable to the issuer and therefore it reduces the price relative to an otherwise comparable option-free bond.³ In Exhibit 10, the value of the embedded call option at a given yield can be measured by the difference between the price of an option-free bond (the price shown on the curve $a-a'$) and the price on the curve $a-b$. Notice that at low yield levels (below y^* on the horizontal axis), the value of the embedded call option is high.

Using the information in Exhibit 10, let's compare the price volatility of a callable bond to that of an option-free bond. Exhibit 11 focuses on the portion of the price/yield relationship for the callable bond where the two curves in Exhibit 10 depart (segment $b'-b$ in Exhibit 10). We know from our earlier discussion that for a large change in yield, the price of an option-free bond increases by more than it decreases (Property 4 above). Is that what happens for a callable bond in the region of the price/yield relationship shown in Exhibit 11? No, it is not. In fact, as can be seen in the exhibit, the opposite is true! That is, for a given large change in yield, the price appreciation is less than the price decline.

This very important characteristic of a callable bond—that its price appreciation is less than its price decline when rates change by a large number of basis points—is referred to as **negative convexity**.⁴ But notice from Exhibit 10 that callable bonds don't exhibit this characteristic at every yield level. When yields are high (relative to the issue's coupon rate), the bond exhibits the same price/yield relationship as an option-free bond; therefore at high yield levels it also has the characteristic that the gain is greater than the loss. Because market participants have referred to the shape of the price/yield relationship shown in Exhibit 11 as negative convexity, market participants refer to the relationship for an option-free bond as **positive convexity**. Consequently, a callable bond exhibits negative convexity at low yield levels and positive convexity at high yield levels. This is depicted in Exhibit 12.

As can be seen from the exhibits, when a bond exhibits negative convexity, the bond compresses in price as rates decline. That is, at a certain yield level there is very little price appreciation when rates decline. When a bond enters this region, the bond is said to exhibit “price compression.”

²This is because there is still some chance that interest rates will decline in the future and the issue will be called.

³For readers who are already familiar with option theory, this characteristic can be restated as follows: When the coupon rate for the issue is below the market yield, the embedded call option is said to be “out-of-the-money.” When the coupon rate for the issue is above the market yield, the embedded call option is said to be “in-the-money.”

⁴Mathematicians refer to this shape as being “concave.”

EXHIBIT 11 Negative Convexity Region of the Price/Yield Relationship for a Callable Bond

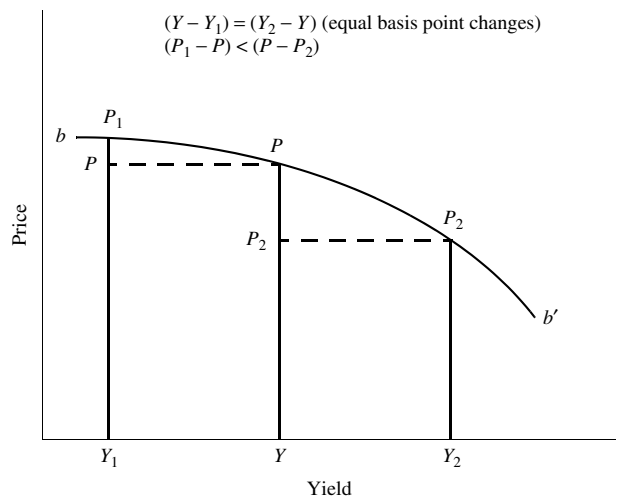
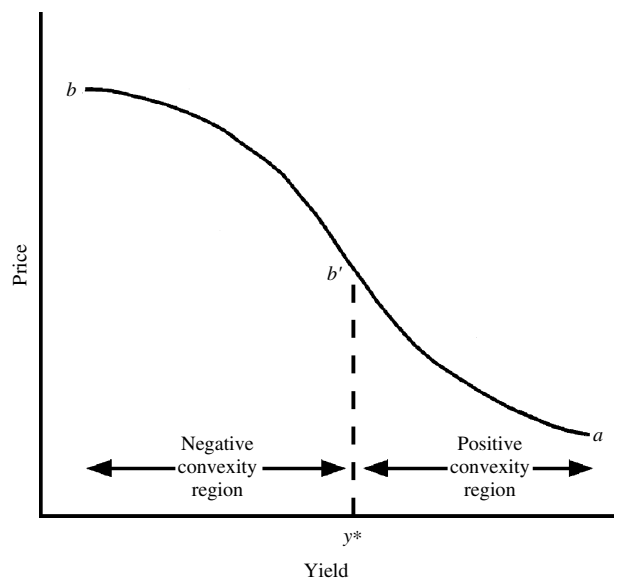
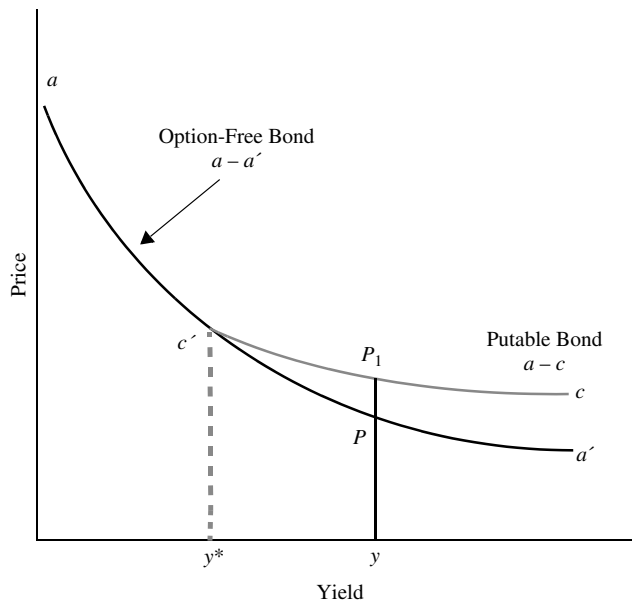


EXHIBIT 12 Negative and Positive Convexity Exhibited by a Callable Bond



2. Bonds with Embedded Put Options Putable bonds may be redeemed by the bondholder on the dates and at the put price specified in the indenture. Typically, the put price is par value. The advantage to the investor is that if yields rise such that the bond's value falls below the put price, the investor will exercise the put option. If the put price is par value, this means that if market yields rise above the coupon rate, the bond's value will fall below par and the investor will then exercise the put option.

EXHIBIT 13 Price/Yield Relationship for a Putable Bond and an Option-Free Bond



The value of a putable bond is equal to the value of an option-free bond plus the value of the put option. Thus, the difference between the value of a putable bond and the value of an otherwise comparable option-free bond is the value of the embedded put option. This can be seen in Exhibit 13 which shows the price/yield relationship for a putable bond is the curve $a-c$ and for an option-free bond is the curve $a-a'$.

At low yield levels (low relative to the issue's coupon rate), the price of the putable bond is basically the same as the price of the option-free bond because the value of the put option is small. As rates rise, the price of the putable bond declines, but the price decline is less than that for an option-free bond. The divergence in the price of the putable bond and an otherwise comparable option-free bond at a given yield level (y) is the value of the put option ($P_1 - P$). When yields rise to a level where the bond's price would fall below the put price, the price at these levels is the put price.

IV. DURATION

With the background about the price volatility characteristics of a bond, we can now turn to an alternate approach to full valuation: the duration/convexity approach. As explained in Chapter 2, *duration is a measure of the approximate price sensitivity of a bond to interest rate changes. More specifically, it is the approximate percentage change in price for a 100 basis point change in rates.* We will see in this section that duration is the first (linear) approximation of the percentage price change. To improve the approximation provided by duration, an adjustment for "convexity" can be made. Hence, using duration combined with convexity to estimate the percentage price change of a bond caused by changes in interest rates is called the **duration/convexity approach**.

A. Calculating Duration

In Chapter 2, we explained that the duration of a bond is estimated as follows:

$$\frac{\text{price if yields decline} - \text{price if yields rise}}{2(\text{initial price})(\text{change in yield in decimal})}$$

If we let

Δy = change in yield in decimal

V_0 = initial price

V_- = price if yields decline by Δy

V_+ = price if yields increase by Δy

then duration can be expressed as

$$\text{duration} = \frac{V_- - V_+}{2(V_0)(\Delta y)} \quad (1)$$

For example, consider a 9% coupon 20-year option-free bond selling at 134.6722 to yield 6% (see Exhibit 4). Let's change (i.e., shock) the yield down and up by 20 basis points and determine what the new prices will be for the numerator. If the yield is decreased by 20 basis points from 6.0% to 5.8%, the price would increase to 137.5888. If the yield increases by 20 basis points, the price would decrease to 131.8439. Thus,

$$\Delta y = 0.002$$

$$V_0 = 134.6722$$

$$V_- = 137.5888$$

$$V_+ = 131.8439$$

Then,

$$\text{duration} = \frac{137.5888 - 131.8439}{2 \times (134.6722) \times (0.002)} = 10.66$$

As explained in Chapter 2, duration is interpreted as the approximate percentage change in price for a 100 basis point change in rates. Consequently, a duration of 10.66 means that the approximate change in price for this bond is 10.66% for a 100 basis point change in rates.

A common question asked about this interpretation of duration is the consistency between the yield change that is used to compute duration using equation (1) and the interpretation of duration. For example, recall that in computing the duration of the 9% coupon 20-year bond, we used a 20 basis point yield change to obtain the two prices to use in the numerator of equation (1). Yet, we interpret the duration computed as the approximate percentage price change for a 100 basis point change in yield. The reason is that regardless of the yield change used to estimate duration in equation (1), the interpretation is the same. If we used a 25 basis point change in yield to compute the prices used in the numerator of equation (1), the resulting duration is interpreted as the approximate percentage price change for a 100 basis point change in yield. Later we will use different changes in yield to illustrate the sensitivity of the computed duration.

B. Approximating the Percentage Price Change Using Duration

In Chapter 2, we explained how to approximate the percentage price change for a given change in yield and a given duration. Here we will express the process using the following formula:

$$\text{approximate percentage price change} = -\text{duration} \times \Delta y_* \times 100 \quad (2)$$

where Δy_* is the yield change (in decimal) for which the estimated percentage price change is sought.⁵ The reason for the negative sign on the right-hand side of equation (2) is due to the inverse relationship between price change and yield change (e.g., as yields increase, bond prices decrease). The following two examples illustrate how to use duration to estimate a bond's price change.

Example #1: small change in basis point yield. For example, consider the 9% 20-year bond trading at 134.6722 whose duration we just showed is 10.66. The approximate percentage price change for a 10 basis point increase in yield (i.e., $\Delta y_* = +0.001$) is:

$$\text{approximate percentage price change} = -10.66 \times (+0.001) \times 100 = -1.066\%$$

How good is this approximation? The actual percentage price change is -1.06% (as shown in Exhibit 6 when yield increases to 6.10%). Duration, in this case, did an excellent job in estimating the percentage price change.

We would come to the same conclusion if we used duration to estimate the percentage price change if the yield declined by 10 basis points (i.e., $\Delta y = -0.001$). In this case, the approximate percentage price change would be $+1.066\%$ (i.e., the direction of the estimated price change is the reverse but the magnitude of the change is the same). Exhibit 6 shows that the actual percentage price change is $+1.07\%$.

In terms of estimating the new price, let's see how duration performed. The initial price is 134.6722. For a 10 basis point increase in yield, duration estimates that the price will decline by 1.066%. Thus, the price will decline to 133.2366 (found by multiplying 134.6722 by one minus 0.01066). The actual price from Exhibit 4 if the yield increases by 10 basis points is 133.2472. Thus, the price estimated using duration is close to the actual price.

For a 10 basis point decrease in yield, the actual price from Exhibit 4 is 136.1193 and the estimated price using duration is 136.1078 (a price increase of 1.066%). Consequently, the new price estimated by duration is close to the actual price for a 10 basis point change in yield.

Example #2: large change in basis point yield. Let's look at how well duration does in estimating the percentage price change if the yield increases by 200 basis points instead of 10 basis points. In this case, Δy is equal to $+0.02$. Substituting into equation (2), we have

$$\text{approximate percentage price change} = -10.66 \times (+0.02) \times 100 = -21.32\%$$

⁵The difference between Δy in the duration formula given by equation (1) and Δy_* in equation (2) to get the approximate percentage change is as follows. In the duration formula, the Δy is used to estimate duration and, as explained later, for reasonably small changes in yield the resulting value for duration will be the same. We refer to this change as the "rate shock." Given the duration, the next step is to estimate the percentage price change for any change in yield. The Δy_* in equation (2) is the specific change in yield for which the approximate percentage price change is sought.

How good is this estimate? From Exhibit 6, we see that the actual percentage price change when the yield increases by 200 basis points to 8% is -18.40% . Thus, the estimate is not as accurate as when we used duration to approximate the percentage price change for a change in yield of only 10 basis points. If we use duration to approximate the percentage price change when the yield decreases by 200 basis points, the approximate percentage price change in this scenario is $+21.32\%$. The actual percentage price change as shown in Exhibit 6 is $+25.04\%$.

Let's look at the use of duration in terms of estimating the new price. Since the initial price is 134.6722 and a 200 basis point increase in yield will decrease the price by 21.32%, the estimated new price using duration is 105.9601 (found by multiplying 134.6722 by one minus 0.2132). From Exhibit 4, the actual price if the yield is 8% is 109.8964. Consequently, the estimate is not as accurate as the estimate for a 10 basis point change in yield. The estimated new price using duration for a 200 basis point decrease in yield is 163.3843 compared to the actual price (from Exhibit 4) of 168.3887. Once again, the estimation of the price using duration is not as accurate as for a 10 basis point change. *Notice that whether the yield is increased or decreased by 200 basis points, duration underestimates what the new price will be.* We will see why shortly.

Summary. Let's summarize what we found in our application of duration to approximate the percentage price change:

Yield change (bp)	Initial price	New price		Percent price change		Comment
		Based on duration	Actual	Based on duration	Actual	
+10	134.6722	133.2366	133.2472	-1.066	-1.06	estimated price close to new price
-10	134.6722	136.1078	136.1193	+1.066	+1.07	estimated price close to new price
+200	134.6722	105.9601	109.8964	-21.320	-18.40	underestimates new price
-200	134.6722	163.3843	168.3887	+21.320	+25.04	underestimates new price

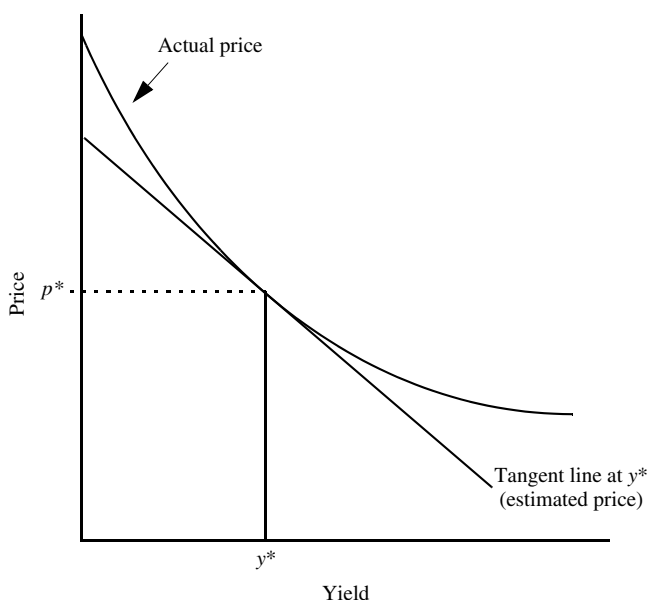
Should any of this be a surprise to you? No, not after reading Section III of this chapter and evaluating equation (2) in terms of the properties for the price/yield relationship discussed in that section. Look again at equation (2). Notice that whether the change in yield is an increase or a decrease, the approximate percentage price change will be the same except that the sign is reversed. This violates Property 3 and Property 4 with respect to the price volatility of option-free bonds when yields change. Recall that Property 3 states that the percentage price change will not be the same for a large increase and decrease in yield by the same number of basis points. Property 4 states the percentage price increase is greater than the percentage price decrease. These are two reasons why the estimate is inaccurate for a 200 basis point yield change.

Why did the duration estimate of the price change do a good job for a small change in yield of 10 basis points? Recall from Property 2 that the percentage price change will be approximately the same whether there is an increase or decrease in yield by a small number of basis points. We can also explain these results in terms of the graph of the price/yield relationship.

C. Graphical Depiction of Using Duration to Estimate Price Changes

In Section III, we used the graph of the price/yield relationship to demonstrate the price volatility properties of bonds. We can also use graphs to illustrate what we observed in our examples about how duration estimates the percentage price change, as well as some other noteworthy points.

EXHIBIT 14 Price/Yield Relationship for an Option-Free Bond with a Tangent Line



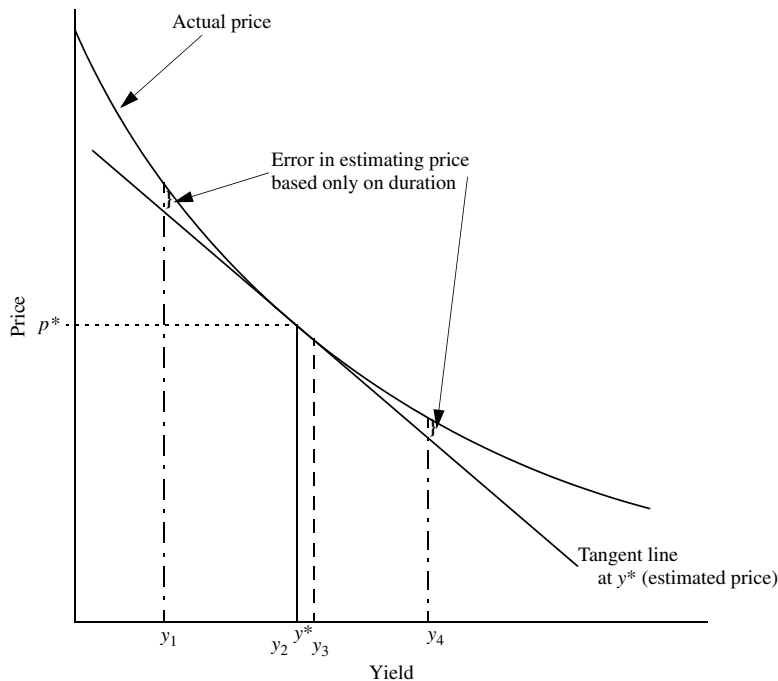
The shape of the price/yield relationship for an option-free bond is convex. Exhibit 14 shows this relationship. In the exhibit, a tangent line is drawn to the price/yield relationship at yield y^* . (For those unfamiliar with the concept of a tangent line, it is a straight line that just touches a curve at one point within a relevant (local) range. In Exhibit 14, the tangent line touches the curve at the point where the yield is equal to y^* and the price is equal to p^* .) The tangent line is used to *estimate* the new price if the yield changes. If we draw a vertical line from any yield (on the horizontal axis), as in Exhibit 14, the distance between the horizontal axis and the tangent line represents the price approximated by using duration starting with the initial yield y^* .

Now how is the tangent line related to duration? Given an initial price and a specific yield change, the tangent line tells us the approximate new price of a bond. The approximate percentage price change can then be computed for this change in yield. But this is precisely what duration [using equation (2)] gives us: the approximate percentage price change for a given change in yield. Thus, using the tangent line, one obtains the same approximate percentage price change as using equation (2).

This helps us understand why duration did an effective job of estimating the percentage price change, or equivalently the new price, when the yield changes by a small number of basis points. Look at Exhibit 15. Notice that for a small change in yield, the tangent line does not depart much from the price/yield relationship. Hence, when the yield changes up or down by 10 basis points, the tangent line does a good job of estimating the new price, as we found in our earlier numerical illustration.

Exhibit 15 shows what happens to the estimate using the tangent line when the yield changes by a large number of basis points. Notice that the error in the estimate gets larger the further one moves from the initial yield. The estimate is less accurate the more convex the bond as illustrated in Exhibit 16.

EXHIBIT 15 Estimating the New Price Using a Tangent Line



Also note that, regardless of the magnitude of the yield change, the tangent line always underestimates what the new price will be for an option-free bond because the tangent line is below the price/yield relationship. This explains why we found in our illustration that when using duration, we underestimated what the actual price will be.

The results reported in Exhibit 17 are for option-free bonds. When we deal with more complicated securities, small rate shocks that do not reflect the types of rate changes that may occur in the market do not permit the determination of how prices can change. This is because expected cash flows may change when dealing with bonds with embedded options. In comparison, if large rate shocks are used, we encounter the asymmetry caused by convexity. Moreover, large rate shocks may cause dramatic changes in the expected cash flows for bonds with embedded options that may be far different from how the expected cash flows will change for smaller rate shocks.

There is another potential problem with using small rate shocks for complicated securities. The prices that are inserted into the duration formula as given by equation (2) are derived from a valuation model. The duration measure depends crucially on the valuation model. If the rate shock is small and the valuation model used to obtain the prices for equation (1) is poor, dividing poor price estimates by a small shock in rates (in the denominator) will have a significant effect on the duration estimate.

D. Rate Shocks and Duration Estimate

In calculating duration using equation (1), it is necessary to shock interest rates (yields) up and down by the same number of basis points to obtain the values for V_- and V_+ . In our

EXHIBIT 16 Estimating the New Price for a Large Yield Change for Bonds with Different Convexities

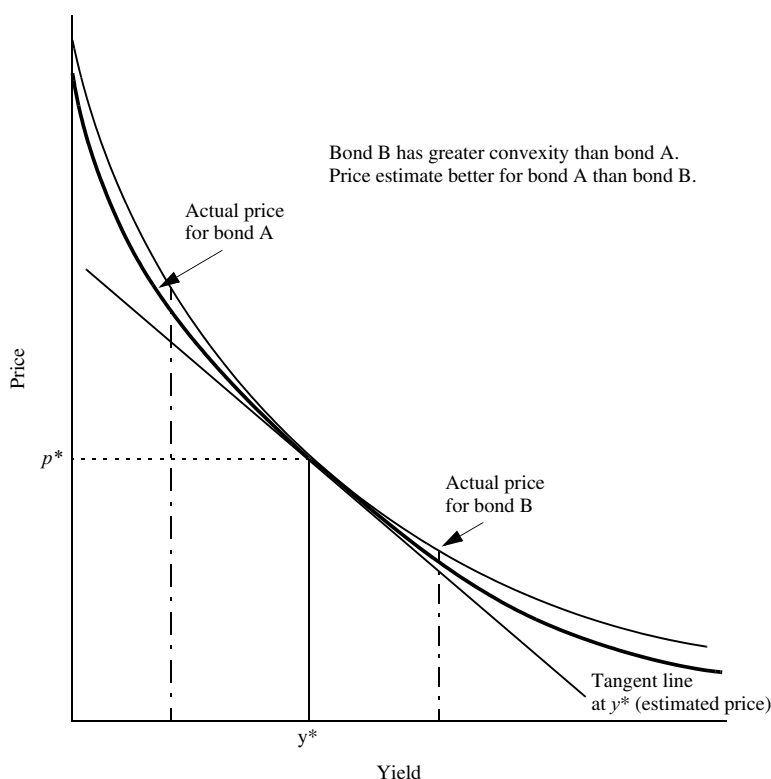


EXHIBIT 17 Duration Estimates for Different Rate Shocks

Assumption: Initial yield is 6%

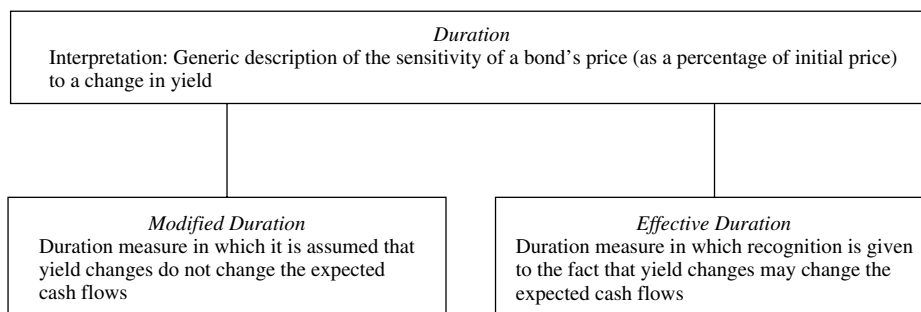
Bond	1 bp	10 bps	20 bps	50 bps	100 bps	150 bps	200 bps
6% 5 year	4.27	4.27	4.27	4.27	4.27	4.27	4.27
6% 20 year	11.56	11.56	11.56	11.57	11.61	11.69	11.79
9% 5 year	4.07	4.07	4.07	4.07	4.07	4.08	4.08
9% 20 year	10.66	10.66	10.66	10.67	10.71	10.77	10.86

illustration, 20 basis points was arbitrarily selected. But how large should the shock be? That is, how many basis points should be used to shock the rate?

In Exhibit 17, the duration estimates for our four hypothetical bonds using equation (1) for rate shocks of 1 basis point to 200 basis points are reported. The duration estimates for the two 5-year bonds are not affected by the size of the shock. The two 5-year bonds are less convex than the two 20-year bonds. But even for the two 20-year bonds, for the size of the shocks reported in Exhibit 17, the duration estimates are not materially affected by the greater convexity.

What is done in practice by dealers and vendors of analytical systems? Each system developer uses rate shocks that they have found to be realistic based on historical rate changes.

EXHIBIT 18 Modified Duration versus Effective Duration



E. Modified Duration versus Effective Duration

One form of duration that is cited by practitioners is **modified duration**. Modified duration is the approximate percentage change in a bond's price for a 100 basis point change in yield *assuming that the bond's expected cash flows do not change when the yield changes*. What this means is that in calculating the values of V_- and V_+ in equation (1), the same cash flows used to calculate V_0 are used. Therefore, the change in the bond's price when the yield is changed is due solely to discounting cash flows at the new yield level.

The assumption that the cash flows will not change when the yield is changed makes sense for option-free bonds such as noncallable Treasury securities. This is because the payments made by the U.S. Department of the Treasury to holders of its obligations do not change when interest rates change. However, the same cannot be said for bonds with embedded options (i.e., callable and puttable bonds and mortgage-backed securities). For these securities, a change in yield may significantly alter the expected cash flows.

In Section III, we showed the price/yield relationship for callable and prepayable bonds. Failure to recognize how changes in yield can alter the expected cash flows will produce two values used in the numerator of equation (1) that are not good estimates of how the price will actually change. The duration is then not a good number to use to estimate how the price will change.

Some valuation models for bonds with embedded options take into account how changes in yield will affect the expected cash flows. Thus, when V_- and V_+ are the values produced from these valuation models, the resulting duration takes into account both the discounting at different interest rates and how the expected cash flows may change. When duration is calculated in this manner, it is referred to as **effective duration** or **option-adjusted duration**. (Lehman Brothers refers to this measure in some of its publications as **adjusted duration**.) Exhibit 18 summarizes the distinction between modified duration and effective duration.

The difference between modified duration and effective duration for bonds with embedded options can be quite dramatic. For example, a callable bond could have a modified duration of 5 but an effective duration of only 3. For certain collateralized mortgage obligations, the modified duration could be 7 and the effective duration 20! Thus, using modified duration as a measure of the price sensitivity for a security with embedded options to changes in yield would be misleading. Effective duration is the more appropriate measure for any bond with an embedded option.

F. Macaulay Duration and Modified Duration

It is worth comparing the relationship between modified duration to the another duration measure, **Macaulay duration**. Modified duration can be written as:⁶

$$\frac{1}{(1 + \text{yield}/k)} \left[\frac{1 \times \text{PVCF}_1 + 2 \times \text{PVCF}_2 \dots + n \times \text{PVCF}_n}{k \times \text{Price}} \right] \quad (3)$$

where

k = number of periods, or payments, per year (e.g., $k = 2$

for semiannual-pay bonds and $k = 12$ for monthly-pay bonds)

n = number of periods until maturity (i.e., number of years to maturity times k)

yield = yield to maturity of the bond

PVCF_t = present value of the cash flow in period t discounted at the yield to maturity

where $t = 1, 2, \dots, n$

We know that duration tells us the approximate percentage price change for a bond if the yield changes.

The expression in the brackets of the modified duration formula given by equation (3) is a measure formulated in 1938 by Frederick Macaulay.⁷ This measure is popularly referred to as Macaulay duration. Thus, modified duration is commonly expressed as:

$$\text{Modified duration} = \frac{\text{Macaulay duration}}{(1 + \text{yield}/k)}$$

The general formulation for duration as given by equation (1) provides a short-cut procedure for determining a bond's modified duration. Because it is easier to calculate the modified duration using the short-cut procedure, most vendors of analytical software will use equation (1) rather than equation (3) to reduce computation time.

However, *modified duration is a flawed measure of a bond's price sensitivity to interest rate changes for a bond with embedded options and therefore so is Macaulay duration*. The duration formula given by equation (3) *misleads* the user because it masks the fact that changes in the expected cash flows must be recognized for bonds with embedded options. Although equation (3) will give the same estimate of percent price change for an option-free bond as equation (1), equation (1) is still better because it acknowledges cash flows and thus value can change due to yield changes.

G. Interpretations of Duration

Throughout this book, the definition provided for duration is: the approximate percentage price change for a 100 basis point change in rates. That definition is the most relevant for how a manager or investor uses duration. In fact, if you understand this definition, you can easily calculate the change in a bond's value.

For example, suppose we want to know the approximate percentage change in price for a 50 basis point change in yield for our hypothetical 9% coupon 20-year bond selling for 134.6722. Since the duration is 10.66, a 100 basis point change in yield would change the price by about 10.66%. For a 50 basis point change in yield, the price will change by

⁶More specifically, this is the formula for the modified duration of a bond on a coupon anniversary date.

⁷Frederick Macaulay, *Some Theoretical Problems Suggested by the Movement of Interest Rates, Bond Yields, and Stock Prices in the U.S. Since 1856* (New York: National Bureau of Economic Research, 1938).

approximately 5.33% ($= 10.66\%/2$). So, if the yield increases by 50 basis points, the price will decrease by about 5.33% from 134.6722 to 127.4942.

Now let's look at some other duration definitions or interpretations that appear in publications and are cited by managers in discussions with their clients.

1. Duration Is the "First Derivative" Sometimes a market participant will refer to duration as the "first derivative of the price/yield function" or simply the "first derivative." Wow! Sounds impressive. First, "derivative" here has nothing to do with "derivative instruments" (i.e., futures, swaps, options, etc.). A derivative as used in this context is obtained by differentiating a mathematical function using calculus. There are first derivatives, second derivatives, and so on. When market participants say that duration is the first derivative, here is what they mean. The first derivative calculates the slope of a line—in this case, the slope of the tangent line in Exhibit 14. If it were possible to write a mathematical equation for a bond in closed form, the first derivative would be the result of differentiating that equation the first time. Even if you don't know how to do the process of differentiation to get the first derivative, it sounds like you are really smart since it suggests you understand calculus! While it is a correct interpretation of duration, it is an interpretation that in no way helps us understand what the interest rate risk is of a bond. That is, it is an operationally meaningless interpretation.

Why is it an operationally meaningless interpretation? Go back to the \$10 million bond position with a duration of 6. Suppose a client is concerned with the exposure of the bond to changes in interest rates. Now, tell that client the duration is 6 and that it is the first derivative of the price function for that bond. What have you told the client? Not much. In contrast, tell that client that the duration is 6 and that duration is the approximate price sensitivity of a bond to a 100 basis point change in rates and you have told the client more relevant information with respect to the bond's interest rate risk.

2. Duration Is Some Measure of Time When the concept of duration was originally introduced by Macaulay in 1938, he used it as a gauge of the time that the bond was outstanding. More specifically, Macaulay defined duration as the weighted average of the time to each coupon and principal payment of a bond. Subsequently, duration has too often been thought of in temporal terms, i.e., years. This is most unfortunate for two reasons.

First, in terms of dimensions, there is nothing wrong with expressing duration in terms of years because that is the proper dimension of this value. But the proper interpretation is that duration is the price volatility of a zero-coupon bond with that number of years to maturity. So, when a manager says a bond has a duration of 4 years, it is not useful to think of this measure in terms of time, but that the bond has the price sensitivity to rate changes of a 4-year zero-coupon bond.

Second, thinking of duration in terms of years makes it difficult for managers and their clients to understand the duration of some complex securities. Here are a few examples. For a mortgage-backed security that is an interest-only security (i.e., receives coupons but not principal repayment) the duration is negative. What does a negative number, say, -4 mean? In terms of our interpretation as a percentage price change, it means that when rates change by 100 basis points, the price of the bond changes by about 4% but the change is in the same direction as the change in rates.

As a second example, consider an inverse floater created in the collateralized mortgage obligation (CMO) market. The underlying collateral for such a security might be loans with 25 years to final maturity. However, an inverse floater can have a duration that easily exceeds 25.

This does not make sense to a manager or client who uses a measure of time as a definition for duration.

As a final example, consider derivative instruments, such as an option that expires in one year. Suppose that it is reported that its duration is 60. What does that mean? To someone who interprets duration in terms of time, does that mean 60 years, 60 days, 60 seconds? It doesn't mean any of these. It simply means that the option tends to have the price sensitivity to rate changes of a 60-year zero-coupon bond.

3. Forget First Derivatives and Temporal Definitions The bottom line is that one should not care if it is technically correct to think of duration in terms of years (volatility of a zero-coupon bond) or in terms of first derivatives. There are even some who interpret duration in terms of the "half life" of a security.⁸ Subject to the limitations that we will describe as we proceed in this book, duration is the measure of a security's price sensitivity to changes in yield. We will fine tune this definition as we move along.

Users of this interest rate risk measure are interested in what it tells them about the price sensitivity of a bond (or a portfolio) to changes in interest rates. Duration provides the investor with a feel for the dollar price exposure or the percentage price exposure to potential interest rate changes. Try the following definitions on a client who has a portfolio with a duration of 4 and see which one the client finds most useful for understanding the interest rate risk of the portfolio when rates change:

Definition 1: The duration of 4 for your portfolio indicates that the portfolio's value will change by approximately 4% if rates change by 100 basis points.

Definition 2: The duration of 4 for your portfolio is the first derivative of the price function for the bonds in the portfolio.

Definition 3: The duration of 4 for your portfolio is the weighted average number of years to receive the present value of the portfolio's cash flows.

Definition 1 is clearly preferable. It would be ridiculous to expect clients to understand the last two definitions better than the first.

Moreover, interpreting duration in terms of a measure of price sensitivity to interest rate changes allows a manager to make comparisons between bonds regarding their interest rate risk under certain assumptions.

H. Portfolio Duration

A portfolio's duration can be obtained by calculating the weighted average of the duration of the bonds in the portfolio. The weight is the proportion of the portfolio that a security comprises. Mathematically, a portfolio's duration can be calculated as follows:

$$w_1D_1 + w_2D_2 + w_3D_3 + \dots + w_KD_K$$

where

w_i = market value of bond i /market value of the portfolio

D_i = duration of bond i

K = number of bonds in the portfolio

⁸"Half-life" is the time required for an element to be reduced to half its initial value.

To illustrate this calculation, consider the following 3-bond portfolio in which all three bonds are option free:

Bond	Price (\$)	Yield (%)	Par amount owned	Market value	Duration
10% 5-year	100.0000	10	\$4 million	\$4,000,000	3.861
8% 15-year	84.6275	10	5 million	4,231,375	8.047
14% 30-year	137.8586	10	1 million	1,378,586	9.168

In this illustration, it is assumed that the next coupon payment for each bond is exactly six months from now (i.e., there is no accrued interest). The market value for the portfolio is \$9,609,961. Since each bond is option free, modified duration can be used. The market price per \$100 par value of each bond, its yield, and its duration are given below:

In this illustration, K is equal to 3 and:

$$w_1 = \$4,000,000 / \$9,609,961 = 0.416 \quad D_1 = 3.861$$

$$w_2 = \$4,231,375 / \$9,609,961 = 0.440 \quad D_2 = 8.047$$

$$w_3 = \$1,378,586 / \$9,609,961 = 0.144 \quad D_3 = 9.168$$

The portfolio's duration is:

$$0.416(3.861) + 0.440(8.047) + 0.144(9.168) = 6.47$$

A portfolio duration of 6.47 means that for a 100 basis point change in the yield for each of the three bonds, the market value of the portfolio will change by approximately 6.47%. But keep in mind, the yield for each of the three bonds must change by 100 basis points for the duration measure to be useful. (In other words, there must be a parallel shift in the yield curve.) This is a *critical assumption* and its importance cannot be overemphasized.

An alternative procedure for calculating the duration of a portfolio is to calculate the dollar price change for a given number of basis points for each security in the portfolio and then add up all the price changes. Dividing the total of the price changes by the initial market value of the portfolio produces a percentage price change that can be adjusted to obtain the portfolio's duration.

For example, consider the 3-bond portfolio shown above. Suppose that we calculate the dollar price change for each bond in the portfolio based on its respective duration for a 50 basis point change in yield. We would then have:

Bond	Market value	Duration	Change in value for 50 bp yield change
10% 5-year	\$4,000,000	3.861	\$77,220
8% 15-year	4,231,375	8.047	170,249
14% 30-year	1,378,586	9.168	63,194
Total			\$310,663

Thus, a 50 basis point change in all rates changes the market value of the 3-bond portfolio by \$310,663. Since the market value of the portfolio is \$9,609,961, a 50 basis point change produced a change in value of 3.23% (\$310,663 divided by \$9,609,961). Since duration is the approximate percentage change for a 100 basis point change in rates, this means that the portfolio duration is 6.46 (found by doubling 3.23). This is essentially the same value for the portfolio's duration as found earlier.

V. CONVEXITY ADJUSTMENT

The duration measure indicates that regardless of whether interest rates increase or decrease, the approximate percentage price change is the same. However, as we noted earlier, this is not consistent with Property 3 of a bond's price volatility. Specifically, while for small changes in yield the percentage price change will be the same for an increase or decrease in yield, for large changes in yield this is not true. This suggests that duration is only a good approximation of the percentage price change for small changes in yield.

We demonstrated this property earlier using a 9% 20-year bond selling to yield 6% with a duration of 10.66. For a 10 basis point change in yield, the estimate was accurate for both an increase or decrease in yield. However, for a 200 basis point change in yield, the approximate percentage price change was off *considerably*.

The reason for this result is that duration is in fact a first (linear) approximation for a small change in yield.⁹ The approximation can be improved by using a second approximation. This approximation is referred to as the "convexity adjustment." It is used to approximate the change in price that is not explained by duration.

The formula for the convexity adjustment to the percentage price change is

$$\text{Convexity adjustment to the percentage price change} = C \times (\Delta y_*)^2 \times 100 \quad (4)$$

where Δy_* = the change in yield for which the percentage price change is sought and

$$C = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2} \quad (5)$$

The notation is the same as used in equation (1) for duration.¹⁰

For example, for our hypothetical 9% 20-year bond selling to yield 6%, we know from Section IV A that for a 20 basis point change in yield ($\Delta y = 0.002$):

$$V_0 = 134.6722, V_- = 137.5888, \text{ and } V_+ = 131.8439$$

Substituting these values into the formula for C :

$$C = \frac{131.8439 + 137.5888 - 2(134.6722)}{2(134.6722)(0.002)^2} = 81.95$$

Suppose that a convexity adjustment is sought for the approximate percentage price change for our hypothetical 9% 20-year bond for a change in yield of 200 basis points. That is, in equation (4), Δy_* is 0.02. Then the convexity adjustment is

$$81.95 \times (0.02)^2 \times 100 = 3.28\%$$

If the yield decreases from 6% to 4%, the convexity adjustment to the percentage price change based on duration would also be 3.28%.

⁹The reason it is a linear approximation can be seen in Exhibit 15 where the tangent line is used to estimate the new price. That is, a straight line is being used to approximate a non-linear (i.e., convex) relationship.

¹⁰See **footnote 5** for the difference between Δy in the formula for C and Δy_* in equation (4).

The approximate percentage price change based on duration and the convexity adjustment is found by adding the two estimates. So, for example, if yields change from 6% to 8%, the estimated percentage price change would be:

Estimated change using duration	= -21.32%
Convexity adjustment	= <u>+3.28%</u>
Total estimated percentage price change	= -18.04%

The actual percentage price change is -18.40%.

For a decrease of 200 basis points, from 6% to 4%, the approximate percentage price change would be as follows:

Estimated change using duration	= +21.32%
Convexity adjustment	= <u>+3.28%</u>
Total estimated percentage price change	= +24.60%

The actual percentage price change is +25.04%. Thus, duration *combined* with the convexity adjustment does a better job of estimating the sensitivity of a bond's price change to large changes in yield (i.e., better than using duration alone).

A. Positive and Negative Convexity Adjustment

Notice that when the convexity adjustment is positive, we have the situation described earlier that the gain is greater than the loss for a given large change in rates. That is, the bond exhibits positive convexity. We can see this in the example above. However, if the convexity adjustment is negative, we have the situation where the loss will be greater than the gain. For example, suppose that a callable bond has an effective duration of 4 and a convexity adjustment for a 200 basis point change of -1.2%.

The bond then exhibits the negative convexity property illustrated in Exhibit 11. The approximate percentage price change after adjusting for convexity is:

Estimated change using duration	= -8.0%
Convexity adjustment	= <u>-1.2%</u>
Total estimated percentage price change	= -9.2%

For a decrease of 200 basis points, the approximate percentage price change would be as follows:

Estimated change using duration	= +8.0%
Convexity adjustment	= <u>-1.2%</u>
Total estimated percentage price change	= +6.8%

Notice that the loss is greater than the gain—a property called negative convexity that we discussed in Section III and illustrated in Exhibit 11.

B. Modified and Effective Convexity Adjustment

The prices used in computing C in equation (4) to calculate the convexity adjustment can be obtained by assuming that, when the yield changes, the expected cash flows either do not change or they do change. In the former case, the resulting convexity is referred to as **modified convexity adjustment**. (Actually, in the industry, convexity adjustment is not qualified by the adjective “modified.”) In contrast, **effective convexity adjustment** assumes that the cash flows change when yields change. This is the same distinction made for duration.

As with duration, there is little difference between a modified convexity adjustment and an effective convexity adjustment for option-free bonds. However, for bonds with embedded options, there can be quite a difference between the calculated modified convexity adjustment and an effective convexity adjustment. In fact, for all option-free bonds, either convexity adjustment will have a positive value. For bonds with embedded options, the calculated effective convexity adjustment can be negative when the calculated modified convexity adjustment is positive.

VI. PRICE VALUE OF A BASIS POINT

Some managers use another measure of the price volatility of a bond to quantify interest rate risk—the **price value of a basis point** (PVBP). This measure, also called the **dollar value of an 01** (DV01), is the absolute value of the change in the price of a bond for a 1 basis point change in yield. That is,

$$\text{PVBP} = |\text{initial price} - \text{price if yield is changed by 1 basis point}|$$

Does it make a difference if the yield is increased or decreased by 1 basis point? It does not because of Property 2—the change will be about the same for a small change in basis points.

To illustrate the computation, let’s use the values in Exhibit 4. If the initial yield is 6%, we can compute the PVBP by using the prices for either the yield at 5.99% or 6.01%. The PVBP for both for each bond is shown below:

Coupon	6.0%	6.0%	9.0%	9.0%
Maturity	5	20	5	20
Initial price	\$100.0000	\$100.0000	\$112.7953	\$134.6722
Price at 5.99%	100.0427	100.1157	112.8412	134.8159
PVBP at 5.99%	\$0.0427	\$0.1157	\$0.0459	\$0.1437
Price at 6.01%	99.9574	99.8845	112.7494	134.5287
PVBP at 6.01%	\$0.0426	\$0.1155	\$0.0459	\$0.1435

The PVBP is related to duration. In fact, PVBP is simply a special case of dollar duration described in Chapter 2. We know that the duration of a bond is the approximate percentage price change for a 100 basis point change in interest rates. We also know how to compute the approximate percentage price change for any number of basis points given a bond’s duration using equation (2). Given the initial price and the approximate percentage price change for 1 basis point, we can compute the change in price for a 1 basis point change in rates.

For example, consider the 9% 20-year bond. The duration for this bond is 10.66. Using equation (2), the approximate percentage price change for a 1 basis point increase in interest rates (i.e., $\Delta y = 0.0001$), ignoring the negative sign in equation (2), is:

$$10.66 \times (0.0001) \times 100 = 0.1066\%$$

Given the initial price of 134.6722, the dollar price change estimated using duration is

$$0.1066\% \times 134.6722 = \$0.1435$$

This is the same price change as shown above for a PVBP for this bond. Below is (1) the PVBP based on a 1 basis point increase for each bond and (2) the estimated price change using duration for a 1 basis point increase for each bond:

Coupon	6.0%	6.0%	9.0%	9.0%
Maturity	5	20	5	20
PVBP for 1 bp increase	\$0.0426	\$0.1155	\$0.0459	\$0.1435
Duration of bond	4.2700	11.5600	4.0700	10.6600
Duration estimate	\$0.0427	\$0.1156	\$0.0459	\$0.1436

VII. THE IMPORTANCE OF YIELD VOLATILITY

What we have not considered thus far is the volatility of interest rates. For example, as we explained in Chapter 2, all other factors equal, the higher the coupon rate, the lower the price volatility of a bond to changes in interest rates. In addition, the higher the level of yields, the lower the price volatility of a bond to changes in interest rates. This is illustrated in Exhibit 19 which shows the price/yield relationship for an option-free bond. When the yield level is high (Y_H , for example, in the exhibit), a change in interest rates does not produce a large change in the initial price. For example, as yields change from Y_H to Y_H'' , the price changes a *small* amount from P_H to P_H' . However, when the yield level is low and changes (Y_L to Y_L' , for example, in the exhibit), a change in interest rates of the same number of basis points as Y_H to Y_H'' produces a *large* change in the initial price (P_L to P_L').

This can also be cast in terms of duration properties: the higher the coupon, the lower the duration; the higher the yield level, the lower the duration. Given these two properties, a 10-year non-investment grade bond has a lower duration than a current coupon 10-year Treasury note since the former has a higher coupon rate and trades at a higher yield level. Does this mean that a 10-year non-investment grade bond has less interest rate risk than a current coupon 10-year Treasury note? Consider also that a 10-year Swiss government bond has a lower coupon rate than a current coupon 10-year U.S. Treasury note and trades at a lower yield level. Therefore, a 10-year Swiss government bond will have a higher duration than a current coupon 10-year Treasury note. Does this mean that a 10-year Swiss government bond has greater interest rate risk than a current coupon 10-year U.S. Treasury note? The missing link is the relative volatility of rates, which we shall refer to as **yield volatility** or **interest rate volatility**.

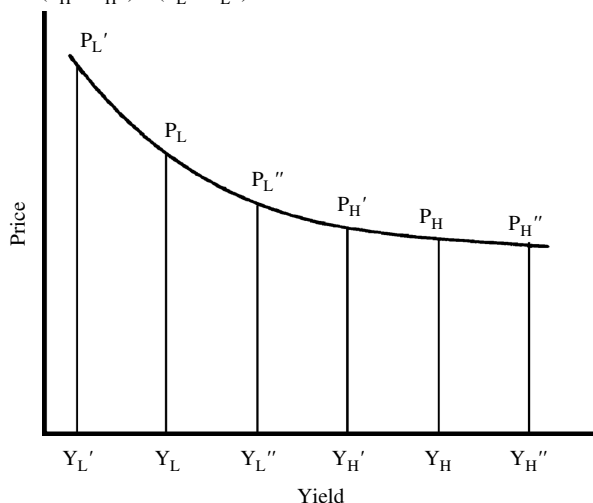
The greater the expected yield volatility, the greater the interest rate risk for a given duration and current value of a position. In the case of non-investment grade bonds, while their durations are less than current coupon Treasuries of the same maturity, the yield volatility

EXHIBIT 19 The Effect of Yield Level on Price
Volatility—Option-Free Bond

$$(Y_H' - Y_H) = (Y_H - Y_H'') = (Y_L' - Y_L) (Y_L - Y_L'')$$

$$(P_H - P_H') < (P_L - P_L') \text{ and}$$

$$(P_H - P_H'') < (P_L - P_L'')$$



of non-investment grade bonds is greater than that of current coupon Treasuries. For the 10-year Swiss government bond, while the duration is greater than for a current coupon 10-year U.S. Treasury note, the yield volatility of 10-year Swiss bonds is considerably less than that of 10-year U.S. Treasury notes.

Consequently, to measure the exposure of a portfolio or position to interest rate changes, it is necessary to measure yield volatility. This requires an understanding of the fundamental principles of probability distributions. The measure of yield volatility is the standard deviation of yield changes. As we will see, depending on the underlying assumptions, there could be a wide range for the yield volatility estimates.

A framework that ties together the price sensitivity of a bond position to interest rate changes and yield volatility is the **value-at-risk (VaR) framework**. Risk in this framework is defined as the maximum estimated loss in market value of a given position that is expected to occur with a specified probability.

TERM STRUCTURE AND VOLATILITY OF INTEREST RATES

I. INTRODUCTION

Market participants pay close attention to yields on Treasury securities. An analysis of these yields is critical because they are used to derive interest rates which are used to value securities. Also, they are benchmarks used to establish the minimum yields that investors want when investing in a non-Treasury security. We distinguish between the on-the-run (i.e., the most recently auctioned Treasury securities) Treasury yield curve and the term structure of interest rates. The on-the-run Treasury yield curve shows the relationship between the yield for on-the-run Treasury issues and maturity. The term structure of interest rates is the relationship between the theoretical yield on zero-coupon Treasury securities and maturity. The yield on a zero-coupon Treasury security is called the Treasury spot rate. The term structure of interest rates is thus the relationship between Treasury spot rates and maturity. The importance of this distinction between the Treasury yield curve and the Treasury spot rate curve is that it is the latter that is used to value fixed-income securities.

In Chapter 6 we demonstrated how to derive the Treasury spot rate curve from the on-the-run Treasury issues using the method of bootstrapping and then how to obtain an arbitrage-free value for an option-free bond. In this chapter we will describe other methods to derive the Treasury spot rates. In addition, we explained that another benchmark that is being used by practitioners to value securities is the swap curve. We discuss the swap curve in this chapter.

In Chapter 4, the theories of the term structure of interest rates were explained. Each of these theories seeks to explain the shape of the yield curve. Then, in Chapter 6, the concept of forward rates was explained. In this chapter, we explain the role that forward rates play in the theories of the term structure of interest rates. In addition, we critically evaluate one of these theories, the pure expectations theory, because of the economic interpretation of forward rates based on this theory.

We also mentioned the role of interest rate volatility or yield volatility in valuing securities and in measuring interest rate exposure of a bond. In the analytical chapters we will continue to see the importance of this measure. Specifically, we will see the role of interest rate volatility in valuing bonds with embedded options, valuing mortgage-backed and certain asset-backed

securities, and valuing derivatives. Consequently, in this chapter, we will explain how interest rate volatility is estimated and the issues associated with computing this measure.

In the opening sections of this chapter we provide some historical information about the Treasury yield curve. In addition, we set the stage for understanding bond returns by looking at empirical evidence on some of the factors that drive returns.

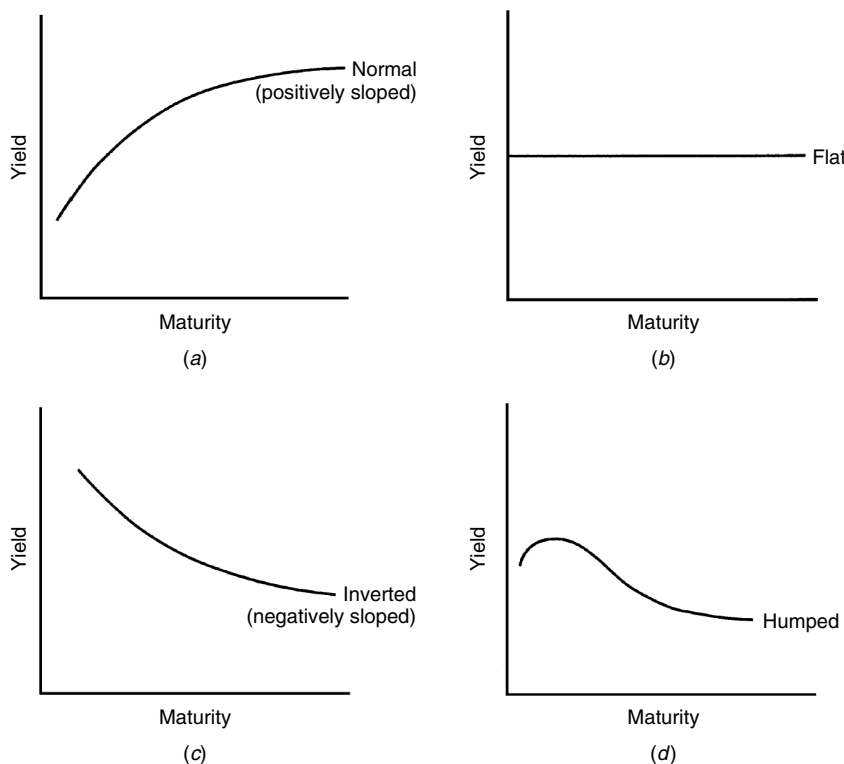
II. HISTORICAL LOOK AT THE TREASURY YIELD CURVE

The yields offered on Treasury securities represent the base interest rate or minimum interest rate that investors demand if they purchase a non-Treasury security. For this reason market participants continuously monitor the yields on Treasury securities, particularly the yields of the on-the-run issues. In this chapter we will discuss the historical relationship that has been observed between the yields offered on on-the-run Treasury securities and maturity (i.e., the yield curve).

A. Shape of the Yield Curve

Exhibit 1 shows some yield curves that have been observed in the U.S. Treasury market and in the government bond market of other countries. Four shapes have been observed. The most

EXHIBIT 1 Yield Curve Shapes



common relationship is a yield curve in which the longer the maturity, the higher the yield as shown in panel a. That is, investors are rewarded for holding longer maturity Treasuries in the form of a higher potential yield. This shape is referred to as a **normal** or **positively sloped yield curve**. A **flat yield curve** is one in which the yield for all maturities is approximately equal, as shown in panel b. There have been times when the relationship between maturities and yields was such that the longer the maturity the lower the yield. Such a downward sloping yield curve is referred to as an **inverted** or a **negatively sloped yield curve** and is shown in panel c. In panel d, the yield curve shows yields increasing with maturity for a range of maturities and then the yield curve becoming inverted. This is called a **humped yield curve**.

Market participants talk about the difference between long-term Treasury yields and short-term Treasury yields. The spread between these yields for two maturities is referred to as the steepness or slope of the yield curve. There is no industrywide accepted definition of the maturity used for the long-end and the maturity used for the short-end of the yield curve. Some market participants define the slope of the yield curve as the difference between the 30-year yield and the 3-month yield. Other market participants define the slope of the yield curve as the difference between the 30-year yield and the 2-year yield. The more common practice is to use the spread between the 30-year and 2-year yield. While as of June 2003 the U.S. Treasury has suspended the issuance of the 30-year Treasury issue, most market participants view the benchmark for the 30-year issue as the last issued 30-year bond which as of June 2003 had a maturity of approximately 27 years. (Most market participants use this issue as a barometer of long-term interest rates; however, it should be noted that in daily conversations and discussions of bond market developments, the 10-year Treasury rate is frequently used as barometer of long-term interest rates.)

The slope of the yield curve varies over time. For example, in the U.S., over the period 1989 to 1999, the slope of the yield curve as measured by the difference between the 30-year Treasury yield and the 2-year Treasury yield was steepest at 348 basis points in September and October 1992. It was negative—that is, the 2-year Treasury yield was greater than the 30-year Treasury yield—for most of 2000. In May 2000, the 2-year Treasury yield exceeded the 30-year Treasury yield by 65 basis points (i.e., the slope of the yield curve was -65 basis points).

It should be noted that not all sectors of the bond market view the slope of the yield curve in the same way. The mortgage sector of the bond—which we cover in Chapter 10—views the yield curve in terms of the spread between the 10-year and 2-year Treasury yields. This is because it is the 10-year rates that affect the pricing and refinancing opportunities in the mortgage market.

Moreover, it is not only within the U.S. bond market that there may be different interpretations of what is meant by the slope of the yield curve, but there are differences across countries. In Europe, the only country with a liquid 30-year government market is the United Kingdom. In European markets, it has become increasingly common to measure the slope in terms of the swap curve (in particular, the euro swap curve) that we will cover later in this chapter.

Some market participants break up the yield curve into a “short end” and “long end” and look at the slope of the short end and long end of the yield curve. Once again, there is no universal consensus that defines the maturity break points. In the United States, it is common for market participants to refer to the short end of the yield curve as up to the 10-year maturity and the long end as from the 10-year maturity to the 30-year maturity. Using the 2-year as the shortest maturity, the slope of the short end of the yield curve is then the difference between the 10-year Treasury yield and the 2-year Treasury yield. The slope of the long end of the yield curve is the difference between the 30-year Treasury yield and the 10-year Treasury yield.

Historically, the long end of the yield curve has been flatter than the short-end of the yield curve. For example, in October 1992 when the slope of the yield curve was the greatest at 348 basis points, the slope of the long end of the yield curve was only 95 basis points.

Market participants often decompose the yield curve into three maturity sectors: short, intermediate, and long. Again, there is no consensus as to what the maturity break points are and those break points can differ by sector and by country. In the United States, a common breakdown has the 1–5 year sector as the short end (ignoring maturities less than 1 year), the 5–10 year sector as the intermediate end, and greater than 10-year maturities as the long end.¹ In Continental Europe where there is little issuance of bonds with a maturity greater than 10 years, the long end of the yield sector is the 10-year sector.

B. Yield Curve Shifts

A shift in the yield curve refers to the relative change in the yield for each Treasury maturity. A **parallel shift in the yield curve** refers to a shift in which the change in the yield for all maturities is the same. A **nonparallel shift in the yield curve** means that the yield for different maturities does not change by the same number of basis points. Both of these shifts are graphically portrayed in Exhibit 2.

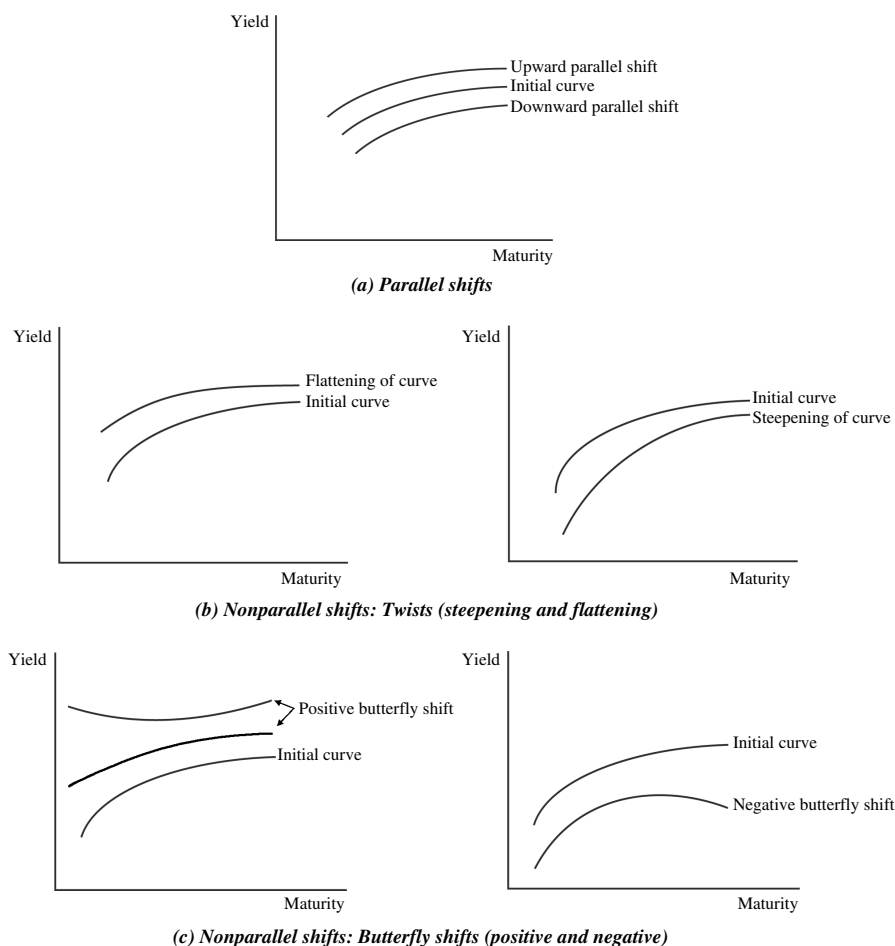
Historically, two types of nonparallel yield curve shifts have been observed: (1) a twist in the slope of the yield curve and (2) a change in the humpedness or curvature of the yield curve. A **twist in the slope of the yield curve** refers to a flattening or steepening of the yield curve. A **flattening of the yield curve** means that the slope of the yield curve (i.e., the spread between the yield on a long-term and short-term Treasury) has decreased; a **steepening of the yield curve** means that the slope of the yield curve has increased. This is depicted in panel b of Exhibit 2.

The other type of nonparallel shift is a change in the curvature or humpedness of the yield curve. This type of shift involves the movement of yields at the short maturity and long maturity sectors of the yield curve relative to the movement of yields in the intermediate maturity sector of the yield curve. Such nonparallel shifts in the yield curve that change its curvature are referred to as **butterfly shifts**. The name comes from viewing the three maturity sectors (short, intermediate, and long) as three parts of a butterfly. Specifically, the intermediate maturity sector is viewed as the body of the butterfly and the short maturity and long maturity sectors are viewed as the wings of the butterfly.

A **positive butterfly** means that the yield curve becomes less humped (i.e., has less curvature). This means that if yields increase, for example, the yields in the short maturity and long maturity sectors increase more than the yields in the intermediate maturity sector. If yields decrease, the yields in the short and long maturity sectors decrease less than the intermediate maturity sector. A **negative butterfly** means the yield curve becomes more humped (i.e., has more curvature). So, if yields increase, for example, yields in the intermediate maturity sector will increase more than yields in the short maturity and long maturity sectors. If, instead, yields decrease, a negative butterfly occurs when yields in the intermediate maturity sector decrease less than the short maturity and long maturity sectors. Butterfly shifts are depicted in panel c of Exhibit 2.

¹Index constructors such as Lehman Brothers when constructing maturity sector indexes define “short-term sector” as up to three years, the “intermediate sector” as maturities greater than three years but less than 10 years (note the overlap with the short-term sector), and the “long-term sector” as greater than 10 years.

EXHIBIT 2 Types of Yield Curve Shifts



Historically, these three types of shifts in the yield curve have not been found to be independent. The two most common types of shifts have been (1) a downward shift in the yield curve combined with a steepening of the yield curve and (2) an upward shift in the yield curve combined with a flattening of the yield curve. Positive butterfly shifts tend to be associated with an upward shift in yields and negative butterfly shifts with a downward shift in yields. Another way to state this is that yields in the short-term sector tend to be more volatile than yields in the long-term sector.

III. TREASURY RETURNS RESULTING FROM YIELD CURVE MOVEMENTS

As we discussed in Chapter 6, a yield measure is a promised return if certain assumptions are satisfied; but total return (return from coupons and price change) is a more appropriate measure of the potential return from investing in a Treasury security. The total return for a

short investment horizon depends critically on how interest rates change, reflected by how the yield curve changes.

There have been several published and unpublished studies of how changes in the shape of the yield curve affect the total return on Treasury securities. The first such study by two researchers at Goldman Sachs (Robert Litterman and José Scheinkman) was published in 1991.² The results reported in more recent studies support the findings of the Litterman-Scheinkman study so we will just discuss their findings. Litterman and Scheinkman found that three factors explained historical returns for zero-coupon Treasury securities for all maturities. The first factor was changes in the level of rates, the second factor was changes in the slope of the yield curve, and the third factor was changes in the curvature of the yield curve.

Litterman and Scheinkman employed regression analysis to determine the relative contribution of these three factors in explaining the returns on zero-coupon Treasury securities of different maturities. They determined the importance of each factor by its coefficient of determination, popularly referred to as the “ R^2 .” In general, the R^2 measures the percentage of the variance in the dependent variable (i.e., the total return on the zero-coupon Treasury security in their study) explained by the independent variables (i.e., the three factors).³ For example, an R^2 of 0.8 means that 80% of the variation of the return on a zero-coupon Treasury security is explained by the three factors. Therefore, 20% of the variation of the return is not explained by these three factors. The R^2 will have a value between 0% and 100%. In the Litterman-Scheinkman study, the R^2 was very high for all maturities, meaning that the three factors had a very strong explanatory power.

The first factor, representing changes in the level of rates, holding all other factors constant (in particular, yield curve slope), had the greatest explanatory power for all the maturities, averaging about 90%. The implication is that the most important factor that a manager of a Treasury portfolio should control for is exposure to changes in the level of interest rates. For this reason it is important to have a way to measure or quantify this risk. Duration is in fact the measure used to quantify exposure to a parallel shift in the yield curve.

The second factor, changes in the yield curve slope, was the second largest contributing factor. The average relative contribution for all maturities was 8.5%. Thus, changes in the yield curve slope was, on average, about one tenth as significant as changes in the level of rates. While the relative contribution was only 8.5%, this can still have a significant impact on the return for a Treasury portfolio and a portfolio manager must control for this risk. We briefly explained in Chapter 2 how a manager can do this using key rate duration and will discuss this further in this chapter.

The third factor, changes in the curvature of the yield curve, contributed relatively little to explaining historical returns for Treasury zero-coupon securities.

IV. CONSTRUCTING THE THEORETICAL SPOT RATE CURVE FOR TREASURIES

Our focus thus far has been on the shape of the Treasury yield curve. In fact, often the financial press in its discussion of interest rates focuses on the Treasury yield curve. However,

²Robert Litterman and José Scheinkman, “Common Factors Affecting Bond Returns,” *Journal of Fixed Income* (June 1991), pp. 54–61.

³For a further explanation of the coefficient of determination, see Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, and David E. Runkle, *Quantitative Methods for Investment Analysis* (Charlottesville, VA: Association for Investment Management and Research, 2002), pp. 388–390.

as explained in Chapter 5, it is the default-free spot rate curve as represented by the Treasury spot rate curve that is used in valuing fixed-income securities. But how does one obtain the default-free spot rate curve? This curve can be constructed from the yields on Treasury securities. The Treasury issues that are candidates for inclusion are:

1. Treasury coupon strips
2. on-the-run Treasury issues
3. on-the-run Treasury issues and selected off-the-run Treasury issues
4. all Treasury coupon securities and bills

Once the securities that are to be included in the construction of the theoretical spot rate curve are selected, the methodology for constructing the curve must be determined. The methodology depends on the securities included. If Treasury coupon strips are used, the procedure is simple since the observed yields are the spot rates. If the on-the-run Treasury issues with or without selected off-the-run Treasury issues are used, then the methodology of bootstrapping is used.

Using an estimated Treasury par yield curve, bootstrapping is a repetitive technique whereby the yields prior to some maturity, same m , are used to obtain the spot rate for year m . For example, suppose that the yields on the par yield curve are denoted by $y_1, \dots, y_T$ where the subscripts denote the time periods. Then the yield for the first period, y_1 , is the spot rate for the first period. Let the first period spot rate be denoted as s_1 . Then y_2 and s_1 can be used to derive s_2 using arbitrage arguments. Next, y_3 , s_1 , and s_2 are used to derive s_3 using arbitrage arguments. The process continues until all the spot rates are derived, $s_1, \dots, s_m$.

In selecting the universe of securities used to construct a default-free spot rate curve, one wants to make sure that the yields are not biased by any of the following: (1) default, (2) embedded options, (3) liquidity, and (4) pricing errors. To deal with default, U.S. Treasury securities are used. Issues with embedded options are avoided because the market yield reflects the value of the embedded options. In the U.S. Treasury market, there are only a few callable bonds so this is not an issue. In other countries, however, there are callable and puttable government bonds. Liquidity varies by issue. There are U.S. Treasury issues that have less liquidity than bonds with a similar maturity. In fact, there are some issues that have extremely high liquidity because they are used by dealers in repurchase agreements. Finally, in some countries the trading of certain government bonds issues is limited, resulting in estimated prices that may not reflect the true price.

Given the theoretical spot rate for each maturity, there are various statistical techniques that are used to create a continuous spot rate curve. A discussion of these statistical techniques is a specialist topic.

A. Treasury Coupon Strips

It would seem simplest to use the observed yield on Treasury coupon strips to construct an actual spot rate curve because there are three problems with using the observed rates on Treasury strips. First, the liquidity of the strips market is not as great as that of the Treasury coupon market. Thus, the observed rates on strips reflect a premium for liquidity.

Second, the tax treatment of strips is different from that of Treasury coupon securities. Specifically, the accrued interest on strips is taxed even though no cash is received by the investor. Thus, they are negative cash flow securities to taxable entities, and, as a result, their yield reflects this tax disadvantage.

Finally, there are maturity sectors where non-U.S. investors find it advantageous to trade off yield for tax advantages associated with a strip. Specifically, certain foreign tax authorities allow their citizens to treat the difference between the maturity value and the purchase price as a capital gain and tax this gain at a favorable tax rate. Some will grant this favorable treatment only when the strip is created from the principal rather than the coupon. For this reason, those who use Treasury strips to represent theoretical spot rates restrict the issues included to coupon strips.

B. On-the-Run Treasury Issues

The on-the-run Treasury issues are the most recently auctioned issues of a given maturity. In the U.S., these issues include the 1-month, 3-month, and 6-month Treasury bills, and the 2-year, 5-year, and 10-year Treasury notes. Treasury bills are zero-coupon instruments; the notes are coupon securities.⁴

There is an observed yield for each of the on-the-run issues. For the coupon issues, these yields are not the yields used in the analysis when the issue is not trading at par. Instead, for each on-the-run coupon issue, the estimated yield necessary to make the issue trade at par is used. The resulting on-the-run yield curve is called the **par coupon curve**. The reason for using securities with a price of par is to eliminate the effect of the tax treatment for securities selling at a discount or premium. The differential tax treatment distorts the yield.

C. On-the-Run Treasury Issues and Selected Off-the-Run Treasury Issues

One of the problems with using just the on-the-run issues is the large gap between maturities, particularly after five years. To mitigate this problem, some dealers and vendors use selected off-the-run Treasury issues. Typically, the issues used are the 20-year issue and 25-year issue.⁵ Given the par coupon curve including any off-the-run selected issues, a linear interpolation method is used to fill in the gaps for the other maturities. The bootstrapping method is then used to construct the theoretical spot rate curve.

D. All Treasury Coupon Securities and Bills

Using only on-the-run issues and a few off-the-run issues fails to recognize the information embodied in Treasury prices that are not included in the analysis. Thus, some market participants argue that it is more appropriate to use all outstanding Treasury coupon securities and bills to construct the theoretical spot rate curve. Moreover, a common practice is to filter the Treasury securities universe to eliminate securities that are on special (trading at a lower yield than their true yield) in the repo market.⁶

⁴At one time, the Department of the Treasury issued 3-year notes, 7-year notes, 15-year bonds, 20-year bonds, and 30-year bonds.

⁵See, for example, Philip H. Galdi and Shenglin Lu, *Analyzing Risk and Relative Value of Corporate and Government Securities*, Merrill Lynch & Co., Global Securities Research & Economics Group, Fixed Income Analytics, 1997, p. 11.

⁶There must also be an adjustment for what is known as the “specials effect.” This has to do with a security trading at a lower yield than its true yield because of its value in the repurchase agreement market. As explained, in a repurchase agreement, a security is used as collateral for a loan. If the security

When all coupon securities and bills are used, methodologies more complex than bootstrapping must be employed to construct the theoretical spot rate curve since there may be more than one yield for each maturity. There are various methodologies for fitting a curve to the points when all the Treasury securities are used. The methodologies make an adjustment for the effect of taxes.⁷ A discussion of the various methodologies is a specialist topic.

V. THE SWAP CURVE (LIBOR CURVE)

In the United States it is common to use the Treasury spot rate curve for purposes of valuation. In other countries, either a government spot rate curve is used (if a liquid market for the securities exists) or the swap curve is used (or as explained shortly, the *LIBOR curve*). LIBOR is the London interbank offered rate and is the interest rate which major international banks offer each other on Eurodollar certificates of deposit (CD) with given maturities. The maturities range from overnight to five years. So, references to “3-month LIBOR” indicate the interest rate that major international banks are offering to pay to other such banks on a CD that matures in three months. A swap curve can be constructed that is unique to a country where there is a swap market for converting fixed cash flows to floating cash flows in that country’s currency.

A. Elements of a Swap and a Swap Curve

To discuss a swap curve, we need the basics of a generic (also called a “plain vanilla” interest rate) swap. In a generic interest rate swap two parties are exchanging cash flows based on a **notional amount** where (1) one party is paying fixed cash flows and receiving floating cash flows and (2) the other party is paying floating cash flows and receiving fixed cash flows. It is called a “swap” because the two parties are “swapping” payments: (1) one party is paying a floating rate and receiving a fixed rate and (2) the other party is paying a fixed rate and receiving a floating rate. While the swap is described in terms of a “rate,” the amount the parties exchange is expressed in terms of a currency and determined by using the notional amount as explained below.

For example, suppose the swap specifies that (1) one party is to pay a fixed rate of 6%, (2) the notional amount is \$100 million, (3) the payments are to be quarterly, and (4) the term of the swap is 7 years. The fixed rate of 6% is called the **swap rate**, or equivalently, the **swap fixed rate**. The swap rate of 6% multiplied by the notional amount of \$100 million gives the amount of the annual payment, \$6 million. If the payment is to be made quarterly, the amount paid each quarter is \$1.5 million (\$6 million/4) and this amount is paid every quarter for the next 7 years.⁸

is one that is in demand by dealers, referred to as “hot collateral” or “collateral on special,” then the borrowing rate is lower if that security is used as collateral. As a result of this favorable feature, a security will offer a lower yield in the market if it is on special so that the investor can finance that security cheaply. As a result, the use of the yield of a security on special will result in a biased yield estimate. The 10-year on-the-run U.S. Treasury issue is typically on special.

⁷See, Oldrich A. Vasicek and H. Gifford Fong, “Term Structure Modeling Using Exponential Splines,” *Journal of Finance* (May 1982), pp. 339–358.

⁸Actually we will see in Chapter 14 that the payments are slightly different each quarter because the amount of the quarterly payment depends on the actual number of days in the quarter.

The floating rate in an interest rate swap can be any short-term interest rate. For example, it could be the rate on a 3-month Treasury bill or the rate on 3-month LIBOR. The most common **reference rate** used in swaps is 3-month LIBOR. When LIBOR is the reference rate, the swap is referred to as a “LIBOR-based swap.”

Consider the swap we just used in our illustration. We will assume that the reference rate is 3-month LIBOR. In that swap, one party is paying a fixed rate of 6% (i.e., the swap rate) and receiving 3-month LIBOR for the next 7 years. Hence, the 7-year swap rate is 6%. But entering into this swap with a swap rate of 6% is equivalent to locking in 3-month LIBOR for 7 years (rolled over on a quarterly basis). So, casting this in terms of 3-month LIBOR, the 7-year maturity rate for 3-month LIBOR is 6%.

So, suppose that the swap rate for the maturities quoted in the swap market are as shown below:

Maturity	Swap rate
2 years	4.2%
3 years	4.6%
4 years	5.0%
5 years	5.3%
6 years	5.7%
7 years	6.0%
8 years	6.2%
9 years	6.4%
10 years	6.5%
15 years	6.7%
30 years	6.8%

This would be the swap curve. But this swap curve is also telling us how much we can lock in 3-month LIBOR for a specified future period. By locking in 3-month LIBOR it is meant that a party that pays the floating rate (i.e., agrees to pay 3-month LIBOR) is locking in a borrowing rate; the party receiving the floating rate is locking in an amount to be received. Because 3-month LIBOR is being exchanged, the swap curve is also called the **LIBOR curve**.

Note that we have not indicated the currency in which the payments are to be made for our hypothetical swap curve. Suppose that the swap curve above refers to swapping U.S. dollars (i.e., the notional amount is in U.S. dollars) from a fixed to a floating (and vice versa). Then the swap curve above would be the U.S. swap curve. If the notional amount was for euros, and the swaps involved swapping a fixed euro amount for a floating euro amount, then it would be the euro swap curve.

Finally, let's look at how the terms of a swap are quoted. Rather than quote a swap rate for a given maturity, the convention in the swap market is to quote a **swap spread**. The spread can be over any benchmark desired, typically a government bond yield. The swap spread is defined as follows for a given maturity:

$$\text{swap spread} = \text{swap rate} - \text{government yield on a bond with the same maturity as the swap}$$

For euro-denominated swaps (i.e., swaps in which the currency in which the payments are made is the euro), the government yield used as the benchmark is the German government bond with the same maturity as the swap.

For example, consider our hypothetical 7-year swap. Suppose that the currency of the swap payments is in U.S. dollars and the estimated 7-year U.S. Treasury yield is 5.4%. Then since the swap rate is 6%, the swap spread is:

$$\text{swap spread} = 6\% - 5.4\% = 0.6\% = 60 \text{ basis points.}$$

Suppose, instead, the swap was denominated in euros and the swap rate is 6%. Also suppose that the estimated 7-year German government bond yield is 5%. Then the swap spread would be quoted as 100 basis points (6%–5%).

Effectively the swap spread reflects the risk of the counterparty to the swap failing to satisfy its obligation. Consequently, it primarily reflects credit risk. Since the counterparty in swaps are typically bank-related entities, the swap spread is a rough indicator of the credit risk of the banking sector. Therefore, the swap rate curve is not a default-free curve. Instead, it is an inter-bank or AA rated curve.

Notice that the swap rate is compared to a government bond yield to determine the swap spread. Why would one want to use a swap curve if a government bond yield curve is available? We answer that question next.

B. Reasons for Increased Use of Swap Curve

Investors and issuers use the swap market for hedging and arbitrage purposes, and the swap curve as a benchmark for evaluating performance of fixed income securities and the pricing of fixed income securities. Since the swap curve is effectively the LIBOR curve and investors borrow based on LIBOR, the swap curve is more useful to funded investors than a government yield curve.

The increased application of the swap curve for these activities is due to its advantages over using the government bond yield curve as a benchmark. Before identifying these advantages, it is important to understand that the drawback of the swap curve relative to the government bond yield curve could be poorer liquidity. In such instances, the swap rates would reflect a liquidity premium. Fortunately, liquidity is not an issue in many countries as the swap market has become highly liquid, with narrow bid-ask spreads for a wide range of swap maturities. In some countries swaps may offer better liquidity than that country's government bond market.

The advantages of the swap curve over a government bond yield curve are:⁹

1. There is almost no government regulation of the swap market. The lack of government regulation makes swap rates across different markets more comparable. In some countries, there are some sovereign issues that offer various tax benefits to investors and, as a result, for global investors it makes comparative analysis of government rates across countries difficult because some market yields do not reflect their true yield.
2. The supply of swaps depends only on the number of counterparties that are seeking or are willing to enter into a swap transaction at any given time. Since there is no underlying government bond, there can be no effect of market technical factors¹⁰ that may result in the yield for a government bond issue being less than its true yield.

⁹See Uri Ron, "A Practical Guide to Swap Curve Construction," Chapter 6 in Frank J. Fabozzi (ed.), *Interest Rate, Term Structure, and Valuation Modeling* (NY: John Wiley & Sons, 2002).

¹⁰For example, a government bond issue being on "special" in the repurchase agreement market.

3. Comparisons across countries of government yield curves is difficult because of the differences in sovereign credit risk. In contrast, the credit risk as reflected in the swaps curve are similar and make comparisons across countries more meaningful than government yield curves. Sovereign risk is not present in the swap curve because, as noted earlier, the swap curve is viewed as an inter-bank yield curve or AA yield curve.
4. There are more maturity points available to construct a swap market than a government bond yield curve. More specifically, what is quoted in the swap market are swap rates for 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, and 30 year maturities. Thus, in the swap market there are 10 market interest rates with a maturity of 2 years and greater. In contrast, in the U.S. Treasury market, for example, there are only three market interest rates for on-the-run Treasuries with a maturity of 2 years or greater (2, 5, and 10 years) and one of the rates, the 10-year rate, may not be a good benchmark because it is often on special in the repo market. Moreover, because the U.S. Treasury has ceased the issuance of 30-year bonds, there is no 30-year yield available.

C. Constructing the LIBOR Spot Rate Curve

In the valuation of fixed income securities, it is not the Treasury yield curve that is used as the basis for determining the appropriate discount rate for computing the present value of cash flows but the Treasury spot rates. The Treasury spot rates are derived from the Treasury yield curve using the bootstrapping process.

Similarly, it is not the swap curve that is used for discounting cash flows when the swap curve is the benchmark but the spot rates. The spot rates are derived from the swap curve in exactly the same way—using the bootstrapping methodology. The resulting spot rate curve is called the **LIBOR spot rate curve**. Moreover, a forward rate curve can be derived from the spot rate curve. The same thing is done in the swap market. The forward rate curve that is derived is called the **LIBOR forward rate curve**. Consequently, if we understand the mechanics of moving from the yield curve to the spot rate curve to the forward rate curve in the Treasury market, there is no reason to repeat an explanation of that process here for the swap market; that is, it is the same methodology, just different yields are used.¹¹

VI. EXPECTATIONS THEORIES OF THE TERM STRUCTURE OF INTEREST RATES

So far we have described the different types of curves that analysts and portfolio managers focus on. The key curve is the spot rate curve because it is the spot rates that are used to value

¹¹The question is what yields are used to construct the swap rate curve. Practitioners use yields from two related markets: the Eurodollar CD futures contract and the swap market. We will not review the Eurodollar CD futures contract here. It is discussed in Chapter 14. For now, the only important fact to note about this contract is that it provides a means for locking in 3-month LIBOR in the future. In fact, it provides a means for doing so for an extended time into the future.

Practitioners use the Eurodollar CD futures rate up to four years to get 3-month LIBOR for every quarter. While there are Eurodollar CD futures contracts that settle further out than four years, for technical reasons (having to do with the convexity of the contract) analysts use only the first four years. (In fact, this actually varies from practitioner to practitioner. Some will use the Eurodollar CD futures from two years up to four years.) For maturities after four years, the swap rates are used to get 3-month LIBOR. As noted above, there is a swap rate for maturities for each year to year 10, and then swap rates for 15 years and 30 years.

the cash flows of a fixed-income security. The spot rate curve is also called the term structure of interest rates, or simply term structure. Now we turn to another potential use of the term structure. Analysts and portfolio managers are interested in knowing if there is information contained in the term structure that can be used in making investment decisions. For this purpose, market participants rely on different theories about the term structure.

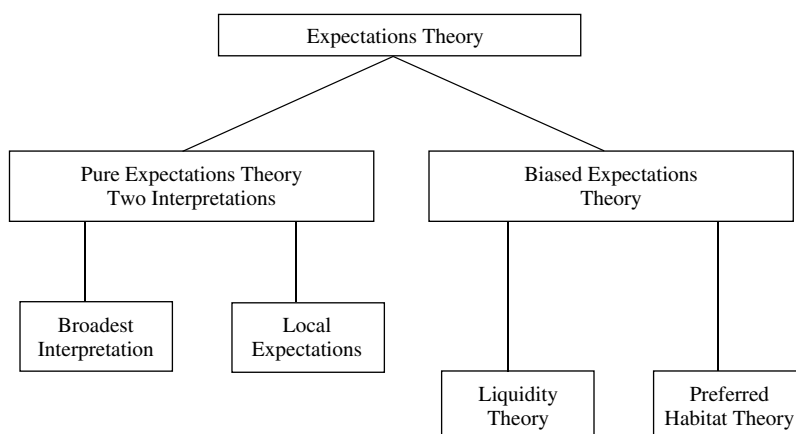
In Chapter 4, we explained four theories of the term structure of interest rates—pure expectations theory, liquidity preference theory, preferred habitat theory, and market segmentation theory. Unlike the market segmentation theory, the first three theories share a hypothesis about the behavior of short-term forward rates and also assume that the forward rates in current long-term bonds are closely related to the market's expectations about future short-term rates. For this reason, the pure expectations theory, liquidity preference theory, and preferred habitat theory are referred to as **expectations theories of the term structure of interest rates**.

What distinguishes these three expectations theories is whether there are systematic factors other than expectations of future interest rates that affect forward rates. The pure expectations theory postulates that no systematic factors other than expected future short-term rates affect forward rates; the liquidity preference theory and the preferred habitat theory assert that there are other factors. Accordingly, the last two forms of the expectations theory are sometimes referred to as **biased expectations theories**. The relationship among the various theories is described below and summarized in Exhibit 3.

A. The Pure Expectations Theory

According to the pure expectations theory, forward rates exclusively represent expected future spot rates. Thus, the entire term structure at a given time reflects the market's current expectations of the family of future short-term rates. Under this view, a rising term structure must indicate that the market expects short-term rates to rise throughout the relevant future. Similarly, a flat term structure reflects an expectation that future short-term rates will be mostly constant, while a falling term structure must reflect an expectation that future short-term rates will decline.

EXHIBIT 3 Expectations Theories of the Term Structure of Interest Rates



1. Drawbacks of the Theory The pure expectations theory suffers from one shortcoming, which, qualitatively, is quite serious. It neglects the risks inherent in investing in bonds. If forward rates were perfect predictors of future interest rates, then the future prices of bonds would be known with certainty. The return over any investment period would be certain and independent of the maturity of the instrument acquired. However, with the uncertainty about future interest rates and, therefore, about future prices of bonds, these instruments become risky investments in the sense that the return over some investment horizon is unknown.

There are two risks that cause uncertainty about the return over some investment horizon. The first is the uncertainty about the price of the bond at the end of the investment horizon. For example, an investor who plans to invest for five years might consider the following three investment alternatives:

Alternative 1: Invest in a 5-year zero-coupon bond and hold it for five years.

Alternative 2: Invest in a 12-year zero-coupon bond and sell it at the end of five years.

Alternative 3: Invest in a 30-year zero-coupon bond and sell it at the end of five years.

The return that will be realized in Alternatives 2 and 3 is not known because the price of each of these bonds at the end of five years is unknown. In the case of the 12-year bond, the price will depend on the yield on 7-year bonds five years from now; and the price of the 30-year bond will depend on the yield on 25-year bonds five years from now. Since forward rates implied in the current term structure for a 7-year bond five years from now and a 25-year bond five years from now are not perfect predictors of the actual future rates, there is uncertainty about the price for both bonds five years from now. Thus, there is interest rate risk; that is, the price of the bond may be lower than currently expected at the end of the investment horizon due to an increase in interest rates. As explained earlier, an important feature of interest rate risk is that it increases with the length of the bond's maturity.

The second risk involves the uncertainty about the rate at which the proceeds from a bond that matures prior to the end of the investment horizon can be reinvested until the maturity date, that is, reinvestment risk. For example, an investor who plans to invest for five years might consider the following three alternative investments:

Alternative 1: Invest in a 5-year zero-coupon bond and hold it for five years.

Alternative 2: Invest in a 6-month zero-coupon instrument and, when it matures, reinvest the proceeds in 6-month zero-coupon instruments over the entire 5-year investment horizon.

Alternative 3: Invest in a 2-year zero-coupon bond and, when it matures, reinvest the proceeds in a 3-year zero-coupon bond.

The risk for Alternatives 2 and 3 is that the return over the 5-year investment horizon is unknown because rates at which the proceeds can be reinvested until the end of the investment horizon are unknown.

2. Interpretations of the Theory There are several interpretations of the pure expectations theory that have been put forth by economists. These interpretations are not exact

equivalents nor are they consistent with each other, in large part because they offer different treatments of the two risks associated with realizing a return that we have just explained.¹²

a. Broadest Interpretation The broadest interpretation of the pure expectations theory suggests that investors expect the return for any investment horizon to be the same, regardless of the maturity strategy selected.¹³ For example, consider an investor who has a 5-year investment horizon. According to this theory, it makes no difference if a 5-year, 12-year, or 30-year bond is purchased and held for five years since the investor expects the return from all three bonds to be the same over the 5-year investment horizon. A major criticism of this very broad interpretation of the theory is that, because of price risk associated with investing in bonds with a maturity greater than the investment horizon, the expected returns from these three very different investments should differ in significant ways.¹⁴

b. Local Expectations Form of the Pure Expectations Theory A second interpretation, referred to as the **local expectations form of the pure expectations theory**, suggests that the return will be the same over a short-term investment horizon starting today. For example, if an investor has a 6-month investment horizon, buying a 1-year, 5-year or 10-year bond will produce the same 6-month return.

To illustrate this, we will use the hypothetical yield curve shown in Exhibit 4. In Chapter 6, we used the yield curve in Exhibit 4 to show how to compute spot rates and forward rates. Exhibit 5 shows all the 6-month forward rates. We will focus on the 1-year, 5-year, and 10-year issues.

Our objective is to look at what happens to the total return over a 6-month investment horizon for the 1-year, 5-year, and 10-year issues if all the 6-month forward rates are realized. Look first at panel a in Exhibit 6. This shows the total return for the 1-year issue. At the end of 6 months, this issue is a 6-month issue. The 6-month forward rate is 3.6%. This means that if the forward rate is realized, the 6-month yield 6 months from now will be 3.6%. Given a 6-month issue that must offer a yield of 3.6% (the 6-month forward rate), the price of this issue will decline from 100 (today) to 99.85265 six months from now. The price must decline because if the 6-month forward rate is realized 6 months from now, the yield increases from 3.3% to 3.6%. The total dollars realized over the 6 months are coupon interest adjusted for the decline in the price. The total return for the 6 months is 3%.

What the local expectations theory asserts is that over the 6-month investment horizon even the 5-year and the 10-year issues will generate a total return of 3% if forward rates are realized. Panels b and c show this to be the case. We need only explain the computation for one of the two issues. Let's use the 5-year issue. The 6-month forward rates are shown in the third column of panel b. Now we apply a few principles discussed in Chapter 6. We demonstrated that to value a security each cash flow should be discounted at the spot rate with the same maturity. We also demonstrated that 6-month forward rates can be used to value the cash flows of a security and that the results will be identical using the forward rates to value a security. For example, consider the cash flow in period 3 for the 5-year issue. The cash flow is \$2.60. The 6-month forward rates are 3.6%, 3.92%, and 5.15%. These are annual rates. So,

¹²These formulations are summarized by John Cox, Jonathan Ingersoll, Jr., and Stephen Ross, "A Re-examination of Traditional Hypotheses About the Term Structure of Interest Rates," *Journal of Finance* (September 1981), pp. 769–799.

¹³F. Lutz, "The Structure of Interest Rates," *Quarterly Journal of Economics* (1940–41), pp. 36–63.

¹⁴Cox, Ingersoll, and Ross, pp. 774–775.

EXHIBIT 4 Hypothetical Treasury Par Yield Curve

Period	Years	Annual yield to maturity (BEY) (%)*	Price	Spot rate (BEY) (%)*
1	0.5	3.00	—	3.0000
2	1.0	3.30	—	3.3000
3	1.5	3.50	100.00	3.5053
4	2.0	3.90	100.00	3.9164
5	2.5	4.40	100.00	4.4376
6	3.0	4.70	100.00	4.7520
7	3.5	4.90	100.00	4.9622
8	4.0	5.00	100.00	5.0650
9	4.5	5.10	100.00	5.1701
10	5.0	5.20	100.00	5.2772
11	5.5	5.30	100.00	5.3864
12	6.0	5.40	100.00	5.4976
13	6.5	5.50	100.00	5.6108
14	7.0	5.55	100.00	5.6643
15	7.5	5.60	100.00	5.7193
16	8.0	5.65	100.00	5.7755
17	8.5	5.70	100.00	5.8331
18	9.0	5.80	100.00	5.9584
19	9.5	5.90	100.00	6.0863
20	10.0	6.00	100.00	6.2169

*The yield to maturity and the spot rate are annual rates. They are reported as bond-equivalent yields. To obtain the semiannual yield or rate, one half the annual yield or annual rate is used.

EXHIBIT 5 Six-Month Forward Rates: The Short-Term Forward Rate Curve (Annualized Rates on a Bond-Equivalent Basis)

Notation	Forward rate	Notation	Forward rate
${}_1f_0$	3.00	${}_1f_{10}$	6.48
${}_1f_1$	3.60	${}_1f_{11}$	6.72
${}_1f_2$	3.92	${}_1f_{12}$	6.97
${}_1f_3$	5.15	${}_1f_{13}$	6.36
${}_1f_4$	6.54	${}_1f_{14}$	6.49
${}_1f_5$	6.33	${}_1f_{15}$	6.62
${}_1f_6$	6.23	${}_1f_{16}$	6.76
${}_1f_7$	5.79	${}_1f_{17}$	8.10
${}_1f_8$	6.01	${}_1f_{18}$	8.40
${}_1f_9$	6.24	${}_1f_{19}$	8.72

half these rates are 1.8%, 1.96%, and 2.575%. The present value of \$2.60 using the 6-month forward is:

$$\frac{\$2.60}{(1.018)(1.0196)(1.02575)} = \$2.44205$$

This is the present value shown in the third column of panel b. In a similar manner, all of the other present values in the third column are computed. The arbitrage-free value for this 5-year issue 6 months from now (when it is a 4.5-year issue) is 98.89954. The total

EXHIBIT 6 Total Return Over 6-Month Investment Horizon if 6-Month Forward Rates Are Realized

a: Total return on 1-year issue if forward rates are realized

Period	Cash flow (\$)	Six-month forward rate (%)	Price at horizon (\$)
1	101.650	3.60	99.85265
Price at horizon: 99.85265		Total proceeds: 101.5027	
Coupon: 1.65		Total return: 3.00%	

b: Total return on 5-year issue if forward rates are realized

Period	Cash flow (\$)	Six-month forward rate (%)	Present value (\$)
1	2.60	3.60	2.55403
2	2.60	3.92	2.50493
3	2.60	5.15	2.44205
4	2.60	6.54	2.36472
5	2.60	6.33	2.29217
6	2.60	6.23	2.22293
7	2.60	5.79	2.16039
8	2.60	6.01	2.09736
9	102.60	6.24	80.26096
		Total:	98.89954
Price at horizon: 98.89954		Total proceeds: 101.4995	
Coupon: 2.60		Total return: 3.00%	

c: Total return on 10-year issue if forward rates are realized

Period	Cash flow (\$)	Six-month forward rate (%)	Present value (\$)
1	3.00	3.60	2.94695
2	3.00	3.92	2.89030
3	3.00	5.15	2.81775
4	3.00	6.54	2.72853
5	3.00	6.33	2.64482
6	3.00	6.23	2.56492
7	3.00	5.79	2.49275
8	3.00	6.01	2.42003
9	3.00	6.24	2.34681
10	3.00	6.48	2.27316
11	3.00	6.72	2.19927
12	3.00	6.97	2.12520
13	3.00	6.36	2.05970
14	3.00	6.49	1.99497
15	3.00	6.62	1.93105
16	3.00	6.76	1.86791
17	3.00	8.10	1.79521
18	3.00	8.40	1.72285
19	103.00	8.72	56.67989
		Total:	98.50208
Price at horizon: 98.50208		Total proceeds: 101.5021	
Coupon: 3.00		Total return: 3.00%	

return (taking into account the coupon interest and the loss due to the decline in price from 100) is 3%. Thus, if the 6-month forward rates are realized, all three issues provide a short-term (6-month) return of 3%.¹⁵

c. Forward Rates and Market Consensus We first introduced forward rates in Chapter 6. We saw how various types of forward rates can be computed. That is, we saw how to compute the forward rate for any length of time beginning at any future period of time. So, it is possible to compute the 2-year forward rate beginning 5 years from now or the 3-year forward rate beginning 8 years from now. We showed how, using arbitrage arguments, forward rates can be derived from spot rates.

Earlier, no interpretation was given to the forward rates. The focus was just on how to compute them from spot rates based on arbitrage arguments. Let's provide two interpretations now with a simple illustration. Suppose that an investor has a 1-year investment horizon and has a choice of investing in either a 1-year Treasury bill or a 6-month Treasury bill and rolling over the proceeds from the maturing 6-month issue in another 6-month Treasury bill. Since the Treasury bills are zero-coupon securities, the rates on them are spot rates and can be used to compute the 6-month forward rate six months from now. For example, if the 6-month Treasury bill rate is 5% and the 1-year Treasury bill rate is 5.6%, then the 6-month forward rate six months from now is 6.2%. To verify this, suppose an investor invests \$100 in a 1-year investment. The \$100 investment in a zero-coupon instrument will grow at a rate of 2.8% (one half 5.6%) for two 6-month periods to:

$$\$100 (1.028)^2 = \$105.68$$

If \$100 is invested in a six month zero-coupon instrument at 2.5% (one-half 5%) and the proceeds reinvested at the 6-month forward rate of 3.1% (one-half 6.2%), the \$100 will grow to:

$$\$100 (1.025)(1.031) = \$105.68$$

Thus, the 6-month forward rate generates the same future dollars for the \$100 investment at the end of 1 year.

One interpretation of the forward rate is that it is a "break-even rate." That is, a forward rate is the rate that will make an investor indifferent between investing for the full investment horizon and part of the investment horizon and rolling over the proceeds for the balance of the investment horizon. So, in our illustration, the forward rate of 6.2% can be interpreted as the break-even rate that will make an investment in a 6-month zero-coupon instrument with a yield of 5% rolled-over into another 6-month zero-coupon instrument equal to the yield on a 1-year zero-coupon instrument with a yield of 5.6%.

Similarly, a 2-year forward rate beginning four years from now can be interpreted as the break-even rate that will make an investor indifferent between investing in (1) a 4-year zero-coupon instrument at the 4-year spot rate and rolling over the investment for two more years in a zero-coupon instrument and (2) investing in a 6-year zero-coupon instrument at the 6-year spot rate.

¹⁵It has been demonstrated that the local expectations formulation, which is narrow in scope, is the only interpretation of the pure expectations theory that can be sustained in equilibrium. See Cox, Ingersoll, and Ross, "A Re-examination of Traditional Hypotheses About the Term Structure of Interest Rates."

A second interpretation of the forward rate is that it is a rate that allows the investor to lock in a rate for some future period. For example, consider once again our 1-year investment. If an investor purchases this instrument rather than the 6-month instrument, the investor has locked in a 6.2% rate six months from now regardless of how interest rates change six months from now. Similarly, in the case of a 6-year investment, by investing in a 6-year zero-coupon instrument rather than a 4-year zero-coupon instrument, the investor has locked in the 2-year zero-coupon rate four years from now. That locked in rate is the 2-year forward rate four years from now. The 1-year forward rate five years from now is the rate that is locked in by buying a 6-year zero-coupon instrument rather than investing in a 5-year zero-coupon instrument and reinvesting the proceeds at the end of five years in a 1-year zero-coupon instrument.

There is another interpretation of forward rates. Proponents of the pure expectations theory argue that forward rates reflect the “market’s consensus” of future interest rates. They argue that forward rates can be used to predict future interest rates. A natural question about forward rates is then how well they do at predicting future interest rates. Studies have demonstrated that forward rates do not do a good job at predicting future interest rates.¹⁶ Then, why is it so important to understand forward rates? The reason is that forward rates indicate how an investor’s expectations must differ from the “break-even rate” or the “lock-in rate” when making an investment decision.

Thus, even if a forward rate may not be realized, forward rates can be highly relevant in deciding between two alternative investments. Specifically, if an investor’s expectation about a rate in the future is less than the corresponding forward rate, then he would be better off investing now to lock in the forward rate.

B. Liquidity Preference Theory

We have explained that the drawback of the pure expectations theory is that it does not consider the risks associated with investing in bonds. We know from Chapter 7 that the interest rate risk associated with holding a bond for one period is greater the longer the maturity of a bond. (Recall that duration increases with maturity.)

Given this uncertainty, and considering that investors typically do not like uncertainty, some economists and financial analysts have suggested a different theory—the liquidity preference theory. This theory states that investors will hold longer-term maturities if they are offered a long-term rate higher than the average of expected future rates by a risk premium that is positively related to the term to maturity.¹⁷ Put differently, the forward rates should reflect both interest rate expectations and a “liquidity” premium (really a risk premium), and the premium should be higher for longer maturities.

According to the liquidity preference theory, forward rates will not be an unbiased estimate of the market’s expectations of future interest rates because they contain a liquidity premium. Thus, an upward-sloping yield curve may reflect expectations that future interest rates either (1) will rise, or (2) will be unchanged or even fall, but with a liquidity premium increasing fast enough with maturity so as to produce an upward-sloping yield curve. That is, any shape for either the yield curve or the term structure of interest rates can be explained by the biased expectations theory.

¹⁶Eugene F. Fama, “Forward Rates as Predictors of Future Spot Rates,” *Journal of Financial Economics* Vol. 3, No. 4, 1976, pp. 361–377.

¹⁷John R. Hicks, *Value and Capital* (London: Oxford University Press, 1946), second ed., pp. 141–145.

C. The Preferred Habitat Theory

Another theory, known as the **preferred habitat theory**, also adopts the view that the term structure reflects the expectation of the future path of interest rates as well as a risk premium. However, the preferred habitat theory rejects the assertion that the risk premium must rise uniformly with maturity.¹⁸ Proponents of the preferred habitat theory say that the latter conclusion could be accepted if all investors intend to liquidate their investment at the shortest possible date while all borrowers are anxious to borrow long. This assumption can be rejected since institutions have holding periods dictated by the nature of their liabilities.

The preferred habitat theory asserts that if there is an imbalance between the supply and demand for funds within a given maturity range, investors and borrowers will not be reluctant to shift their investing and financing activities out of their preferred maturity sector to take advantage of any imbalance. However, to do so, investors must be induced by a yield premium in order to accept the risks associated with shifting funds out of their preferred sector. Similarly, borrowers can only be induced to raise funds in a maturity sector other than their preferred sector by a sufficient cost savings to compensate for the corresponding funding risk.

Thus, this theory proposes that the shape of the yield curve is determined by both expectations of future interest rates and a risk premium, positive or negative, to induce market participants to shift out of their preferred habitat. Clearly, according to this theory, yield curves that slope up, down, or flat are all possible.

VII. MEASURING YIELD CURVE RISK

We now know how to construct the term structure of interest rates and the potential information content contained in the term structure that can be used for making investment decisions under different theories of the term structure. Next we look at how to measure exposure of a portfolio or position to a change in the term structure. This risk is referred to as **yield curve risk**.

Yield curve risk can be measured by changing the spot rate for a particular key maturity and determining the sensitivity of a security or portfolio to this change holding the spot rate for the other key maturities constant. The sensitivity of the change in value to a particular change in spot rate is called **rate duration**. There is a rate duration for every point on the spot rate curve. Consequently, there is not one rate duration, but a vector of durations representing each maturity on the spot rate curve. The total change in value if all rates change by the same number of basis points is simply the effective duration of a security or portfolio to a parallel shift in rates. Recall that effective duration measures the exposure of a security or portfolio to a parallel shift in the term structure, taking into account any embedded options.

This rate duration approach was first suggested by Donald Chambers and Willard Carleton in 1988¹⁹ who called it “duration vectors.” Robert Reitano suggested a similar

¹⁸Franco Modigliani and Richard Sutch, “Innovations in Interest Rate Policy,” *American Economic Review* (May 1966), pp. 178–197.

¹⁹Donald Chambers and Willard Carleton, “A Generalized Approach to Duration,” *Research in Finance* 7(1988).

approach in a series of papers and referred to these durations as “partial durations.”²⁰ The most popular version of this approach is that developed by Thomas Ho in 1992.²¹

Ho’s approach focuses on 11 key maturities of the spot rate curve. These rate durations are called **key rate durations**. The specific maturities on the spot rate curve for which a key rate duration is measured are 3 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, 15 years, 20 years, 25 years, and 30 years. Changes in rates between any two key rates are calculated using a linear approximation.

The impact of any type of yield curve shift can be quantified using key rate durations. A level shift can be quantified by changing all key rates by the same number of basis points and determining, based on the corresponding key rate durations, the effect on the value of a portfolio. The impact of a steepening of the yield curve can be found by (1) decreasing the key rates at the short end of the yield curve and determining the positive change in the portfolio’s value using the corresponding key rate durations, and (2) increasing the key rates at the long end of the yield curve and determining the negative change in the portfolio’s value using the corresponding key rate durations.

To simplify the key rate duration methodology, suppose that instead of a set of 11 key rates, there are only three key rates—2 years, 16 years, and 30 years.²² The duration of a zero-coupon security is approximately the number of years to maturity. Thus, the three key rate durations are 2, 16, and 30. Consider the following two \$100 portfolios composed of 2-year, 16-year, and 30-year issues:

Portfolio	2-year issue	16-year issue	30-year issue
I	\$50	\$0	\$50
II	\$0	\$100	\$0

The key rate durations for these three points will be denoted by $D(1)$, $D(2)$, and $D(3)$ and defined as follows:

$D(1)$ = key rate duration for the 2-year part of the curve

$D(2)$ = key rate duration for the 16-year part of the curve

$D(3)$ = key rate duration for the 30-year part of the curve

The key rate durations for the three issues and the duration are as follows:

Issue	$D(1)$	$D(2)$	$D(3)$	Crash duration
2-year	2	0	0	2
16-year	0	16	0	16
30-year	0	0	30	30

²⁰See, for example, Robert R. Reitano, “Non-Parallel Yield Curve Shifts and Durational Leverage,” *Journal of Portfolio Management* (Summer 1990), pp. 62–67, and “A Multivariate Approach to Duration Analysis,” *ARCH* 2(1989).

²¹Thomas S.Y. Ho, “Key Rate Durations: Measures of Interest Rate Risk,” *The Journal of Fixed Income* (September 1992), pp. 29–44.

²²This is the numerical example used by Ho, “Key Rate Durations,” p. 33.

A portfolio's key rate duration is the weighted average of the key rate durations of the securities in the portfolio. The key rate duration and the effective duration for each portfolio are calculated below:

Portfolio I

$$D(1) = (50/100) \times 2 + (0/100) \times 0 + (50/100) \times 0 = 1$$

$$D(2) = (50/100) \times 0 + (0/100) \times 16 + (50/100) \times 0 = 0$$

$$D(3) = (50/100) \times 0 + (0/100) \times 0 + (50/100) \times 30 = 15$$

$$\text{Effective duration} = (50/100) \times 2 + (0/100) \times 16 + (50/100) \times 30 = 16$$

Portfolio II

$$D(1) = (0/100) \times 2 + (100/100) \times 0 + (0/100) \times 0 = 0$$

$$D(2) = (0/100) \times 0 + (100/100) \times 16 + (0/100) \times 0 = 16$$

$$D(3) = (0/100) \times 0 + (100/100) \times 0 + (0/100) \times 30 = 0$$

$$\text{Effective duration} = (0/100) \times 2 + (100/100) \times 16 + (0/100) \times 30 = 16$$

Thus, the key rate durations differ for the two portfolios. However, the effective duration for each portfolio is the same. Despite the same effective duration, the performance of the two portfolios will not be the same for a nonparallel shift in the spot rates. Consider the following three scenarios:

Scenario 1: All spot rates shift down 10 basis points.

Scenario 2: The 2-year key rate shifts up 10 basis points and the 30-year rate shifts down 10 basis points.

Scenario 3: The 2-year key rate shifts down 10 basis points and the 30-year rate shifts up 10 basis points.

Let's illustrate how to compute the estimated total return based on the key rate durations for Portfolio I for scenario 2. The 2-year key rate duration [D(1)] for Portfolio I is 1. For a 100 basis point increase in the 2-year key rate, the portfolio's value will decrease by approximately 1%. For a 10 basis point increase (as assumed in scenario 2), the portfolio's value will decrease by approximately 0.1%. Now let's look at the change in the 30-year key rate in scenario 2. The 30-year key rate duration [D(3)] is 15. For a 100 basis point decrease in the 30-year key rate, the portfolio's value will increase by approximately 15%. For a 10 basis point decrease (as assumed in scenario 2), the increase in the portfolio's value will be approximately 1.5%. Consequently, for Portfolio I in scenario 2 we have:

change in portfolio's value due to 2-year key rate change	-0.1%
change in portfolio's value due to 30-year key rate change	+1.5%
change in portfolio value	+1.4%

In the same way, the total return for both portfolios can be estimated for the three scenarios. The estimated total returns are as follows:

Portfolio	Scenario 1	Scenario 2	Scenario 3
I	1.6%	1.4%	−1.4%
II	1.6%	0%	0%

Thus, only for the parallel yield curve shift (scenario 1) do the two portfolios have identical performance based on their durations.

Key rate durations are different for ladder, barbell, and bullet portfolios. A **ladder portfolio** is one with approximately equal dollar amounts (market values) in each maturity sector. A **barbell portfolio** has considerably greater weights given to the shorter and longer maturity bonds than to the intermediate maturity bonds. A **bullet portfolio** has greater weights concentrated in the intermediate maturity relative to the shorter and longer maturities.

The key rate duration profiles for a ladder, a barbell, and a bullet portfolio are graphed in Exhibit 7.²³ All these portfolios have the same effective duration. As can be seen, the ladder portfolio has roughly the same key rate duration for all the key maturities from year 2 on. For the barbell portfolio, the key rate durations are much greater for the 5-year and 20-year key maturities and much smaller for the other key maturities. For the bullet portfolio, the key rate duration is substantially greater for the 10-year maturity than the duration for other key maturities.

VIII. YIELD VOLATILITY AND MEASUREMENT

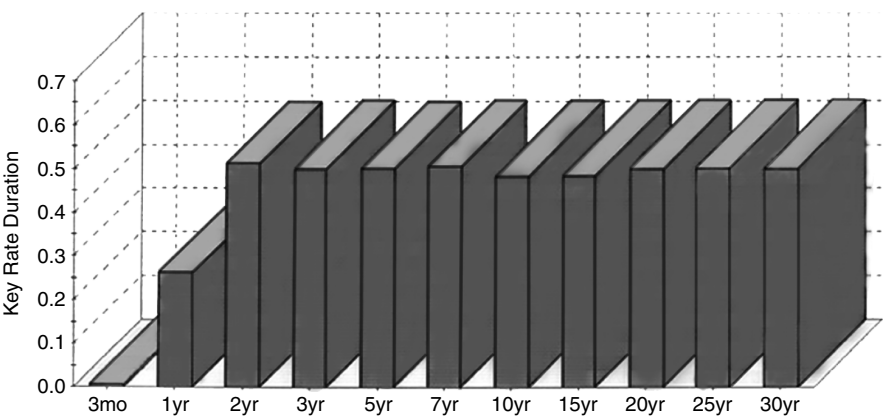
In assessing the interest rate exposure of a security or portfolio one should combine effective duration with yield volatility because effective duration alone is not sufficient to measure interest rate risk. The reason is that effective duration says that if interest rates change, a security's or portfolio's market value will change by approximately the percentage projected by its effective duration. However, the risk exposure of a portfolio to rate changes depends on how likely and how much interest rates may change, a parameter measured by yield volatility. For example, consider a U.S. Treasury security with an effective duration of 6 and a government bond of an emerging market country with an effective duration of 4. Based on effective duration alone, it would seem that the U.S. Treasury security has greater interest rate risk than the emerging market government bond. Suppose that yield volatility is substantial in the emerging market country relative to in the United States. Then the effective durations alone are not sufficient to identify the interest rate risk.

There is another reason why it is important to be able to measure yield or interest rate volatility: it is a critical input into a valuation model. An assumption of yield volatility is needed to value bonds with embedded options and structured products. The same measure is also needed in valuing some interest rate derivatives (i.e., options, caps, and floors).

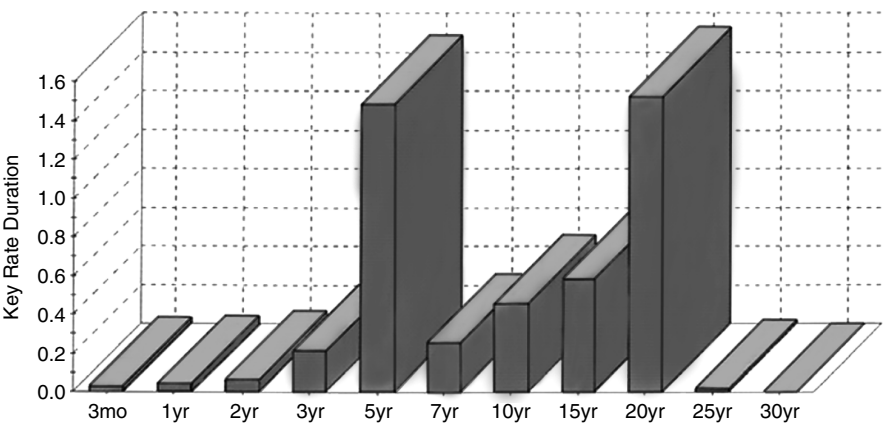
In this section, we look at how to measure yield volatility and discuss some techniques used to estimate it. Volatility is measured in terms of the standard deviation or variance. We will see how yield volatility as measured by the daily percentage change in yields is calculated from historical yields. We will see that there are several issues confronting an investor in measuring historical yield volatility. Then we turn to modeling and forecasting yield volatility.

²³The portfolios whose key rate durations are shown in Exhibit 7 were hypothetical Treasury portfolios constructed on April 23, 1997.

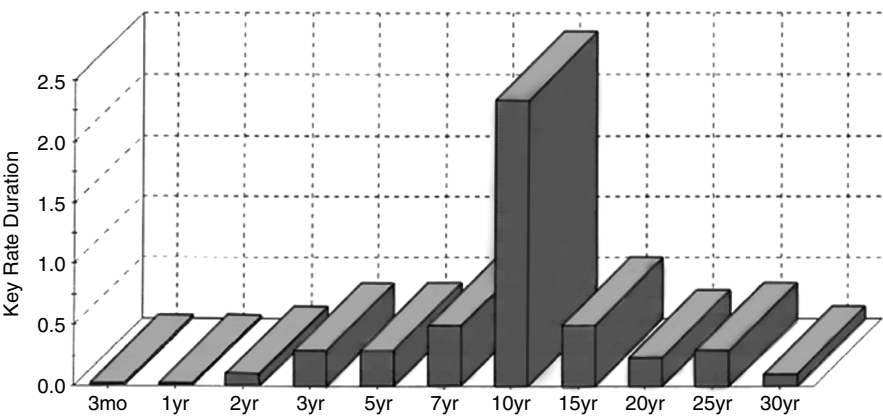
EXHIBIT 7 Key Rate Duration Profile for Three Treasury Portfolios (April 23, 1997): Ladder, Barbell, and Bullet



(a) Ladder Portfolio



(b) Barbell Portfolio



(c) Bullet Portfolio

Source: Barra

A. Measuring Historical Yield Volatility

Market participants seek a measure of yield volatility. The measure used is the standard deviation or variance. Here we will see how to compute yield volatility using historical data.

The sample variance of a random variable using historical data is calculated using the following formula:

$$\text{variance} = \frac{\sum_{t=1}^T (X_t - \bar{X})^2}{T - 1} \quad (1)$$

and then

$$\text{standard deviation} = \sqrt{\text{variance}}$$

where

X_t = observation t of variable X

$\bar{X}$ = the sample mean for variable X

T = the number of observations in the sample

Our focus is on yield volatility. More specifically, we are interested in the change in the daily yield relative to the previous day's yield. So, for example, suppose the yield on a zero-coupon Treasury bond was 6.555% on Day 1 and 6.593% on Day 2. The relative change in yield would be:

$$\frac{6.593\% - 6.555\%}{6.555\%} = 0.005797$$

This means if the yield is 6.555% on Day 1 and grows by 0.005797 in one day, the yield on Day 2 will be:

$$6.555\%(1.005797) = 6.593\%$$

If instead of assuming simple compounding it is assumed that there is continuous compounding, the relative change in yield can be computed as the natural logarithm of the ratio of the yield for two days. That is, the relative yield change can be computed as follows:

$$\text{Ln}(6.593\%/6.555\%) = 0.0057804$$

where "Ln" stands for the natural logarithm. There is not much difference between the relative change of daily yields computed assuming simple compounding and continuous compounding.²⁴ In practice, continuous compounding is used. Multiplying the natural logarithm of the ratio of the two yields by 100 scales the value to a percentage change in daily yields.

²⁴See Chapter 2 in DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*.

Therefore, letting y_t be the yield on day t and y_{t-1} be the yield on day $t-1$, the percentage change in yield, X_t , is found as follows:

$$X_t = 100[\text{Ln}(y_t/y_{t-1})]$$

In our example, y_t is 6.593% and y_{t-1} is 6.555%. Therefore,

$$X_t = 100[\text{Ln}(6.593/6.555)] = 0.57804\%$$

To illustrate how to calculate a daily standard deviation from historical data, consider the data in Exhibit 8 which show the yield on a Treasury zero for 26 consecutive days. From the 26 observations, 25 days of percentage yield changes are calculated in Column (3). Column (4) shows the square of the deviations of the observations from the mean. The bottom of Exhibit 8 shows the calculation of the daily mean for 25 yield changes, the variance, and the standard deviation. The daily standard deviation is 0.6360%.

The daily standard deviation will vary depending on the 25 days selected. It is important to understand that the daily standard deviation is dependent on the period selected, a point we return to later in this chapter.

1. Determining the Number of Observations In our illustration, we used 25 observations for the daily percentage change in yield. The appropriate number of observations depends on the situation at hand. For example, traders concerned with overnight positions might use the 10 most recent trading days (i.e., two weeks). A bond portfolio manager who is concerned with longer term volatility might use 25 trading days (about one month). The selection of the number of observations can have a significant effect on the calculated daily standard deviation.

2. Annualizing the Standard Deviation The daily standard deviation can be annualized by multiplying it by the square root of the number of days in a year.²⁵ That is,

$$\text{daily standard deviation} \times \sqrt{\text{number of days in a year}}$$

Market practice varies with respect to the number of days in the year that should be used in the annualizing formula above. Some investors and traders use the number of days in the year, 365 days, to annualize the daily standard deviation. Some investors and traders use only either 250 days or 260 days to annualize. The latter is simply the number of trading days in a year based on five trading days per week for 52 weeks. The former reduces the number of trading days of 260 for 10 non-trading holidays.

Thus, in calculating an annual standard deviation, the investor must decide on:

1. the number of daily observations to use
2. the number of days in the year to use to annualize the daily standard deviation.

²⁵For any probability distribution, it is important to assess whether the value of a random variable in one period is affected by the value that the random variable took on in a prior period. Casting this in terms of yield changes, it is important to know whether the yield today is affected by the yield in a prior period. The term *serial correlation* is used to describe the correlation between the yield in different periods. Annualizing the daily yield by multiplying the daily standard deviation by the square root of the number of days in a year assumes that serial correlation is not significant.

EXHIBIT 8 Calculation of Daily Standard Deviation Based on 26 Daily Observations for a Treasury Zero

(1) t	(2) y_t	(3) $X_t = 100[\ln(y_t/y_{t-1})]$	(4) $(X_t - \bar{X})^2$
0	6.6945		
1	6.699	0.06720	0.02599
2	6.710	0.16407	0.06660
3	6.675	-0.52297	0.18401
4	6.555	-1.81411	2.95875
5	6.583	0.42625	0.27066
6	6.569	-0.21290	0.01413
7	6.583	0.21290	0.09419
8	6.555	-0.42625	0.11038
9	6.593	0.57804	0.45164
10	6.620	0.40869	0.25270
11	6.568	-0.78860	0.48246
12	6.575	0.10652	0.04021
13	6.646	1.07406	1.36438
14	6.607	-0.58855	0.24457
15	6.612	0.07565	0.02878
16	6.575	-0.56116	0.21823
17	6.552	-0.35042	0.06575
18	6.515	-0.56631	0.22307
19	6.533	0.27590	0.13684
20	6.543	0.15295	0.06099
21	6.559	0.24424	0.11441
22	6.500	-0.90360	0.65543
23	6.546	0.70520	0.63873
24	6.589	0.65474	0.56063
25	6.539	-0.76173	0.44586
Total		-2.35020	9.7094094

$$\text{sample mean} = \bar{X} = \frac{-2.35020\%}{25} = -0.09401\%$$

$$\text{variance} = \frac{9.7094094\%}{25 - 1} = 0.4045587\%$$

$$\text{std dev} = \sqrt{0.4045587\%} = 0.6360493\%$$

The annual standard deviation for the daily standard deviation based on the 25-daily yield changes shown in Exhibit 8 (0.6360493%) using 250 days, 260 days, and 365 days to annualize are as follows:

250 days	260 days	365 days
10.06%	10.26%	12.15%

Now keep in mind that all of these decisions regarding the number of days to use in the daily standard deviation calculation, which set of days to use, and the number of days to use to annualize are not merely an academic exercise. Eventually, the standard deviation will be used in either the valuation of a security or in the measurement of risk exposure and can have a significant impact on the resulting value.

3. Using the Standard Deviation with Yield Estimation What does it mean if the annual standard deviation for the *change in* the Treasury zero yield is 12%? It means that if the prevailing yield is 8%, then the annual standard deviation of the yield change is 96 basis points. This is found by multiplying the annual standard deviation of the yield change of 12% by the prevailing yield of 8%.

Assuming that yield volatility is approximately normally distributed, we can use the normal distribution to construct a confidence interval for the future yield.²⁶ For example, we know that there is a 68.3% probability that an interval between *one* standard deviation below and above the sample expected value will bracket the future yield. The sample expected value is the prevailing yield. If the annual standard deviation is 96 basis points and the prevailing yield is 8%, then there is a 68.3% probability that the range between 7.04% (8% minus 96 basis points) and 8.96% (8% plus 96 basis points) will include the future yield. For *three* standard deviations below and above the prevailing yield, there is a 99.7% probability. Using the numbers above, three standard deviations is 288 basis points (3 times 96 basis points). The interval is then 5.12% (8% minus 288 basis points) and 10.88% (8% plus 288 basis points).

The interval or range constructed is called a “confidence interval.”²⁷ Our first interval of 7.04% to 8.96% is a 68.3% confidence interval. Our second interval of 5.12% to 10.88% is a 99.7% confidence interval. A confidence interval with any probability can be constructed.

B. Historical versus Implied Volatility

Market participants estimate yield volatility in one of two ways. The first way is by estimating historical yield volatility. This is the method that we have thus far described in this chapter. The resulting volatility is called **historical volatility**. The second way is to estimate yield volatility based on the observed prices of interest rate options and caps. Yield volatility calculated using this approach is called **implied volatility**.

The implied volatility is based on some option pricing model. One of the inputs to any option pricing model in which the underlying is a Treasury security or Treasury futures contract is expected yield volatility. If the observed price of an option is assumed to be the fair price and the option pricing model is assumed to be the model that would generate that fair price, then the implied yield volatility is the yield volatility that, when used as an input into the option pricing model, would produce the observed option price.

There are several problems with using implied volatility. First, it is assumed the option pricing model is correct. Second, option pricing models typically assume that volatility is constant over the life of the option. Therefore, interpreting an implied volatility becomes difficult.²⁸

C. Forecasting Yield Volatility

As has been seen, the yield volatility as measured by the standard deviation can vary based on the time period selected and the number of observations. Now we turn to the issue of forecasting yield volatility. There are several methods. Before describing these methods, let's

²⁶ See Chapter 4 in DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*.

²⁷ See Chapter 6 in DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*.

²⁸ For a further discussion, see Frank J. Fabozzi and Wai Lee, “Measuring and Forecasting Yield Volatility,” Chapter 16 in Frank J. Fabozzi (ed.), *Perspectives on Interest Rate Risk Management for Money Managers and Traders* (New Hope, PA: Frank J. Fabozzi Associates, 1998).

address the question of what mean value should be used in the calculation of the forecasted standard deviation.

Suppose at the end of Day 12 a trader was interested in a forecast for volatility using the 10 most recent days of trading and updating that forecast at the end of each trading day. What mean value should be used?

The trader can calculate a 10-day moving average of the daily percentage yield change. Exhibit 8 shows the daily percentage change in yield for the Treasury zero from Day 1 to Day 25. To calculate a moving average of the daily percentage yield change at the end of Day 12, the trader would use the 10 trading days from Day 3 to Day 12. At the end of Day 13, the trader will calculate the 10-day average by using the percentage yield change on Day 13 and would exclude the percentage yield change on Day 3. The trader will use the 10 trading days from Day 4 to Day 13.

Exhibit 9 shows the 10-day moving average calculated from Day 12 to Day 25. Notice the considerable variation over this period. The 10-day moving average ranged from -0.20324% to 0.07902% .

Thus far, it is assumed that the moving average is the appropriate value to use for the expected value of the change in yield. However, there are theoretical arguments that suggest it is more appropriate to assume that the expected value of the change in yield will be zero.²⁹ In the equation for the variance given by equation (1), instead of using for $\bar{X}$ the moving average, the value of zero is used. If zero is substituted into equation (1), the equation for the variance becomes:

$$\text{variance} = \frac{\sum_{t=1}^T X_t^2}{T - 1} \quad (2)$$

There are various methods for forecasting daily volatility. The daily standard deviation given by equation (2) assigns an equal weight to all observations. So, if a trader is calculating volatility based on the most recent 10 days of trading, each day is given a weight of 0.10.

EXHIBIT 9 10-Day Moving Average of Daily Yield
Change for Treasury Zero

10-Trading days ending	Daily average (%)
Day 12	-0.20324
Day 13	-0.04354
Day 14	0.07902
Day 15	0.04396
Day 16	0.00913
Day 17	-0.04720
Day 18	-0.06121
Day 19	-0.09142
Day 20	-0.11700
Day 21	-0.01371
Day 22	-0.11472
Day 23	-0.15161
Day 24	-0.02728
Day 25	-0.11102

²⁹Jacques Longerstacey and Peter Zangari, *Five Questions about RiskMetrics™*, JP Morgan Research Publication 1995.

EXHIBIT 10 Moving Averages of Daily Standard Deviations
Based on 10 Days of Observations

10-Trading days ending	Moving average Daily standard deviation (%)
Day 12	0.75667
Day 13	0.81874
Day 14	0.58579
Day 15	0.56886
Day 16	0.59461
Day 17	0.60180
Day 18	0.61450
Day 19	0.59072
Day 20	0.57705
Day 21	0.52011
Day 22	0.59998
Day 23	0.53577
Day 24	0.54424
Day 25	0.60003

For example, suppose that a trader is interested in the daily volatility of our hypothetical Treasury zero yield and decides to use the 10 most recent trading days. Exhibit 10 reports the 10-day volatility for various days using the data in Exhibit 8 and the standard deviation derived from the formula for the variance given by equation (2).

There is reason to suspect that market participants give greater weight to recent movements in yield or price when determining volatility. To give greater importance to more recent information, observations farther in the past should be given less weight. This can be done by revising the variance as given by equation (2) as follows:

$$\text{variance} = \frac{\sum_{t=1}^T W_t X_t^2}{T - 1} \quad (3)$$

where W_t is the weight assigned to observation t such that the sum of the weights is equal to T (i.e., $\sum W_t = T$) and the farther the observation is from today, the lower the weight. The weights should be assigned so that the forecasted volatility reacts faster to a recent major market movement and declines gradually as we move away from any major market movement.

Finally, a time series characteristic of financial assets suggests that a period of high volatility is followed by a period of high volatility. Furthermore, a period of relative stability in returns appears to be followed by a period that can be characterized in the same way. This suggests that volatility today may depend upon recent prior volatility. This can be modeled and used to forecast volatility. The statistical model used to estimate this time series property of volatility is called an autoregressive conditional heteroskedasticity (ARCH) model.³⁰ The term “conditional” means that the value of the variance depends on or is conditional on the value of the random variable. The term heteroskedasticity means that the variance is not equal for all values of the random variable. The foundation for ARCH models is a specialist topic.³¹

³⁰See Robert F. Engle, “Autoregressive Conditional Heteroskedasticity with Estimates of Variance of U.K. Inflation,” *Econometrica* 50 (1982), pp. 987–1008.

³¹See Chapter 9 in DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*.

VALUING BONDS WITH EMBEDDED OPTIONS

I. INTRODUCTION

The presence of an embedded option in a bond structure makes the valuation of such bonds complicated. In this chapter, we present a model to value bonds that have one or more embedded options and where the value of the embedded options depends on future interest rates. Examples of such embedded options are call and put provisions and caps (i.e., maximum interest rate) in floating-rate securities. While there are several models that have been proposed to value bonds with embedded options, our focus will be on models that provide an “arbitrage-free value” for a security. At the end of this chapter, we will discuss the valuation of convertible bonds. The complexity here is that these bonds are typically callable and may be puttable. Thus, the valuation of convertible bonds must take into account not only embedded options that depend on future interest rates (i.e., the call and the put options) but also the future price movement of the common stock (i.e., the call option on the common stock).

In order to understand how to value a bond with an embedded option, there are several fundamental concepts that must be reviewed. We will do this in Sections II, III, IV, and V. In Section II, the key elements involved in developing a bond valuation model are explained. In Section III, an overview of the bond valuation process is provided. Since the valuation of bonds requires benchmark interest rates, the various benchmarks are described in Section IV. In this section we also explain how to interpret spread measures relative to a particular benchmark. In Section V, the valuation of an option-free bond is reviewed using a numerical illustration. We first introduced the concepts described in this section in Chapter 5. The bond used in the illustration in this section to show how to value an option-free bond is then used in the remainder of the chapter to show how to value that bond if there is one or more embedded options.

II. ELEMENTS OF A BOND VALUATION MODEL

The valuation process begins with determining benchmark interest rates. As will be explained later in this section, there are three potential markets where benchmark interest rates can be obtained:

- the Treasury market
- a sector of the bond market
- the market for the issuer’s securities

An arbitrage-free value for an option-free bond is obtained by first generating the spot rates (or forward rates). When used to discount cash flows, the spot rates are the rates that would produce a model value equal to the observed market price for each on-the-run security in the benchmark. For example, if the Treasury market is the benchmark, an arbitrage-free model would produce a value for each on-the-run Treasury issue that is equal to its observed market price. In the Treasury market, the on-the-run issues are the most recently auctioned issues. (Note that all such securities issued by the U.S. Department of the Treasury are option free.) If the market used to establish the benchmark is a sector of the bond market or the market for the issuer's securities, the on-the-run issues are estimates of what the market price would be if newly issued *option-free* securities with different maturities are sold.

In deriving the interest rates that should be used to value a bond with an embedded option, the same principle must be maintained. No matter how complex the valuation model, when each on-the-run issue for a benchmark security is valued using the model, the value produced should be equal to the on-the-run issue's market price. The on-the-run issues for a given benchmark are assumed to be fairly priced.¹

The first complication in building a model to value bonds with embedded options is that the future cash flows will depend on what happens to interest rates in the future. This means that future interest rates must be considered. This is incorporated into a valuation model by considering how interest rates can change based on some assumed interest rate volatility. In the previous chapter, we explained what interest rate volatility is and how it is estimated. Given the assumed interest rate volatility, an interest rate "tree" representing possible future interest rates consistent with the volatility assumption can be constructed. It is from the interest rate tree that two important elements in the valuation process are obtained. First, the interest rates on the tree are used to generate the cash flows taking into account the embedded option. Second, the interest rates on the tree are used to compute the present value of the cash flows.

For a given interest rate volatility, there are several interest rate models that have been used in practice to construct an interest rate tree. An **interest rate model** is a probabilistic description of how interest rates can change over the life of the bond. An interest rate model does this by making an assumption about the relationship between the level of short-term interest rates and the interest rate volatility as measured by the standard deviation. A discussion of the various interest rate models that have been suggested in the finance literature and that are used by practitioners in developing valuation models is beyond the scope of this chapter.² What is important to understand is that the interest rate models commonly used are based on how short-term interest rates can evolve (i.e., change) over time. Consequently, these interest rate models are referred to as **one-factor models**, where "factor" means only one interest rate is being modeled over time. More complex models would consider how more than one interest rate changes over time. For example, an interest rate model can specify how the short-term interest rate and the long-term interest rate can change over time. Such a model is called a **two-factor model**.

Given an interest rate model and an interest rate volatility assumption, it can be assumed that interest rates can realize one of two possible rates in the next period. A valuation model

¹Market participants also refer to this characteristic of a model as one that "calibrates to the market."

²An excellent source for further explanation of many of these models is Gerald W. Buetow Jr. and James Sochacki, *Term Structure Models Using Binomial Trees: Demystifying the Process* (Charlottesville, VA: Association of Investment Management and Research, 2000).

that makes this assumption in creating an interest rate tree is called a **binomial model**. There are valuation models that assume that interest rates can take on three possible rates in the next period and these models are called **trinomial models**. There are even more complex models that assume in creating an interest rate tree that more than three possible rates in the next period can be realized. These models that assume discrete change in interest rates are referred to as “discrete-time option pricing models.” It makes sense that option valuation technology is employed to value a bond with an embedded option because the valuation requires an estimate of what the value of the embedded option is worth. However, a discussion of the underlying theory of discrete-time pricing models in general and the binomial model in particular are beyond the scope of this chapter.³

As we will see later in this chapter, when a discrete-time option pricing model is portrayed in graph form, it shows the different paths that interest rates can take. The graphical presentation looks like a lattice.⁴ Hence, discrete-time option pricing models are sometimes referred to as “lattice models.” Since the pattern of the interest rate paths also look like the branches of a tree, the graphical presentation is referred to as an **interest rate tree**.

Regardless of the assumption about how many possible rates can be realized in the next period, the interest rate tree generated must produce a value for the securities in the benchmark that is equal to their observed market price—that is, it must produce an arbitrage-free value. Consequently, if the Treasury market is used for the benchmark interest rates, the interest rate tree generated must produce a value for each on-the-run Treasury issue that is equal to its observed market price. Moreover, the intuition and the methodology for using the interest rate tree (i.e., the backward induction methodology described later) are the same. Once an interest rate tree is generated that (1) is consistent with both the interest rate volatility assumption and the interest rate model and (2) generates the observed market price for the securities in the benchmark, the next step is to use the interest rate tree to value a bond with an embedded option. The complexity here is that a set of rules must be introduced to determine, for any period, when the embedded option will be exercised. For a callable bond, these rules are called the “call rules.” The rules vary from model builder to model builder.

While the building of a model to value bonds with embedded options is more complex than building a model to value option-free bonds, the basic principles are the same. In the case of valuing an option-free bond, the model that is built is simply a set of spot rates that are used to value cash flows. The spot rates will produce an arbitrage-free value. For a model to value a bond with embedded options, the interest rate tree is used to value future cash flows and the interest rate tree is combined with the call rules to generate the future cash flows. Again, the interest rate tree will produce an arbitrage-free value.

Let’s move from theory to practice. Only a few practitioners will develop their own model to value bonds with embedded options. Instead, it is typical for a portfolio manager or analyst to use a model developed by either a dealer firm or a vendor of analytical systems. A fair question is then: Why bother covering a valuation model that is readily available from a third-party? The answer is that a valuation model should not be a black box to portfolio managers and analysts. *The models in practice share all of the principles described in this chapter,*

³For a discussion of the binomial model and the underlying theory, see Chapter 4 in Don M. Chance, *Analysis of Derivatives for the CFA Program* (Charlottesville, VA: Association for Investment Management and Research, 2003).

⁴A lattice is an arrangement of points in a regular periodic pattern.

but differ with respect to certain assumptions that can produce quite different values. The reasons for these differences in valuation must be understood. Moreover, third-party models give the user a choice of changing the assumptions. A user who has not “walked through” a valuation model has no appreciation of the significance of these assumptions and therefore how to assess the impact of these assumptions on the value produced by the model. Earlier, we discussed “modeling risk.” This is the risk that the underlying assumptions of a model may be incorrect. Understanding a valuation model permits the user to effectively determine the significance of an assumption.

As an example of the importance of understanding the assumptions of a model, consider interest rate volatility. Suppose that the market price of a bond is \$89. Suppose further that a valuation model produces a value for a bond with an embedded option of \$90 based on a 12% interest rate volatility assumption. Then, according to the valuation model, this bond is cheap by one point. However, suppose that the same model produces a value of \$87 if a 15% volatility is assumed. This tells the portfolio manager or analyst that the bond is two points rich. Which is correct? The answer clearly depends on what the investor believes interest rate volatility will be in the future.

In this chapter, we will use the binomial model to demonstrate all of the issues and assumptions associated with valuing a bond with embedded options. This model is available on Bloomberg, as well as from other commercial vendors and several dealer firms.⁵ We show how to create an interest rate tree (more specifically, a binomial interest rate tree) given a volatility assumption and how the interest rate tree can be used to value an option-free bond. Given the interest rate tree, we then show how to value several types of bonds with an embedded option—a callable bond, a puttable bond, a step-up note, and a floating-rate note with a cap. We postpone until Chapter 12 an explanation of why the binomial model is not used to value mortgage-backed and asset-backed securities. The binomial model is used to value options, caps, and floors as will be explained in Chapter 14.

Once again, it must be emphasized that while the binomial model is used in this chapter to demonstrate how to value bonds with embedded options, other models that allow for more than one interest rate in the next period all follow the same principles—they begin with on-the-run yields, they produce an interest rate tree that generates an arbitrage-free value, and they depend on assumptions regarding the volatility of interest rates and rules for when an embedded option will be exercised.

III. OVERVIEW OF THE BOND VALUATION PROCESS

In this section we review the bond valuation process and the key concepts that were introduced earlier. This will help us tie together the concepts that have already been covered and how they relate to the valuation of bonds with embedded options.

Regardless if a bond has an embedded option, we explained that the following can be done:

⁵The model described in this chapter was first presented in Andrew J. Kalotay, George O. Williams, and Frank J. Fabozzi, “A Model for the Valuation of Bonds and Embedded Options,” *Financial Analysts Journal* (May–June 1993), pp. 35–46.

1. Given a required yield to maturity, we can compute the value of a bond. For example, if the required yield to maturity of a 9-year, 8% coupon bond that pays interest semiannually is 7%, its price is 106.59.
2. Given the observed market price of a bond we can calculate its yield to maturity. For example, if the price of a 5-year, 6% coupon bond that pays interest semiannually is 93.84, its yield to maturity is 7.5%.
3. Given the yield to maturity, a yield spread can be computed. For example, if the yield to maturity for a 5-year, 6% coupon bond that pays interest semiannually is 7.5% and its yield is compared to a benchmark yield of 6.5%, then the yield spread is 100 basis points (7.5% minus 6.5%). We referred to the yield spread as the *nominal spread*.

The problem with using a single interest rate when computing the value of a bond (as in (1) above) or in computing a yield to maturity (as in (2) above) is that it fails to recognize that each cash flow is unique and warrants its own discount rate. Failure to discount each cash flow at an appropriate interest rate unique to when that cash flow is expected to be received results in an arbitrage opportunity as described in Chapter 8.

It is at this point in the valuation process that the notion of theoretical spot rates are introduced to overcome the problem associated with using a single interest rate. The spot rates are the appropriate rates to use to discount cash flows. There is a theoretical spot rate that can be obtained for each maturity. The procedure for computing the spot rate curve (i.e., the spot rate for each maturity) was explained and discussed further in the previous chapter.

Using the spot rate curve, one obtains the bond price. However, how is the spot rate curve used to compute the yield to maturity? Actually, there is no equivalent concept to a yield to maturity in this case. Rather, there is a yield spread measure that is used to overcome the problem of a single interest rate. This measure is the zero-volatility spread that was explained in Chapter 4. The zero-volatility spread, also called the Z-spread and the static spread, is the spread that when added to all of the spot rates will make the present value of the bond's cash flow equal to the bond's market price.

At this point, we have not introduced any notion of how to handle bonds with embedded options. We have simply dealt with the problem of using a single interest rate for discounting cash flows. But there is still a critical issue that must be resolved. When a bond has an embedded option, a portion of the yield, and therefore a portion of the spread, is attributable to the embedded option. When valuing a bond with an embedded option, it is necessary to adjust the spread for the value of the embedded option. The measure that does this is called the option-adjusted spread (OAS). We mentioned this measure in Chapter 4 but did not provide any details. In this chapter, we show of how this spread measure is computed for bonds with embedded options.

A. The Benchmark Interest Rates and Relative Value Analysis

Yield spread measures are used in assessing the **relative value** of securities. Relative value analysis involves identifying securities that can potentially enhance return relative to a benchmark. Relative value analysis can be used to identify securities as being overpriced ("rich"), underpriced ("cheap"), or fairly priced. A portfolio manager can use relative value analysis in ranking issues within a sector or sub-sector of the bond market or different issues of a specific issuer.

Two questions that need to be asked in order to understand spread measures were identified:

1. What is the benchmark for computing the spread? That is, what is the spread measured relative to?
2. What is the spread measuring?

As explained, the different spread measures begin with benchmark interest rates. The benchmark interest rates can be one of the following:

- the Treasury market
- a specific bond sector with a given credit rating
- a specific issuer

A specific bond sector with a given credit rating, for example, would include single-A rated corporate bonds or double-A rated banks. The LIBOR curve discussed in the previous chapter is an example, since it is viewed by the market as an inter-bank or AA rated benchmark.

Moreover, the benchmark interest rates can be based on either

- an estimated yield curve
- an estimated spot rate curve

A yield curve shows the relationship between yield and maturity for coupon bonds; a spot rate curve shows the relationship between spot rates and maturity.

Consequently, there are six potential benchmark interest rates as summarized below:

	Treasury market	Specific bond sector with a given credit rating	Specific issuer
Yield curve	Treasury yield curve	Sector yield curve	Issuer yield curve
Spot rate curve	Treasury spot rate curve	Sector spot rate curve	Issuer spot rate curve

We illustrated and explained further in Chapter 8 how the Treasury spot rate curve can be constructed from the Treasury yield curve. Rather than start with yields in the Treasury market as the benchmark interest rates, an estimated on-the-run yield curve for a bond sector with a given credit rating or a specific issuer can be obtained. To obtain a sector with a given credit rating or a specific issuer's on-the-run yield curve, an appropriate credit spread is added to each on-the-run Treasury issue. The credit spread need not be constant for all maturities. For example, as explained in Chapter 5, the credit spread may increase with maturity. Given the on-the-run yield curve, the theoretical spot rates for the bond sector with a given credit rating or issuer can be constructed using the same methodology to construct the Treasury spot rates given the Treasury yield curve.

B. Interpretation of Spread Measures

Given the alternative benchmark interest rates, in this section we will see how to interpret the three spread measures that were described nominal spread, zero-volatility spread, and option-adjusted spread.

1. Treasury Market Benchmark In the United States, yields in the U.S. Treasury market are typically used as the benchmark interest rates. The benchmark can be either the Treasury yield curve or the Treasury spot rate curve. As explained earlier, the nominal spread is a spread measured relative to the Treasury yield curve and the zero-volatility spread is a spread relative to the Treasury spot rate curve. As we will see in this chapter, the OAS is a spread relative to the Treasury spot rate curve.

If the Treasury market rates are used, then the benchmark for the three spread measures and the risks for which the spread is compensating are summarized below:

Spread measure	Benchmark	Reflects compensation for . . .
Nominal	Treasury yield curve	Credit risk, option risk, liquidity risk
Zero-volatility	Treasury spot rate curve	Credit risk, option risk, liquidity risk
Option-adjusted	Treasury spot rate curve	Credit risk, liquidity risk

where “credit risk” is relative to the default-free rate since the Treasury market is viewed as a default-free market.

In the case of an OAS, if the computed OAS is greater than what the market requires for credit risk and liquidity risk, then the security is undervalued. If the computed OAS is less than what the market requires for credit risk and liquidity risk, then the security is overvalued. Only using the nominal spread or zero-volatility spread, masks the compensation for the embedded option.

For example, assume the following for a non-Treasury security, Bond W, a triple B rated corporate bond with an embedded call option:

Benchmark: Treasury market

Nominal spread based on Treasury yield curve: 170 basis points

Zero-volatility spread based on Treasury spot rate curve: 160 basis points

OAS based on Treasury spot rate curve: 125 basis points

Suppose that in the market *option-free* bonds with the same credit rating, maturity, and liquidity as Bond W trade at a nominal spread of 145 basis points. It would seem, based solely on the nominal spread, Bond W is undervalued (i.e., cheap) since its nominal spread is greater than the nominal spread for comparable bonds (170 versus 145 basis points). Even comparing Bond W's zero-volatility spread of 160 basis points to the market's 145 basis point nominal spread for option-free bonds (not a precise comparison since the Treasury benchmarks are different), the analysis would suggest that Bond W is cheap. However, after removing the value of the embedded option—which as we will see is precisely what the OAS measure does—the OAS tells us that the bond is trading at a spread that is less than the nominal spread of otherwise comparable option-free bonds. Again, while the benchmarks are different, the OAS tells us that Bond W is overvalued.

C. Specific Bond Sector with a Given Credit Rating Benchmark

Rather than use the Treasury market as the benchmark, the benchmark can be a specific bond sector *with a given credit rating*. The interpretation for the spread measures would then be:

Spread measure	Benchmark	Reflects compensation for . . .
Nominal	Sector yield curve	Credit risk, option risk, liquidity risk
Zero-volatility	Sector spot rate curve	Credit risk, option risk, liquidity risk
Option-adjusted	Sector spot rate curve	Credit risk, liquidity risk

where “Sector” means the sector with a specific credit rating. “Credit risk” in this case means the *credit risk of a security under consideration relative to the credit risk of the sector used as the benchmark* and “liquidity risk” is the *liquidity risk of a security under consideration relative to the liquidity risk of the sector used as the benchmark*.

Let’s again use Bond W, a triple B rated corporate bond with an embedded call option to illustrate. Assume the following spread measures were computed:

Benchmark: double A rated corporate bond sector

Nominal spread based on benchmark: 110 basis points

Zero-volatility spread based benchmark spot rate curve: 100 basis points

OAS based on benchmark spot rate curve: 80 basis points

Suppose that in the market *option-free* bonds with the same credit rating, maturity, and liquidity as Bond W trade at a nominal spread *relative to the double A corporate bond sector* of 90 basis points. Based solely on the nominal spread as a relative yield measure using the same benchmark, Bond W is undervalued (i.e., cheap) since its nominal spread is greater than the nominal spread for comparable bonds (110 versus 90 basis points). Even naively comparing Bond W’s zero-volatility spread of 100 basis points (relative to the double A corporate spot rate curve) to the market’s 90 basis point nominal spread for option-free bonds relative to the double A corporate bond yield curve, the analysis would suggest that Bond W is cheap. However, the proper assessment of Bond W’s relative value will depend on what its OAS is in comparison to the OAS (relative to the same double A corporate benchmark) of other triple B rated bonds. For example, if the OAS of other triple B rated corporate bonds is less than 80 basis points, then Bond W is cheap.

D. Issuer-Specific Benchmark

Instead of using as a benchmark the Treasury market or a bond market sector to measure relative value for a specific issue, one can use an estimate of the issuer’s yield curve or an estimate of the issuer’s spot rate curve as the benchmark. Then we would have the following interpretation for the three spread measures:

Spread measure	Benchmark	Reflects compensation for . . .
Nominal	Issuer yield curve	Optional risk, liquidity risk
Zero-volatility	Issuer spot rate curve	Optional risk, liquidity risk
Option-adjusted	Issuer spot rate curve	Liquidity risk

Note that there is no credit risk since it is assumed that the specific issue analyzed has the same credit risk as the embedded in the issuer benchmark. Using the nominal spread, a value that is positive indicates that, ignoring any embedded option, the issue is cheap relative to how the market is pricing other bonds of the issuer. A negative value would indicate that the security is expensive. The same interpretation holds for the zero-volatility spread, ignoring

any embedded option. For the OAS, a positive spread means that even after adjusting for the embedded option, the value of the security is cheap. If the OAS is zero, the security is fairly priced and if it is negative, the security is expensive.

Once again, let's use our hypothetical Bond W, a triple B rated corporate bond with an embedded call option. Assume this bond is issued by RJK Corporation. Then suppose for Bond W:

Benchmark: RJK Corporation's bond issues

Nominal spread based on RJK Corporation's yield curve: 30 basis points

Zero-volatility spread based on RJK Corporation's spot rate curve: 20 basis points

OAS based on RJK Corporation's spot rate curve: -25 basis points

Both the nominal spread and the zero-volatility spread would suggest that Bond W is cheap (i.e., both spread measures have a positive value). However, once the embedded option is taken into account, the appropriate spread measure, the OAS, indicates that there is a negative spread. This means that Bond W is expensive and should be avoided.

E. OAS, The Benchmark, and Relative Value

Our focus in this chapter is the valuation of bonds with an embedded option. While we have yet to describe how an OAS is calculated, here we summarize how to interpret OAS as a relative value measure based on the benchmark.

Consider first when the benchmark is the Treasury spot rate curve. A zero OAS means that the security offers no spread over Treasuries. Hence, a security with a zero OAS in this case should be avoided. A negative OAS means that the security is offering a spread that is less than Treasuries. Therefore, it should be avoided. A positive value alone does not mean a security is fairly priced or cheap. It depends on what spread relative to the Treasury market the market is demanding for comparable issues. Whether the security is rich, fairly priced, or cheap depends on the OAS for the security compared to the OAS for comparable securities. We will refer to the OAS offered on comparable securities as the "required OAS" and the OAS computed for the security under consideration as the "security OAS." Then,

if security OAS is greater than required OAS, the security is cheap

if security OAS is less than required OAS, the security is rich

if security OAS is equal to the required OAS, the security is fairly priced

When a sector of the bond market with the same credit rating is the benchmark, the credit rating of the sector relative to the credit rating of the security being analyzed is important. In the discussion, *it is assumed that the credit rating of the bond sector that is used as a benchmark is higher than the credit rating of the security being analyzed.* A zero OAS means that the security offers no spread over the bond sector benchmark and should therefore be avoided. A negative OAS means that the security is offering a spread that is less than the bond sector benchmark and hence should be avoided. As with the Treasury benchmark, when there is a positive OAS, relative value depends on the security OAS compared to the required OAS. Here the required OAS is the OAS of comparable securities relative to the bond sector benchmark. Given the security OAS and the required OAS, then

if security OAS is greater than required OAS, the security is cheap
 if security OAS is less than required OAS, the security is rich
 if security OAS is equal to the required OAS, the security is fairly priced

The terms “rich,” “cheap,” and “fairly priced” are only relative to the benchmark. If an investor is a funded investor who is assessing a security relative to his or her borrowing costs, then a different set of rules exists. For example, suppose that the bond sector used as the benchmark is the LIBOR spot rate curve. Also assume that the funding cost for the investor is a spread of 40 basis points over LIBOR. Then the decision to invest in the security depends on whether the OAS exceeds the 40 basis point spread by a sufficient amount to compensate for the credit risk.

Finally, let's look at relative valuation when the issuer's spot rate curve is the benchmark. If a particular security by the issuer is fairly priced, its OAS should be equal to zero. Thus, unlike when the Treasury benchmark or bond sector benchmark are used, a zero OAS is fairly valued security. A positive OAS means that the security is trading cheap relative to other

EXHIBIT 1 Relationship Between the Benchmark, OAS, and Relative Value

Benchmark	Negative OAS	Zero OAS	Positive OAS
Treasury market	Overpriced (rich) security	Overpriced (rich) security	Comparison must be made between security OAS and OAS of comparable securities (required OAS): if security OAS > required OAS, security is cheap if security OAS < required OAS, security is rich if security OAS = required OAS, security is fairly priced
Bond sector with a given credit rating (assumes credit rating higher than security being analyzed)	Overpriced (rich) security (assumes credit rating higher than security being analyzed)	Overpriced (rich) security (assumes credit rating higher than security being analyzed)	Comparison must be made between security OAS and OAS of comparable securities (required OAS): if security OAS > required OAS, security is cheap if security OAS < required OAS, security is rich if security OAS = required OAS, security is fairly priced
Issuer's own securities	Overpriced (rich) security	Fairly valued	Underpriced (cheap) security

securities of the issuer and a negative OAS means that the security is trading rich relative to other securities of the same issuer.

The relationship between the benchmark, OAS, and relative value are summarized in Exhibit 1.

IV. REVIEW OF HOW TO VALUE AN OPTION-FREE BOND

Before we illustrate how to value a bond with an embedded option, we will review how to value an option-free bond. We will then take the same bond and explain how it would be valued if it has an embedded option.

In Chapter 5, we explained how to compute an arbitrage-free value for an option-free bond using spot rates. At Level I (Chapter 6), we showed the relationship between spot rates and forward rates, and then how forward rates can be used to derive the same arbitrage-free value as using spot rates. What we will review in this section is how to value an option-free bond using both spot rates and forward rates. We will use as our benchmark in the rest of this chapter, the securities of the issuer whose bond we want to value. Hence, *we will start with the issuer's on-the-run yield curve.*

To obtain a particular issuer's on-the-run yield curve, an appropriate credit spread is added to each on-the-run Treasury issue. The credit spread need not be constant for all maturities. In our illustration, we use the following hypothetical *on-the-run issue for the issuer whose bond we want to value:*

Maturity	Yield to maturity	Market price
1 year	3.5%	100
2 years	4.2%	100
3 years	4.7%	100
4 years	5.2%	100

Each bond is trading at par value (100) so the coupon rate is equal to the yield to maturity. We will simplify the illustration by assuming annual-pay bonds.

Using the bootstrapping methodology explained in Chapter 6, the spot rates are given below:

Year	Spot rate
1	3.5000%
2	4.2148%
3	4.7352%
4	5.2706%

we will use the above spot rates shortly to value a bond.

In Chapter 6, we explained how to derive forward rates from spot rates. Recall that forward rates can have different interpretations based on the theory of the term structure to which one subscribes. However, in the valuation process, *we are not relying on any theory.* The forward rates below are mathematically derived from the spot rates and, as we will see, when used to value a bond will produce the same value as the spot rates. The 1-year forward rates are:

Current 1-year forward rate	3.500%
1-year forward rate one year from now	4.935%
1-year forward rate two years from now	5.784%
1-year forward rate three years from now	6.893%

Now consider an option-free bond with four years remaining to maturity and a coupon rate of 6.5%. The value of this bond can be calculated in one of two ways, both producing the same value. First, the cash flows can be discounted at the spot rates as shown below:

$$\frac{\$6.5}{(1.035)^1} + \frac{\$6.5}{(1.042148)^2} + \frac{\$6.5}{(1.047352)^3} + \frac{\$100 + \$6.5}{(1.052706)^4} = \$104.643$$

The second way is to discount by the 1-year forward rates as shown below:

$$\begin{aligned} & \frac{\$6.5}{(1.035)} + \frac{\$6.5}{(1.035)(1.04935)} + \frac{\$6.5}{(1.035)(1.04935)(1.05784)} \\ & + \frac{\$100 + \$6.5}{(1.035)(1.04935)(1.05784)(1.06893)} = \$104.643 \end{aligned}$$

As can be seen, discounting by spot rates or forward rates will produce the same value for a bond.

Remember this value for the option-free bond, \$104.643. When we value the same bond using the binomial model later in this chapter, that model should produce a value of \$104.643 or else our model is flawed.

V. VALUING A BOND WITH AN EMBEDDED OPTION USING THE BINOMIAL MODEL

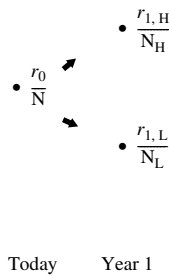
As explained in Section II, there are various models that have been developed to value a bond with embedded options. The one that we will use to illustrate the issues and assumptions associated with valuing bonds with embedded options is the binomial model. The interest rates that are used in the valuation process are obtained from a **binomial interest rate tree**. We'll explain the general characteristics of this tree first. Then we see how to value a bond using the binomial interest rate tree. We will then see how to construct this tree from an on-the-run yield curve. Basically, the derivation of a binomial interest rate tree is the same in principle as deriving the spot rates using the bootstrapping method described in Chapter 6—that is, there is no arbitrage.

A. Binomial Interest Rate Tree

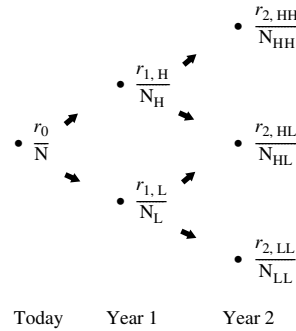
Once we allow for embedded options, consideration must be given to interest rate volatility. The reason is, the decision of the issuer or the investor (depending upon who has the option) will be affected by what interest rates are in the future. This means that the valuation model must explicitly take into account how interest rates may change in the future. In turn, this recognition is achieved by incorporating interest rate volatility into the valuation model. In the previous chapter, we explained what interest rate volatility is and how it can be measured.

EXHIBIT 2 Binomial Interest Rate Tree

Panel a: One-Year Binomial Interest Rate Tree



Panel b: Two-Year Binomial Interest Rate Tree



Let's see how interest rate volatility is introduced into the valuation model. More specifically, let's see how this can be done in the binomial model using Exhibit 2. Look at panel a of the exhibit which shows the beginning or *root* of the interest rate tree. The time period shown is "Today." At the dot, denoted N , in the exhibit is an interest rate denoted by r_0 which represents the interest rate today.

Notice that there are two arrows as we move to the right of N . Here is where we are introducing interest rate volatility. The dot in the exhibit referred to as a *node*. What takes place at a node is either a *random event* or a *decision*. We will see that in building a binomial interest rate tree, at each node there is a random event. The change in interest rates represents a random event. Later when we show how to use the binomial interest rate tree to determine the value of a bond with an embedded option, at each node there will be a decision. Specifically, the decision will be whether or not the issuer or bondholders (depending on the type of embedded option) will exercise the option.

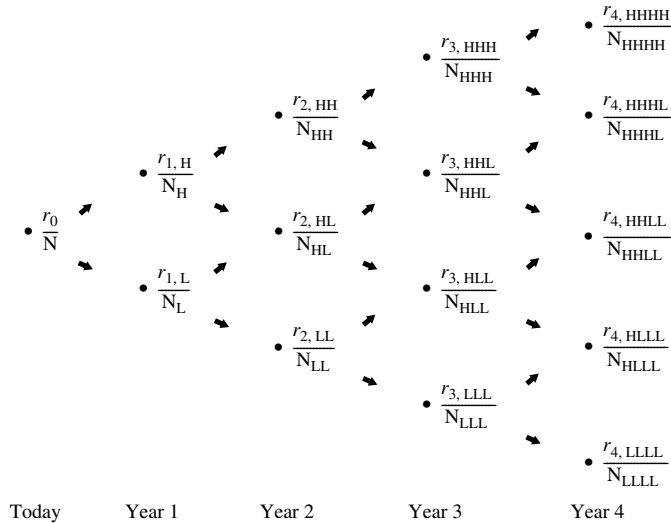
In the binomial model, it is assumed that the random event (i.e., the change in interest rates) will take on only two possible values. Moreover, it is assumed that the probability of realizing either value is equal. The two possible values are the interest rates shown by $r_{1,H}$ and $r_{1,L}$ in panel a.⁶ If you look at the time frame at the bottom of panel a, you will notice that it is in years.⁷ What this means is that the interest rate at r_0 is the current (i.e., today's) 1-year rate and at year 1, the two possible 1-year interest rates are $r_{1,H}$ and $r_{1,L}$. Notice the notation that is used for the two subscripts. The first subscript, 1, means that it is the interest rate starting in year 1. The second subscript indicates whether it is the higher (H) or lower (L) of the two interest rates in year 1.

Now we will grow the binomial interest rate tree. Look at panel b of Exhibit 2 which shows today, year 1, and year 2. There are two nodes at year 1 depending on whether the higher or the lower interest rate is realized. At both of the nodes a random event occurs. N_H is the node if the higher interest rate ($r_{1,H}$) is realized. In the binomial model, the interest rate that can occur in the next year (i.e., year 2) can be one of two values: $r_{2,HH}$ or $r_{2,HL}$. The subscript 2 indicates year 2. This would get us to either the node N_{HH} or N_{HL} . The subscript "HH" means that the path to get to node N_{HH} is the higher interest rate in year 1 and in

⁶If we were using a trinomial model, there would be three possible interest rates shown in the next year.

⁷In practice, much shorter time periods are used to construct an interest rate tree.

EXHIBIT 3 Four-Year Binomial Interest Rate Tree



year 2. The subscript “HL” means that the path to get to node N_{HL} is the higher interest rate in year 1 and the lower interest rate in year 2.

Similarly, N_L is the node if the lower interest rate ($r_{1,L}$) is realized in year 1. The interest rate that can occur in year 2 is either $r_{2,LH}$ or $r_{2,LL}$. This would get us to either the node N_{LH} or N_{LL} . The subscript “LH” means that the path to get to node N_{LH} is the lower interest rate in year 1 and the higher interest rate in year 2. The subscript “LL” means that the path to get to node N_{LL} is the lower interest rate in year 1 and in year 2.

Notice that in panel b, at year 2 only N_{HL} is shown but no N_{LH} . The reason is that if the higher interest rate is realized in year 1 and the lower interest rate is realized in year 2, we would get to the same node as if the lower interest rate is realized in year 1 and the higher interest rate is realized in year 2. Rather than clutter up the interest rate tree with notation, only one of the two paths is shown.

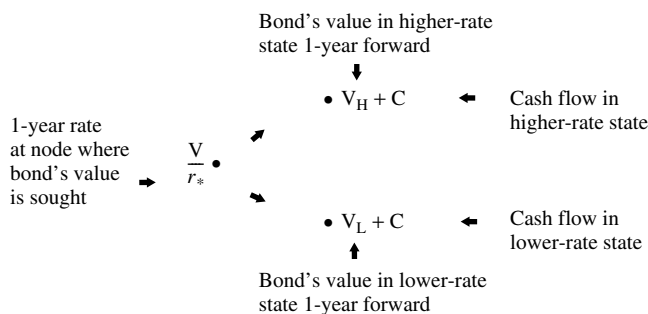
In our illustration of valuing a bond with an embedded option, we will use a 4-year bond. Consequently, we will need a 4-year binomial interest rate tree to value this bond. Exhibit 3 shows the tree and the notation used.

The interest rates shown in the binomial interest rate tree are actually forward rates. Basically, they are the one-period rates starting in period t . (A period in our illustration is one year.) Thus, in valuing an option-free bond we know that it is valued using forward rates and we have illustrated this by using 1-period forward rates. For each period, there is a unique forward rate. When we value bonds with embedded options, we will see that we continue to use forward rates but there is not just one forward rate for a given period but a set of forward rates.

There will be a relationship between the rates in the binomial interest rate tree. The relationship depends on the interest rate model assumed. Based on some interest rate volatility assumption, the interest rate model selected would show the relationship between:

- $r_{1,L}$ and $r_{1,H}$ for year 1
- $r_{2,LL}$, $r_{2,HL}$, and $r_{2,HH}$ for year 2
- etc.

EXHIBIT 4 Calculating a Value at a Node



For our purpose of understanding the valuation model, it is not necessary that we show the mathematical relationships here.

B. Determining the Value at a Node

Now we want to see how to use the binomial interest rate tree to value a bond. To do this, we first have to determine the value of the bond at each node. To find the value of the bond at a node, we begin by calculating the bond's value at the high and low nodes to the right of the node for which we are interested in obtaining a value. For example, in Exhibit 4, suppose we want to determine the bond's value at node N_H . The bond's value at node N_{HH} and N_{HL} must be determined. Hold aside for now how we get these two values because, as we will see, the process involves starting from the last (right-most) year in the tree and working backwards to get the final solution we want. Because the procedure for solving for the final solution in any interest rate tree involves moving backwards, the methodology is known as **backward induction**.

Effectively what we are saying is that if we are at some node, then the value at that node will depend on the future cash flows. In turn, the future cash flows depend on (1) the coupon payment one year from now and (2) the bond's value one year from now. The former is known. The bond's value depends on whether the rate is the higher or lower rate reported at the two nodes to the right of the node that is the focus of our attention. So, the cash flow at a node will be either (1) the bond's value if the 1-year rate is the higher rate plus the coupon payment, or (2) the bond's value if the 1-year rate is the lower rate plus the coupon payment. Let's return to the bond's value at node N_H . The cash flow will be either the bond's value at N_{HH} plus the coupon payment, or the bond's value at N_{HL} plus the coupon payment.

In general, to get the bond's value at a node we follow the fundamental rule for valuation: the value is the present value of the expected cash flows. The appropriate discount rate to use is the 1-year rate at the node where we are computing the value. Now there are two present values in this case: the present value if the 1-year rate is the higher rate and one if it is the lower rate. Since it is assumed that the probability of both outcomes is equal (i.e., there is a 50% probability for each), an average of the two present values is computed. This is illustrated in Exhibit 4 for any node assuming that the 1-year rate is r_* at the node where the valuation is sought and letting:

V_H = the bond's value for the higher 1-year rate

V_L = the bond's value for the lower 1-year rate

C = coupon payment

Using our notation, the cash flow at a node is either:

$V_H + C$ for the higher 1-year rate

$V_L + C$ for the lower 1-year rate

The present value of these two cash flows using the 1-year rate at the node, r_* , is:

$$\frac{V_H + C}{(1 + r_*)} = \text{present value for the higher 1-year rate}$$

$$\frac{V_L + C}{(1 + r_*)} = \text{present value for the lower 1-year rate}$$

Then, the value of the bond at the node is found as follows:

$$\text{Value at a node} = \frac{1}{2} \left[\frac{V_H + C}{(1 + r_*)} + \frac{V_L + C}{(1 + r_*)} \right]$$

C. Constructing the Binomial Interest Rate Tree

The construction of any interest rate tree is complicated, although the principle is simple to understand. This applies to the binomial interest rate tree or a tree based on more than two future rates in the next period. *The fundamental principle is that when a tree is used to value an on-the-run issue for the benchmark, the resulting value should be arbitrage free.* That is, the tree should generate a value for an on-the-run issue equal to its observed market value. Moreover, the interest rate tree should be consistent with the interest rate volatility assumed.

Here is a brief overview of the process for constructing the interest rate tree. It is not essential to know how to derive the interest rate tree; rather, it should be understood how to value a bond given the rates on the tree. The interest rate at the first node (i.e., the root of the tree) is the one year interest rate for the on-the-run issue. (This is because in our simplified illustration we are assuming that the length of the time between nodes is one year.) The tree is grown just the same way that the spot rates were obtained using the bootstrapping method based on arbitrage arguments.

The interest rates for year 1 (there are two of them and remember they are forward rates) are obtained from the following information:

1. the coupon rate for the 2-year on-the-run issue
2. the interest rate volatility assumed
3. the interest rate at the root of the tree (i.e., the current 1-year on-the-run rate)

Given the above, a *guess* is then made of the lower rate at node N_L , which is $r_{1,L}$. The upper rate, $r_{1,H}$, is not guessed at. Instead, it is determined by the assumed volatility of the 1-year rate ($r_{1,L}$). The formula for determining $r_{1,H}$ given $r_{1,L}$ is specified by the interest rate model used. Using the $r_{1,L}$ that was guessed and the corresponding $r_{1,H}$, the 2-year on-the-run issue can be valued. If the resulting value computed using the backward induction method is not

equal to the market value of the 2-year on-the-run issue, then the $r_{1,L}$ that was tried is not the rate that should be used in the tree. If the value is too high, then a higher rate guess should be tried; if the value is too low, then a lower rate guess should be tried. The process continues in an iterative (i.e., trial and error) process until a value for $r_{1,L}$ and the corresponding $r_{1,H}$ produce a value for the 2-year on-the-run issue equal to its market value.

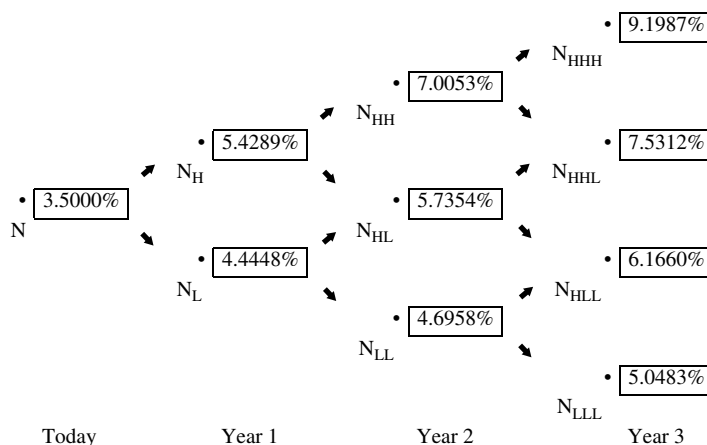
After this stage, we have the rate at the root of the tree and the two rates for year 1— $r_{1,L}$ and $r_{1,H}$. Now we need the three rates for year 2— $r_{2,LL}$, $r_{2,HL}$, and $r_{2,HH}$. These rates are determined from the following information:

1. the coupon rate for the 3-year on-the-run issue
2. the interest rate model assumed
3. the interest rate volatility assumed
4. the interest rate at the root of the tree (i.e., the current 1-year on-the-run rate)
5. the two 1-year rates (i.e., $r_{1,L}$ and $r_{1,H}$)

A guess is made for $r_{2,LL}$. The interest rate model assumed specifies how to obtain $r_{2,HL}$, and $r_{2,HH}$ given $r_{2,LL}$ and the assumed volatility for the 1-year rate. This gives the rates in the interest rate tree that are needed to value the 3-year on-the-run issue. The 3-year on-the-run issue is then valued. If the value generated is not equal to the market value of the 3-year on-the-run issue, then the $r_{2,LL}$ value tried is not the rate that should be used in the tree. At iterative process is again followed until a value for $r_{2,LL}$ produces rates for year 2 that will make the value of the 3-year on-the-run issue equal to its market value.

The tree is grown using the same procedure as described above to get $r_{1,L}$ and $r_{1,H}$ for year 1 and $r_{2,LL}$, $r_{2,HL}$, and $r_{2,HH}$ for year 2. Exhibit 5 shows the binomial interest rate tree for this issuer for valuing issues up to four years of maturity assuming volatility for the 1-year rate of 10%. The interest rate model used is not important. How can we be sure that the interest rates shown in Exhibit 5 are the correct rates? Verification involves using the interest rate tree to value an on-the-run issue and showing that the value obtained from the binomial model is equal to the observed market value. For example, let's just show that the interest rates in the tree for years 0, 1, and 2 in Exhibit 5 are correct. To do this, we use the 3-year on-the-run

EXHIBIT 5 Binomial Interest Rate Tree for Valuing an Issuer's Bond with a Maturity Up to 4 Years (10% Volatility Assumed)



yield curve is the one given earlier and hence the appropriate binomial interest rate tree is the one in Exhibit 5. Exhibit 7 shows the various values in the discounting process, and produces a bond value of \$104.643.

It is important to note that this value is identical to the bond value found earlier when we discounted at either the spot rates or the 1-year forward rates. We should expect to find this result since our bond is option free. This clearly demonstrates that the valuation model is consistent with the arbitrage-free valuation model for an option-free bond.

VI. VALUING AND ANALYZING A CALLABLE BOND

Now we will demonstrate how the binomial interest rate tree can be applied to value a callable bond. The valuation process proceeds in the same fashion as in the case of an option-free bond, but with one exception: when the call option may be exercised by the issuer, the bond value at a node must be changed to reflect the lesser of its values if it is not called (i.e., the value obtained by applying the backward induction method described above) and the call price. As explained earlier, at a node either a random event or a decision must be made. In constructing the binomial interest rate tree, there is a random event at a node. When valuing a bond with an embedded option, at a node there will be a decision made as to whether or not an option will be exercised. In the case of a callable bond, the issuer must decide whether or not to exercise the call option.

For example, consider a 6.5% bond with four years remaining to maturity that is callable in one year at \$100. Exhibit 8 shows two values at each node of the binomial interest rate tree. The discounting process explained above is used to calculate the first of the two values at each node. The second value is the value based on whether the issue will be called. For simplicity, let's assume that this issuer calls the issue if it exceeds the call price.

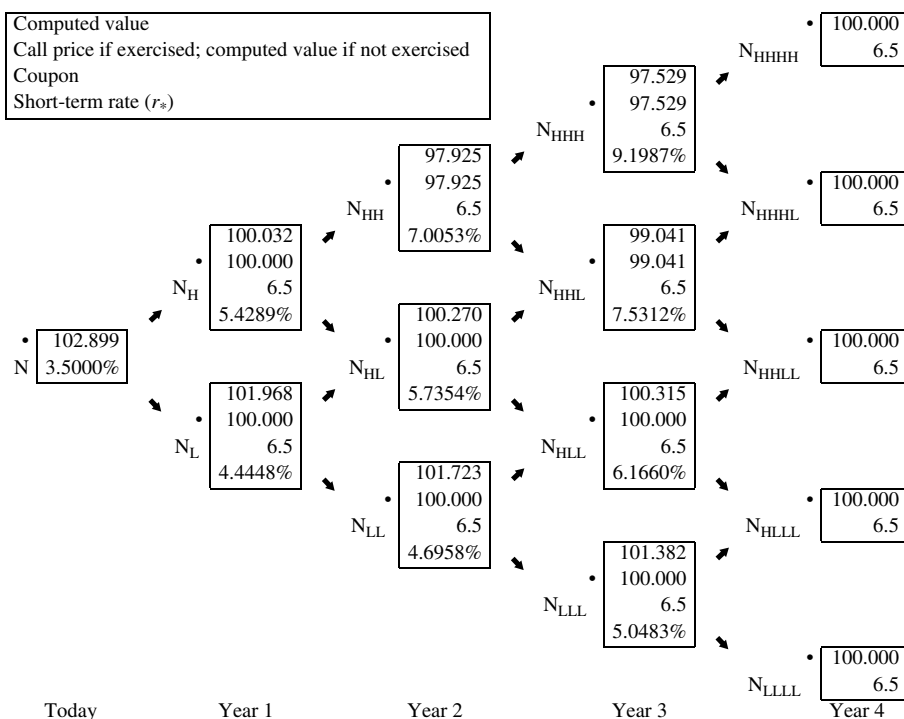
In Exhibit 9 two portions of Exhibit 8 are highlighted. Panel a of the exhibit shows nodes where the issue is not called (based on the simple call rule used in the illustration) in year 2 and year 3. The values reported in this case are the same as in the valuation of an option-free bond. Panel b of the exhibit shows some nodes where the issue is called in year 2 and year 3. Notice how the methodology changes the cash flows. In year 3, for example, at node N_{HLL} the backward induction method produces a value (i.e., cash flow) of 100.315. However, given the simplified call rule, this issue would be called. Therefore, 100 is shown as the second value at the node and it is this value that is then used in the backward induction methodology. From this we can see how the binomial method changes the cash flow based on future interest rates and the embedded option.

The root of the tree, shown in Exhibit 8, indicates that the value for this callable bond is \$102.899.

The question that we have not addressed in our illustration, which is nonetheless important, is the circumstances under which the issuer will actually call the bond. A detailed explanation of the call rule is beyond the scope of this chapter. Basically, it involves determining when it would be economical for the issuer on an after-tax basis to call the issue.

Suppose instead that the call price schedule is 102 in year 1, 101 in year 2, and 100 in year 3. Also assume that the bond will not be called unless it exceeds the call price for that year. Exhibit 10 shows the value at each node and the value of the callable bond. The call price schedule results in a greater value for the callable bond, \$103.942 compared to \$102.899 when the call price is 100 in each year.

EXHIBIT 8 Valuing a Callable Bond with Four Years to Maturity, a Coupon Rate of 6.5%, and Callable in One Year at 100 (10% Volatility Assumed)



A. Determining the Call Option Value

As explained in Chapter 2, the value of a callable bond is equal to the value of an option-free bond minus the value of the call option. This means that:

$$\text{value of a call option} = \text{value of an option-free bond} - \text{value of a callable bond}$$

We have just seen how the value of an option-free bond and the value of a callable bond can be determined. The difference between the two values is therefore the value of the call option.

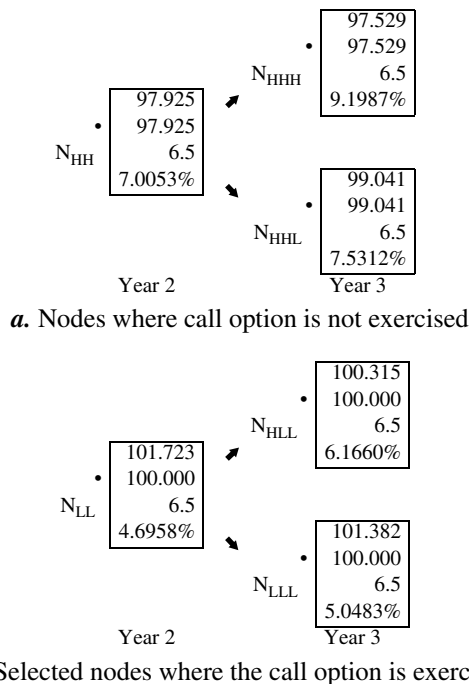
In our illustration, the value of the option-free bond is \$104.643. If the call price is \$100 in each year and the value of the callable bond is \$102.899 assuming 10% volatility for the 1-year rate, the value of the call option is \$1.744 ($= \$104.643 - \$102.899$).

B. Volatility and the Arbitrage-Free Value

In our illustration, interest rate volatility was assumed to be 10%. The volatility assumption has an important impact on the arbitrage-free value. More specifically, the higher the expected volatility, the higher the value of an option. The same is true for an option embedded in a bond. Correspondingly, this affects the value of a bond with an embedded option.

For example, for a callable bond, a higher interest rate volatility assumption means that the value of the call option increases and, since the value of the option-free bond is not affected, the value of the callable bond must be lower.

EXHIBIT 9 Highlighting Nodes in Years 2 and 3 for a Callable Bond

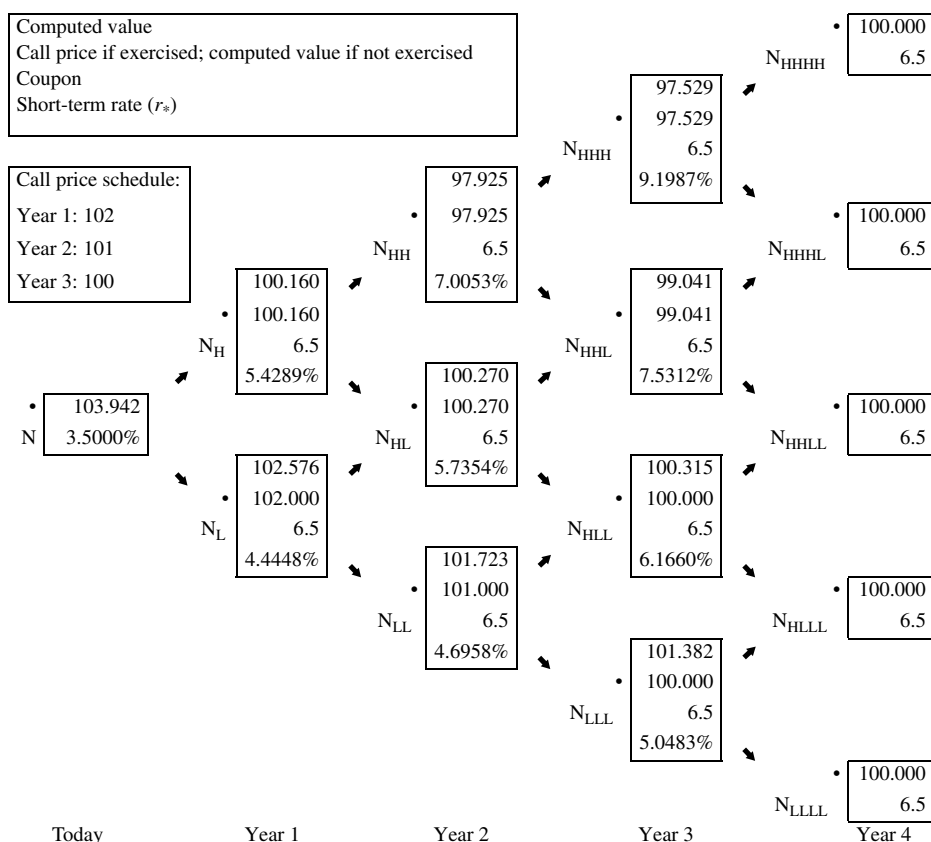


We can see this using the on-the-run yield curve in our previous illustrations. Where we assumed an interest rate volatility of 10%. To see the effect of higher volatility, assume an interest rate volatility of 20%. It can be demonstrated that at this higher level of volatility, the value of the option-free bond is unchanged at \$104.643. This is as expected since there is no embedded option. For the callable bond where it is assumed that the issue is callable at par beginning in Year 1, it can be demonstrated that the value is \$102.108 at a 20% volatility, a value that is less than when a 10% volatility is assumed (\$102.899). The reason for this is that the value of an option increases with the higher assumed volatility. So, at 20% volatility the value of the embedded call option is higher than at 10% volatility. But the embedded call option is subtracted from the option-free value to obtain the value of the callable bond. Since a higher value for the embedded call option is subtracted from the option-free value at 20% volatility than at 10% volatility, the value of the callable bond is lower at 20% volatility.

C. Option-Adjusted Spread

Suppose the market price of the 4-year 6.5% callable bond is \$102.218 and the theoretical value assuming 10% volatility is \$102.899. This means that this bond is cheap by \$0.681 according to the valuation model. Bond market participants prefer to think not in terms of a bond's price being cheap or expensive in dollar terms but rather in terms of a yield spread—a cheap bond trades at a higher yield spread and an expensive bond at a lower yield spread.

EXHIBIT 10 Valuing a Callable Bond with Four Years to Maturity, a Coupon Rate of 6.5%, and with a Call Price Schedule (10% Volatility Assumed)

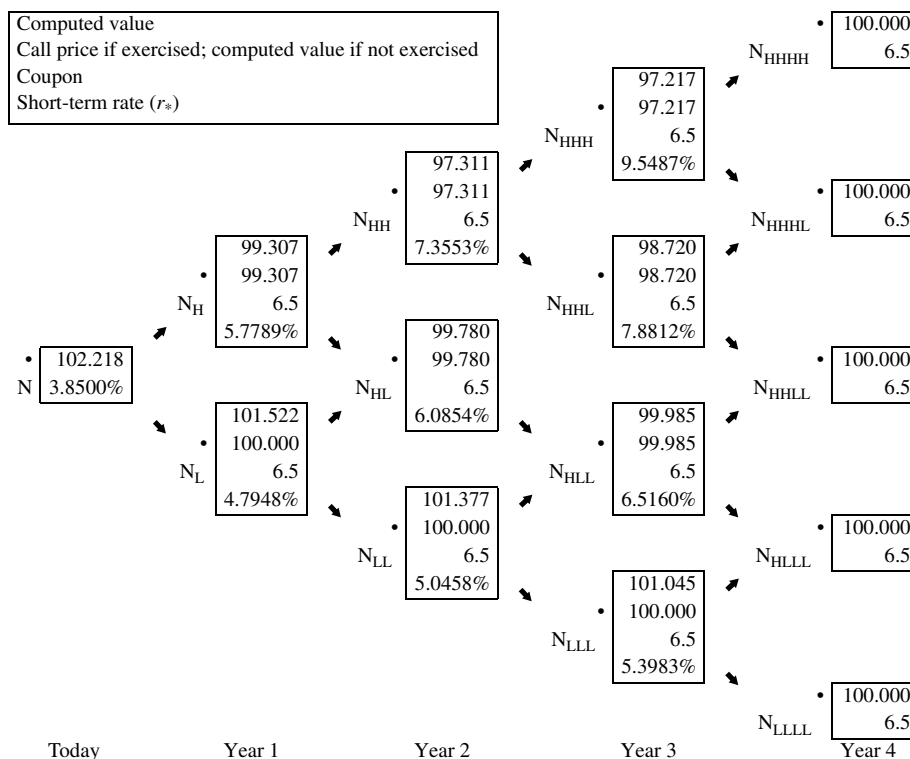


The **option-adjusted spread** is the constant spread that when added to all the 1-year rates on the binomial interest rate tree that will make the arbitrage-free value (i.e., the value produced by the binomial model) equal to the market price. In our illustration, if the market price is \$102.218, the OAS would be the constant spread added to every rate in Exhibit 5 that will make the arbitrage-free value equal to \$102.218. The solution in this case would be 35 basis points. This can be verified in Exhibit 11 which shows the value of this issue by adding 35 basis points to each rate.

As with the value of a bond with an embedded option, the OAS will depend on the volatility assumption. For a given bond price, the higher the interest rate volatility assumed, the lower the OAS for a callable bond. For example, if volatility is 20% rather than 10%, the OAS would be -6 basis points. This illustration clearly demonstrates the importance of the volatility assumption. Assuming volatility of 10%, the OAS is 35 basis points. At 20% volatility, the OAS declines and, in this case is negative and therefore the bond is overvalued relative to the model.

What the OAS seeks to do is remove from the nominal spread the amount that is due to the option risk. The measure is called an OAS because (1) it is a spread and (2) it adjusts the cash flows for the option when computing the spread to the benchmark interest rates. The

EXHIBIT 11 Demonstration that the Option-Adjusted Spread is 35 Basis Points for a 6.5% Callable Bond Selling at 102.218 (Assuming 10% Volatility)



*Each 1-year rate is 35 basis points greater than in Exhibit 5.

second point can be seen from Exhibits 8 and 9. Notice that at each node the value obtained from the backward induction method is adjusted based on the call option and the call rule. Thus, the resulting spread is “option adjusted.”

What does the OAS tell us about the relative value for our callable bond? As explained in Section III, the answer depends on the benchmark used. Exhibit 1 provides a summary of how to interpret the OAS. In valuing the callable bond in our illustration, the benchmark is the issuer’s own securities. As can be seen in Exhibit 1, a positive OAS means that the callable bond is cheap (i.e., underpriced). At a 10% volatility, the OAS is 35 basis points. Consequently, assuming a 10% volatility, on a relative value basis the callable bond is attractive. However, and this is critical to remember, the OAS depends on the assumed interest rate volatility. When a 20% interest rate volatility is assumed, the OAS is -6 basis points. Hence, if an investor assumes that this is the appropriate interest rate volatility that should be used in valuing the callable bond, the issue is expensive (overvalued) on a relative value basis.

D. Effective Duration and Effective Convexity

At Level I (Chapter 7), we explained the meaning of duration and convexity measures and explained how these two measures can be computed. Specifically, duration is the approximate percentage change in the value of a security for a 100 basis point change in interest rates (assuming a parallel shift in the yield curve). The convexity measure allows for an adjustment

to the estimated price change obtained by using duration. The formula for duration and convexity are repeated below:

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)}$$

$$\text{convexity} = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2}$$

where

Δy = change in rate used to calculate new values

V_+ = estimated value if yield is increased by Δy

V_- = estimated value if yield is decreased by Δy

V_0 = initial price (per \$100 of par value)

We also made a distinction between “modified” duration and convexity and “effective” duration and convexity.⁸ Modified duration and convexity do not allow for the fact that the cash flows for a bond with an embedded option may change due to the exercise of the option. In contrast, effective duration and convexity do take into consideration how changes in interest rates in the future may alter the cash flows due to the exercise of the option. But, we did not demonstrate how to compute effective duration and convexity because they require a model for valuing bonds with embedded options and we did not introduce such models until this chapter.

So, let’s see how effective duration and convexity are computed using the binomial model. With effective duration and convexity, the values V_- and V_+ are obtained from the binomial model. Recall that in using the binomial model, the cash flows at a node are adjusted for the embedded call option as was demonstrated in Exhibit 8 and highlighted in the lower panel of Exhibit 9.

The procedure for calculating the value of V_+ is as follows:

Step 1: Given the market price of the issue calculate its OAS using the procedure described earlier.

Step 2: Shift the on-the-run yield curve up by a small number of basis points (Δy).

Step 3: Construct a binomial interest rate tree based on the new yield curve in Step 2.

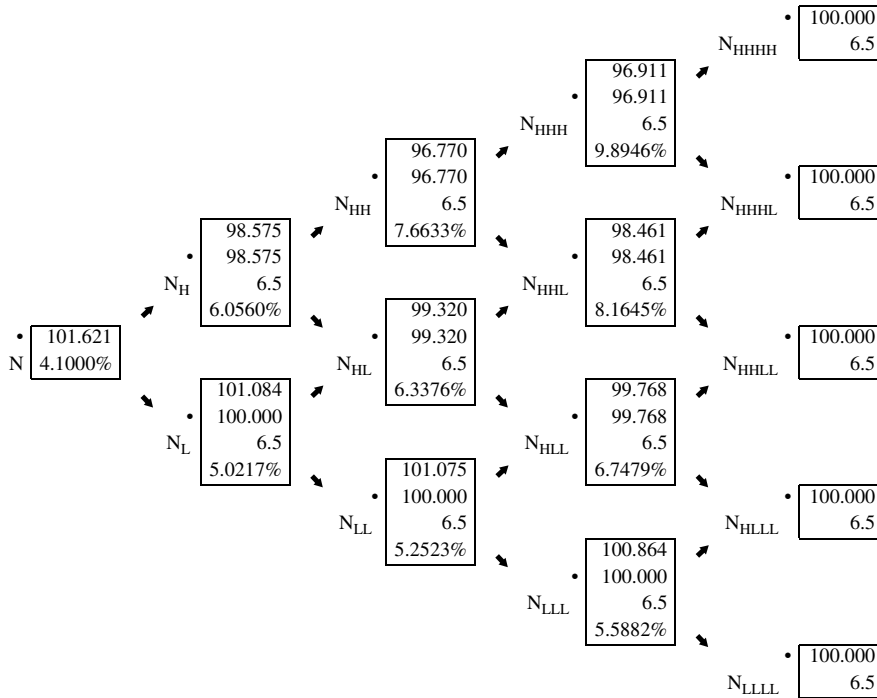
Step 4: To each of the 1-year rates in the binomial interest rate tree, add the OAS to obtain an “adjusted tree.” That is, the calculation of the effective duration and convexity assumes that the OAS will not change when interest rates change.

Step 5: Use the adjusted tree found in Step 4 to determine the value of the bond, which is V_+ .

To determine the value of V_- , the same five steps are followed except that in Step 2, the on-the-run yield curve is shifted down by a small number of basis points (Δy).

To illustrate how V_+ and V_- are determined in order to calculate effective duration and effective convexity, we will use the same on-the-run yield curve that we have used in our previous illustrations assuming a volatility of 10%. The 4-year callable bond with a coupon

⁸See Exhibit 18 in Chapter 7.

EXHIBIT 12 Determination of V_+ for Calculating Effective Duration and Convexity*

*+25 basis point shift in on-the-run yield curve.

rate of 6.5% and callable at par selling at 102.218 will be used in this illustration. The OAS for this issue is 35 basis points.

Exhibit 12 shows the adjusted tree by shifting the yield curve up by an arbitrarily small number of basis points, 25 basis points, and then adding 35 basis points (the OAS) to each 1-year rate. The adjusted tree is then used to value the bond. The resulting value, V_+ , is 101.621. Exhibit 13 shows the adjusted tree by shifting the yield curve down by 25 basis points and then adding 35 basis points to each 1-year rate. The resulting value, V_- , is 102.765.

The results are summarized below:

$$\Delta y = 0.0025$$

$$V_+ = 101.621$$

$$V_- = 102.765$$

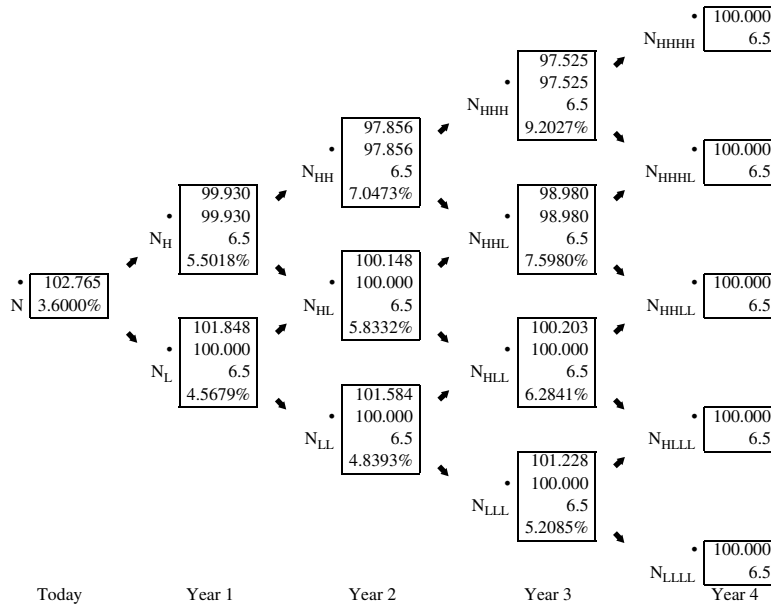
$$V_0 = 102.218$$

Therefore,

$$\text{effective duration} = \frac{102.765 - 101.621}{2(102.218)(0.0025)} = 2.24$$

$$\text{effective convexity} = \frac{101.621 + 102.765 - 2(102.218)}{2(102.218)(0.0025)^2} = -39.1321$$

EXHIBIT 13 Determination of V_- for Calculating Effective Duration and Convexity*



*—25 basis point shift in on-the-run yield curve.

Notice that this callable bond exhibits negative convexity. The characteristic of negative convexity for a bond with an embedded option was explained in Chapter 7.

VII. VALUING A PUTABLE BOND

A putable bond is one in which the bondholder has the right to force the issuer to pay off the bond prior to the maturity date. To illustrate how the binomial model can be used to value a putable bond, suppose that a 6.5% bond with four years remaining to maturity is putable in one year at par (\$100). Also assume that the appropriate binomial interest rate tree for this issuer is the one in Exhibit 5 and the bondholder exercises the put if the bond's price is less than par.

Exhibit 14 shows the binomial interest rate tree with the values based on whether or not the investor exercises the option at a node. Exhibit 15 highlights selected nodes for year 2 and year 3 just as we did in Exhibit 9. The lower part of the exhibit shows the nodes where the put option is not exercised and therefore the value at each node is the same as when the bond is option free. In contrast, the upper part of the exhibit shows where the value obtained from the backward induction method is overridden and 100 is used because the put option is exercised.

The value of the putable bond is \$105.327, a value that is greater than the value of the corresponding option-free bond. The reason for this can be seen from the following relationship:

$$\text{value of a putable bond} = \text{value of an option-free bond} + \text{value of the put option}$$

EXHIBIT 14 Valuing a Putable Bond with Four Years to Maturity, a Coupon Rate of 6.5%, and Putable in One Year at 100 (10% Volatility Assumed)

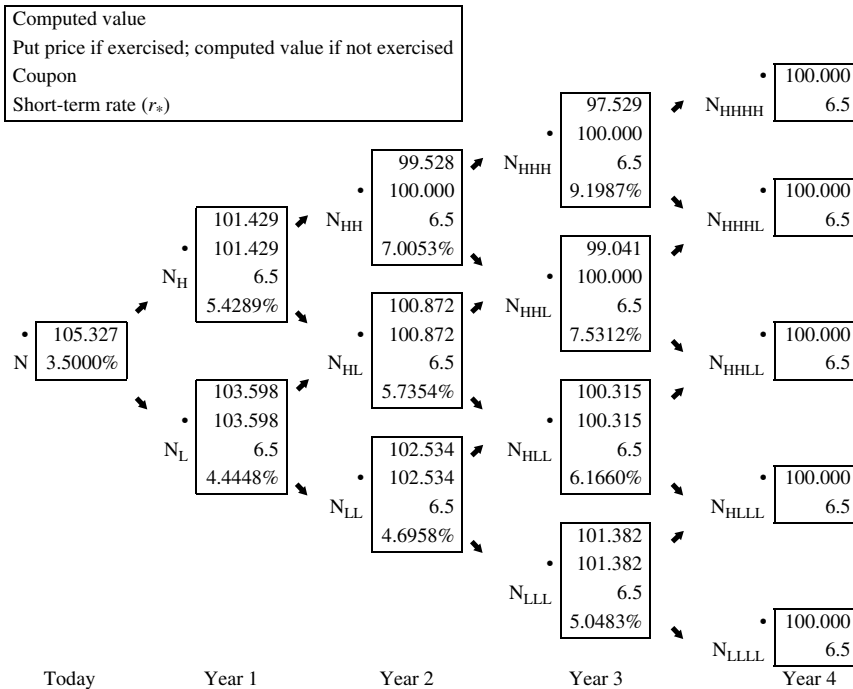
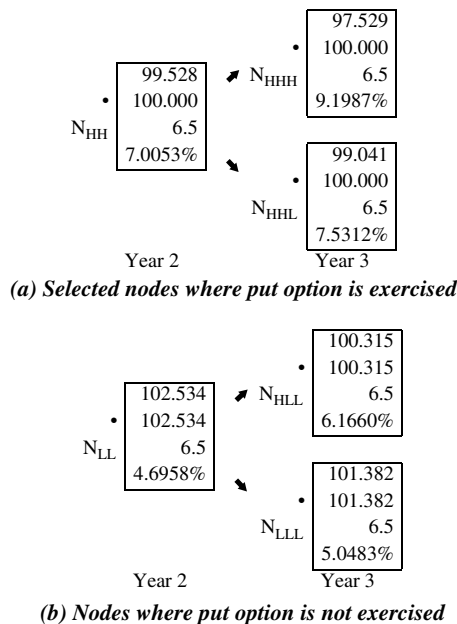


EXHIBIT 15 Highlighting Nodes in Years 2 and 3 for a Putable Bond



The reason for adding the value of the put option is that the investor has purchased the put option.

We can rewrite the above relationship to determine the value of the put option:

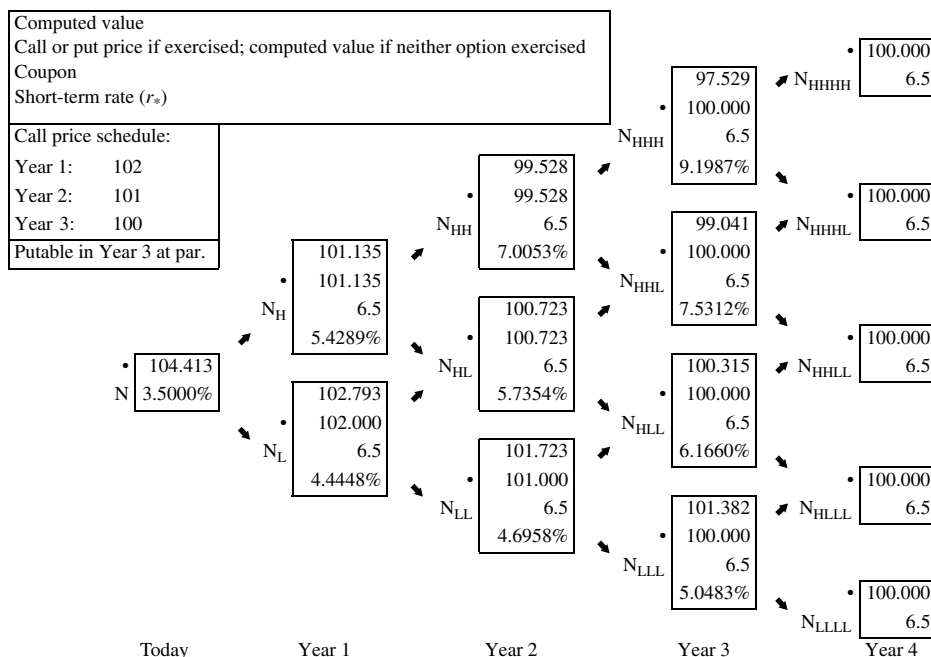
$$\text{value of the put option} = \text{value of a putable bond} - \text{value of an option-free bond}$$

In our example, since the value of the putable bond is \$105.327 and the value of the corresponding option-free bond is \$104.643, the value of the put option is $-\$0.684$. The negative sign indicates the issuer has sold the option, or equivalently, the investor has purchased the option.

We have stressed that the value of a bond with an embedded option is affected by the volatility assumption. Unlike a callable bond, the value of a putable bond increases if the assumed volatility increases. It can be demonstrated that if a 20% volatility is assumed the value of this putable bond increases from 105.327 at 10% volatility to 106.010.

Suppose that a bond is both putable and callable. The procedure for valuing such a structure is to adjust the value at each node to reflect whether the issue would be put or called. To illustrate this, consider the 4-year callable bond analyzed earlier that had a call schedule. The valuation of this issue is shown in Exhibit 10. Suppose the issue is putable in year 3 at par value. Exhibit 16 shows how to value this callable/putable issue. At each node there are two decisions about the exercising of an option that must be made. First, given the valuation from the backward induction method at a node, the call rule is invoked to determine whether the issue will be called. If it is called, the value at the node is replaced by the call price. The valuation procedure then continues using the call price at that node. Second, if the call option is not exercised at a node, it must be determined whether or not the put option is exercised. If

EXHIBIT 16 Valuing a Putable/Callable Issue (10% Volatility Assumed)



it is exercised, then the value from the backward induction method is overridden and the put price is substituted at that node and is used in subsequent calculations.

VIII. VALUING A STEP-UP CALLABLE NOTE

Step-up callable notes are callable instruments whose coupon rate is increased (i.e., “stepped up”) at designated times. When the coupon rate is increased only once over the security’s life, it is said to be a **single step-up callable note**. A **multiple step-up callable note** is a step-up callable note whose coupon is increased more than one time over the life of the security. Valuation using the binomial model is similar to that for valuing a callable bond except that the cash flows are altered at each node to reflect the coupon changing characteristics of a step-up note.

To illustrate how the binomial model can be used to value step-up callable notes, let’s begin with a single step-up callable note. Suppose that a 4-year step-up callable note pays 4.25% for two years and then 7.5% for two more years. Assume that this note is callable at par at the end of Year 2 and Year 3. We will use the binomial interest rate tree given in Exhibit 5 to value this note.

Exhibit 17 shows the value of a corresponding single step-up *noncallable* note. The valuation procedure is identical to that performed in Exhibit 8 except that the coupon in the box at each node reflects the step-up terms. The value is \$102.082. Exhibit 18 shows that the value of the single step-up callable note is \$100.031. The value of the embedded call option is equal to the difference in the step-up noncallable note value and the step-up callable note value, \$2.051.

The procedure is the same for a multiple step-up callable note. Suppose that a multiple step-up callable note has the following coupon rates: 4.2% in Year 1, 5% in Year 2, 6% in

EXHIBIT 17 Valuing a Single Step-Up Noncallable Note with Four Years to Maturity (10% Volatility Assumed)

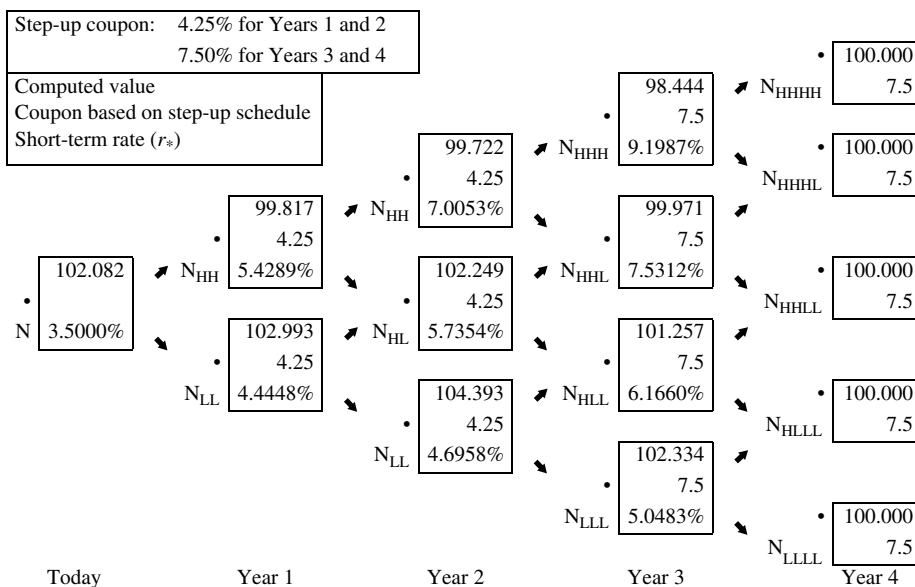
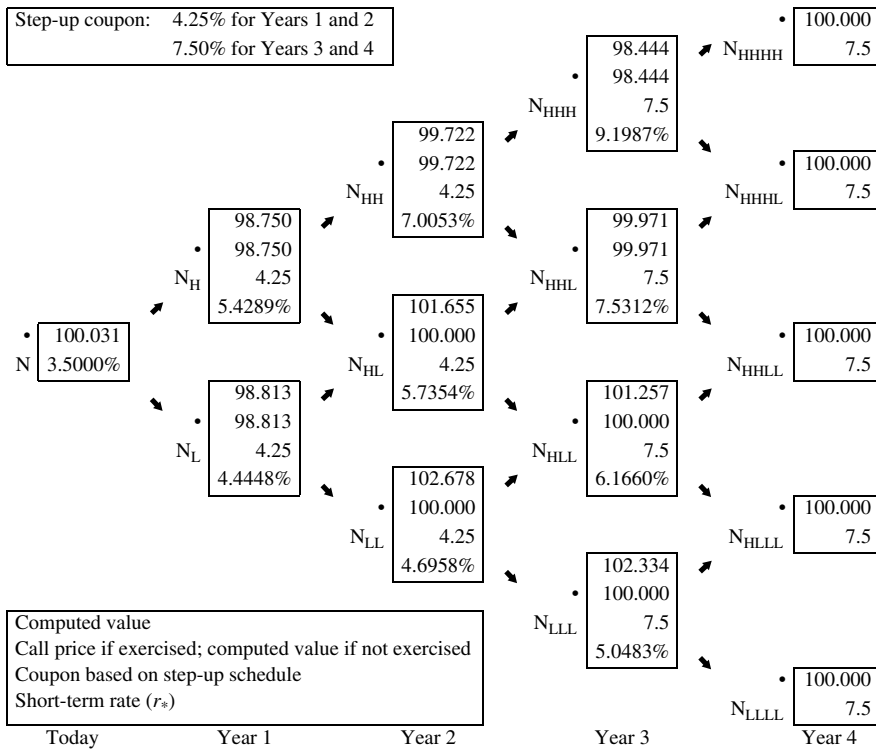


EXHIBIT 18 Valuing a Single Step-Up Callable Note with Four Years to Maturity, Callable in Two Years at 100 (10% Volatility Assumed)



Year 3, and 7% in Year 4. Also assume that the note is callable at the end of Year 1 at par. Exhibit 19 shows that the value of this note if it *is* noncallable is \$101.012. The value of the multiple step-up callable note is \$99.996 as shown in Exhibit 20. Therefore, the value of the embedded call option is \$1.016 (= 101.012 – 99.996).

IX. VALUING A CAPPED FLOATER

The valuation of a floating-rate note with a cap (i.e., a capped floater) using the binomial model requires that the coupon rate be adjusted based on the 1-year rate (which is assumed to be the reference rate). Exhibit 21 shows the binomial tree and the relevant values at each node for a floater whose coupon rate is the 1-year rate flat (i.e., no margin over the reference rate) and in which there are no restrictions on the coupon rate.

What is important to recall about floaters is that the coupon rate is set at the beginning of the period but paid at the end of the period (i.e., beginning of the next period). That is, the coupon interest is paid in arrears. We discussed this feature of floaters at Chapter 1.

The valuation procedure is identical to that for the other structures described above except that an adjustment is made for the characteristic of a floater that the coupon rate is set at the beginning of the year and paid in arrears. Here is how the payment in arrears characteristic affects the backward induction method. Look at the top node for year 2 in Exhibit 21. The

EXHIBIT 19 Valuing a Multiple Step-Up Noncallable Note with Four Years to Maturity (10% Volatility Assumed)

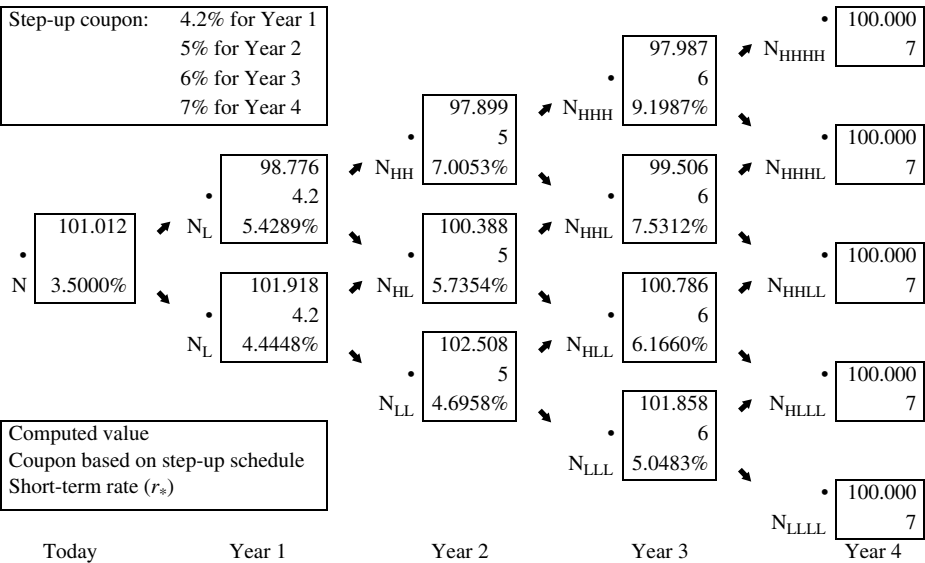


EXHIBIT 20 Valuing a Multiple Step-Up Callable Note with Four Years to Maturity, and Callable in One Year at 100 (10% Volatility Assumed)

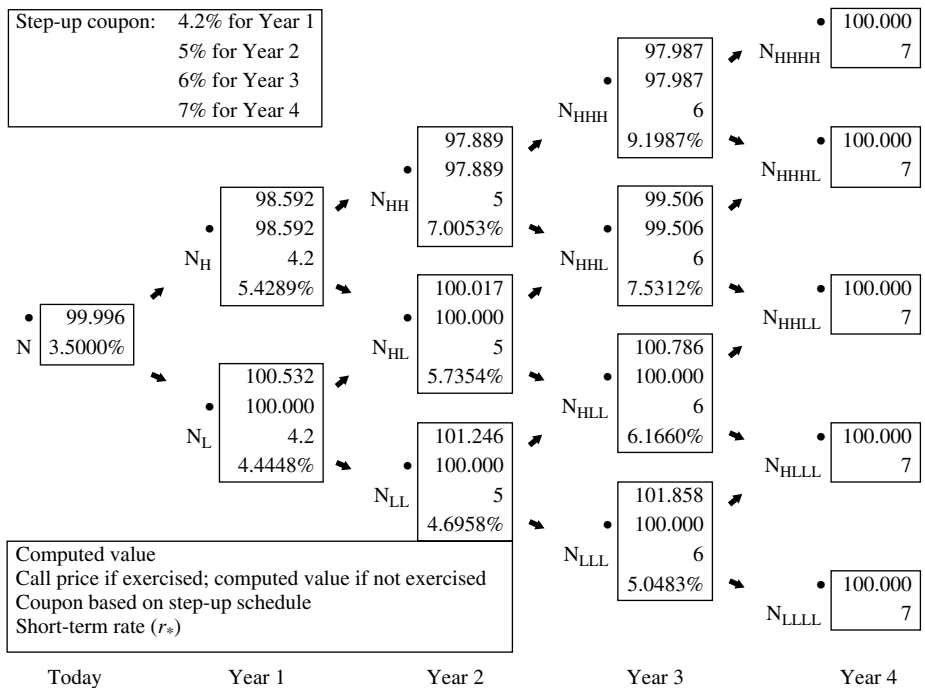
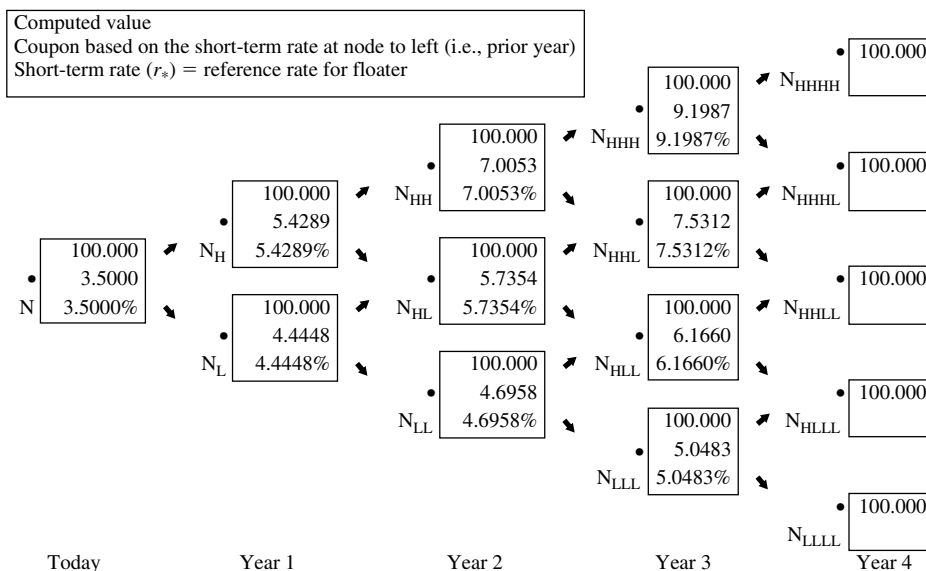


EXHIBIT 21 Valuing a Floater with No Cap (10% Volatility Assumed)



Note: The coupon rate shown at a node is the coupon rate to be received in the next year.

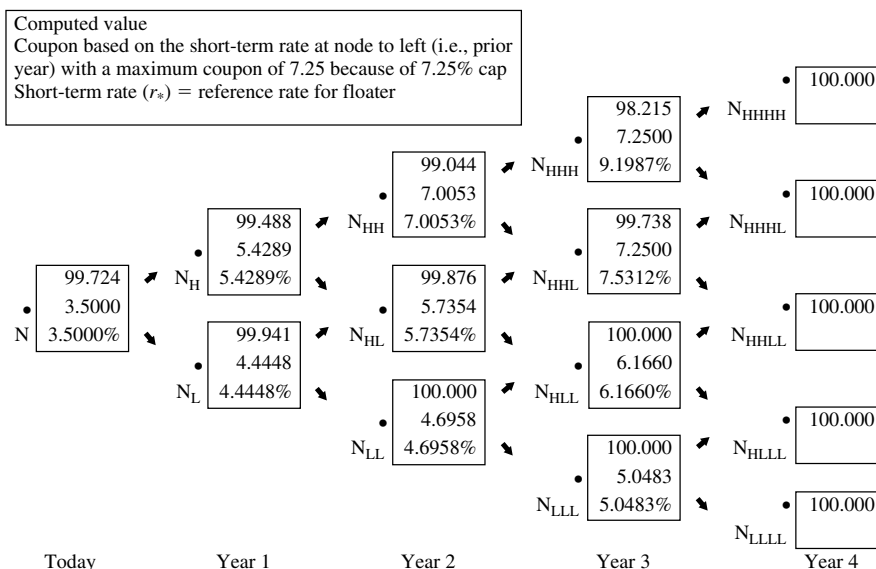
coupon rate shown at that node is 7.0053% as determined by the 1-year rate at that node. Since the coupon payment will not be made until year 3 (i.e., paid in arrears), the value of 100 shown at the node is determined using the backward induction method but discounting the coupon rate shown at the node. For example, let's see how we get the value of 100 in the top box in year 2. The procedure is to calculate the average of the two present values of the bond value and coupon. Since the bond values and coupons are the same, the present value is simply:

$$\frac{100 + 7.0053}{1.070053} = 100$$

Suppose that the floater has a cap of 7.25%. Exhibit 22 shows how this floater would be valued. At each node where the 1-year rate exceeds 7.25%, a coupon of \$7.25 is substituted. The value of this capped floater is 99.724. Thus, the cost of the cap is the difference between par and 99.724. If the cap for this floater was 7.75% rather than 7.25%, it can be shown that the value of this floater would be 99.858. That is, the higher the cap, the closer the capped floater will trade to par.

Thus, it is important to emphasize that the valuation mechanics are being modified slightly only to reflect the characteristics of the floater's cash flow. All of the other principles regarding valuation of bonds with embedded options are the same. For a capped floater there is a rule for determining whether or not to override the cash flow at a node based on the cap. Since a cap embedded in a floater is effectively an option granted by the investor to the issuer, it should be no surprise that the valuation model described in this chapter can be used to value a capped floater.

EXHIBIT 22 Valuing a Floating Rate Note with a 7.25% Cap (10% Volatility Assumed)



Note: The coupon rate shown at a node is the coupon rate to be received in the next year.

X. ANALYSIS OF CONVERTIBLE BONDS

A convertible bond is a security that can be converted into common stock at the option of the investor. Hence, it is a bond with an embedded option where the option is granted to the investor. Moreover, since a convertible bond may be callable and puttable, it is a complex bond because the value of the bond will depend on both how interest rates change (which affects the value of the call and any put option) *and* how changes in the market price of the stock affects the value of the option to convert to common stock.

A. Basic Features of Convertible Securities

The conversion provision of a convertible security grants the securityholder the right to convert the security into a predetermined number of shares of common stock of the issuer. A convertible security is therefore a security with an embedded call option to buy the common stock of the issuer. An **exchangeable security** grants the securityholder the right to exchange the security for the common stock of a firm *other* than the issuer of the security. Throughout this chapter we use the term convertible security to refer to both convertible and exchangeable securities.

In illustrating the calculation of the various concepts described below, we will use a hypothetical convertible bond issue. The issuer is the All Digital Component Corporation (ADC) 5 $\frac{3}{4}$ % convertible issue due in 9+ years. Information about this hypothetical bond issue and the stock of this issuer is provided in Exhibit 23.

The number of shares of common stock that the securityholder will receive from exercising the call option of a convertible security is called the **conversion ratio**. The conversion privilege may extend for all or only some portion of the security's life, and the stated conversion ratio may change over time. It is always adjusted proportionately for stock splits and stock

EXHIBIT 23 Information About All Digital Component Corporation (ADC) 5 $\frac{3}{4}$ % Convertible Bond Due in 9+ Years and Common Stock

Convertible bond

Current market price: \$106.50

Maturity date: 9+ years

Non-call for 3 years

Call price schedule	
In Year 4	103.59
In Year 5	102.88
In Year 6	102.16
In Year 7	101.44
In Year 8	100.72
In Year 9	100.00
In Year 10	100.00

Coupon rate: 5 $\frac{3}{4}$ %

Conversion ratio: 25.320 shares of ADC shares per \$1,000 par value

Rating: A3/A-

ADC common stock

Expected volatility: 17%

Current dividend yield: 2.727%

Dividend per share: \$0.90 per year

Stock price: \$33

dividends. For the ADC convertible issue, the conversion ratio is 25.32 shares. This means that for each \$1,000 of par value of this issue the securityholder exchanges for ADC common stock, he will receive 25.32 shares.

At the time of issuance of a convertible bond, the effective price at which the buyer of the convertible bond will pay for the stock can be determined as follows. The prospectus will specify the number of shares that the investor will receive by exchanging the bond for the common stock. The number of shares is called the **conversion ratio**. So, for example, assume the conversion ratio is 20. If the investor converts the bond for stock the investor will receive 20 shares of common stock. Now, suppose that the par value for the convertible bond is \$1,000 and is sold to investors at issuance at that price. Then effectively by buying the convertible bond for \$1,000 at issuance, investors are purchasing the common stock for \$50 per share (\$1,000/20 shares). This price is referred to in the prospectus as the **conversion price** and some investors refer to it as the **stated conversion price**. For a bond not issued at par (for example, a zero-coupon bond), the market or effective conversion price is determined by dividing the issue price per \$1,000 of par value by the conversion ratio.

The ADC convertible was issued for \$1,000 per \$1,000 of par value and the conversion ratio is 25.32. Therefore, the conversion price at issuance for the ADC convertible issue is \$39.49 (\$1,000/25.32 shares).

Almost all convertible issues are callable. The ADC convertible issue has a non-call period of three years. The call price schedule for the ADC convertible issue is shown in Exhibit 23. There are some issues that have a provisional call feature that allows the issuer to call the issue during the non-call period if the price of the stock reaches a certain price.

Some convertible bonds are puttable. Put options can be classified as “hard” puts and “soft” puts. A **hard put** is one in which the convertible security must be redeemed by the

issuer for cash. In the case of a **soft put**, while the investor has the option to exercise the put, the *issuer* may select how the payment will be made. The issuer may redeem the convertible security for cash, common stock, subordinated notes, or a combination of the three.

B. Traditional Analysis of Convertible Securities

Traditional analysis of convertible bonds relies on measures that do not attempt to directly value the embedded call, put, or common stock options. We present and illustrate these measures below and later discuss an option-based approach to valuation of convertible bonds.

1. **Minimum Value of a Convertible Security** The **conversion value** or **parity value** of a convertible security is the value of the security if it is converted immediately.⁹ That is,

$$\text{conversion value} = \text{market price of common stock} \times \text{conversion ratio}$$

The minimum price of a convertible security is the greater of¹⁰

1. Its conversion value, or
2. Its value as a security without the conversion option—that is, based on the convertible security’s cash flows if not converted (i.e., a plain vanilla security). This value is called its **straight value** or **investment value**. The straight value is found by using the valuation model described earlier in this chapter because almost all issues are callable.

If the convertible security does not sell for the greater of these two values, arbitrage profits could be realized. For example, suppose the conversion value is greater than the straight value, and the security trades at its straight value. An investor can buy the convertible security at the straight value and immediately convert it. By doing so, the investor realizes a gain equal to the difference between the conversion value and the straight value. Suppose, instead, the straight value is greater than the conversion value, and the security trades at its conversion value. By buying the convertible at the conversion value, the investor will realize a higher yield than a comparable straight security.

Consider the ADC convertible issue. Suppose that the straight value of the bond is \$98.19 per \$100 of par value. Since the market price per share of common stock is \$33, the conversion value per \$1,000 of par value is:

$$\text{conversion value} = \$33 \times 25.32 = \$835.56$$

Consequently, the conversion value is 83.556% of par value. Per \$100 of par value the conversion value is \$83.556. Since the straight value is \$98.19 and the conversion value is \$83.556, the minimum value for the ADC convertible has to be \$98.19.

⁹Technically, the standard textbook definition of conversion value given here is theoretically incorrect because as bondholders convert, the price of the stock will decline. The theoretically correct definition for the conversion value is that it is the product of the conversion ratio and the stock price *after* conversion.

¹⁰If the conversion value is the greater of the two values, it is possible for the convertible bond to trade below the conversion value. This can occur for the following reasons: (1) there are restrictions that prevent the investor from converting, (2) the underlying stock is illiquid, and (3) an anticipated forced conversion will result in loss of accrued interest of a high coupon issue. See, Mihir Bhattacharya, “Convertible Securities and Their Valuation,” Chapter 51 in Frank J. Fabozzi (ed.), *The Handbook of Fixed Income Securities: Sixth Edition* (New York: McGraw Hill, 2001), p. 1128.

2. Market Conversion Price The price that an investor effectively pays for the common stock if the convertible bond is purchased and then converted into the common stock is called the **market conversion price** or **conversion parity price**. It is found as follows:

$$\text{market conversion price} = \frac{\text{market price of convertible security}}{\text{conversion ratio}}$$

The market conversion price is a useful benchmark because once the actual market price of the stock rises above the market conversion price, any further stock price increase is certain to increase the value of the convertible bond by at least the same percentage. Therefore, the market conversion price can be viewed as a break-even price.

An investor who purchases a convertible bond rather than the underlying stock, effectively pays a premium over the current market price of the stock. This premium per share is equal to the difference between the market conversion price and the current market price of the common stock. That is,

$$\text{market conversion premium per share} = \text{market conversion price} - \text{current market price}$$

The market conversion premium per share is usually expressed as a percentage of the current market price as follows:

$$\text{market conversion premium ratio} = \frac{\text{market conversion premium per share}}{\text{market price of common stock}}$$

Why would someone be willing to pay a premium to buy the stock? Recall that the minimum price of a convertible security is the greater of its conversion value or its straight value. Thus, as the common stock price declines, the price of the convertible bond will not fall below its straight value. The straight value therefore acts as a floor for the convertible security's price. However, it is a moving floor as the straight value will change with changes in interest rates.

Viewed in this context, the market conversion premium per share can be seen as the price of a call option. As will be explained in Chapter 13, the buyer of a call option limits the downside risk to the option price. In the case of a convertible bond, for a premium, the securityholder limits the downside risk to the straight value of the bond. The difference between the buyer of a call option and the buyer of a convertible bond is that the former knows precisely the dollar amount of the downside risk, while the latter knows only that the most that can be lost is the difference between the convertible bond's price and the straight value. The straight value at some future date, however, is unknown; the value will change as market interest rates change or if the issuer's credit quality changes.

The calculation of the market conversion price, market conversion premium per share, and market conversion premium ratio for the ADC convertible issue is shown below:

$$\text{market conversion price} = \frac{\$1,065}{25.32} = \$42.06$$

Thus, if the investor purchased the convertible and then converted it to common stock, the effective price that the investor paid per share is \$42.06.

$$\text{market conversion premium per share} = \$42.06 - \$33 = \$9.06$$

The investor is effectively paying a premium per share of \$9.06 by buying the convertible rather than buying the stock for \$33.

$$\text{market conversion premium ratio} = \frac{\$9.06}{\$33} = 0.275 = 27.5\%$$

The premium per share of \$9.06 means that the investor is paying 27.5% above the market price of \$33 by buying the convertible.

3. Current Income of Convertible Bond versus Common Stock As an offset to the market conversion premium per share, investing in the convertible bond rather than buying the stock directly, generally means that the investor realizes higher current income from the coupon interest from a convertible bond than would be received from common stock dividends based on the number of shares equal to the conversion ratio. Analysts evaluating a convertible bond typically compute the time it takes to recover the premium per share by computing the **premium payback period** (which is also known as the **break-even time**). This is computed as follows:

$$\text{premium payback period} = \frac{\text{market conversion premium per share}}{\text{favorable income differential per share}}$$

where the favorable income differential per share is equal to the following:

$$\frac{\text{coupon interest} - (\text{conversion ratio} \times \text{common stock dividend per share})}{\text{conversion ratio}}$$

The numerator of the formula is the difference between the coupon interest for the issue and the dividends that would be received if the investor converted the issue into common stock. Since the investor would receive the number of shares specified by the conversion ratio, then multiplying the conversion ratio by the dividend per share of common stock gives the total dividends that would be received if the investor converted. Dividing the difference between the coupon interest and the total dividends that would be received if the issue is converted by the conversion ratio gives the favorable income differential on a per share basis by owning the convertible rather than the common stock or changes in the dividend over the period.

Notice that the premium payback period does *not* take into account the time value of money or changes in the dividend over the period.

For the ADC convertible issue, the market conversion premium per share is \$9.06. The favorable income differential per share is found as follows:

$$\text{coupon interest from bond} = 0.0575 \times \$1,000 = \$57.50$$

$$\text{conversion ratio} \times \text{dividend per share} = 25.32 \times \$0.90 = \$22.79$$

Therefore,

$$\text{favorable income differential per share} = \frac{\$57.50 - \$22.79}{25.32} = \$1.37$$

and

$$\text{premium payback period} = \frac{\$9.06}{\$1.37} = 6.6 \text{ years}$$

Without considering the time value of money, the investor would recover the market conversion premium per share assuming unchanged dividends in about 6.6 years.

4. Downside Risk with a Convertible Bond Unfortunately, investors usually use the straight value as a measure of the downside risk of a convertible security, because it is assumed that the price of the convertible cannot fall below this value. Thus, some investors view the straight value as the floor for the price of the convertible bond. The downside risk is measured as a percentage of the straight value and computed as follows:

$$\text{premium over straight value} = \frac{\text{market price of convertible bond}}{\text{straight value}} - 1$$

The higher the premium over straight value, all other factors constant, the less attractive the convertible bond.

Despite its use in practice, this measure of downside risk is flawed because the straight value (the floor) changes as interest rates change. If interest rates rise (fall), the straight value falls (rises) making the floor fall (rise). Therefore, the downside risk changes as interest rates change.

For the ADC convertible issue, since the market price of the convertible issue is 106.5 and the straight value is 98.19, the premium over straight value is

$$\text{premium over straight value} = \frac{\$106.50}{\$98.19} - 1 = 0.085 = 8.5\%$$

5. The Upside Potential of a Convertible Security The evaluation of the upside potential of a convertible security depends on the prospects for the underlying common stock. Thus, the techniques for analyzing common stocks discussed in books on equity analysis should be employed.

C. Investment Characteristics of a Convertible Security

The investment characteristics of a convertible bond depend on the common stock price. If the price is low, so that the straight value is considerably higher than the conversion value, the security will trade much like a straight security. The convertible security in such instances is referred to as a **fixed income equivalent** or a **busted convertible**.

When the price of the stock is such that the conversion value is considerably higher than the straight value, then the convertible security will trade as if it were an equity instrument; in this case it is said to be a **common stock equivalent**. In such cases, the market conversion premium per share will be small.

Between these two cases, fixed income equivalent and common stock equivalent, the convertible security trades as a **hybrid security**, having the characteristics of both a fixed income security and a common stock instrument.

D. An Option-Based Valuation Approach

In our discussion of convertible bonds, we did not address the following questions:

1. What is a fair value for the conversion premium per share?
2. How do we handle convertible bonds with call and/or put options?
3. How does a change in interest rates affect the stock price?

Consider first a noncallable/nonputable convertible bond. The investor who purchases this security would be effectively entering into two separate transactions: (1) buying a noncallable/nonputable straight security and (2) buying a call option (or warrant) on the stock, where the number of shares that can be purchased with the call option is equal to the conversion ratio.

The question is: What is the fair value for the call option? The fair value depends on the factors to be discussed in Chapter 14 that affect the price of a call option. While the discussion in that chapter will focus on options where the underlying is a fixed income instrument, the principles apply also to options on common stock. One key factor is the expected price volatility of the stock: the higher the expected price volatility, the greater the value of the call option. The theoretical value of a call option can be valued using the Black-Scholes option pricing model. This model will be discussed in Chapter 14 and is explained in more detail in investment textbooks. As a first approximation to the value of a convertible bond, the formula would be:

$$\text{convertible security value} = \text{straight value} + \text{value of the call option on the stock}$$

The value of the call option is added to the straight value because the investor has purchased a call option on the stock.

Now let's add in a common feature of a convertible bond: the issuer's right to call the issue. Therefore, the value of a convertible bond that is callable is equal to:

$$\begin{aligned} \text{convertible bond value} &= \text{straight value} + \text{value of the call option on the stock} \\ &\quad - \text{value of the call option on the bond} \end{aligned}$$

Consequently, the analysis of convertible bonds must take into account the value of the issuer's right to call. This depends, in turn, on (1) future interest rate volatility and (2) economic factors that determine whether or not it is optimal for the issuer to call the security. The Black-Scholes option pricing model cannot handle this situation.

Let's add one more wrinkle. Suppose that the callable convertible bond is also putable. Then the value of such a convertible would be equal to:

$$\begin{aligned} \text{convertible bond value} &= \text{straight value} + \text{value of the call option on the stock} \\ &\quad - \text{value of the call option on the bond} \\ &\quad + \text{value of the put option on the bond} \end{aligned}$$

To link interest rates and stock prices together (the third question we raise above), statistical analysis of historical movements of these two variables must be estimated and incorporated into the model.

Valuation models based on an option pricing approach have been suggested by several researchers.¹¹ These models can generally be classified as one-factor or multi-factor models. By "factor" we mean the stochastic (i.e., random) variables that are assumed to drive the

¹¹See, for example: Michael Brennan and Eduardo Schwartz, "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," *Journal of Finance* (December 1977), pp. 1699–1715; Jonathan Ingersoll, "A Contingent-Claims Valuation of Convertible Securities," *Journal of Financial Economics* (May 1977), pp. 289–322; Michael Brennan and Eduardo Schwartz, "Analyzing Convertible

value of a convertible or bond. The obvious candidates for factors are the price movement of the underlying common stock and the movement of interest rates. According to Mihir Bhattacharya and Yu Zhu, the most widely used convertible valuation model has been the one-factor model and the factor is the price movement of the underlying common stock.¹²

E. The Risk/Return Profile of a Convertible Security

Let's use the ADC convertible issue and the valuation model to look at the risk/return profile by investing in a convertible issue or the underlying common stock.

Suppose an investor is considering the purchase of either the common stock of ADC or the convertible issue. The stock can be purchased in the market for \$33. By buying the convertible bond, the investor is effectively purchasing the stock for \$42.06 (the market conversion price per share). Exhibit 24 shows the total return for both alternatives one year later assuming (1) the stock price does not change, (2) it changes by $\pm 10\%$, and (3) it changes by $\pm 25\%$. The convertible's theoretical value is based on some valuation model not discussed here.

If the ADC's stock price is unchanged, the stock position will underperform the convertible position despite the fact that a premium was paid to purchase the stock by acquiring the convertible issue. The reason is that even though the convertible's theoretical value decreased, the income from coupon more than compensates for the capital loss. In the two scenarios where the price of ADC stock declines, the convertible position outperforms the stock position because the straight value provides a floor for the convertible.

EXHIBIT 24 Comparison of 1-Year Return for ADC Stock and Convertible Issue for Assumed Changes in Stock Price

Beginning of horizon: October 7, 1993

End of horizon: October 7, 1994

Price of ADC stock on October 7, 1993: \$33.00

Assumed volatility of ADC stock return: 17%

Stock price change (%)	GSX stock return (%)	Convertible's theoretical value	Convertible's return (%)
-25	-22.27	100.47	-0.26
-10	-7.27	102.96	2.08
0	2.73	105.27	4.24
10	12.73	108.12	6.92
25	27.73	113.74	12.20

Bonds," *Journal of Financial and Quantitative Analysis* (November 1980), pp. 907-929; and, George Constantinides, "Warrant Exercise and Bond Conversion in Competitive Markets," *Journal of Financial Economics* (September 1984), pp. 371-398.

¹²Mihir Bhattacharya and Yu Zhu, "Valuation and Analysis of Convertible Securities," Chapter 42 in Frank J. Fabozzi (ed.), *The Handbook of Fixed Income Securities: Fifth Edition* (Chicago: Irwin Professional Publishing, 1997).

One of the critical assumptions in this analysis is that the straight value does not change except for the passage of time. If interest rates rise, the straight value will decline. Even if interest rates do not rise, the perceived creditworthiness of the issuer may deteriorate, causing investors to demand a higher yield. The illustration clearly demonstrates that there are benefits and drawbacks of investing in convertible securities. The disadvantage is the upside potential give-up because a premium per share must be paid. An advantage is the reduction in downside risk (as determined by the straight value).

Keep in mind that the major reason for the acquisition of the convertible bond is the potential price appreciation due to the increase in the price of the stock. An analysis of the growth prospects of the issuer's earnings and stock price is beyond the scope of this book but is described in all books on equity analysis.

CHAPTER 10

MORTGAGE-BACKED SECTOR OF THE BOND MARKET

I. INTRODUCTION

In this chapter and the next we will discuss securities backed by a pool of loans or receivables—mortgage-backed securities and asset-backed securities. We described these securities briefly in Chapter 3. The mortgage-backed securities sector, simply referred to as the **mortgage sector** of the bond market, includes securities backed by a pool of mortgage loans. There are securities backed by residential mortgage loans, referred to as **residential mortgage-backed securities**, and securities backed by commercial loans, referred to as **commercial mortgage-backed securities**.

In the United States, the securities backed by residential mortgage loans are divided into two sectors: (1) those issued by federal agencies (one federally related institution and two government sponsored enterprises) and (2) those issued by private entities. The former securities are called **agency mortgage-backed securities** and the latter **nonagency mortgage-backed securities**.

Securities backed by loans other than traditional residential mortgage loans or commercial mortgage loans and backed by receivables are referred to as **asset-backed securities**. There is a long and growing list of loans and receivables that have been used as collateral for these securities. Together, mortgage-backed securities and asset-backed securities are referred to as **structured financial products**.

It is important to understand the classification of these sectors in terms of bond market indexes. A popular bond market index, the Lehman Aggregate Bond Index, has a sector that it refers to as the “mortgage passthrough sector.” Within the “mortgage passthrough sector,” Lehman Brothers includes only agency mortgage-backed securities that are mortgage passthrough securities. To understand why it is essential to understand this sector, consider that the “mortgage passthrough sector” represents more than one third of the Lehman Aggregate Bond Index. It is the largest sector in the bond market index. The commercial mortgage-backed securities sector represents about 2% of the bond market index. The mortgage sector of the Lehman Aggregate Bond Index includes the mortgage passthrough sector and the commercial mortgage-backed securities.

In this chapter, our focus will be on the mortgage sector. Although many countries have developed a mortgage-backed securities sector, our focus in this chapter is the U.S. mortgage sector because of its size and its important role in U.S. bond market indexes.

Credit risk does not exist for agency mortgage-backed securities issued by a federally related institution and is viewed as minimal for securities issued by government sponsored enterprises. The significant risk is prepayment risk and there are ways to redistribute prepayment risk among the different bond classes created. Historically, it is important to note that the agency mortgage-backed securities market developed first. The technology developed for creating agency mortgage-backed security was then transferred to the securitization of other types of loans and receivables. In transferring the technology to create securities that expose investors to credit risk, mechanisms had to be developed to create securities that could receive investment grade credit ratings sought by the issuer. In the next chapter, we will discuss these mechanisms. We postpone a discussion of how to value and estimate the interest rate risk of both mortgage-backed and asset-backed securities until Chapter 12.

Outside the United States, market participants treat asset-backed securities more generically. Specifically, asset-backed securities include mortgage-backed securities as a subsector. While that is actually the proper way to classify these securities, it was not the convention adopted in the United States. In the next chapter, the development of the asset-backed securities (including mortgage-backed securities) outside the United States will be covered.

Residential mortgage-backed securities include: (1) mortgage passthrough securities, (2) collateralized mortgage obligations, and (3) stripped mortgage-backed securities. The latter two mortgage-backed securities are referred to as **derivative mortgage-backed securities** because they are created from mortgage passthrough securities.

II. RESIDENTIAL MORTGAGE LOANS

A **mortgage** is a loan secured by the collateral of some specified real estate property which obliges the borrower to make a predetermined series of payments. The mortgage gives the lender the right to “foreclose” on the loan if the borrower defaults and to seize the property in order to ensure that the debt is paid off. The interest rate on the mortgage loan is called the **mortgage rate** or **contract rate**. Our focus in this section is on residential mortgage loans.

When the lender makes the loan based on the credit of the borrower and on the collateral for the mortgage, the mortgage is said to be a **conventional mortgage**. The lender may require that the borrower obtain mortgage insurance to guarantee the fulfillment of the borrower’s obligations. Some borrowers can qualify for mortgage insurance which is guaranteed by one of three U.S. government agencies: the Federal Housing Administration (FHA), the Veteran’s Administration (VA), and the Rural Housing Service (RHS). There are also private mortgage insurers. The cost of mortgage insurance is paid by the borrower in the form of a higher mortgage rate.

There are many types of **mortgage designs** used throughout the world. A mortgage design is a specification of the interest rate, term of the mortgage, and the manner in which the borrowed funds are repaid. In the United States, the alternative mortgage designs include (1) fixed rate, level-payment fully amortized mortgages, (2) adjustable-rate mortgages, (3) balloon mortgages, (4) growing equity mortgages, (5) reverse mortgages, and (6) tiered payment mortgages. Other countries have developed mortgage designs unique to their housing finance market. Some of these mortgage designs relate the mortgage payment to the country’s rate of inflation. Below we will look at the most common mortgage design in the United States—the fixed-rate, level-payment, fully amortized mortgage. All of the principles we need to know regarding the risks associated with investing in mortgage-backed securities and the difficulties associated with their valuation can be understood by just looking at this mortgage design.

A. Fixed-Rate, Level-Payment, Fully Amortized Mortgage

A **fixed-rate, level-payment, fully amortized mortgage** has the following features:

- the mortgage rate is fixed for the life of the mortgage loan
- the dollar amount of each monthly payment is the same for the life of the mortgage loan (i.e., there is a “level payment”)
- when the last scheduled monthly mortgage payment is made the remaining mortgage balance is zero (i.e., the loan is fully amortized).

The monthly mortgage payments include principal repayment and interest. The frequency of payment is typically monthly. Each monthly mortgage payment for this mortgage design is due on the first of each month and consists of:

1. interest of $\frac{1}{12}$ of the fixed annual interest rate times the amount of the outstanding mortgage balance at the beginning of the previous month, and
2. a repayment of a portion of the outstanding mortgage balance (principal).

The difference between the monthly mortgage payment and the portion of the payment that represents interest equals the amount that is applied to reduce the outstanding mortgage balance. The monthly mortgage payment is designed so that after the last scheduled monthly mortgage payment is made, the amount of the outstanding mortgage balance is zero (i.e., the mortgage is fully repaid).

To illustrate this mortgage design, consider a 30-year (360-month), \$100,000 mortgage with an 8.125% mortgage rate. The monthly mortgage payment would be \$742.50. Exhibit 1 shows for selected months how each monthly mortgage payment is divided between interest and scheduled principal repayment. At the beginning of month 1, the mortgage balance is \$100,000, the amount of the original loan. The mortgage payment for month 1 includes interest on the \$100,000 borrowed for the month. Since the interest rate is 8.125%, the monthly interest rate is 0.0067708 (0.08125 divided by 12). Interest for month 1 is therefore \$677.08 (\$100,000 times 0.0067708). The \$65.41 difference between the monthly mortgage payment of \$742.50 and the interest of \$677.08 is the portion of the monthly mortgage payment that represents the scheduled principal repayment. It is also referred to as the **scheduled amortization** and we shall use the terms scheduled principal repayment and scheduled amortization interchangeably throughout this chapter. This \$65.41 in month 1 reduces the mortgage balance.

The mortgage balance at the end of month 1 (beginning of month 2) is then \$99,934.59 (\$100,000 minus \$65.41). The interest for the second monthly mortgage payment is \$676.64, the monthly interest rate (0.0067708) times the mortgage balance at the beginning of month 2 (\$99,934.59). The difference between the \$742.50 monthly mortgage payment and the \$676.64 interest is \$65.86, representing the amount of the mortgage balance paid off with that monthly mortgage payment. Notice that the mortgage payment in month 360—the final payment—is sufficient to pay off the remaining mortgage balance.

As Exhibit 1 clearly shows, *the portion of the monthly mortgage payment applied to interest declines each month and the portion applied to principal repayment increases*. The reason for this is that as the mortgage balance is reduced with each monthly mortgage payment, the interest on the mortgage balance declines. Since the monthly mortgage payment is a fixed dollar amount, an increasingly larger portion of the monthly payment is applied to reduce the mortgage balance outstanding in each subsequent month.

EXHIBIT 1 Amortization Schedule for a Level-Payment, Fixed-Rate, Fully Amortized Mortgage (Selected Months)

Mortgage loan: \$100,000			Monthly payment: \$742.50		
Mortgage rate: 8.125%			Term of loan: 30 years (360 months)		
Month	Beginning of Month Mortgage Balance	Mortgage Payment	Interest	Scheduled Repayment	End of Month Mortgage Balance
1	\$100,000.00	\$742.50	\$677.08	\$65.41	\$99,934.59
2	99,934.59	742.50	676.64	65.86	99,868.73
3	99,868.73	742.50	676.19	66.30	99,802.43
4	99,802.43	742.50	675.75	66.75	99,735.68
...	...	...	...	...	...
25	98,301.53	742.50	665.58	76.91	98,224.62
26	98,224.62	742.50	665.06	77.43	98,147.19
27	98,147.19	742.50	664.54	77.96	98,069.23
...	...	...	...	...	...
74	93,849.98	742.50	635.44	107.05	93,742.93
75	93,742.93	742.50	634.72	107.78	93,635.15
76	93,635.15	742.50	633.99	108.51	93,526.64
...	...	...	...	...	...
141	84,811.77	742.50	574.25	168.25	84,643.52
142	84,643.52	742.50	573.11	169.39	84,474.13
143	84,474.13	742.50	571.96	170.54	84,303.59
...	...	...	...	...	...
184	76,446.29	742.50	517.61	224.89	76,221.40
185	76,221.40	742.50	516.08	226.41	75,994.99
186	75,994.99	742.50	514.55	227.95	75,767.04
...	...	...	...	...	...
233	63,430.19	742.50	429.48	313.02	63,117.17
234	63,117.17	742.50	427.36	315.14	62,802.03
235	62,802.03	742.50	425.22	317.28	62,484.75
...	...	...	...	...	...
289	42,200.92	742.50	285.74	456.76	41,744.15
290	41,744.15	742.50	282.64	459.85	41,284.30
291	41,284.30	742.50	279.53	462.97	40,821.33
...	...	...	...	...	...
321	25,941.42	742.50	175.65	566.85	25,374.57
322	25,374.57	742.50	171.81	570.69	24,803.88
323	24,803.88	742.50	167.94	574.55	24,229.32
...	...	...	...	...	...
358	2,197.66	742.50	14.88	727.62	1,470.05
359	1,470.05	742.50	9.95	732.54	737.50
360	737.50	742.50	4.99	737.50	0.00

1. **Servicing Fee** Every mortgage loan must be serviced. Servicing of a mortgage loan involves collecting monthly payments and forwarding proceeds to owners of the loan; sending payment notices to mortgagors; reminding mortgagors when payments are overdue; maintaining records of principal balances; initiating foreclosure proceedings if necessary; and, furnishing tax information to borrowers (i.e., mortgagors) when applicable.

The servicing fee is a portion of the mortgage rate. If the mortgage rate is 8.125% and the servicing fee is 50 basis points, then the investor receives interest of 7.625%. The interest

rate that the investor receives is said to be the **net interest** or **net coupon**. The servicing fee is commonly called the **servicing spread**.

The dollar amount of the servicing fee declines over time as the mortgage amortizes. This is true for not only the mortgage design that we have just described, but for all mortgage designs.

2. Prepayments and Cash Flow Uncertainty Our illustration of the cash flow from a level-payment, fixed-rate, fully amortized mortgage assumes that the homeowner does not pay off any portion of the mortgage balance prior to the scheduled due date. But homeowners can pay off all or part of their mortgage balance prior to the maturity date. A payment made in excess of the monthly mortgage payment is called a **prepayment**. The prepayment could be to pay off the entire outstanding balance or a partial payoff of the mortgage balance. When a prepayment is not for the entire outstanding balance it is called a **curtailment**.

The effect of prepayments is that the amount and timing of the cash flow from a mortgage loan are not known with certainty. This risk is referred to as **prepayment risk**. For example, all that the lender in a \$100,000, 8.125% 30-year mortgage knows is that as long as the loan is outstanding and the borrower does not default, interest will be received and the principal will be repaid at the scheduled date each month; then at the end of the 30 years, the investor would have received \$100,000 in principal payments. What the investor does not know—the uncertainty—is for how long the loan will be outstanding, and therefore what the timing of the principal payments will be. This is true for all mortgage loans, not just the level-payment, fixed-rate, fully amortized mortgage. Factors affecting prepayments will be discussed later in this chapter.

Most mortgages have no prepayment penalty. The outstanding loan balance can be repaid at par. However, there are mortgages with prepayment penalties. The purpose of the penalty is to deter prepayment when interest rates decline. A prepayment penalty mortgage has the following structure. There is a period of time over which if the loan is prepaid in full or in excess of a certain amount of the outstanding balance, there is a prepayment penalty. This period is referred to as the **lockout period** or **penalty period**. During the penalty period, the borrower may prepay up to a specified amount of the outstanding balance without a penalty. Over that specified amount, the penalty is set in terms of the number of months of interest that must be paid.

III. MORTGAGE PASSTHROUGH SECURITIES

A **mortgage passthrough security** is a security created when one or more holders of mortgages form a collection (pool) of mortgages and sell shares or participation certificates in the pool. A pool may consist of several thousand or only a few mortgages. When a mortgage is included in a pool of mortgages that is used as collateral for a mortgage passthrough security, the mortgage is said to be **securitized**.

A. Cash Flow Characteristics

The cash flow of a mortgage passthrough security depends on the cash flow of the underlying pool of mortgages. As we explained in the previous section, the cash flow consists of monthly mortgage payments representing interest, the scheduled repayment of principal, and any prepayments.

Payments are made to security holders each month. However, neither the amount nor the timing of the cash flow from the pool of mortgages is identical to that of the cash flow passed through to investors. The monthly cash flow for a passthrough is less than the monthly cash flow of the underlying pool of mortgages by an amount equal to servicing and other fees. The other fees are those charged by the issuer or guarantor of the passthrough for guaranteeing the issue (discussed later). The coupon rate on a passthrough is called the **passthrough rate**. The passthrough rate is less than the mortgage rate on the underlying pool of mortgages by an amount equal to the servicing and guaranteeing fees.

The timing of the cash flow is also different. The monthly mortgage payment is due from each mortgagor on the first day of each month, but there is a delay in passing through the corresponding monthly cash flow to the security holders. The length of the delay varies by the type of passthrough security.

Not all of the mortgages that are included in a pool of mortgages that are securitized have the same mortgage rate and the same maturity. Consequently, when describing a passthrough security, a weighted average coupon rate and a weighted average maturity are determined. A **weighted average coupon rate**, or WAC, is found by weighting the mortgage rate of each mortgage loan in the pool by the percentage of the mortgage outstanding relative to the outstanding amount of all the mortgages in the pool. A **weighted average maturity**, or WAM, is found by weighting the remaining number of months to maturity for each mortgage loan in the pool by the amount of the outstanding mortgage balance.

For example, suppose a mortgage pool has just five loans and the outstanding mortgage balance, mortgage rate, and months remaining to maturity of each loan are as follows:

Loan	Outstanding mortgage balance	Weight in pool	Mortgage rate	Months remaining
1	\$125,000	22.12%	7.50%	275
2	\$85,000	15.04%	7.20%	260
3	\$175,000	30.97%	7.00%	290
4	\$110,000	19.47%	7.80%	285
5	\$70,000	12.39%	6.90%	270
Total	\$565,000	100.00%	7.28%	279

The WAC for this mortgage pool is:

$$0.2212 (7.5\%) + 0.1504 (7.2\%) + 0.3097 (7.0\%) \\ + 0.1947 (7.8\%) + 0.1239 (6.90\%) = 7.28\%$$

The WAM for this mortgage pool is

$$0.2212 (275) + 0.1504 (260) + 0.3097 (290) + 0.1947 (285) + 0.1239 (270) \\ = 279 \text{ months (rounded)}$$

B. Types of Mortgage Passthrough Securities

In the United States, the three major types of passthrough securities are guaranteed by agencies created by Congress to increase the supply of capital to the residential mortgage market. Those agencies are the Government National Mortgage Association (Ginnie Mae), the Federal Home

Loan Mortgage Corporation (Freddie Mac), and the Federal National Mortgage Association (Fannie Mae).

While Freddie Mac and Fannie Mae are commonly referred to as “agencies” of the U.S. government, both are corporate instrumentalities of the U.S. government. That is, they are government sponsored enterprises; therefore, their guarantee does not carry the full faith and credit of the U.S. government. In contrast, Ginnie Mae is a federally related institution; it is part of the Department of Housing and Urban Development. As such, its guarantee carries the full faith and credit of the U.S. government. The passthrough securities issued by Fannie Mae and Freddie Mac are called **conventional passthrough securities**. However, in this book we shall refer to those passthrough securities issued by all three entities (Ginnie Mae, Fannie Mae, and Freddie Mac) as **agency passthrough securities**. It should be noted, however, that market participants do reserve the term “agency passthrough securities” for those issued only by Ginnie Mae.¹

In order for a loan to be included in a pool of loans backing an agency security, it must meet specified underwriting standards. These standards set forth the maximum size of the loan, the loan documentation required, the maximum loan-to-value ratio, and whether or not insurance is required. If a loan satisfies the underwriting standards for inclusion as collateral for an agency mortgage-backed security, it is called a **conforming mortgage**. If a loan fails to satisfy the underwriting standards, it is called a **nonconforming mortgage**.

Nonconforming mortgages used as collateral for mortgage passthrough securities are privately issued. These securities are called **nonagency mortgage passthrough securities** and are issued by thrifts, commercial banks, and private conduits. Private conduits may purchase nonconforming mortgages, pool them, and then sell passthrough securities whose collateral is the underlying pool of nonconforming mortgages. Nonagency passthrough securities are rated by the nationally recognized statistical rating organizations. These securities are supported by credit enhancements so that they can obtain an investment grade rating. We shall describe these securities in the next chapter.

C. Trading and Settlement Procedures

Agency passthrough securities are identified by a pool prefix and pool number provided by the agency. The prefix indicates the type of passthrough. There are specific rules established by the Bond Market Association for the trading and settlement of mortgage-backed securities. Many trades occur while a pool is still unspecified, and therefore no pool information is known at the time of the trade. This kind of trade is known as a **TBA trade (to-be-announced trade)**. In a TBA trade the two parties agree on the agency type, the agency program, the coupon rate, the face value, the price, and the settlement date. The actual pools of mortgage loans underlying the agency passthrough are not specified in a TBA trade. However, this information is provided by the seller to the buyer before delivery. There are trades where more specific requirements are established for the securities to be delivered. An example is a Freddie

¹The name of the passthrough issued by Ginnie Mae and Fannie Mae is a *Mortgage-Backed Security* or *MBS*. So, when a market participant refers to a Ginnie Mae MBS or Fannie Mae MBS, what is meant is a passthrough issued by these two entities. The name of the passthrough issued by Freddie Mac is a *Participation Certificate* or *PC*. So, when a market participant refers to a Freddie Mac PC, what is meant is a passthrough issued by Freddie Mac. Every agency has different “programs” under which passthroughs are issued with different types of mortgage pools (e.g., 30-year fixed-rate mortgages, 15-year fixed-rate mortgages, adjustable-rate mortgages). We will not review the different programs here.

Mac with a coupon rate of 8.5% and a WAC between 9.0% and 9.2%. There are also specified pool trades wherein the actual pool numbers to be delivered are specified.

Passthrough prices are quoted in the same manner as U.S. Treasury coupon securities. A quote of 94-05 means 94 and 5 32nds of par value, or 94.15625% of par value. The price that the buyer pays the seller is the agreed upon sale price plus accrued interest. Given the par value, the dollar price (excluding accrued interest) is affected by the amount of the pool mortgage balance outstanding. The **pool factor** indicates the percentage of the initial mortgage balance still outstanding. So, a pool factor of 90 means that 90% of the original mortgage pool balance is outstanding. The pool factor is reported by the agency each month.

The dollar price paid for just the principal is found as follows given the agreed upon price, par value, and the month's pool factor provided by the agency:

$$\text{price} \times \text{par value} \times \text{pool factor}$$

For example, if the parties agree to a price of 92 for \$1 million par value for a passthrough with a pool factor of 0.85, then the dollar price paid by the buyer in addition to accrued interest is:

$$0.92 \times \$1,000,000 \times 0.85 = \$782,000$$

The buyer does not know what he will get unless he specifies a pool number. There are many seasoned issues of the same agency with the same coupon rate outstanding at a given point in time. For example, in early 2000 there were more than 30,000 pools of 30-year Ginnie Mae MBSs outstanding with a coupon rate of 9%. One passthrough may be backed by a pool of mortgage loans in which all the properties are located in California, while another may be backed by a pool of mortgage loans in which all the properties are in Minnesota. Yet another may be backed by a pool of mortgage loans in which the properties are from several regions of the country. So which pool are dealers referring to when they talk about Ginnie Mae 9s? They are not referring to any specific pool but instead to a generic security, despite the fact that the prepayment characteristics of passthroughs with underlying pools from different parts of the country are different. Thus, the projected prepayment rates for passthroughs reported by dealer firms (discussed later) are for generic passthroughs. A particular pool purchased may have a materially different prepayment rate from the generic. Moreover, when an investor purchases a passthrough without specifying a pool number, the seller has the option to deliver the worst-paying pools as long as the pools delivered satisfy good delivery requirements.

D. Measuring the Prepayment Rate

A prepayment is any payment toward the repayment of principal that is in excess of the scheduled principal payment. In describing prepayments, market participants refer to the **prepayment rate** or **prepayment speed**. In this section we will see how the historical prepayment rate is computed for a month. We then look at how to annualize a monthly prepayment rate and then explain the convention in the residential mortgage market for describing a pattern of prepayment rates over the life of a mortgage pool.

There are three points to keep in mind in the discussion in this section. First, we will look at how the actual or historical prepayment rate of a mortgage pool is calculated. Second, we will see later how in projecting the cash flow of a mortgage pool, an investor uses the same

prepayment measures to project prepayments given a prepayment rate. The third point is that we are just describing the mechanics of calculating prepayment measures. The difficult task of projecting the prepayment rate is not discussed here. In fact, this task is beyond the scope of this chapter. However, the factors that investors use in prepayment models (i.e., statistical models used to project prepayments) will be described in Section III F.

1. Single Monthly Mortality Rate Given the amount of the prepayment for a month and the amount that was available to prepay that month, a monthly prepayment rate can be computed. The amount available to prepay in a month is *not* the outstanding mortgage balance of the pool in the previous month. The reason is that there will be scheduled principal payments for the month and therefore by definition this amount cannot be prepaid. Thus, the amount available to prepay in a given month, say month t , is the beginning mortgage balance in month t reduced by the scheduled principal payment in month t .

The ratio of the prepayment in a month and the amount available to prepay that month is called the **single monthly mortality rate**² or simply SMM. That is, the SMM for month t is computed as follows

$$\text{SMM}_t = \frac{\text{prepayment in month } t}{\text{beginning mortgage balance for month } t - \text{scheduled principal payment in month } t}$$

Let's illustrate the calculation of the SMM. Assume the following:

$$\begin{aligned}\text{beginning mortgage balance in month 33} &= \$358,326,766 \\ \text{scheduled principal payment in month 33} &= \$297,825 \\ \text{prepayment in month 33} &= \$1,841,347\end{aligned}$$

The SMM for month 33 is therefore:

$$\text{SMM}_{33} = \frac{\$1,841,347}{\$358,326,766 - \$297,825} = 0.005143 = 0.5143\%$$

The SMM_{33} of 0.5143% is interpreted as follows: In month 33, 0.5143% of the outstanding mortgage balance available to prepay in month 33 prepaid.

Let's make sure we understand the two ways in which the SMM can be used. First, given the prepayment for a month for a mortgage pool, an investor can calculate the SMM as we just did in our illustration to determine the SMM for month 33. Second given an *assumed* SMM, an investor will use it to project the prepayment for a month. The prepayment for a month will then be used to determine the cash flow of a mortgage pool for the month. We'll see this later in this section when we illustrate how to calculate the cash flow for a passthrough security. For now, it is important to understand that given an assumed SMM for month t , the prepayment for month t is found as follows:

$$\begin{aligned}\text{prepayment for month } t &= \text{SMM} \times (\text{beginning mortgage balance for month } t \\ &\quad - \text{scheduled principal payment for month } t)\end{aligned}\tag{1}$$

²It may seem strange that the term "mortality" is used to describe this prepayment measure. This term reflects the influence of actuaries who in the early years of the development of the mortgage market migrated to dealer firms to assist in valuing mortgage-backed securities. Actuaries viewed the prepayment of a mortgage loan as the "death" of a mortgage.

For example, suppose that an investor owns a passthrough security in which the remaining mortgage balance at the beginning of some month is \$290 million and the scheduled principal payment for that month is \$3 million. The investor believes that the SMM next month will be 0.5143%. Then the projected prepayment for the month is:

$$0.005143 \times (\$290,000,000 - \$3,000,000) = \$1,476,041$$

2. Conditional Prepayment Rate Market participants prefer to talk about prepayment rates on an annual basis rather than a monthly basis. This is handled by annualizing the SMM. The annualized SMM is called the **conditional prepayment rate** or CPR.³ Given the SMM for a given month, the CPR can be demonstrated to be:⁴

$$\text{CPR} = 1 - (1 - \text{SMM})^{12} \quad (2)$$

For example, suppose that the SMM is 0.005143. Then the CPR is

$$\begin{aligned} \text{CPR} &= 1 - (1 - 0.005143)^{12} \\ &= 1 - (0.994857)^{12} = 0.06 = 6\% \end{aligned}$$

A CPR of 6% means that, ignoring scheduled principal payments, approximately 6% of the outstanding mortgage balance at the beginning of the year will be prepaid by the end of the year.

Given a CPR, the corresponding SMM can be computed by solving equation (2) for the SMM:

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12} \quad (3)$$

To illustrate equation (3), suppose that the CPR is 6%, then the SMM is

$$\text{SMM} = 1 - (1 - 0.06)^{1/12} = 0.005143 = 0.5143\%$$

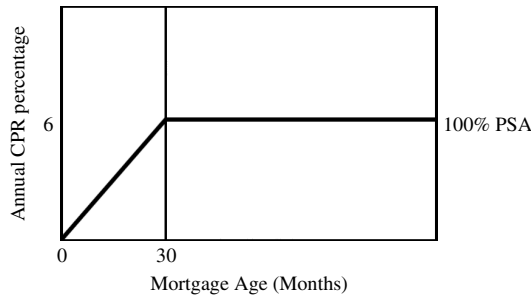
3. PSA Prepayment Benchmark An SMM is the prepayment rate for a month. A CPR is a prepayment rate for a year. Market participants describe prepayment rates (historical/actual prepayment rates and those used for projecting future prepayments) in terms of a prepayment pattern or benchmark over the life of a mortgage pool. In the early 1980s, the Public Securities Association (PSA), later renamed the Bond Market Association, undertook a study to look at the pattern of prepayments over the life of a typical mortgage pool. Based on the study, the PSA established a prepayment benchmark which is referred to as the **PSA prepayment benchmark**. Although sometimes referred to as a “prepayment model,” it is a convention and not a model to predict prepayments.

The PSA prepayment benchmark is expressed as a monthly series of CPRs. The PSA benchmark assumes that prepayment rates are low for newly originated mortgages and then will speed up as the mortgages become seasoned. The PSA benchmark assumes the following prepayment rates for 30-year mortgages: (1) a CPR of 0.2% for the first month, increased by

³It is referred to as a “conditional” prepayment rate because the prepayments in one year depend upon (i.e., are conditional upon) the amount available to prepay in the previous year. Sometimes market participants refer to the CPR as the “constant” prepayment rate.

⁴The derivation of the CPR for a given SMM is beyond the scope of this chapter. The proof is provided in Lakhbir S. Hayre and Cyrus Mohebbi, “Mortgage Mathematics,” in Frank J. Fabozzi (ed.), *Handbook of Mortgage-Backed Securities: Fifth Edition* (New York, NY: McGraw-Hill, 2001), pp. 844–845.

EXHIBIT 2 Graphical Depiction of 100 PSA



0.2% per year per month for the next 30 months until it reaches 6% per year, and (2) a 6% CPR for the remaining months.

This benchmark, referred to as “100% PSA” or simply “100 PSA,” is graphically depicted in Exhibit 2. Mathematically, 100 PSA can be expressed as follows:

$$\text{if } t < 30 \text{ then CPR} = 6\% (t/30)$$

$$\text{if } t \geq 30 \text{ then CPR} = 6\%$$

where t is the number of months since the mortgages were originated.

It is important to emphasize that the CPRs and corresponding SMMs apply to a mortgage pool *based on the number of months since origination*. For example, if a mortgage pool has loans that were originally 30-year (360-month) mortgage loans and the WAM is currently 357 months, this means that the mortgage pool is seasoned three months. So, in determining prepayments for the next month, the CPR and SMM that are applicable are those for month 4.

Slower or faster speeds are then referred to as some percentage of PSA. For example, “50 PSA” means one-half the CPR of the PSA prepayment benchmark; “150 PSA” means 1.5 times the CPR of the PSA prepayment benchmark; “300 PSA” means three times the CPR of the prepayment benchmark. A prepayment rate of 0 PSA means that no *prepayments* are assumed. While there are no prepayments at 0 PSA, there are scheduled principal repayments.

In constructing a schedule for monthly prepayments, the CPR (an annual rate) must be converted into a monthly prepayment rate (an SMM) using equation (3). For example, the SMMs for month 5, month 20, and months 31 through 360 assuming 100 PSA are calculated as follows:

for month 5:

$$\text{CPR} = 6\% (5/30) = 1\% = 0.01$$

$$\text{SMM} = 1 - (1 - 0.01)^{1/12} = 1 - (0.99)^{0.083333} = 0.000837$$

for month 20:

$$\text{CPR} = 6\% (20/30) = 4\% = 0.04$$

$$\text{SMM} = 1 - (1 - 0.04)^{1/12} = 1 - (0.96)^{0.083333} = 0.003396$$

for months 31–360:

$$\text{CPR} = 6\%$$

$$\text{SMM} = 1 - (1 - 0.06)^{1/12} = 1 - (0.94)^{0.083333} = 0.005143$$

What if the PSA were 165 instead? The SMMs for month 5, month 20, and months 31 through 360 assuming 165 PSA are computed as follows:

for month 5:

$$\text{CPR} = 6\% (5/30) = 1\% = 0.01$$

$$165 \text{ PSA} = 1.65(0.01) = 0.0165$$

$$\text{SMM} = 1 - (1 - 0.0165)^{1/12} = 1 - (0.9835)^{0.08333} = 0.001386$$

for month 20:

$$\text{CPR} = 6\% (20/30) = 4\% = 0.04$$

$$165 \text{ PSA} = 1.65 (0.04) = 0.066$$

$$\text{SMM} = 1 - (1 - 0.066)^{1/12} = 1 - (0.934)^{0.08333} = 0.005674$$

for months 31–360:

$$\text{CPR} = 6\%$$

$$165 \text{ PSA} = 1.65 (0.06) = 0.099$$

$$\text{SMM} = 1 - (1 - 0.099)^{1/12} = 1 - (0.901)^{0.08333} = 0.008650$$

Notice that the SMM assuming 165 PSA is not just 1.65 times the SMM assuming 100 PSA. It is the CPR that is a multiple of the CPR assuming 100 PSA.

4. Illustration of Monthly Cash Flow Construction As our first step in valuing a hypothetical passthrough given a PSA assumption, we must construct a monthly cash flow. For the purpose of this illustration, the underlying mortgages for this hypothetical passthrough are assumed to be fixed-rate, level-payment, fully amortized mortgages with a weighted average coupon (WAC) rate of 8.125%. It will be assumed that the passthrough rate is 7.5% with a weighted average maturity (WAM) of 357 months.

Exhibit 3 shows the cash flow for selected months assuming 100 PSA. The cash flow is broken down into three components: (1) interest (based on the passthrough rate), (2) the scheduled principal repayment (i.e., scheduled amortization), and (3) prepayments based on 100 PSA.

Let's walk through Exhibit 3 column by column.

Column 1: This is the number of months from now when the cash flow will be received.

Column 2: This is the number of months of seasoning. Since the WAM for this mortgage pool is 357 months, this means that the loans are seasoned an average of 3 months (360 months – 357 months) now.

Column 3: This column gives the outstanding mortgage balance at the beginning of the month. It is equal to the outstanding balance at the beginning of the previous month reduced by the total principal payment in the previous month.

Column 4: This column shows the SMM based on the number of months the loans are seasoned—the number of months shown in Column (2). For example, for the first month shown in the exhibit, the loans are seasoned three months going into that month. Therefore, the CPR used is the CPR that corresponds to four months. From the PSA benchmark, the CPR is 0.8% (4 times 0.2%). The corresponding SMM is

EXHIBIT 3 Monthly Cash Flow for a \$400 Million Passthrough with a 7.5% Passthrough Rate, a WAC of 8.125%, and a WAM of 357 Months Assuming 100 PSA

Months from now (1)	Months seasoned* (2)	Outstanding balance (3)	SMM (4)	Mortgage payment (5)	Net interest (6)	Scheduled principal (7)	Prepayment (8)	Total principal (9)	Cash flow (10)
1	4	\$400,000,000	0.00067	\$2,975,868	\$2,500,000	\$267,535	\$267,470	\$535,005	\$3,035,005
2	5	399,464,995	0.00084	2,973,877	2,496,656	269,166	334,198	603,364	3,100,020
3	6	398,861,631	0.00101	2,971,387	2,492,885	270,762	400,800	671,562	3,164,447
4	7	398,190,069	0.00117	2,968,399	2,488,688	272,321	467,243	739,564	3,228,252
5	8	397,450,505	0.00134	2,964,914	2,484,066	273,843	533,493	807,335	3,291,401
6	9	396,643,170	0.00151	2,960,931	2,479,020	275,327	599,514	874,841	3,353,860
7	10	395,768,329	0.00168	2,956,453	2,473,552	276,772	665,273	942,045	3,415,597
8	11	394,826,284	0.00185	2,951,480	2,467,664	278,177	730,736	1,008,913	3,476,577
9	12	393,817,371	0.00202	2,946,013	2,461,359	279,542	795,869	1,075,410	3,536,769
10	13	392,741,961	0.00219	2,940,056	2,454,637	280,865	860,637	1,141,502	3,596,140
11	14	391,600,459	0.00236	2,933,608	2,447,503	282,147	925,008	1,207,155	3,654,658
27	30	364,808,016	0.00514	2,766,461	2,280,050	296,406	1,874,688	2,171,094	4,451,144
28	31	362,636,921	0.00514	2,752,233	2,266,481	296,879	1,863,519	2,160,398	4,426,879
29	32	360,476,523	0.00514	2,738,078	2,252,978	297,351	1,852,406	2,149,758	4,402,736
30	33	358,326,766	0.00514	2,723,996	2,239,542	297,825	1,841,347	2,139,173	4,378,715
100	103	231,249,776	0.00514	1,898,682	1,445,311	332,928	1,187,608	1,520,537	2,965,848
101	104	229,729,239	0.00514	1,888,917	1,435,808	333,459	1,179,785	1,513,244	2,949,052
102	105	228,215,995	0.00514	1,879,202	1,426,350	333,990	1,172,000	1,505,990	2,932,340
103	106	226,710,004	0.00514	1,869,538	1,416,938	334,522	1,164,252	1,498,774	2,915,712
104	107	225,211,230	0.00514	1,859,923	1,407,570	335,055	1,156,541	1,491,596	2,899,166
105	108	223,719,634	0.00514	1,850,357	1,398,248	335,589	1,148,867	1,484,456	2,882,703
200	203	109,791,339	0.00514	1,133,751	686,196	390,372	562,651	953,023	1,639,219
201	204	108,838,316	0.00514	1,127,920	680,239	390,994	557,746	948,740	1,628,980
202	205	107,889,576	0.00514	1,122,119	674,310	391,617	552,863	944,480	1,618,790
203	206	106,945,096	0.00514	1,116,348	668,407	392,241	548,003	940,243	1,608,650
300	303	32,383,611	0.00514	676,991	202,398	457,727	164,195	621,923	824,320
301	304	31,761,689	0.00514	673,510	198,511	458,457	160,993	619,449	817,960
302	305	31,142,239	0.00514	670,046	194,639	459,187	157,803	616,990	811,629
303	306	30,525,249	0.00514	666,600	190,783	459,918	154,626	614,545	805,328
352	355	3,034,311	0.00514	517,770	18,964	497,226	13,048	510,274	529,238
353	356	2,524,037	0.00514	515,107	15,775	498,018	10,420	508,437	524,213
354	357	2,015,600	0.00514	512,458	12,597	498,811	7,801	506,612	519,209
355	358	1,508,988	0.00514	509,823	9,431	499,606	5,191	504,797	514,228
356	359	1,004,191	0.00514	507,201	6,276	500,401	2,591	502,992	509,269
357	360	501,199	0.00514	504,592	3,132	501,199	0	501,199	504,331

*Since the WAM is 357 months, the underlying mortgage pool is seasoned an average of three months, and therefore based on 100 PSA, the CPR is 0.8% in month 1 and the pool seasons at 6% in month 27.

0.00067. The mortgage pool becomes fully seasoned in Column (1) corresponding to month 27 because by that time the loans are seasoned 30 months. When the loans are fully seasoned the CPR at 100 PSA is 6% and the corresponding SMM is 0.00514.

Column 5: The total monthly mortgage payment is shown in this column. Notice that the total monthly mortgage payment declines over time as prepayments reduce the mortgage balance outstanding. There is a formula to determine what the monthly mortgage balance will be for each month given prepayments.⁵

Column 6: The *net* monthly interest (i.e., amount available to pay bondholders after the servicing fee) is found in this column. This value is determined by multiplying the

⁵The formula is presented in Chapter 19 of Frank J. Fabozzi, *Fixed Income Mathematics* (Chicago: Irwin Professional Publishing, 1997).

outstanding mortgage balance at the beginning of the month by the passthrough rate of 7.5% and then dividing by 12.

Column 7: This column gives the scheduled principal repayment (i.e., scheduled amortization). This is the difference between the total monthly mortgage payment [the amount shown in Column (5)] and the gross coupon interest for the month. The gross coupon interest is found by multiplying 8.125% by the outstanding mortgage balance at the beginning of the month and then dividing by 12.

Column 8: The prepayment for the month is reported in this column. The prepayment is found by using equation (1). For example, in month 100, the beginning mortgage balance is \$231,249,776, the scheduled principal payment is \$332,928, and the SMM at 100 PSA is 0.00514301 (only 0.00514 is shown in the exhibit to save space), so the prepayment is:

$$0.00514301 \times (\$231,249,776 - \$332,928) = \$1,187,608$$

Column 9: The total principal payment, which is the sum of columns (7) and (8), is shown in this column.

Column 10: The projected monthly cash flow for this passthrough is shown in this last column. The monthly cash flow is the sum of the interest paid [Column (6)] and the total principal payments for the month [Column (9)].

Let's look at what happens to the cash flows for this passthrough if a different PSA assumption is made. Suppose that instead of 100 PSA, 165 PSA is assumed. Prepayments are assumed to be faster. Exhibit 4 shows the cash flow for this passthrough based on 165 PSA. Notice that the cash flows are greater in the early years compared to Exhibit 3 because prepayments are higher. The cash flows in later years are less for 165 PSA compared to 100 PSA because of the higher prepayments in the earlier years.

E. Average Life

It is standard practice in the bond market to refer to the maturity of a bond. If a bond matures in five years, it is referred to as a "5-year bond." However, the typical bond repays principal only once: at the maturity date. Bonds with this characteristic are referred to as "bullet bonds." We know that the maturity of a bond affects its interest rate risk. More specifically, for a given coupon rate, the greater the maturity the greater the interest rate risk.

For a mortgage-backed security, we know that the principal repayments (scheduled payments and prepayments) are made over the life of the security. While a mortgage-backed has a "legal maturity," which is the date when the last scheduled principal payment is due, the legal maturity does not tell us much about the characteristic of the security as it pertains to interest rate risk. For example, it is incorrect to think of a 30-year corporate bond and a mortgage-backed security with a 30-year legal maturity with the same coupon rate as being equivalent in terms of interest rate risk. Of course, duration can be computed for both the corporate bond and the mortgage-backed security. (We will see how this is done for a mortgage-backed security in Chapter 12.) Instead of duration, another measure widely used by market participants is the **weighted average life** or simply **average life**. This is the convention-based average time to receipt of principal payments (scheduled principal payments and projected prepayments).

EXHIBIT 4 Monthly Cash Flow for a \$400 Million Passthrough with a 7.5% Passthrough Rate, a WAC of 8.125%, and a WAM of 357 Months Assuming 165 PSA

Month (1)	Months seasoned* (2)	Outstanding Balance (3)	SMM (4)	Mortgage payment (5)	Net interest (6)	Scheduled principal (7)	prepayment (8)	Total principal (9)	Cash flow (10)
1	4	\$400,000,000	0.00111	\$2,975,868	\$2,500,000	\$267,535	\$442,389	\$709,923	\$3,209,923
2	5	399,290,077	0.00139	2,972,575	2,495,563	269,048	552,847	821,896	3,317,459
3	6	398,468,181	0.00167	2,968,456	2,490,426	270,495	663,065	933,560	3,423,986
4	7	397,534,621	0.00195	2,963,513	2,484,591	271,873	772,949	1,044,822	3,529,413
5	8	396,489,799	0.00223	2,957,747	2,478,061	273,181	882,405	1,155,586	3,633,647
6	9	395,334,213	0.00251	2,951,160	2,470,839	274,418	991,341	1,265,759	3,736,598
7	10	394,068,454	0.00279	2,943,755	2,462,928	275,583	1,099,664	1,375,246	3,838,174
8	11	392,693,208	0.00308	2,935,534	2,454,333	276,674	1,207,280	1,483,954	3,938,287
9	12	391,209,254	0.00336	2,926,503	2,445,058	277,690	1,314,099	1,591,789	4,036,847
10	13	389,617,464	0.00365	2,916,666	2,435,109	278,631	1,420,029	1,698,659	4,133,769
11	14	387,918,805	0.00393	2,906,028	2,424,493	279,494	1,524,979	1,804,473	4,228,965
27	30	347,334,116	0.00865	2,633,950	2,170,838	282,209	3,001,955	3,284,164	5,455,002
28	31	344,049,952	0.00865	2,611,167	2,150,312	281,662	2,973,553	3,255,215	5,405,527
29	32	340,794,737	0.00865	2,588,581	2,129,967	281,116	2,945,400	3,226,516	5,356,483
30	33	337,568,221	0.00865	2,566,190	2,109,801	280,572	2,917,496	3,198,067	5,307,869
100	103	170,142,350	0.00865	1,396,958	1,063,390	244,953	1,469,591	1,714,544	2,777,933
101	104	168,427,806	0.00865	1,384,875	1,052,674	244,478	1,454,765	1,699,243	2,751,916
102	105	166,728,563	0.00865	1,372,896	1,042,054	244,004	1,440,071	1,684,075	2,726,128
103	106	165,044,489	0.00865	1,361,020	1,031,528	243,531	1,425,508	1,669,039	2,700,567
104	107	163,375,450	0.00865	1,349,248	1,021,097	243,060	1,411,075	1,654,134	2,675,231
105	108	161,721,315	0.00865	1,337,577	1,010,758	242,589	1,396,771	1,639,390	2,650,118
200	203	56,746,664	0.00865	585,990	354,667	201,767	489,106	690,874	1,045,540
201	204	56,055,790	0.00865	580,921	350,349	201,377	483,134	684,510	1,034,859
202	205	55,371,280	0.00865	575,896	346,070	200,986	477,216	678,202	1,024,273
203	206	54,693,077	0.00865	570,915	341,832	200,597	471,353	671,950	1,013,782
300	303	11,758,141	0.00865	245,808	73,488	166,196	100,269	266,465	339,953
301	304	11,491,677	0.00865	243,682	71,823	165,874	97,967	263,841	335,664
302	305	11,227,836	0.00865	241,574	70,174	165,552	95,687	261,240	331,414
303	306	10,966,596	0.00865	239,485	68,541	165,232	93,430	258,662	327,203
352	355	916,910	0.00865	156,460	5,731	150,252	6,631	156,883	162,614
353	356	760,027	0.00865	155,107	4,750	149,961	5,277	155,238	159,988
354	357	604,789	0.00865	153,765	3,780	149,670	3,937	153,607	157,387
355	358	451,182	0.00865	152,435	2,820	149,380	2,611	151,991	154,811
356	359	299,191	0.00865	151,117	1,870	149,091	1,298	150,389	152,259
357	360	148,802	0.00865	149,809	930	148,802	0	148,802	149,732

*Since the WAM is 357 months, the underlying mortgage pool is seasoned an average of three months, and therefore based on 165 PSA, the CPR is $0.8\% \times 1.65$ in month 1 and the pool seasons at $6\% \times 1.65$ in month 27.

Mathematically, the average life is expressed as follows:

$$\text{Average life} = \sum_{t=1}^T \frac{t \times \text{Projected principal received at time } t}{12 \times \text{Total principal}}$$

where T is the number of months.

The average life of a passthrough depends on the prepayment assumption. To see this, the average life is shown below for different prepayment speeds for the pass-through we used to illustrate the cash flow for 100 PSA and 165 PSA in Exhibits 3 and 4:

PSA speed	50	100	165	200	300	400	500	600	700
Average life (years)	15.11	11.66	8.76	7.68	5.63	4.44	3.68	3.16	2.78

F. Factors Affecting Prepayment Behavior

The factors that affect prepayment behavior are:

1. prevailing mortgage rate
2. housing turnover
3. characteristics of the underlying residential mortgage loans

The current mortgage rate affects prepayments. The spread between the prevailing mortgage rate in the market and the rate paid by the homeowner affects the **incentive to refinance**. Moreover, the path of mortgage rates since the loan was originated affects prepayments through a phenomenon referred to as **refinancing burnout**. Both the spread and path of mortgage rates affect prepayments that are the product of refinancing.

By far, the single most important factor affecting prepayments because of refinancing is the current level of mortgage rates relative to the borrower's contract rate. The greater the difference between the two, the greater the incentive to refinance the mortgage loan. For refinancing to make economic sense, the interest savings must be greater than the costs associated with refinancing the mortgage. These costs include legal expenses, origination fees, title insurance, and the value of the time associated with obtaining another mortgage loan. Some of these costs will vary proportionately with the amount to be financed. Other costs such as the application fee and legal expenses are typically fixed.

Historically it had been observed that mortgage rates had to decline by between 250 and 350 basis points below the contract rate in order to make it worthwhile for borrowers to refinance. However, the creativity of mortgage originators in designing mortgage loans such that the refinancing costs are folded into the amount borrowed has changed the view that mortgage rates must drop dramatically below the contract rate to make refinancing economic. Moreover, mortgage originators now do an effective job of advertising to make homeowners cognizant of the economic benefits of refinancing.

The historical pattern of prepayments and economic theory suggests that it is not only the level of mortgage rates that affects prepayment behavior but also the path that mortgage rates take to get to the current level. To illustrate why, suppose the underlying contract rate for a pool of mortgage loans is 11% and that three years after origination, the prevailing mortgage rate declines to 8%. Let's consider two possible paths of the mortgage rate in getting to the 8% level. In the first path, the mortgage rate declines to 8% at the end of the first year, then rises to 13% at the end of the second year, and then falls to 8% at the end of the third year. In the second path, the mortgage rate rises to 12% at the end of the first year, continues its rise to 13% at the end of the second year, and then falls to 8% at the end of the third year.

If the mortgage rate follows the first path, those who can benefit from refinancing will more than likely take advantage of this opportunity when the mortgage rate drops to 8% in the first year. When the mortgage rate drops again to 8% at the end of the third year, the likelihood is that prepayments because of refinancing will not surge; those who want to benefit by taking advantage of the refinancing opportunity will have done so already when the mortgage rate declined for the first time. This is the prepayment behavior referred to as the refinancing burnout (or simply, burnout) phenomenon. In contrast, the expected prepayment behavior when the mortgage rate follows the second path is quite different. Prepayment rates are expected to be low in the first two years. When the mortgage rate declines to 8% in the third year, refinancing activity and therefore prepayments are expected to surge. Consequently, the burnout phenomenon is related to the path of mortgage rates.

There is another way in which the prevailing mortgage rate affects prepayments: through its effect on the affordability of housing and **housing turnover**. The level of mortgage rates affects housing turnover to the extent that a lower rate increases the affordability of homes. However, even without lower interest rates, there is a normal amount of housing turnover. This is attribute to economic growth. The link is as follows: a growing economy results in a rise in personal income and in opportunities for worker migration; this increases family mobility and as a result increases housing turnover. The opposite holds for a weak economy.

Two characteristics of the underlying residential mortgage loans that affect prepayments are the amount of seasoning and the geographical location of the underlying properties. **Seasoning** refers to the aging of the mortgage loans. Empirical evidence suggests that prepayment rates are low after the loan is originated and increase after the loan is somewhat seasoned. Then prepayment rates tend to level off, in which case the loans are referred to as fully seasoned. This is the underlying theory for the PSA prepayment benchmark discussed earlier in this chapter. In some regions of the country the prepayment behavior tends to be faster than the average national prepayment rate, while other regions exhibit slower prepayment rates. This is caused by differences in local economies that affect housing turnover.

G. Contraction Risk and Extension Risk

An investor who owns passthrough securities does not know what the cash flow will be because that depends on actual prepayments. As we noted earlier, this risk is called prepayment risk.

To understand the significance of prepayment risk, suppose an investor buys a 9% coupon passthrough security at a time when mortgage rates are 10%. Let's consider what will happen to prepayments if mortgage rates decline to, say, 6%. There will be two adverse consequences. First, a basic property of fixed income securities is that the price of an option-free bond will rise. But in the case of a passthrough security, the rise in price will not be as large as that of an option-free bond because a fall in interest rates will give the borrower an incentive to prepay the loan and refinance the debt at a lower rate. This results in the same adverse consequence faced by holders of callable bonds. As in the case of those instruments, the upside price potential of a passthrough security is compressed because of prepayments. (This is the negative convexity characteristic explained in Chapter 7.) The second adverse consequence is that the cash flow must be reinvested at a lower rate. Basically, the faster prepayments resulting from a decline in interest rates causes the passthrough to shorten in terms of the timing of its cash flows. Another way of saying this is that "shortening" results in a decline in the average life. Consequently, the two adverse consequences from a decline in interest rates for a passthrough security are referred to as **contraction risk**.

Now let's look at what happens if mortgage rates rise to 15%. The price of the passthrough, like the price of any bond, will decline. But again it will decline more because the higher rates will tend to slow down the rate of prepayment, in effect increasing the amount invested at the coupon rate, which is lower than the market rate. Prepayments will slow down, because homeowners will not refinance or partially prepay their mortgages when mortgage rates are higher than the contract rate of 10%. Of course this is just the time when investors want prepayments to speed up so that they can reinvest the prepayments at the higher market interest rate. Basically, the slower prepayments associated with a rise in interest rates that causes these adverse consequences are due to the passthrough lengthening in terms of the timing of its cash flows. Another way of saying this is that "lengthening" results in an increase in the average life. Consequently, the adverse consequence from a rise in interest rates for a passthrough security is referred to as **extension risk**.

Therefore, prepayment risk encompasses contraction risk and extension risk. Prepayment risk makes passthrough securities unattractive for certain financial institutions to hold from an asset/liability management perspective. Some institutional investors are concerned with extension risk and others with contraction risk when they purchase a passthrough security. This applies even for assets supporting specific types of insurance contracts. Is it possible to alter the cash flow of a passthrough so as to reduce the contraction risk or extension risk for institutional investors? This can be done, as we shall see, when we describe collateralized mortgage obligations.

IV. COLLATERALIZED MORTGAGE OBLIGATIONS

As we noted, there is prepayment risk associated with investing in a mortgage passthrough security. Some institutional investors are concerned with extension risk and others with contraction risk. This problem can be mitigated by redirecting the cash flows of mortgage-related products (passthrough securities or a pool of loans) to different bond classes, called **tranches**,⁶ so as to create securities that have different exposure to prepayment risk and therefore different risk/return patterns than the mortgage-related product from which they are created.

When the cash flows of mortgage-related products are redistributed to different bond classes, the resulting securities are called **collateralized mortgage obligations** (CMO). The mortgage-related products from which the cash flows are obtained are referred to as the collateral. Since the typical mortgage-related product used in a CMO is a pool of passthrough securities, sometimes market participants will use the terms “collateral” and “passthrough securities” interchangeably. The creation of a CMO cannot eliminate prepayment risk; it can only distribute the various forms of this risk among different classes of bondholders. The CMO’s major financial innovation is that the securities created more closely satisfy the asset/liability needs of institutional investors, thereby broadening the appeal of mortgage-backed products.

There is a wide range of CMO structures.⁷ We review the major ones below.

A. Sequential-Pay Tranches

The first CMO was structured so that each class of bond would be retired sequentially. Such structures are referred to as **sequential-pay** CMOs. The rule for the monthly distribution of the principal payments (scheduled principal plus prepayments) to the tranches would be as follows:

- Distribute all principal payments to Tranche 1 until the principal balance for Tranche 1 is zero. After Tranche 1 is paid off,

⁶“Tranche” is from an old French word meaning “slice.” In the case of a collateralized mortgage obligation it refers to a “slice of the cash flows.”

⁷The issuer of a CMO wants to be sure that the trust created to pass through the interest and principal payments is not treated as a taxable entity. A provision of the Tax Reform Act of 1986, called the Real Estate Mortgage Investment Conduit (REMIC), specifies the requirements that an issuer must fulfill so that the legal entity created to issue a CMO is not taxable. Most CMOs today are created as REMICs. While it is common to hear market participants refer to a CMO as a REMIC, not all CMOs are REMICs.

EXHIBIT 5 FJF-01—A Hypothetical 4-Tranche Sequential-Pay Structure

Tranche	Par amount	Coupon rate (%)
A	194,500,000	7.5
B	36,000,000	7.5
C	96,500,000	7.5
D	73,000,000	7.5
Total	400,000,000	

Payment rules:

1. *For payment of monthly coupon interest:* Disburse monthly coupon interest to each tranche on the basis of the amount of principal outstanding for each tranche at the beginning of the month.

2. *For disbursement of principal payments:* Disburse principal payments to tranche A until it is completely paid off. After tranche A is completely paid off, disburse principal payments to tranche B until it is completely paid off. After tranche B is completely paid off, disburse principal payments to tranche C until it is completely paid off. After tranche C is completely paid off, disburse principal payments to tranche D until it is completely paid off.

- distribute all principal payments to Tranche 2 until the principal balance for Tranche 2 is zero; After Tranche 2 is paid off,
- distribute all principal payments to Tranche 3 until the principal balance for Tranche 3 is zero; After Tranche 3 is paid off, . . .
and so on.

To illustrate a sequential-pay CMO, we discuss FJF-01, a hypothetical deal made up to illustrate the basic features of the structure. The collateral for this hypothetical CMO is a hypothetical passthrough with a total par value of \$400 million and the following characteristics: (1) the passthrough coupon rate is 7.5%, (2) the weighted average coupon (WAC) is 8.125%, and (3) the weighted average maturity (WAM) is 357 months. This is the same passthrough that we used in Section III to describe the cash flow of a passthrough based on some PSA assumption.

From this \$400 million of collateral, four bond classes or tranches are created. Their characteristics are summarized in Exhibit 5. The total par value of the four tranches is equal to the par value of the collateral (i.e., the passthrough security).⁸ In this simple structure, the coupon rate is the same for each tranche and also the same as the coupon rate on the collateral. There is no reason why this must be so, and, in fact, typically the coupon rate varies by tranche.

Now remember that a CMO is created by redistributing the cash flow—interest and principal—to the different tranches based on a set of payment rules. The payment rules at the bottom of Exhibit 5 describe how the cash flow from the passthrough (i.e., collateral) is to be distributed to the four tranches. There are separate rules for the distribution of the coupon interest and the payment of principal (the principal being the total of the scheduled principal payment and any prepayments).

While the payment rules for the disbursement of the principal payments are known, the precise amount of the principal in each month is not. This will depend on the cash flow, and therefore principal payments, of the collateral, which depends on the actual prepayment

⁸Actually, a CMO is backed by a pool of passthrough securities.

rate of the collateral. An assumed PSA speed allows the cash flow to be projected. Exhibit 6 shows the cash flow (interest, scheduled principal repayment, and prepayments) assuming 165 PSA. Assuming that the collateral does prepay at 165 PSA, the cash flow available to all four tranches of FJF-01 will be precisely the cash flow shown in Exhibit 6.

To demonstrate how the payment rules for FJF-01 work, Exhibit 6 shows the cash flow for selected months assuming the collateral prepays at 165 PSA. For each tranche, the exhibit shows: (1) the balance at the end of the month, (2) the principal paid down (scheduled principal repayment plus prepayments), and (3) interest. In month 1, the cash flow for the collateral consists of a principal payment of \$709,923 and an interest payment of \$2.5 million (0.075 times \$400 million divided by 12). The interest payment is distributed to the four tranches based on the amount of the par value outstanding. So, for example, tranche A receives \$1,215,625 (0.075 times \$194,500,000 divided by 12) of the \$2.5 million. The principal, however, is all distributed to tranche A. Therefore, the cash flow for tranche A in month 1 is \$1,925,548. The principal balance at the end of month 1 for tranche A is \$193,790,076 (the original principal balance of \$194,500,000 less the principal payment of \$709,923). No principal payment is distributed to the three other tranches because there is still a principal balance outstanding for tranche A. This will be true for months 2 through 80. The cash flow for tranche A for each month is found by adding the amounts shown in the “Principal” and “Interest” columns. So, for tranche A, the cash flow in month 8 is \$1,483,954 plus \$1,169,958, or \$2,653,912. The cash flow from months 82 on is zero based on 165 PSA.

After month 81, the principal balance will be zero for tranche A. For the collateral, the cash flow in month 81 is \$3,318,521, consisting of a principal payment of \$2,032,197 and interest of \$1,286,325. At the beginning of month 81 (end of month 80), the principal balance for tranche A is \$311,926. Therefore, \$311,926 of the \$2,032,196 of the principal payment from the collateral will be disbursed to tranche A. After this payment is made, no additional principal payments are made to this tranche as the principal balance is zero. The remaining principal payment from the collateral, \$1,720,271, is distributed to tranche B. Based on an assumed prepayment speed of 165 PSA, tranche B then begins receiving principal payments in month 81. The cash flow for tranche B for each month is found by adding the amounts shown in the “Principal” and “Interest” columns. For months 1 through 80, the cash flow is just the interest. There is no cash flow after month 100 for tranche B.

Exhibit 6 shows that tranche B is fully paid off by month 100, when tranche C begins to receive principal payments. Tranche C is not fully paid off until month 178, at which time tranche D begins receiving the remaining principal payments. The maturity (i.e., the time until the principal is fully paid off) for these four tranches assuming 165 PSA would be 81 months for tranche A, 100 months for tranche B, 178 months for tranche C, and 357 months for tranche D. The cash flow for each month for tranches C and D is found by adding the principal and the interest for the month.

The **principal pay down window** or **principal window** for a tranche is the time period between the beginning and the ending of the principal payments to that tranche. So, for example, for tranche A, the principal pay down window would be month 1 to month 81 assuming 165 PSA. For tranche B it is from month 81 to month 100.⁹ In confirmation of trades involving CMOs, the principal pay down window is specified in terms of the initial

⁹The window is also specified in terms of the length of the time from the beginning of the principal pay down window to the end of the principal pay down window. For tranche A, the window would be stated as 81 months, for tranche B 20 months.

month that principal is expected to be received to the final month that principal is expected to be received.

Let's look at what has been accomplished by creating the CMO. Earlier we saw that the average life of the passthrough is 8.76 years assuming a prepayment speed of 165 PSA. Exhibit 7 reports the average life of the collateral and the four tranches assuming different prepayment speeds. Notice that the four tranches have average lives that are both shorter and longer than the collateral, thereby attracting investors who have a preference for an average life different from that of the collateral.

There is still a major problem: there is considerable variability of the average life for the tranches. We'll see how this can be handled later on. However, there is some protection provided for each tranche against prepayment risk. This is because prioritizing the distribution of principal (i.e., establishing the payment rules for principal) effectively protects the shorter-term tranche A in this structure against extension risk. This protection must come from somewhere, so it comes from the three other tranches. Similarly, tranches C and D provide protection against extension risk for tranches A and B. At the same time, tranches C and D benefit because they are provided protection against contraction risk, the protection coming from tranches A and B.

EXHIBIT 6 Monthly Cash Flow for Selected Months for FJF-01 Assuming 165 PSA

Month	Tranche A			Tranche B		
	Balance (\$)	Principal (\$)	Interest (\$)	Balance (\$)	Principal (\$)	Interest (\$)
1	194,500,000	709,923	1,215,625	36,000,000	0	225,000
2	193,790,077	821,896	1,211,188	36,000,000	0	225,000
3	192,968,181	933,560	1,206,051	36,000,000	0	225,000
4	192,034,621	1,044,822	1,200,216	36,000,000	0	225,000
5	190,989,799	1,155,586	1,193,686	36,000,000	0	225,000
6	189,834,213	1,265,759	1,186,464	36,000,000	0	225,000
7	188,568,454	1,375,246	1,178,553	36,000,000	0	225,000
8	187,193,208	1,483,954	1,169,958	36,000,000	0	225,000
9	185,709,254	1,591,789	1,160,683	36,000,000	0	225,000
10	184,117,464	1,698,659	1,150,734	36,000,000	0	225,000
11	182,418,805	1,804,473	1,140,118	36,000,000	0	225,000
12	180,614,332	1,909,139	1,128,840	36,000,000	0	225,000
75	12,893,479	2,143,974	80,584	36,000,000	0	225,000
76	10,749,504	2,124,935	67,184	36,000,000	0	225,000
77	8,624,569	2,106,062	53,904	36,000,000	0	225,000
78	6,518,507	2,087,353	40,741	36,000,000	0	225,000
79	4,431,154	2,068,807	27,695	36,000,000	0	225,000
80	2,362,347	2,050,422	14,765	36,000,000	0	225,000
81	311,926	311,926	1,950	36,000,000	1,720,271	225,000
82	0	0	0	34,279,729	2,014,130	214,248
83	0	0	0	32,265,599	1,996,221	201,660
84	0	0	0	30,269,378	1,978,468	189,184
85	0	0	0	28,290,911	1,960,869	176,818
95	0	0	0	9,449,331	1,793,089	59,058
96	0	0	0	7,656,242	1,777,104	47,852
97	0	0	0	5,879,138	1,761,258	36,745
98	0	0	0	4,117,879	1,745,550	25,737
99	0	0	0	2,372,329	1,729,979	14,827
100	0	0	0	642,350	642,350	4,015
101	0	0	0	0	0	0

*Continued on next page.

EXHIBIT 6 (Continued)

Month	Tranche C			Tranche D		
	Balance (\$)	Principal (\$)	Interest (\$)	Balance (\$)	Principal (\$)	Interest (\$)
1	96,500,000	0	603,125	73,000,000	0	456,250
2	96,500,000	0	603,125	73,000,000	0	456,250
3	96,500,000	0	603,125	73,000,000	0	456,250
4	96,500,000	0	603,125	73,000,000	0	456,250
5	96,500,000	0	603,125	73,000,000	0	456,250
6	96,500,000	0	603,125	73,000,000	0	456,250
7	96,500,000	0	603,125	73,000,000	0	456,250
8	96,500,000	0	603,125	73,000,000	0	456,250
9	96,500,000	0	603,125	73,000,000	0	456,250
10	96,500,000	0	603,125	73,000,000	0	456,250
11	96,500,000	0	603,125	73,000,000	0	456,250
12	96,500,000	0	603,125	73,000,000	0	456,250
95	96,500,000	0	603,125	73,000,000	0	456,250
96	96,500,000	0	603,125	73,000,000	0	456,250
97	96,500,000	0	603,125	73,000,000	0	456,250
98	96,500,000	0	603,125	73,000,000	0	456,250
99	96,500,000	0	603,125	73,000,000	0	456,250
100	96,500,000	1,072,194	603,125	73,000,000	0	456,250
101	95,427,806	1,699,243	596,424	73,000,000	0	456,250
102	93,728,563	1,684,075	585,804	73,000,000	0	456,250
103	92,044,489	1,669,039	575,278	73,000,000	0	456,250
104	90,375,450	1,654,134	564,847	73,000,000	0	456,250
105	88,721,315	1,639,359	554,508	73,000,000	0	456,250
175	3,260,287	869,602	20,377	73,000,000	0	456,250
176	2,390,685	861,673	14,942	73,000,000	0	456,250
177	1,529,013	853,813	9,556	73,000,000	0	456,250
178	675,199	675,199	4,220	73,000,000	170,824	456,250
179	0	0	0	72,829,176	838,300	455,182
180	0	0	0	71,990,876	830,646	449,943
181	0	0	0	71,160,230	823,058	444,751
182	0	0	0	70,337,173	815,536	439,607
183	0	0	0	69,521,637	808,081	434,510
184	0	0	0	68,713,556	800,690	429,460
185	0	0	0	67,912,866	793,365	424,455
350	0	0	0	1,235,674	160,220	7,723
351	0	0	0	1,075,454	158,544	6,722
352	0	0	0	916,910	156,883	5,731
353	0	0	0	760,027	155,238	4,750
354	0	0	0	604,789	153,607	3,780
355	0	0	0	451,182	151,991	2,820
356	0	0	0	299,191	150,389	1,870
357	0	0	0	148,802	148,802	930

Note: The cash flow for a tranche in each month is the sum of the principal and interest.

B. Accrual Tranches

In our previous example, the payment rules for interest provided for all tranches to be paid interest each month. In many sequential-pay CMO structures, at least one tranche does not receive current interest. Instead, the interest for that tranche would accrue and be added to the principal balance. Such a tranche is commonly referred to as an **accrual tranche** or a **Z bond**. The interest that would have been paid to the accrual tranche is used to pay off the principal balance of earlier tranches.

EXHIBIT 7 Average Life for the Collateral and the Four Tranches of FJF-01

Prepayment speed (PSA)	Average life (in years) for				
	Collateral	Tranche A	Tranche B	Tranche C	Tranche D
50	15.11	7.48	15.98	21.02	27.24
100	11.66	4.90	10.86	15.78	24.58
165	8.76	3.48	7.49	11.19	20.27
200	7.68	3.05	6.42	9.60	18.11
300	5.63	2.32	4.64	6.81	13.36
400	4.44	1.94	3.70	5.31	10.34
500	3.68	1.69	3.12	4.38	8.35
600	3.16	1.51	2.74	3.75	6.96
700	2.78	1.38	2.47	3.30	5.95

To see this, consider FJF-02, a hypothetical CMO structure with the same collateral as our previous example and with four tranches, each with a coupon rate of 7.5%. The last tranche, Z, is an accrual tranche. The structure for FJF-02 is shown in Exhibit 8.

Exhibit 9 shows cash flows for selected months for tranches A and B. Let's look at month 1 and compare it to month 1 in Exhibit 6. Both cash flows are based on 165 PSA. The principal payment from the collateral is \$709,923. In FJF-01, this is the principal paydown for tranche A. In FJF-02, the interest for tranche Z, \$456,250, is not paid to that tranche but instead is used to pay down the principal of tranche A. So, the principal payment to tranche A in Exhibit 9 is \$1,166,173, the collateral's principal payment of \$709,923 plus the interest of \$456,250 that was diverted from tranche Z.

The expected final maturity for tranches A, B, and C has shortened as a result of the inclusion of tranche Z. The final payout for tranche A is 64 months rather than 81 months;

EXHIBIT 8 FJF-02—A Hypothetical 4-Tranche Sequential-Pay Structure with an Accrual Tranche

Tranche	Par amount (\$)	Coupon rate (%)
A	194,500,000	7.5
B	36,000,000	7.5
C	96,500,000	7.5
Z (Accrual)	73,000,000	7.5
Total	400,000,000	

Payment rules:

1. *For payment of monthly coupon interest:* Disburse monthly coupon interest to tranches A, B, and C on the basis of the amount of principal outstanding for each tranche at the beginning of the month. For tranche Z, accrue the interest based on the principal plus accrued interest in the previous month. The interest for tranche Z is to be paid to the earlier tranches as a principal paydown.
2. *For disbursement of principal payments:* Disburse principal payments to tranche A until it is completely paid off. After tranche A is completely paid off, disburse principal payments to tranche B until it is completely paid off. After tranche B is completely paid off, disburse principal payments to tranche C until it is completely paid off. After tranche C is completely paid off, disburse principal payments to tranche Z until the original principal balance plus accrued interest is completely paid off.

EXHIBIT 9 Monthly Cash Flow for Selected Months for Tranches A and B for FJF-02
Assuming 165 PSA

Month	Tranche A			Tranche B		
	Balance (\$)	Principal (\$)	Interest (\$)	Balance (\$)	Principal (\$)	Interest (\$)
1	194,500,000	1,166,173	1,215,625	36,000,000	0	225,000
2	193,333,827	1,280,997	1,208,336	36,000,000	0	225,000
3	192,052,829	1,395,531	1,200,330	36,000,000	0	225,000
4	190,657,298	1,509,680	1,191,608	36,000,000	0	225,000
5	189,147,619	1,623,350	1,182,173	36,000,000	0	225,000
6	187,524,269	1,736,446	1,172,027	36,000,000	0	225,000
7	185,787,823	1,848,875	1,161,174	36,000,000	0	225,000
8	183,938,947	1,960,543	1,149,618	36,000,000	0	225,000
9	181,978,404	2,071,357	1,137,365	36,000,000	0	225,000
10	179,907,047	2,181,225	1,124,419	36,000,000	0	225,000
11	177,725,822	2,290,054	1,110,786	36,000,000	0	225,000
12	175,435,768	2,397,755	1,096,474	36,000,000	0	225,000
60	15,023,406	3,109,398	93,896	36,000,000	0	225,000
61	11,914,007	3,091,812	74,463	36,000,000	0	225,000
62	8,822,195	3,074,441	55,139	36,000,000	0	225,000
63	5,747,754	3,057,282	35,923	36,000,000	0	225,000
64	2,690,472	2,690,472	16,815	36,000,000	349,863	225,000
65	0	0	0	35,650,137	3,023,598	222,813
66	0	0	0	32,626,540	3,007,069	203,916
67	0	0	0	29,619,470	2,990,748	185,122
68	0	0	0	26,628,722	2,974,633	166,430
69	0	0	0	23,654,089	2,958,722	147,838
70	0	0	0	20,695,367	2,943,014	129,346
71	0	0	0	17,752,353	2,927,508	110,952
72	0	0	0	14,824,845	2,912,203	92,655
73	0	0	0	11,912,642	2,897,096	74,454
74	0	0	0	9,015,546	2,882,187	56,347
75	0	0	0	6,133,358	2,867,475	38,333
76	0	0	0	3,265,883	2,852,958	20,412
77	0	0	0	412,925	412,925	2,581
78	0	0	0	0	0	0
79	0	0	0	0	0	0
80	0	0	0	0	0	0

for tranche B it is 77 months rather than 100 months; and, for tranche C it is 113 months rather than 178 months.

The average lives for tranches A, B, and C are shorter in FJF-02 compared to our previous non-accrual, sequential-pay tranche example, FJF-01, because of the inclusion of the accrual tranche. For example, at 165 PSA, the average lives are as follows:

Structure	Tranche A	Tranche B	Tranche C
FJF-02	2.90	5.86	7.87
FJF-01	3.48	7.49	11.19

The reason for the shortening of the non-accrual tranches is that the interest that would be paid to the accrual tranche is being allocated to the other tranches. Tranche Z in FJF-02 will have a longer average life than tranche D in FJF-01 because in tranche Z the interest payments are being diverted to tranches A, B, and C.

EXHIBIT 10 FJF-03—A Hypothetical 5-Tranche Sequential-Pay Structure with Floater, Inverse Floater, and Accrual Bond Tranches

Tranche	Par amount (\$)	Coupon rate (%)
A	194,500,000	7.50
B	36,000,000	7.50
FL	72,375,000	1-month LIBOR + 0.50
IFL	24,125,000	$28.5 - 3 \times (1\text{-month LIBOR})$
Z (Accrual)	73,000,000	7.50
Total	400,000,000	

Payment rules:

1. *For Payment of monthly coupon interest:* Disburse monthly coupon interest to tranches A, B, FL, and IFL on the basis of the amount of principal outstanding at the beginning of the month. For tranche Z, accrue the interest based on the principal plus accrued interest in the previous month. The interest for tranche Z is to be paid to the earlier tranches as a principal paydown. The maximum coupon rate for FL is 10%; the minimum coupon rate for IFL is 0%.
2. *For disbursement of principal payments:* Disburse principal payments to tranche A until it is completely paid off. After tranche A is completely paid off, disburse principal payments to tranche B until it is completely paid off. After tranche B is completely paid off, disburse principal payments to tranches FL and IFL until they are completely paid off. The principal payments between tranches FL and IFL should be made in the following way: 75% to tranche FL and 25% to tranche IFL. After tranches FL and IFL are completely paid off, disburse principal payments to tranche Z until the original principal balance plus accrued interest are completely paid off.

Thus, shorter-term tranches and a longer-term tranche are created by including an accrual tranche in FJF-02 compared to FJF-01. The accrual tranche has appeal to investors who are concerned with reinvestment risk. Since there are no coupon payments to reinvest, reinvestment risk is eliminated until all the other tranches are paid off.

C. Floating-Rate Tranches

The tranches described thus far have a fixed rate. There is a demand for tranches that have a floating rate. The problem is that the collateral pays a fixed rate and therefore it would be difficult to create a tranche with a floating rate. However, a floating-rate tranche can be created. This is done by creating from any fixed-rate tranche a floater and an inverse floater combination. We will illustrate the creation of a **floating-rate tranche** and an **inverse floating-rate tranche** using the hypothetical CMO structure—the 4-tranche sequential-pay structure with an accrual tranche (FJF-02).¹⁰ We can select any of the tranches from which to create a floating-rate and inverse floating-rate tranche. In fact, we can create these two securities for more than one of the four tranches or for only a portion of one tranche.

In this case, we create a floater and an inverse floater from tranche C. A floater could have been created from any of the other tranches. The par value for this tranche is \$96.5 million, and we create two tranches that have a combined par value of \$96.5 million. We refer to this CMO structure with a floater and an inverse floater as FJF-03. It has five tranches, designated A, B, FL, IFL, and Z, where FL is the floating-rate tranche and IFL is the inverse floating-rate tranche. Exhibit 10 describes FJF-03. Any reference rate can be used to create a floater and the

¹⁰The same principle for creating a floating-rate tranche and inverse-floating rate tranche could have been accomplished using the 4-tranche sequential-pay structure without an accrual tranche (FJF-01).

corresponding inverse floater. The reference rate for setting the coupon rate for FL and IFL in FJF-03 is 1-month LIBOR.

The amount of the par value of the floating-rate tranche will be some portion of the \$96.5 million. There are an infinite number of ways to slice up the \$96.5 million between the floater and inverse floater, and final partitioning will be driven by the demands of investors. In the FJF-03 structure, we made the floater from \$72,375,000 or 75% of the \$96.5 million. The coupon formula for the floater is 1-month LIBOR plus 50 basis points. So, for example, if LIBOR is 3.75% at the reset date, the coupon rate on the floater is 3.75% + 0.5%, or 4.25%. There is a cap on the coupon rate for the floater (discussed later).

Unlike a floating-rate note in the corporate bond market whose principal is unchanged over the life of the instrument, the floater's principal balance declines over time as principal payments are made. The principal payments to the floater are determined by the principal payments from the tranche from which the floater is created. In our CMO structure, this is tranche C.

Since the floater's par value is \$72,375,000 of the \$96.5 million, the balance is par value for the inverse floater. Assuming that 1-month LIBOR is the reference rate, the coupon formula for the inverse floater takes the following form:

$$K - L \times (1\text{-month LIBOR})$$

where K and L are constants whose interpretation will be explained shortly.

In FJF-03, K is set at 28.50% and L at 3. Thus, if 1-month LIBOR is 3.75%, the coupon rate for the month is:

$$28.50\% - 3 \times (3.75\%) = 17.25\%$$

K is the cap or maximum coupon rate for the inverse floater. In FJF-03, the cap for the inverse floater is 28.50%. The determination of the inverse floater's cap rate is based on (1) the amount of interest that would have been paid to the tranche from which the floater and the inverse floater were created, tranche C in our hypothetical deal, and (2) the coupon rate for the floater if 1-month LIBOR is zero.

We will explain the determination of K by example. Let's see how the 28.5% for the inverse floater is determined. The total interest to be paid to tranche C if it was not split into the floater and the inverse floater is the principal of \$96,500,000 times 7.5%, or \$7,237,500. The maximum interest for the inverse floater occurs if 1-month LIBOR is zero. In that case, the coupon rate for the floater is

$$1\text{-month LIBOR} + 0.5\% = 0.5\%$$

Since the floater receives 0.5% on its principal of \$72,375,000, the floater's interest is \$361,875. The remainder of the interest of \$7,237,500 from tranche C goes to the inverse floater. That is, the inverse floater's interest is \$6,875,625 ($= \$7,237,500 - \$361,875$). Since the inverse floater's principal is \$24,125,000, the cap rate for the inverse floater is

$$\frac{\$6,875,625}{\$24,125,000} = 28.5\%$$

In general, the formula for the cap rate on the inverse floater, K , is

$$K = \frac{\text{inverse floater interest when reference rate for floater is zero}}{\text{principal for inverse floater}}$$

The L or multiple in the coupon formula to determine the coupon rate for the inverse floater is called the *leverage*. The higher the leverage, the more the inverse floater's coupon rate changes for a given change in 1-month LIBOR. For example, a coupon leverage of 3 means that a 1-basis point change in 1-month LIBOR will change the coupon rate on the inverse floater by 3 basis points.

As in the case of the floater, the principal paydown of an inverse floater will be a proportionate amount of the principal paydown of tranche C.

Because 1-month LIBOR is always positive, the coupon rate paid to the floater cannot be negative. If there are no restrictions placed on the coupon rate for the inverse floater, however, it is possible for its coupon rate to be negative. To prevent this, a floor, or minimum, is placed on the coupon rate. In most structures, the floor is set at zero. Once a floor is set for the inverse floater, a cap or ceiling is imposed on the floater.

In FJF-03, a floor of zero is set for the inverse floater. The floor results in a cap or maximum coupon rate for the floater of 10%. This is determined as follows. If the floor for the inverse floater is zero, this means that the inverse floater receives no interest. All of the interest that would have been paid to tranche C, \$7,237,500, would then be paid to the floater. Since the floater's principal is \$72,375,000, the cap rate on the floater is \$7,237,500/\$72,375,000, or 10%.

In general, the cap rate for the floater *assuming a floor of zero for inverse floater* is determined as follows:

$$\text{cap rate for floater} = \frac{\text{collateral tranche interest}}{\text{principal for floater}}$$

The cap for the floater and the inverse floater, the floor for the inverse floater, the leverage, and the floater's spread are not determined independently. Any cap or floor imposed on the coupon rate for the floater and the inverse floater must be selected so that the weighted average coupon rate does not exceed the collateral tranche's coupon rate.

D. Structured Interest-Only Tranches

CMO structures can be created so that a tranche receives only interest. Interest only (IO) tranches in a CMO structure are commonly referred to as **structured IOs** to distinguish them from IO mortgage strips that we will describe later in this chapter. The basic principle in creating a structured IO is to set the coupon rate below the collateral's coupon rate so that excess interest can be generated. It is the excess interest that is used to create one or more structured IOs.

Let's look at how a structured IO is created using an illustration. Thus far, we used a simple CMO structure in which all the tranches have the same coupon rate (7.5%) and that coupon rate is the same as the collateral. A structured IO is created from a CMO structure where the coupon rate for at least one tranche is different from the collateral's coupon rate. This is seen in FJF-04 shown in Exhibit 11. In this structure, notice that the coupon interest rate for each tranche is less than the coupon interest rate for the collateral. That means that there is excess interest from the collateral that is not being paid to all the tranches. At one time, all of that excess interest not paid to the tranches was paid to a bond class called a "residual." Eventually (due to changes in the tax law that do not concern us here), structurers of CMO began allocating the excess interest to the tranche that receives only interest. This is tranche IO in FJF-04.

Notice that for this structure the par amount for the IO tranche is shown as \$52,566,667 and the coupon rate is 7.5%. Since this is an IO tranche there is no par amount. The amount shown is the amount upon which the interest payments will be determined, not the amount

EXHIBIT 11 FJF-04—A Hypothetical Five Tranche Sequential Pay with an Accrual Tranche, an Interest-Only Tranche, and a Residual Class

Tranche	Par amount	Coupon rate (%)
A	\$194,500,000	6.00
B	36,000,000	6.50
C	96,500,000	7.00
Z	73,000,000	7.25
IO	52,566,667 (Notional)	7.50
Total	\$400,000,000	

Payment rules:

1. *For payment of monthly coupon interest:* Disburse monthly coupon interest to tranches A, B, and C on the basis of the amount of principal outstanding for each class at the beginning of the month. For tranche Z, accrue the interest based on the principal plus accrued interest in the previous month. The interest for tranche Z is to be paid to the earlier tranches as a principal pay down. Disburse periodic interest to the IO tranche based on the notional amount for all tranches at the beginning of the month.
2. *For disbursement of principal payments:* Disburse monthly principal payments to tranche A until it is completely paid off. After tranche A is completely paid off, disburse principal payments to tranche B until it is completely paid off. After tranche B is completely paid off, disburse principal payments to tranche C until it is completely paid off. After tranche C is completely paid off, disburse principal payments to tranche Z until the original principal balance plus accrued interest is completely paid off.
3. *No principal is to be paid to the IO tranche:* The notional amount of the IO tranche declines based on the principal payments to all other tranches.

that will be paid to the holder of this tranche. Therefore, it is called a **notional amount**. The resulting IO is called a **notional IO**.

Let's look at how the notional amount is determined. Consider tranche A. The par value is \$194.5 million and the coupon rate is 6%. Since the collateral's coupon rate is 7.5%, the excess interest is 150 basis points (1.5%). Therefore, an IO with a 1.5% coupon rate and a notional amount of \$194.5 million can be created from tranche A. But this is equivalent to an IO with a notional amount of \$38.9 million and a coupon rate of 7.5%. Mathematically, this notional amount is found as follows:

$$\text{notional amount for 75\% IO} = \frac{\text{original tranche's par value} \times \text{excess interest}}{0.075}$$

where

$$\text{excess interest} = \text{collateral tranche's coupon rate} - \text{tranche coupon rate}$$

For example, for tranche A:

$$\begin{aligned} \text{excess interest} &= 0.075 - 0.060 = 0.015 \\ \text{tranche's par value} &= \$194,500,000 \\ \text{notional amount for 7.5\% IO} &= \frac{\$194,500,000 \times 0.015}{0.075} = \$38,900,000 \end{aligned}$$

EXHIBIT 12 Creating a Notional IO Tranche

Tranche	Par amount	Excess interest (%)	Notional amount for a 7.5% coupon rate IO
A	\$194,500,000	1.50	\$38,900,000
B	36,000,000	1.00	4,800,000
C	96,500,000	0.50	6,433,333
Z	73,000,000	0.25	2,433,333
Notional amount for 7.5% IO = \$52,566,667			

Similarly, from tranche B with a par value of \$36 million, the excess interest is 100 basis points (1%) and therefore an IO with a coupon rate of 1% and a notional amount of \$36 million can be created. But this is equivalent to creating an IO with a notional amount of \$4.8 million and a coupon rate of 7.5%. This procedure is shown in Exhibit 12 for all four tranches.

E. Planned Amortization Class Tranches

The CMO structures discussed above attracted many institutional investors who had previously either avoided investing in mortgage-backed securities or allocated only a nominal portion of their portfolio to this sector of the bond market. While some traditional corporate bond buyers shifted their allocation to CMOs, a majority of institutional investors remained on the sidelines, concerned about investing in an instrument they continued to perceive as posing significant prepayment risk. This concern was based on the substantial average life variability, despite the innovations designed to mitigate prepayment risk.

In 1987, several structures came to market that shared the following characteristic: if the prepayment speed is within a specified band over the collateral's life, the cash flow pattern is known. The greater predictability of the cash flow for these classes of bonds, now referred to as **planned amortization class (PAC) bonds**, occurs because there is a principal repayment schedule that must be satisfied. PAC bondholders have priority over all other classes in the CMO structure in receiving principal payments from the collateral. The greater certainty of the cash flow for the PAC bonds comes at the expense of the non-PAC tranches, called the **support tranches** or **companion tranches**. It is these tranches that absorb the prepayment risk. Because PAC tranches have protection against both extension risk and contraction risk, they are said to provide **two-sided prepayment protection**.

To illustrate how to create a PAC bond, we will use as collateral the \$400 million passthrough with a coupon rate of 7.5%, an 8.125% WAC, and a WAM of 357 months. The creation requires the specification of two PSA prepayment rates—a *lower PSA prepayment assumption* and an *upper PSA prepayment assumption*. In our illustration the lower PSA prepayment assumption will be 90 PSA and the upper PSA prepayment assumption will be 300 PSA. A natural question is: How does one select the lower and upper PSA prepayment assumptions? These are dictated by market conditions. For our purpose here, how they are determined is not important. The lower and upper PSA prepayment assumptions are referred to as the **initial PAC collar** or the **initial PAC band**. In our illustration the initial PAC collar is 90–300 PSA.

The second column of Exhibit 13 shows the principal payment (scheduled principal repayment plus prepayments) for selected months assuming a prepayment speed of 90 PSA, and the next column shows the principal payments for selected months assuming that the passthrough prepays at 300 PSA.

The last column of Exhibit 13 gives the minimum principal payment if the collateral prepays at 90 PSA or 300 PSA for months 1 to 349. (After month 349, the outstanding principal balance will be paid off if the prepayment speed is between 90 PSA and 300 PSA.) For example, in the first month, the principal payment would be \$508,169 if the collateral prepays at 90 PSA and \$1,075,931 if the collateral prepays at 300 PSA. Thus, the minimum

EXHIBIT 13 Monthly Principal Payment for \$400 Million, 7.5% Coupon
Passthrough with an 8.125% WAC and a 357 WAM Assuming Prepayment Rates of 90
PSA and 300 PSA

Month	At 90 PSA (\$)	At 300 PSA	Minimum principal payment available to PAC investors—the PAC schedule (\$)
1	508,169	1,075,931	508,169
2	569,843	1,279,412	569,843
3	631,377	1,482,194	631,377
4	692,741	1,683,966	692,741
5	753,909	1,884,414	753,909
6	814,850	2,083,227	814,850
7	875,536	2,280,092	875,536
8	935,940	2,474,700	935,940
9	996,032	2,666,744	996,032
10	1,055,784	2,855,920	1,055,784
11	1,115,170	3,041,927	1,115,170
12	1,174,160	3,224,472	1,174,160
13	1,232,727	3,403,265	1,232,727
14	1,290,844	3,578,023	1,290,844
15	1,348,484	3,748,472	1,348,484
16	1,405,620	3,914,344	1,405,620
17	1,462,225	4,075,381	1,462,225
18	1,518,274	4,231,334	1,518,274
101	1,458,719	1,510,072	1,458,719
102	1,452,725	1,484,126	1,452,725
103	1,446,761	1,458,618	1,446,761
104	1,440,825	1,433,539	1,433,539
105	1,434,919	1,408,883	1,408,883
211	949,482	213,309	213,309
212	946,033	209,409	209,409
213	942,601	205,577	205,577
346	618,684	13,269	13,269
347	617,071	12,944	12,944
348	615,468	12,626	12,626
349	613,875	12,314	3,432
350	612,292	12,008	0
351	610,719	11,708	0
352	609,156	11,414	0
353	607,603	11,126	0
354	606,060	10,843	0
355	604,527	10,567	0
356	603,003	10,295	0
357	601,489	10,029	0

EXHIBIT 14 FJF-05—CMO Structure with One PAC Tranche and One Support Tranche

Tranche	Par amount (\$)	Coupon rate (%)
P (PAC)	243,800,000	7.5
S (Support)	156,200,000	7.5
Total	400,000,000	

Payment rules:

1. *For payment of monthly coupon interest:* Disburse monthly coupon interest to each tranche on the basis of the amount of principal outstanding for each tranche at the beginning of the month.
2. *For disbursement of principal payments:* Disburse principal payments to tranche P based on its schedule of principal repayments. Tranche P has priority with respect to current and future principal payments to satisfy the schedule. Any excess principal payments in a month over the amount necessary to satisfy the schedule for tranche P are paid to tranche S. When tranche S is completely paid off, all principal payments are to be made to tranche P regardless of the schedule.

principal payment is \$508,169, as reported in the last column of Exhibit 13. In month 103, the minimum principal payment is also the amount if the prepayment speed is 90 PSA, \$1,446,761, compared to \$1,458,618 for 300 PSA. In month 104, however, a prepayment speed of 300 PSA would produce a principal payment of \$1,433,539, which is less than the principal payment of \$1,440,825 assuming 90 PSA. So, \$1,433,539 is reported in the last column of Exhibit 13. From month 104 on, the minimum principal payment is the one that would result assuming a prepayment speed of 300 PSA.

In fact, if the collateral prepays at any one speed between 90 PSA and 300 PSA over its life, the minimum principal payment would be the amount reported in the last column of Exhibit 13. For example, if we had included principal payment figures assuming a prepayment speed of 200 PSA, the minimum principal payment would not change: from month 1 through month 103, the minimum principal payment is that generated from 90 PSA, but from month 104 on, the minimum principal payment is that generated from 300 PSA.

This characteristic of the collateral allows for the creation of a PAC tranche, assuming that the collateral prepays over its life at a speed between 90 PSA to 300 PSA. A schedule of principal repayments that the PAC bondholders are entitled to receive before any other tranche in the CMO structure is specified. The monthly schedule of principal repayments is as specified in the last column of Exhibit 13, which shows the minimum principal payment. That is, this minimum principal payment in each month is the principal repayment schedule (i.e., planned amortization schedule) for investors in the PAC tranche. While there is no assurance that the collateral will prepay at a constant speed between these two speeds over its life, a PAC tranche can be structured to assume that it will.

Exhibit 14 shows a CMO structure, FJF-05, created from the \$400 million, 7.5% coupon passthrough with a WAC of 8.125% and a WAM of 357 months. There are just two tranches in this structure: a 7.5% coupon PAC tranche created assuming 90 to 300 PSA with a par value of \$243.8 million, and a support tranche with a par value of \$156.2 million.

Exhibit 15 reports the average life for the PAC tranche and the support tranche in FJF-05 assuming various *actual* prepayment speeds. Notice that between 90 PSA and 300 PSA, the average life for the PAC bond is stable at 7.26 years. However, at slower or faster PSA speeds, the schedule is broken, and the average life changes, extending when the prepayment speed is less than 90 PSA and contracting when it is greater than 300 PSA. Even so, there is much greater variability for the average life of the support tranche.

EXHIBIT 15 Average Life for PAC Tranche and Support Tranche in FJF-05 Assuming Various Prepayment Speeds (Years)

Prepayment rate (PSA)	PAC bond (P)	Support bond (S)
0	15.97	27.26
50	9.44	24.00
90	7.26	20.06
100	7.26	18.56
150	7.26	12.57
165	7.26	11.16
200	7.26	8.38
250	7.26	5.37
300	7.26	3.13
350	6.56	2.51
400	5.92	2.17
450	5.38	1.94
500	4.93	1.77
700	3.70	1.37

EXHIBIT 16 FJF-06—CMO Structure with Six PAC Tranches and a Support Tranche

Tranche	Par amount	Coupon rate (%)
P-A	\$85,000,000	7.5
P-B	8,000,000	7.5
P-C	35,000,000	7.5
P-D	45,000,000	7.5
P-E	40,000,000	7.5
P-F	30,800,000	7.5
S	156,200,000	7.5
Total	\$400,000,000	

Payment rules:

1. *For payment of monthly coupon interest:* Disburse monthly coupon interest to each tranche on the basis of the amount of principal outstanding of each tranche at the beginning of the month.
2. *For disbursement of principal payments:* Disburse monthly principal payments to tranches P-A to PF based on their respective schedules of principal repayments. Tranche P-A has priority with respect to current and future principal payments to satisfy the schedule. Any excess principal payments in a month over the amount necessary to satisfy the schedule for tranche P-A are paid to tranche S. Once tranche P-A is completely paid off, tranche PB has priority, then tranche PC, etc. When tranche S is completely paid off, all principal payments are to be made to the remaining PAC tranches in order of priority regardless of the schedule.

1. **Creating a Series of PAC Tranches** Most CMO PAC structures have more than one class of PAC tranches. A sequence of six PAC tranches (i.e., PAC tranches paid off in sequence as specified by a principal schedule) is shown in Exhibit 16 and is called FJF-06. The total par value of the six PAC tranches is equal to \$243.8 million, which is the amount of the single PAC tranche in FJF-05. The schedule of principal repayments for selected months for each PAC bond is shown in Exhibit 17.

Exhibit 18 shows the average life for the six PAC tranches and the support tranche in FJF-06 at various prepayment speeds. From a PAC bond in FJF-05 with an average life of

EXHIBIT 17 Mortgage Balance for Selected Months for FJF-06 Assuming 165 PSA

Month	Tranche						Support
	A	B	C	D	E	F	
1	85,000,000	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	156,200,000
2	84,491,830	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	155,998,246
3	83,921,987	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	155,746,193
4	83,290,609	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	155,444,011
5	82,597,868	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	155,091,931
6	81,843,958	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	154,690,254
7	81,029,108	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	154,239,345
8	80,153,572	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	153,739,635
9	79,217,631	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	153,191,621
10	78,221,599	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	152,595,864
11	77,165,814	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	151,952,989
12	76,050,644	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	151,263,687
13	74,876,484	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	150,528,708
52	5,170,458	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	109,392,664
53	3,379,318	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	108,552,721
54	1,595,779	8,000,000	35,000,000	45,000,000	40,000,000	30,800,000	107,728,453
55	0	7,819,804	35,000,000	45,000,000	40,000,000	30,800,000	106,919,692
56	0	6,051,358	35,000,000	45,000,000	40,000,000	30,800,000	106,126,275
57	0	4,290,403	35,000,000	45,000,000	40,000,000	30,800,000	105,348,040
58	0	2,536,904	35,000,000	45,000,000	40,000,000	30,800,000	104,584,824
59	0	790,826	35,000,000	45,000,000	40,000,000	30,800,000	103,836,469
60	0	0	34,052,132	45,000,000	40,000,000	30,800,000	103,102,817
61	0	0	32,320,787	45,000,000	40,000,000	30,800,000	102,383,711
62	0	0	30,596,756	45,000,000	40,000,000	30,800,000	101,678,995
78	0	0	3,978,669	45,000,000	40,000,000	30,800,000	92,239,836
79	0	0	2,373,713	45,000,000	40,000,000	30,800,000	91,757,440
80	0	0	775,460	45,000,000	40,000,000	30,800,000	91,286,887
81	0	0	44,183,878	40,000,000	30,800,000	90,828,046	
82	0	0	0	42,598,936	40,000,000	30,800,000	90,380,792
83	0	0	0	41,020,601	40,000,000	30,800,000	89,944,997
108	0	0	0	3,758,505	40,000,000	30,800,000	82,288,542
109	0	0	0	2,421,125	40,000,000	30,800,000	82,030,119
110	0	0	0	1,106,780	40,000,000	30,800,000	81,762,929
111	0	0	0	0	39,815,082	30,800,000	81,487,234
112	0	0	0	0	38,545,648	30,800,000	81,203,294
113	0	0	0	0	37,298,104	30,800,000	80,911,362
153	0	0	0	0	1,715,140	30,800,000	65,030,732
154	0	0	0	0	1,107,570	30,800,000	64,575,431
155	0	0	0	0	510,672	30,800,000	64,119,075
156	0	0	0	0	0	30,724,266	63,661,761
157	0	0	0	0	0	30,148,172	63,203,587
158	0	0	0	0	0	29,582,215	62,744,644
347	0	0	0	0	0	29,003	1,697,536
348	0	0	0	0	0	16,058	1,545,142
349	0	0	0	0	0	3,432	1,394,152
350	0	0	0	0	0	0	1,235,674
351	0	0	0	0	0	0	1,075,454
352	0	0	0	0	0	0	916,910
353	0	0	0	0	0	0	760,026
354	0	0	0	0	0	0	604,789
355	0	0	0	0	0	0	451,182
356	0	0	0	0	0	0	299,191
357	0	0	0	0	0	0	148,801

EXHIBIT 18 Average Life for the Six PAC Tranches in FJF-06
Assuming Various Prepayment Speeds

Prepayment rate (PSA)	PAC Bonds					
	P-A	P-B	P-C	P-D	P-E	P-F
0	8.46	14.61	16.49	19.41	21.91	23.76
50	3.58	6.82	8.36	11.30	14.50	18.20
90	2.58	4.72	5.78	7.89	10.83	16.92
100	2.58	4.72	5.78	7.89	10.83	16.92
150	2.58	4.72	5.78	7.89	10.83	16.92
165	2.58	4.72	5.78	7.89	10.83	16.92
200	2.58	4.72	5.78	7.89	10.83	16.92
250	2.58	4.72	5.78	7.89	10.83	16.92
300	2.58	4.72	5.78	7.89	10.83	16.92
350	2.58	4.72	5.44	6.95	9.24	14.91
400	2.57	4.37	4.91	6.17	8.33	13.21
450	2.50	3.97	4.44	5.56	7.45	11.81
500	2.40	3.65	4.07	5.06	6.74	10.65
700	2.06	2.82	3.10	3.75	4.88	7.51

7.26, six tranches have been created with an average life as short as 2.58 years (P-A) and as long as 16.92 years (P-F) if prepayments stay within 90 PSA and 300 PSA.

As expected, the average lives are stable if the prepayment speed is between 90 PSA and 300 PSA. Notice that even outside this range the average life is stable for several of the PAC tranches. For example, the PAC P-A tranche is stable even if prepayment speeds are as high as 400 PSA. For the PAC P-B, the average life does not vary when prepayments are in the initial collar until prepayments are greater than 350 PSA. Why is it that the shorter the PAC, the more protection it has against faster prepayments?

To understand this phenomenon, remember there are \$156.2 million in support tranches that are protecting the \$85 million of PAC P-A. Thus, even if prepayments are faster than the initial upper collar, there may be sufficient support tranches to assure the satisfaction of the schedule. In fact, as can be seen from Exhibit 18, even if prepayments are 400 PSA over the life of the collateral, the average life is unchanged.

Now consider PAC P-B. The support tranches provide protection for both the \$85 million of PAC P-A and \$93 million of PAC P-B. As can be seen from Exhibit 18, prepayments could be 350 PSA and the average life is still unchanged. From Exhibit 18 it can be seen that the degree of protection against extension risk increases the shorter the PAC. Thus, while the initial collar may be 90 to 300 PSA, the **effective collar** is wider for the shorter PAC tranches.

2. PAC Window The length of time over which expected principal repayments are made is referred to as the window. For a PAC tranche it is referred to as the **PAC window**. A PAC window can be wide or narrow. The narrower a PAC window, the more it resembles a corporate bond with a bullet payment. For example, if the PAC schedule calls for just one principal payment (the narrowest window) in month 120 and only interest payments up to month 120, this PAC tranche would resemble a 10-year (120-month) corporate bond.

PAC buyers appear to prefer tight windows, although institutional investors facing a liability schedule are generally better off with a window that more closely matches their liabilities. Investor demand dictates the PAC windows that dealers will create. Investor demand in turn is governed by the nature of investor liabilities.

3. Effective Collars and Actual Prepayments The creation of a mortgage-backed security cannot make prepayment risk disappear. This is true for both a passthrough and a CMO. Thus, the reduction in prepayment risk (both extension risk and contraction risk) that a PAC offers investors must come from somewhere.

Where does the prepayment protection come from? It comes from the support tranches. It is the support tranches that defer principal payments to the PAC tranches if the collateral prepayments are slow; support tranches do not receive any principal until the PAC tranches receive the scheduled principal repayment. This reduces the risk that the PAC tranches will extend. Similarly, it is the support tranches that absorb any principal payments in excess of the scheduled principal payments that are made. This reduces the contraction risk of the PAC tranches. Thus, the key to the prepayment protection offered by a PAC tranche is the amount of support tranches outstanding. If the support tranches are paid off quickly because of faster-than-expected prepayments, then there is no longer any protection for the PAC tranches. *In fact, in FJF-06, if the support tranche is paid off, the structure effectively becomes a sequential-pay CMO.*

The support tranches can be thought of as bodyguards for the PAC bondholders. When the bullets fly—i.e., prepayments occur—it is the bodyguards that get killed off first. The bodyguards are there to absorb the bullets. Once all the bodyguards are killed off (i.e., the support tranches paid off with faster-than-expected prepayments), the PAC tranches must fend for themselves: they are exposed to all the bullets. A PAC tranche in which all the support tranches have been paid off is called a **busted PAC** or **broken PAC**.

With the bodyguard metaphor for the support tranches in mind, let's consider two questions asked by investors in PAC tranches:

1. Will the schedule of principal repayments be satisfied if prepayments are faster than the initial upper collar?
2. Will the schedule of principal repayments be satisfied as long as prepayments stay within the initial collar?

a. Actual Prepayments Greater than the Initial Upper Collar Let's address the first question. The initial upper collar for FJF-06 is 300 PSA. Suppose that actual prepayments are 500 PSA for seven consecutive months. Will this disrupt the schedule of principal repayments? The answer is: It depends!

There are two pieces of information we will need to answer this question. First, when does the 500 PSA occur? Second, what has been the actual prepayment experience up to the time that prepayments are 500 PSA? For example, suppose six years from now is when the prepayments reach 500 PSA, and also suppose that for the past six years the actual prepayment speed has been 90 PSA every month. What this means is that there are more bodyguards (i.e., support tranches) around than were expected when the PAC was structured at the initial collar. In establishing the schedule of principal repayments, it is assumed that the bodyguards would be killed off at 300 PSA. (Recall that 300 PSA is the upper collar prepayment assumption used in creating FJF-06.) But the actual prepayment experience results in them being killed off at only 90 PSA. Thus, six years from now when the 500 PSA is assumed to occur, there are more bodyguards than expected. In turn, a 500 PSA for seven consecutive months may have no effect on the ability of the schedule of principal repayments to be met.

In contrast, suppose that the actual prepayment experience for the first six years is 300 PSA (the upper collar of the initial PAC collar). In this case, there are no extra bodyguards

around. As a result, any prepayment speeds faster than 300 PSA, such as 500 PSA in our example, jeopardize satisfaction of the principal repayment schedule and increase contraction risk. This does not mean that the schedule will be “busted”—the term used in the CMO market when the support tranches are fully paid off. What it does mean is that the prepayment protection is reduced.

It should be clear from these observations that the initial collars are not particularly useful in assessing the prepayment protection for a seasoned PAC tranche. This is most important to understand, as it is common for CMO buyers to compare prepayment protection of PACs in different CMO structures and conclude that the greater protection is offered by the one with the wider initial collar. This approach is inadequate because it is actual prepayment experience that determines the degree of prepayment protection, as well as the expected future prepayment behavior of the collateral.

The way to determine this protection is to calculate the **effective collar** for a seasoned PAC bond. An effective collar for a seasoned PAC is the lower and the upper PSA that can occur in the future and still allow maintenance of the schedule of principal repayments. For example, consider two seasoned PAC tranches in two CMO structures where the two PAC tranches have the same average life and the prepayment characteristics of the remaining collateral (i.e., the remaining mortgages in the mortgage pools) are similar. The information about these PAC tranches is as follows:

	PAC tranche X	PAC tranche Y
Initial PAC collar	180 PSA–350 PSA	170 PSA–410 PSA
Effective PAC collar	160 PSA–450 PSA	240 PSA–300 PSA

Notice that at issuance PAC tranche Y offered greater prepayment protection than PAC tranche X as indicated by the *wider* initial PAC collar. However, that protection is irrelevant for an investor who is considering the purchase of one of these two tranches today. Despite PAC tranche Y's greater prepayment protection at issuance than PAC tranche X, tranche Y has a much narrower effective PAC collar than PAC tranche X and therefore less prepayment protection.

The effective collar changes every month. An extended period over which actual prepayments are below the upper range of the initial PAC collar will result in an increase in the upper range of the effective collar. This is because there will be more bodyguards around than anticipated. An extended period of prepayments slower than the lower range of the initial PAC collar will raise the lower range of the effective collar. This is because it will take faster prepayments to make up the shortfall of the scheduled principal payments not made plus the scheduled future principal payments.

b. Actual Prepayments within the Initial Collar The PAC schedule may not be satisfied even if the actual prepayments never fall outside of the *initial* collar. This may seem surprising since our previous analysis indicated that the average life would not change if prepayments are at either extreme of the initial collar. However, recall that all of our previous analysis has been based on a single PSA speed for the life of the structure.

The following table shows for FJF-05 what happens to the effective collar if prepayments are 300 PSA for the first 24 months but another prepayment speed for the balance of the life of the structure:

PSA from year 2 on	Average life
95	6.43
105	6.11
115	6.01
120	6.00
125	6.00
300	6.00
305	5.62

Notice that the average life is stable at six years if the prepayments for the subsequent months are between 115 PSA and 300 PSA. That is, the effective PAC collar is no longer the initial collar. Instead, the lower collar has shifted upward. This means that the protection from year 2 on is for 115 to 300 PSA, a narrower band than initially (90 to 300 PSA), even though the earlier prepayments did not exceed the initial upper collar.

F. Support Tranches

The support tranches are the bonds that provide prepayment protection for the PAC tranches. Consequently, support tranches expose investors to the greatest level of prepayment risk. Because of this, investors must be particularly careful in assessing the cash flow characteristics of support tranches to reduce the likelihood of adverse portfolio consequences due to prepayments.

The support tranche typically is divided into different tranches. All the tranches we have discussed earlier are available, including sequential-pay support tranches, floater and inverse floater support tranches, and accrual support tranches.

The support tranche can even be partitioned to create support tranches with a schedule of principal payments. That is, support tranches that are PAC tranches can be created. In a structure with a PAC tranche and a support tranche with a PAC schedule of principal payments, the former is called a **PAC I tranche** or **Level I PAC tranche** and the latter a **PAC II tranche** or **Level II PAC tranche** or **scheduled tranche** (denoted SCH in a prospectus). While PAC II tranches have greater prepayment protection than the support tranches without a schedule of principal repayments, the prepayment protection is less than that provided PAC I tranches.

The support tranche without a principal repayment schedule can be used to create any type of tranche. In fact, a portion of the non-PAC II support tranche can be given a schedule of principal repayments. This tranche would be called a **PAC III tranche** or a **Level III PAC tranche**. While it provides protection against prepayments for the PAC I and PAC II tranches and is therefore subject to considerable prepayment risk, such a tranche has greater protection than the support tranche without a schedule of principal repayments.

G. An Actual CMO Structure

Thus far, we have presented some hypothetical CMO structures in order to demonstrate the characteristics of the different types of tranches. Now let's look at an actual CMO structure, one that we will look at further in Chapter 12 when we discuss how to analyze a CMO deal.

The CMO structure we will discuss is the Freddie Mac (FHLMC) Series 1706 issued in early 1994. The collateral for this structure is Freddie Mac 7% coupon passthroughs. A summary of the deal is provided in Exhibit 19.

EXHIBIT 19 Summary of Federal Home Loan Mortgage Corporation—Multiclass Mortgage Participation Certificates (Guaranteed), Series 1706

Total Issue: \$300,000,000 Original Settlement Date: 3/30/94
Issue Date: 2/18/94

Tranche	Original Balance (\$)	Coupon (%)	Average life (yrs)
A (PAC Bond)	24,600,000	4.50	1.3
B (PAC Bond)	11,100,000	5.00	2.5
C (PAC Bond)	25,500,000	5.25	3.5
D (PAC Bond)	9,150,000	5.65	4.5
E (PAC Bond)	31,650,000	6.00	5.8
G (PAC Bond)	30,750,000	6.25	7.9
H (PAC Bond)	27,450,000	6.50	10.9
J (PAC Bond)	5,220,000	6.50	14.4
K (PAC Bond)	7,612,000	7.00	18.4
LA (SCH Bond)	26,673,000	7.00	3.5
LB (SCH Bond)	36,087,000	7.00	3.5
M (SCH Bond)	18,738,000	7.00	11.2
O (TAC Bond)	13,348,000	7.00	2.5
OA (TAC Bond)	3,600,000	7.00	7.2
IA (IO, PAC Bond)	30,246,000	7.00	7.1
PF (FLTR, Support Bond)	21,016,000	6.75*	17.5
PS (INV FLTR, Support Bond)	7,506,000	7.70*	17.5

*Coupon at issuance.

Structural Features

Cash Flow Allocation: Commencing on the first principal payment date of the Class A Bonds, principal equal to the amount specified in the Prospectus will be applied to the Class A, B, C, D, E, G, H, J, K, LB, M, O, OA, PF, and PS Bonds. After all other Classes have been retired, any remaining principal will be used to retire the Class O, OA, LA, LB, M, A, B, C, D, G, H, J, and K Bonds. The notional Class IA Bond will have its notional principal amount retired along with the PAC Bonds.

Other: The PAC Range is 95% to 300% PSA for the A–K Bonds, 190% to 250% PSA for the LA, LB, and M Bonds, and 225% PSA for the O and OA Bonds.

There are 17 tranches in this structure: 10 PAC tranches, three scheduled tranches, a floating-rate support tranche, and an inverse floating-rate support tranche.¹¹ There are also two “TAC” support tranches. We will explain a TAC tranche below. Let’s look at all tranches.

First, we know what a PAC tranche is. There are 10 of them: tranches A, B, C, D, E, G, H, J, K, and IA. The initial collar used to create the PAC tranches was 95 PSA to 300 PSA. The PAC tranches except for tranche IA are simply PACs that pay off in sequence. Tranche IA is structured such that the underlying collateral’s interest not allocated to the other PAC tranches is paid to the IO tranche. This is a notional IO tranche and we described earlier in this section how it is created. In this deal the tranches from which the interest is stripped are the PAC tranches. So, tranche IA is referred to as a **PAC IO**. (As of the time of this writing, tranches A and B had already paid off all of their principal.)

The prepayment protection for the PAC bonds is provided by the support tranches. The support tranches in this deal are tranches LA, LB, M, O, OA, PF, and PS. Notice that the

¹¹Actually there were two other tranches, R and RS, called “residuals.” These tranches were not described in the chapter. They receive any excess cash flows remaining after the payment of all the tranches. The residual is actually the equity part of the deal.

support tranches have been carved up in different ways. First, there are scheduled (SCH) tranches. These are what we have called the PAC II tranches earlier in this section. The scheduled tranches are LA, LB, and M. The initial PAC collar used to create the scheduled tranches was 190 PSA to 250 PSA.

There are two support tranches that are designed such that they are created with a schedule that provides protection against contraction risk but not against extension. We did not discuss these tranches in this chapter. They are called **target amortization class (TAC) tranches**. The support tranches O and OA are TAC tranches. The schedule of principal payments is created by using just a single PSA. In this structure the single PSA is 225 PSA.

Finally, the support tranche without a schedule (that must provide support for the scheduled bonds and the PACs) was carved into two tranches—a floater (tranche PF) and an inverse floater (tranche PS). In this structure the creation of the floater and inverse floater was from a support tranche.

Now that we know what all these tranches are, the next step is to analyze them in terms of their relative value and their price volatility characteristics when rates change. We will do this in Chapter 12.

V. STRIPPED MORTGAGE-BACKED SECURITIES

In a CMO, there are multiple bond classes (tranches) and separate rules for the distribution of the interest and the principal to the bond classes. There are mortgage-backed securities where there are only two bond classes and the rule for the distribution for interest and principal is simple: one bond class receives all of the principal and one bond class receives all of the interest. This mortgage-backed security is called a **stripped mortgage-backed security**. The bond class that receives all of the principal is called the **principal-only class** or **PO class**. The bond class that receives all of the interest is called the **interest-only class** or **IO class**. These securities are also called **mortgage strips**. The POs are called **principal-only mortgage strips** and the IOs are called **interest-only mortgage strips**.

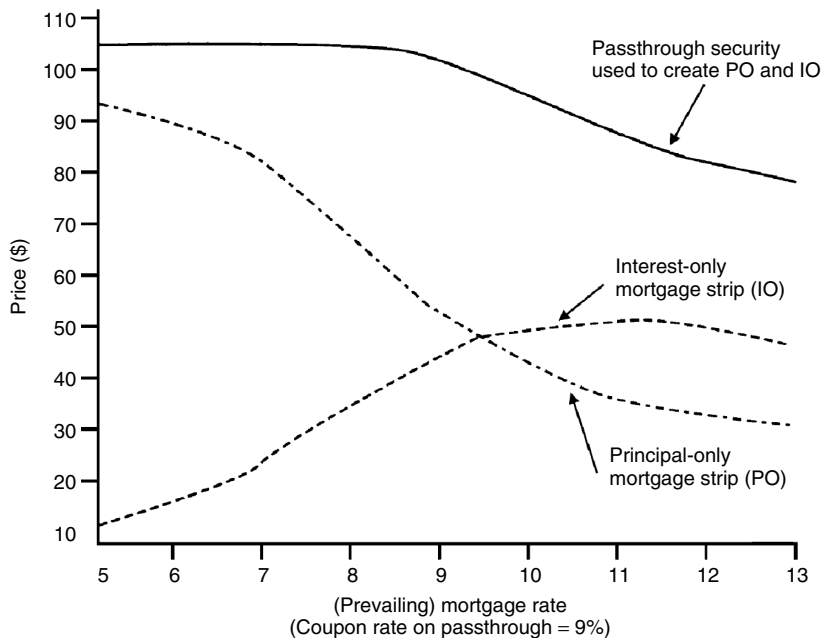
We have already seen interest-only type mortgage-backed securities: the structured IO. This is a product that is created within a CMO structure. A structured IO is created from the excess interest (i.e., the difference between the interest paid on the collateral and the interest paid to the bond classes). There is no corresponding PO class within the CMO structure. In contrast, in a stripped mortgage-backed security, the IO class is created by simply specifying that all interest payments be made to that class.

A. Principal-Only Strips

A principal-only mortgage strip is purchased at a substantial discount from par value. The return an investor realizes depends on the speed at which prepayments are made. The faster the prepayments, the higher the investor's return. For example, suppose that a pool of 30-year mortgages has a par value of \$400 million and the market value of the pool of mortgages is also \$400 million. Suppose further that the market value of just the principal payments is \$175 million. The dollar return from this investment is the difference between the par value of \$400 million that will be repaid to the investor in the principal mortgage strip and the \$175 million paid. That is, the dollar return is \$225 million.

Since there is no interest that will be paid to the investor in a principal-only mortgage strip, the investor's return is determined solely by the speed at which he or she receives

EXHIBIT 20 Relationship between Price and Mortgage Rates for a Passthrough, PO, and IO



the \$225 million. In the extreme case, if all homeowners in the underlying mortgage pool decide to prepay their mortgage loans immediately, PO investors will realize the \$225 million immediately. At the other extreme, if all homeowners decide to remain in their homes for 30 years and make no prepayments, the \$225 million will be spread out over 30 years, which would result in a lower return for PO investors.

Let's look at how the price of the PO would be expected to change as mortgage rates in the market change. When mortgage rates decline below the contract rate, prepayments are expected to speed up, accelerating payments to the PO investor. Thus, the cash flow of a PO improves (in the sense that principal repayments are received earlier). The cash flow will be discounted at a lower interest rate because the mortgage rate in the market has declined. The result is that the PO price will increase when mortgage rates decline. When mortgage rates rise above the contract rate, prepayments are expected to slow down. The cash flow deteriorates (in the sense that it takes longer to recover principal repayments). Couple this with a higher discount rate, and the price of a PO will fall when mortgage rates rise.

Exhibit 20 shows the general relationship between the price of a principal-only mortgage strip when interest rates change and compares it to the relationship for the underlying passthrough from which it is created.

B. Interest-Only Strips

An interest-only mortgage strip has no par value. In contrast to the PO investor, the IO investor wants prepayments to be slow. The reason is that the IO investor receives interest only on the amount of the principal outstanding. When prepayments are made, less dollar interest will be received as the outstanding principal declines. In fact, *if prepayments are too fast, the IO investor may not recover the amount paid for the IO even if the security is held to maturity.*

Let's look at the expected price response of an IO to changes in mortgage rates. If mortgage rates decline below the contract rate, prepayments are expected to accelerate. This would result in a deterioration of the expected cash flow for an IO. While the cash flow will be discounted at a lower rate, the net effect typically is a decline in the price of an IO. If mortgage rates rise above the contract rate, the expected cash flow improves, but the cash flow is discounted at a higher interest rate. The net effect may be either a rise or fall for the IO.

Thus, we see an interesting characteristic of an IO: its price tends to move in the same direction as the change in mortgage rates (1) when mortgage rates fall below the contract rate and (2) for some range of mortgage rates above the contract rate. Both POs and IOs exhibit substantial price volatility when mortgage rates change. The greater price volatility of the IO and PO compared to the passthrough from which they were created is due to the fact that the combined price volatility of the IO and PO must be equal to the price volatility of the passthrough.

Exhibit 20 shows the general relationship between the price of an interest-only mortgage strip when interest rates change and compares it to the relationship for the corresponding principal-only mortgage strip and underlying passthrough from which it is created.

An average life for a PO can be calculated based on some prepayment assumption. However, an IO receives no principal payments, so technically an average life cannot be computed. Instead, for an IO a **cash flow average life** is computed, using the projected interest payments in the average life formula instead of principal.

C. Trading and Settlement Procedures

The trading and settlement procedures for stripped mortgage-backed securities are similar to those set by the Public Securities Association for agency passthroughs described in Section III C. IOs and POs are extreme premium and discount securities and consequently are very sensitive to prepayments, which are driven by the specific characteristics (weighted average coupon, weighted average maturity, geographic concentration, average loan size) of the underlying loans. Therefore, almost all secondary trades in IOs and POs are on a specified pool basis rather than on a TBA basis.

All IOs and POs are given a trust number. For instance, Fannie Mae Trust 1 is a IO/PO trust backed by specific pools of Fannie Mae 9% mortgages. Fannie Mae Trust 2 is backed by Fannie Mae 10% mortgages. Fannie Mae Trust 23 is another IO/PO trust backed by Fannie Mae 10% mortgages. Therefore, a portfolio manager must specify which trust he or she is buying.

The total proceeds of a PO trade are calculated the same way as with a passthrough trade except that there is no accrued interest. The market trades IOs based on notional principal. The proceeds include the price on the notional amount and the accrued interest.

VI. NONAGENCY RESIDENTIAL MORTGAGE-BACKED SECURITIES

In the previous sections we looked at agency mortgage-backed securities in which the underlying mortgages are 1- to 4-single family residential mortgages. The mortgage-backed securities market includes other types of securities. These securities are called **nonagency mortgage-backed securities** (referred to as **nonagency securities** hereafter).

The underlying mortgage loans for nonagency securities can be for any type of real estate property. There are securities backed by 1- to 4-single family residential mortgages with a first lien (i.e., the lender has a first priority or first claim) on the mortgaged property. There are nonagency securities backed by other types of single family residential loans. These include home equity loan-backed securities and manufactured housing-loan backed securities. Our focus in this section is on nonagency securities in which the underlying loans are first-lien mortgages for 1- to 4-single-family residential properties.

As with an agency mortgage-backed security, the servicer is responsible for the collection of interest and principal. The servicer also handles delinquencies and foreclosures. Typically, there will be a master servicer and subservicers. The servicer plays a key role. In fact, in assessing the credit risk of a nonagency security, rating companies look carefully at the quality of the servicers.

A. Underlying Mortgage Loans

The underlying loans for agency securities are those that conform to the underwriting standards of the agency issuing or guaranteeing the issue. That is, only conforming loans are included in pools that are collateral for an agency mortgage-backed security. The three main underwriting standards deal with

1. the maximum loan-to-value ratio
2. the maximum payment-to-income ratio
3. the maximum loan amount

The **loan-to-value ratio** (LTV) is the ratio of the amount of the loan to the market value or appraised value of the property. The *lower* the LTV, the *greater* the protection afforded the lender. For example, an LTV of 0.90 means that if the lender has to repossess the property and sell it, the lender must realize at least 90% of the market value in order to recover the amount lent. An LTV of 0.80 means that the lender only has to sell the property for 80% of its market value in order to recover the amount lent.¹² Empirical studies of residential mortgage loans have found that the LTV is a key determinant of whether a borrower will default: the higher the LTV, the greater the likelihood of default.

As mentioned earlier in this chapter, a nonconforming mortgage loan is one that does not conform to the underwriting standards established by any of the agencies. Typically, the loans for a nonagency security are nonconforming mortgage loans that fail to qualify for inclusion because the amount of the loan exceeds the limit established by the agencies. Such loans are referred to as jumbo loans. Jumbo loans do not necessarily have greater credit risk than conforming mortgages.

Loans that fail to qualify because of the first two underwriting standards expose the lender to greater credit risk than conforming loans. There are specialized lenders who provide mortgage loans to individuals who fail to qualify for a conforming loan because of their credit history. These specialized lenders classify borrowers by credit quality. Borrowers are classified as A borrowers, B borrowers, C borrowers, and D borrowers. A borrowers are those that are viewed as having the best credit record. Such borrowers are referred to as **prime borrowers**. Borrowers rated below A are viewed as **subprime borrowers**. However, there is no industry-wide classification system for prime and subprime borrowers.

¹²This ignores the costs of repossession and selling the property.

B. Differences Between Agency and Nonagency Securities

Nonagency securities can be either passthroughs or CMOs. In the agency market, CMOs are created from pools of passthrough securities. In the nonagency market, CMOs are created from unsecuritized mortgage loans. Since a mortgage loan not securitized as a passthrough is called a whole loan, nonagency CMOs are commonly referred to as **whole-loan CMOs**.

The major difference between agency and nonagency securities has to do with guarantees. With a nonagency security there is no explicit or implicit government guarantee of payment of interest and principal as there is with an agency security. The absence of any such guarantee means that the investor in a nonagency security is exposed to credit risk. The nationally recognized statistical rating organizations rate nonagency securities.

Because of the credit risk, all nonagency securities are **credit enhanced**. By credit enhancement it means that additional support against defaults must be obtained. The amount of credit enhancement needed is determined relative to a specific rating desired for a security rating agency. There are two general types of credit enhancement mechanisms: external and internal. We describe each of these types of credit enhancement in the next chapter where we cover asset-backed securities.

VII. COMMERCIAL MORTGAGE-BACKED SECURITIES

Commercial mortgage-backed securities (CMBSs) are backed by a pool of commercial mortgage loans on income-producing property—multifamily properties (i.e., apartment buildings), office buildings, industrial properties (including warehouses), shopping centers, hotels, and health care facilities (i.e., senior housing care facilities). The basic building block of the CMBS transaction is a commercial loan that was originated either to finance a commercial purchase or to refinance a prior mortgage obligation.

There are two types of CMBS deal structures that have been of primary interest to bond investors: (1) multiproperty single borrower and (2) multiproperty conduit. Conduits are commercial-lending entities that are established for the sole purpose of generating collateral to securitize.

CMBS have been issued outside the United States. The dominant issues have been U.K. based (more than 80% in 2000) with the primary property types being retail and office properties. Starting in 2001, there was dramatic increase in the number of CMBS deals issued by German banks. An increasing number of deals include multi-country properties. The first pan-European securitization was Pan European Industrial Properties in 2001.¹³

A. Credit Risk

Unlike residential mortgage loans where the lender relies on the ability of the borrower to repay and has recourse to the borrower if the payment terms are not satisfied, commercial mortgage loans are **nonrecourse loans**. This means that the lender can only look to the income-producing property backing the loan for interest and principal repayment. If there is

¹³Christopher Flanagan and Edward Reardon, *European Structures Products: 2001 Review and 2002 Outlook*, Global Structured Finance Research, J.P. Morgan Securities Inc. (January 11, 2002), pp. 12–13.

a default, the lender looks to the proceeds from the sale of the property for repayment and has no recourse to the borrower for any unpaid balance. The lender must view each property as a stand-alone business and evaluate each property using measures that have been found useful in assessing credit risk.

While fundamental principles of assessing credit risk apply to all property types, traditional approaches to assessing the credit risk of the collateral differs between CMBS and nonagency mortgage-backed securities and real estate-backed securities that fall into the asset-backed securities sector described in Chapter 11 (those backed by home equity loans and manufactured housing loans). For mortgage-backed securities and asset backed securities in which the collateral is residential property, typically the loans are lumped into buckets based on certain loan characteristics and then assumptions regarding default rates are made regarding each bucket. In contrast, for commercial mortgage loans, the unique economic characteristics of each income-producing property in a pool backing a CMBS require that credit analysis be performed on a loan-by-loan basis not only at the time of issuance, but monitored on an ongoing basis.

Regardless of the type of commercial property, the two measures that have been found to be key indicators of the potential credit performance is the debt-to-service coverage ratio and the loan-to-value ratio.

The **debt-to-service coverage ratio** (DSC) is the ratio of the property's net operating income (NOI) divided by the debt service. The NOI is defined as the rental income reduced by cash operating expenses (adjusted for a replacement reserve). A ratio greater than 1 means that the cash flow from the property is sufficient to cover debt servicing. The higher the ratio, the more likely that the borrower will be able to meet debt servicing from the property's cash flow.

For all properties backing a CMBS deal, a weighted average DSC ratio is computed. An analysis of the credit quality of an issue will also look at the dispersion of the DSC ratios for the underlying loans. For example, one might look at the percentage of a deal with a DSC ratio below a certain value.

As explained in Section VI.A, in computing the LTV, the figure used for "value" in the ratio is either market value or appraised value. In valuing commercial property, it is typically the appraised value. There can be considerable variation in the estimates of the property's appraised value. Thus, analysts tend to be skeptical about estimates of appraised value and the resulting LTVs reported for properties.

B. Basic CMBS Structure

As with any structured finance transaction, a rating agency will determine the necessary level of credit enhancement to achieve a desired rating level. For example, if certain DSC and LTV ratios are needed, and these ratios cannot be met at the loan level, then "subordination" is used to achieve these levels. By subordination it is meant that there will be bond classes in the structure whose claims on the cash flow of the collateral are subordinated to that of other bond classes in the structure.

The rating agencies will require that the CMBS transaction be retired sequentially, with the highest-rated bonds paying off first. Therefore, any return of principal caused by amortization, prepayment, or default will be used to repay the highest-rated tranche.

Interest on principal outstanding will be paid to all tranches. In the event of a delinquency resulting in insufficient cash to make all scheduled payments, the transaction's servicer will advance both principal and interest. Advancing will continue from the servicer for as long as these amounts are deemed recoverable.

Losses arising from loan defaults will be charged against the principal balance of the lowest-rated CMBS tranche outstanding. The total loss charged will include the amount previously advanced as well as the actual loss incurred in the sale of the loan's underlying property.

1. Call Protection A critical investment feature that distinguishes residential MBS and commercial MBS is the call protection afforded an investor. An investor in a residential MBS is exposed to considerable prepayment risk because the borrower has the right to prepay a loan, in whole or in part, before the scheduled principal repayment date. Typically, the borrower does not pay any penalty for prepayment. When we discussed CMOs, we saw how certain types of tranches (e.g., sequential-pay and PAC tranches) can be purchased by an investor to reduce prepayment risk.

With CMBS, there is considerable call protection afforded investors. In fact, it is this protection that results in CMBS trading in the market more like corporate bonds than residential MBS. This call protection comes in two forms: (1) call protection at the loan level and (2) call protection at the structure level. We discuss both below.

a. Protection at the Loan Level At the commercial loan level, call protection can take the following forms:

1. prepayment lockout
2. defeasance
3. prepayment penalty points
4. yield maintenance charges

A **prepayment lockout** is a contractual agreement that prohibits any prepayments during a specified period of time, called the lockout period. The lockout period at issuance can be from 2 to 5 years. After the lockout period, call protection comes in the form of either prepayment penalty points or yield maintenance charges. Prepayment lockout and defeasance are the strongest forms of prepayment protection.

With **defeasance**, rather than loan prepayment, the borrower provides sufficient funds for the servicer to invest in a portfolio of Treasury securities that replicates the cash flows that would exist in the absence of prepayments. Unlike the other call protection provisions discussed next, there is no distribution made to the bondholders when the defeasance takes place. So, since there are no penalties, there is no issue as to how any penalties paid by the borrower are to be distributed amongst the bondholders in a CMBS structure. Moreover, the substitution of the cash flow of a Treasury portfolio for that of the borrower improves the credit quality of the CMBS deal.

Prepayment penalty points are predetermined penalties that must be paid by the borrower if the borrower wishes to refinance. (A point is equal to 1% of the outstanding loan balance.) For example, 5-4-3-2-1 is a common prepayment penalty point structure. That is, if the borrower wishes to prepay during the first year, the borrower must pay a 5% penalty for a total of \$105 rather than \$100 (which is the norm in the residential market). Likewise, during the second year, a 4% penalty would apply, and so on.

When there are prepayment penalty points, there are rules for distributing the penalty among the tranches. Prepayment penalty points are not common in new CMBS structures. Instead, the next form of call protection discussed, yield maintenance charges, is more commonly used.

Yield maintenance charge, in its simplest terms, is designed to make the lender indifferent as to the timing of prepayments. The yield maintenance charge, also called the **make-whole charge**, makes it uneconomical to refinance solely to get a lower mortgage rate. While there are several methods used in practice for calculating the yield maintenance charge, the key principle is to make the lender whole. However, when a commercial loan is included as part of a CMBS deal, there must be an allocation of the yield maintenance charge amongst the tranches. Several methods are used in practice for distributing the yield maintenance charge and, depending on the method specified in a deal, not all tranches may be made whole.

b. Structural Protection The other type of call protection available in CMBS transactions is structural. Because the CMBS bond structures are sequential-pay (by rating), the AA-rated tranche cannot pay down until the AAA is completely retired, and the AA-rated bonds must be paid off before the A-rated bonds, and so on. However, principal losses due to defaults are impacted from the bottom of the structure upward.

2. Balloon Maturity Provisions Many commercial loans backing CMBS transactions are balloon loans that require substantial principal payment at the end of the term of the loan. If the borrower fails to make the balloon payment, the borrower is in default. The lender may extend the loan, and in so doing may modify the original loan terms. During the workout period for the loan, a higher interest rate will be charged, called the default interest rate.

The risk that a borrower will not be able to make the balloon payment because either the borrower cannot arrange for refinancing at the balloon payment date or cannot sell the property to generate sufficient funds to pay off the balloon balance is called **balloon risk**. Since the term of the loan will be extended by the lender during the workout period, balloon risk is a type of “extension risk.” This is the same risk that we referred to earlier in describing residential mortgage-backed securities.

Although many investors like the “bullet bond-like” pay down of the balloon maturities, it does present difficulties from a structural standpoint. That is, if the deal is structured to completely pay down on a specified date, an event of default will occur if any delays occur. However, how such delays impact CMBS investors is dependent on the bond type (premium, par, or discount) and whether the servicer will advance to a particular tranche after the balloon default.

Another concern for CMBS investors in multitranche transactions is the fact that all loans must be refinanced to pay off the most senior bondholders. Therefore, the balloon risk of the most senior tranche (i.e., AAA) may be equivalent to that of the most junior tranche (i.e., B).

CHAPTER 11

ASSET-BACKED SECTOR OF THE BOND MARKET

I. INTRODUCTION

As an alternative to the issuance of a bond, a corporation can issue a security backed by loans or receivables. Debt instruments that have as their collateral loans or receivables are referred to as asset-backed securities. The transaction in which asset-backed securities are created is referred to as a **securitization**.

While the major issuers of asset-backed securities are corporations, municipal governments use this form of financing rather than issuing municipal bonds and several European central governments use this form of financing. In the United States, the first type of asset-backed security (ABS) was the residential mortgage loan. We discussed the resulting securities, referred to as mortgage-backed securities, in the previous chapter. Securities backed by other types of assets (consumer and business loans and receivables) have been issued throughout the world. The largest sectors of the asset-backed securities market in the United States are securities backed by credit card receivables, auto loans, home equity loans, manufactured housing loans, student loans, Small Business Administration loans, corporate loans, and bonds (corporate, emerging market, and structured financial products). Since home equity loans and manufactured housing loans are backed by real estate property, the securities backed by them are referred to as **real estate-backed asset-backed securities**. Other asset-backed securities include securities backed by home improvement loans, health care receivables, agricultural equipment loans, equipment leases, music royalty receivables, movie royalty receivables, and municipal parking ticket receivables. Collectively, these products are called **credit-sensitive structured products**.

In this chapter, we will discuss the securitization process, the basic features of a securitization transaction, and the major asset types that have been securitized. In the last section of this chapter, we look at collateralized debt obligations. While this product has traditionally been classified as part of the ABS market, we will see how the structure of this product differs from that of a typical securitization.

There are two topics not covered in this chapter. The first is the valuation of an ABS. This topic is covered in Chapter 12. Second, the factors considered by rating agencies in rating an ABS transaction are not covered here but are covered in Chapter 15. In that chapter we also compare the factors considered by rating agencies in rating an asset-backed security and a corporate bond.

II. THE SECURITIZATION PROCESS AND FEATURES OF ABS

The issuance of an asset-backed security is more complicated than the issuance of a corporate bond. In this section, we will describe the securitization process and the parties to a securitization. We will do so using a hypothetical securitization.

A. The Basic Securitization Transaction

Quality Home Theaters Inc. (QHT) manufactures high-end equipment for home theaters. The cost of one of QHT's home theaters ranges from \$20,000 to \$200,000. Some of its sales are for cash, but the bulk of its sales are by installment sales contracts. Effectively, an installment sales contract is a loan to the buyer of the home theater who agrees to repay QHT over a specified period of time. For simplicity we will assume that the loans are typically for four years. The collateral for the loan is the home theater purchased by the borrower. The loan specifies an interest rate that the buyer pays.

The credit department of QHT makes the decision as to whether or not to extend credit to a customer. That is, the credit department will request a credit loan application form be completed by a customer and based on criteria established by QHT will decide on whether to extend a loan. The criteria for extending credit are referred to as **underwriting standards**. Because QHT is extending the loan, it is referred to as the **originator** of the loan. Moreover, QHT may have a department that is responsible for servicing the loan. **Servicing** involves collecting payments from borrowers, notifying borrowers who may be delinquent, and, when necessary, recovering and disposing of the collateral (i.e., home theater equipment in our illustration) if the borrower does not make loan repayments by a specified time. While the servicer of the loans need not be the originator of the loans, in our illustration we are assuming that QHT will be the servicer.

Now let's see how these loans can be used in a securitization. We will assume that QHT has \$100 million of installment sales contracts. This amount is shown on QHT's balance sheet as an asset. We will further assume that QHT wants to raise \$100 million. Rather than issuing corporate bonds for \$100 million, QHT's treasurer decides to raise the funds via a securitization. To do so, QHT will set up a legal entity referred to as a **special purpose vehicle** (SPV). In our discussion of asset-backed securities we described the critical role of this legal entity; its role will become clearer in our illustration. In our illustration, the SPV that is set up is called *Homeview Asset Trust* (HAT). QHT will then sell to HAT \$100 million of the loans. QHT will receive from HAT \$100 million in cash, the amount it wanted to raise. But where does HAT get \$100 million? It obtains those funds by selling securities that are backed by the \$100 million of loans. These securities are the asset-backed securities we referred to earlier and we will discuss these further in Section II.C.

In the prospectus, HAT (the SPV) would be referred to as either the "issuer" or the "trust." QHT, the seller of the collateral to HAT, would be referred to as the "seller." The prospectus might then state: "The securities represent obligations of the issuer only and do not represent obligations of or interests in Quality Home Theaters Inc. or any of its affiliates."

The transaction is diagramed in panel a of Exhibit 1. In panel b, the parties to the transaction are summarized.

The payments that are received from the collateral are distributed to pay servicing fees, other administrative fees, and principal and interest to the security holders. The legal documents in a securitization (prospectus or private placement memorandum) will set forth

in considerable detail the priority and amount of payments to be made to the servicer, administrators, and the security holders of each bond class. The priority and amount of payments is commonly referred to as the “waterfall” because the flow of payments in a structure is depicted as a waterfall.

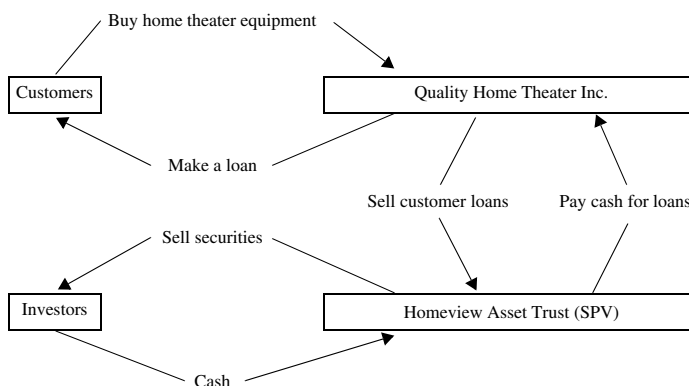
B. Parties to a Securitization

Thus far we have discussed three parties to a securitization: the seller of the collateral (also sometimes referred to as the originator), the special purpose vehicle (referred to in a prospectus or private placement memorandum as the issuer or the trust), and the servicer. There are other parties involved in a securitization: attorneys, independent accountants, trustees, underwriters, rating agencies, and guarantors. All of these parties plus the servicer are referred to as “third parties” to the transaction.

There is a good deal of legal documentation involved in a securitization transaction. The attorneys are responsible for preparing the legal documents. The first is the *purchase agreement* between the seller of the assets (QHT in our illustration) and the SPV (HAT in our illustration).¹ The purchase agreement sets forth the representations and warranties that the seller is making about the assets. The second is one that sets forth how the cash flows

EXHIBIT 1 Securitization Illustration for QHT

Panel a: Securitization Process



Panel b: Parties to the Securitization

Party	Description	Party in illustration
Seller	Originates the loans and sells loans to the SPV	Quality Home Theaters Inc.
Issuer/Trust	The SPV that buys the loans from the seller and issues the asset-backed securities	Homeview Asset Trust
Servicer	Serves the loans	Quality Home Theaters Inc.

¹There are concerns that both the creditors to the seller of the collateral (QHT’s creditors in our illustration) and the investors in the securities issued by the SPV have about the assets. Specifically, QHT’s creditors will be concerned that the assets are being sold to the SPV at less than fair market value, thereby weakening their credit position. The buyers of the asset-backed securities will be concerned that the assets were purchased at less than fair market value, thereby weakening their credit position. Because of this concern, the attorney will issue an opinion that the assets were sold at a fair market value.

are divided among the bond classes (i.e., the structure's waterfall). Finally, the attorneys create the *servicing agreement* between the entity engaged to service the assets (in our illustration QHT retained the servicing of the loans) and the SPV.

An independent accounting firm will verify the accuracy of all numerical information placed in either the prospectus or private placement memorandum.² The result of this task results in a *comfort letter* for a securitization.

The **trustee** or **trustee agent** is the entity that safeguards the assets after they have been placed in the trust, receives the payments due to the bond holders, and provides periodic information to the bond holders. The information is provided in the form of *remittance reports* that may be issued monthly, quarterly or whenever agreed to by the terms of the prospectus or the private placement memorandum.

The underwriters and rating agencies perform the same function in a securitization as they do in a standard corporate bond offering. The rating agencies make an assessment of the collateral and the proposed structure to determine the amount of credit enhancement required to achieve a target credit rating for each bond class.

Finally, a securitization may have an entity that guarantees part of the obligations issued by the SPV. These entities are called guarantors and we will discuss their role in a securitization later.

C. Bonds Issued

Now let's take a closer look at the securities issued, what we refer to as the asset-backed securities.

A simple transaction can involve the sale of just one bond class with a par value of \$100 million in our illustration. We will call this Bond Class A. Suppose HAT issues 100,000 certificates for Bond Class A with a par value of \$1,000 per certificate. Then, each certificate holder would be entitled to 1/100,000 of the payment from the collateral after payment of fees and expenses. Each payment made by the borrowers (i.e., the buyers of the home theater equipment) consists of principal repayment and interest.

A structure can be more complicated. For example, there can be rules for distribution of principal and interest other than on a pro rata basis to different bond classes. As an example, suppose HAT issues Bond Classes A1, A2, A3, and A4 whose total par value is \$100 million as follows:

Bond class	Par value (million)
A1	\$40
A2	30
A3	20
A4	10
Total	\$100

As with a collateralized mortgage obligation (CMO) structure described in the previous chapter, there are different rules for the distribution of principal and interest to these four

²The way this is accomplished is that a copy of the transaction's payment structure, underlying collateral, average life, and yield are supplied to the accountants for verification. In turn, the accountants reverse engineer the deal according to the deal's payment rules (i.e., the waterfall). Following the rules and using the same collateral that will actually generate the cash flows for the transaction, the accountants reproduce the yield and average life tables that are put into the prospectus or private placement memorandum.

bond classes or tranches. A simple structure would be a sequential-pay one. As explained in the previous chapter, in a basic sequential-pay structure, each bond class receives periodic interest. However, the principal is repaid as follows: all principal received from the collateral is paid first to Bond Class A1 until it is fully paid off its \$40 million par value. After Bond Class A1 is paid off, all principal received from the collateral is paid to Bond Class A2 until it is fully paid off. All principal payments from the collateral are then paid to Bond Class A3 until it is fully paid off and then all principal payments are made to Bond Class A4.

The reason for the creation of the structure just described, as explained in the previous chapter, is to redistribute the prepayment risk among different bond classes. Prepayment risk is the uncertainty about the cash flow due to prepayments. This risk can be decomposed into contraction risk (i.e., the undesired shortening in the average life of a security) or extension risk (i.e., the undesired lengthening in the average life of a security). The creation of these bond classes is referred to as **prepayment tranching** or **time tranching**.

Now let's look at a more common structure in a transaction. As will be explained later, there are structures where there is more than one bond class and the bond classes differ as to how they will share any losses resulting from defaults of the borrowers. In such a structure, the bond classes are classified as **senior bond classes** and **subordinate bond classes**. This structure is called a **senior-subordinate structure**. Losses are realized by the subordinate bond classes before there are any losses realized by the senior bond classes. For example, suppose that HAT issued \$90 million par value of Bond Class A, the senior bond class, and \$10 million par value of Bond Class B, the subordinate bond class. So the structure is as follows:

Bond class	Par value (million)
A (senior)	\$90
B (subordinate)	10
Total	\$100

In this structure, as long as there are no defaults by the borrower greater than \$10 million, then Bond Class A will be repaid fully its \$90 million.

The purpose of this structure is to redistribute the credit risk associated with the collateral. This is referred to as **credit tranching**. As explained later, the senior-subordinate structure is a form of credit enhancement for a transaction.

There is no reason why only one subordinate bond class is created. Suppose that HAT issued the following structure

Bond class	Par value (million)
A (senior)	\$90
B (subordinate)	7
C (subordinate)	3
Total	\$100

In this structure, Bond Class A is the senior bond class while both Bond Classes B and C are subordinate bond classes from the perspective of Bond Class A. The rules for the distribution of losses would be as follows. All losses on the collateral are absorbed by Bond Class C before any losses are realized by Bond Classes A or B. Consequently, if the losses on the collateral do not exceed \$3 million, no losses will be realized by Bond Classes A and B. If the losses exceed \$3 million, Bond Class B absorbs the loss up to \$7 million (its par value). As an example, if the total loss on the collateral is \$8 million, Bond Class C loses its entire par value (\$3 million) and Bond Class B realizes a loss of \$5 million of its \$7 million par value. Bond Class

A does not realize any loss in this scenario. It should be clear that Bond Class A only realizes a loss if the loss from the collateral exceeds \$10 million. The bond class that must absorb the losses first is referred to as the **first loss piece**. In our hypothetical structure, Bond Class C is the first loss piece.

Now we will add just one more twist to the structure. Often in larger transactions, the senior bond class will be carved into different bond classes in order to redistribute the prepayment risk. For example, HAT might issue the following structure:

Bond class	Par value (million)
A1 (senior)	\$35
A2 (senior)	28
A3 (senior)	15
A4 (senior)	12
B (subordinate)	7
C (subordinate)	3
Total	\$100

In this structure there is both prepayment tranching for the senior bond class (creation of Bond Classes A1, A2, A3, and A4) and credit tranching (creation of the senior bond classes and the two subordinate bond classes, Bond Classes B and C).

As explained in the previous chapter, a bond class in a securitization is also referred to as a “tranche.” Consequently, throughout this chapter the terms “bond class” and “tranche” are used interchangeably.

D. General Classification of Collateral and Transaction Structure

Later in this chapter, we will describe some of the major assets that have been securitized. In general, the collateral can be classified as either amortizing or non-amortizing assets. **Amortizing assets** are loans in which the borrower’s periodic payment consists of scheduled principal and interest payments over the life of the loan. The schedule for the repayment of the principal is called an **amortization schedule**. The standard residential mortgage loan falls into this category. Auto loans and certain types of home equity loans (specifically, closed-end home equity loans discussed later in this chapter) are amortizing assets. Any excess payment over the scheduled principal payment is called a **prepayment**. Prepayments can be made to pay off the entire balance or a partial prepayment, called a **curtailment**.

In contrast to amortizing assets, non-amortizing assets require only minimum periodic payments with no scheduled principal repayment. If that payment is less than the interest on the outstanding loan balance, the shortfall is added to the outstanding loan balance. If the periodic payment is greater than the interest on the outstanding loan balance, then the difference is applied to the reduction of the outstanding loan balance. Since there is no schedule of principal payments (i.e., no amortization schedule) for a non-amortizing asset, the concept of a prepayment does not apply. A credit card receivable is an example of a non-amortizing asset.

The type of collateral—amortizing or non-amortizing—has an impact on the structure of the transaction. Typically, when amortizing assets are securitized, there is no change in the composition of the collateral over the life of the securities except for loans that have been removed due to defaults and full principal repayment due to prepayments or full amortization. For example, if at the time of issuance the collateral for an ABS consists of 3,000 four-year amortizing loans, then the same 3,000 loans will be in the collateral six months from now

assuming no defaults and no prepayments. If, however, during the first six months, 200 of the loans prepay and 100 have defaulted, then the collateral at the end of six months will consist of 2,700 loans ($3,000 - 200 - 100$). Of course, the remaining principal of the 2,700 loans will decline because of scheduled principal repayments and any partial prepayments. All of the principal repayments from the collateral will be distributed to the security holders.

In contrast, for an ABS transaction backed by non-amortizing assets, the composition of the collateral changes. The funds available to pay the security holders are principal repayments and interest. The interest is distributed to the security holders. However, the principal repayments can be either (1) paid out to security holders or (2) reinvested by purchasing additional loans. What will happen to the principal repayments depends on the time since the transaction was originated. For a certain amount of time after issuance, all principal repayments are reinvested in additional loans. The period of time for which principal repayments are reinvested rather than paid out to the security holders is called the **lockout period** or **revolving period**. At the end of the lockout period, principal repayments are distributed to the security holders. The period when the principal repayments are not reinvested is called the **principal amortization period**. Notice that unlike the typical transaction that is backed by amortizing assets, the collateral backed by non-amortizing assets changes over time. A structure in which the principal repayments are reinvested in new loans is called a **revolving structure**.

While the receivables in a revolving structure may not be prepaid, all the bonds issued by the trust may be retired early if certain events occur. That is, during the lockout period, the trustee is required to use principal repayments to retire the securities rather than reinvest principal in new collateral if certain events occur. The most common trigger is the poor performance of the collateral. This provision that specifies the redirection of the principal repayments during the lockout period to retire the securities is referred to as the **early amortization provision** or **rapid amortization provision**.

Not all transactions that are revolving structures are backed by non-amortizing assets. There are some transactions in which the collateral consists of amortizing assets but during a lockout period, the principal repayments are reinvested in additional loans. For example, there are transactions in the European market in which the collateral consists of residential mortgage loans but during the lockout period principal repayments are used to acquire additional residential mortgage loans.

E. Collateral Cash Flow

For an amortizing asset, projection of the cash flows requires projecting prepayments. One factor that may affect prepayments is the prevailing level of interest rates relative to the interest rate on the loan. In projecting prepayments it is critical to determine the extent to which borrowers take advantage of a decline in interest rates below the loan rate in order to refinance the loan.

As with nonagency mortgage-backed securities, described in the previous chapter, modeling defaults for the collateral is critical in estimating the cash flows of an asset-backed security. Proceeds that are recovered in the event of a default of a loan prior to the scheduled principal repayment date of an amortizing asset represent a prepayment and are referred to as an **involuntary prepayment**. Projecting prepayments for amortizing assets requires an assumption about the default rate and the recovery rate. For a non-amortizing asset, while the concept of a prepayment does not exist, a projection of defaults is still necessary to project how much will be recovered and when.

The analysis of prepayments can be performed on a pool level or a loan level. In **pool-level analysis** it is assumed that all loans comprising the collateral are identical. For an amortizing

asset, the amortization schedule is based on the **gross weighted average coupon** (GWAC) and **weighted average maturity** (WAM) for that single loan. We explained in the previous chapter what the WAC and WAM of a pool of mortgage loans is and illustrated how it is computed. In this chapter, we refer to the WAC as gross WAC. Pool-level analysis is appropriate where the underlying loans are homogeneous. **Loan-level analysis** involves amortizing each loan (or group of homogeneous loans).

The **expected final maturity** of an asset-backed security is the maturity date based on expected prepayments at the time of pricing of a deal. The **legal final maturity** can be two or more years after the expected final maturity. The **average life**, or **weighted average life**, was explained in the previous chapter.

Also explained in the previous chapter is a tranche's **principal window** which refers to the time period over which the principal is expected to be paid to the bondholders. A principal window can be wide or narrow. When there is only one principal payment that is scheduled to be made to a bondholder, the bond is referred to as having a **bullet maturity**. Due to prepayments, an asset-backed security that is expected to have a bullet maturity may have an actual maturity that differs from that specified in the prospectus. Hence, asset-backed securities bonds that have an expected payment of only one principal are said to have a **soft bullet**.

F. Credit Enhancements

All asset-backed securities are credit enhanced. That means that support is provided for one or more of the bondholders in the structure. Credit enhancement levels are determined relative to a specific rating desired by the issuer for a security by each rating agency. Specifically, an investor in a triple A rated security expects to have “minimal” (virtually no) chance of losing any principal due to defaults. For example, a rating agency may require credit enhancement equal to four times expected losses to obtain a triple A rating or three times expected losses to obtain a double A rating. The amount of credit enhancement necessary depends on rating agency requirements.

There are two general types of credit enhancement structures: external and internal. We describe each type below.

1. External Credit Enhancements In an ABS, there are two principal parties: the issuer and the security holder. The issuer in our hypothetical securitization is HAT. If another entity is introduced into the structure to guarantee any payments to the security holders, that entity is referred to as a “third party.”

The most common third party in a securitization is a monoline insurance company (also referred to as a monoline insurer). A monoline insurance company is an insurance company whose business is restricted to providing guarantees for financial products such as municipal securities and asset-backed securities.³ When a securitization has external credit enhancement that is provided by a monoline insurer, the securities are said to be “wrapped.” The insurance works as follows. The monoline insurer agrees to make timely payment of interest and principal up to a specified amount should the issuer fail to make the payment. Unlike municipal bond insurance which guarantees the entire principal amount, the guarantee in a securitization is

³The major monoline insurance companies in the United States are Capital Markets Assurance Corporation (CapMAC), Financial Security Assurance Inc. (FSA), Financial Guaranty Insurance Corporation (FGIC), and Municipal Bond Investors Assurance Corporation (MBIA).

only for a percentage of the par value at origination. For example, a \$100 million securitization may have only \$5 million guaranteed by the monoline insurer.

Two less common forms of external credit enhancement are a letter of credit from a bank and a guarantee by the seller of the assets (i.e., the entity that sold the assets to the SPV—QHT in our hypothetical illustration).⁴ The reason why these two forms of credit enhancement are less commonly used is because of the “weak link approach” employed by rating agencies when they rate securitizations. According to this approach, when rating a proposed structure, the credit quality of a security is only as good as the weakest link in its credit enhancement regardless of the quality of underlying assets. Consequently, if an issuer seeks a triple A rating for one of the bond classes in the structure, it would be unlikely to be awarded such a rating if the external credit enhancer has a rating that is less than triple A. Since few corporations and banks that issue letters of credit have a sufficiently high rating themselves to achieve the rating that may be sought in a securitization, these two forms of external credit enhancement are not as common as insurance.

There is credit risk in a securitization when there is a third-party guarantee because the downgrading of the third party *could* result in the downgrading of the securities in a structure.

2. Internal Credit Enhancements **Internal credit enhancements** come in more complicated forms than external credit enhancements. The most common forms of internal credit enhancement are reserve funds, overcollateralization, and senior/subordinate structures.

a. Reserve Funds **Reserve funds** come in two forms:

- cash reserve funds
- excess spread accounts

Cash reserve funds are straight deposits of cash generated from issuance proceeds. In this case, part of the underwriting profits from the deal are deposited into a fund which typically invests in money market instruments. Cash reserve funds are typically used in conjunction with external credit enhancements.

Excess spread accounts involve the allocation of excess spread or cash into a separate reserve account after paying out the net coupon, servicing fee, and all other expenses on a monthly basis. The excess spread is a design feature of the structure. For example, suppose that:

1. gross weighted average coupon (gross WAC) is 8.00%—this is the interest rate paid by the borrowers
2. servicing and other fees are 0.25%
3. net weighted average coupon (net WAC) is 7.25%—this is the rate that is paid to all the tranches in the structure

So, for this hypothetical deal, 8.00% is available to make payments to the tranches, to cover servicing fees, and to cover other fees. Of that amount, 0.25% is paid for servicing and other fees and 7.25% is paid to the tranches. This means that only 7.50% must be paid out, leaving 0.50% (8.00% – 7.50%). This 0.50% or 50 basis points is called the **excess spread**. This amount is placed in a reserve account—the excess servicing account—and it will gradually increase and can be used to pay for possible future losses.

⁴As noted earlier, the seller is not a party to the transaction once the assets are sold to the SPV who then issues the securities. Hence, if the seller provides a guarantee, it is viewed as a third-party guarantee.

b. Overcollateralization **Overcollateralization** in a structure refers to a situation in which the value of the collateral exceeds the amount of the par value of the outstanding securities issued by the SPV. For example, if \$100 million par value of securities are issued and at issuance the collateral has a market value of \$105, there is \$5 million in overcollateralization. Over time, the amount of overcollateralization changes due to (1) defaults, (2) amortization, and (3) prepayments. For example, suppose that two years after issuance, the par value of the securities outstanding is \$90 million and the value of the collateral at the time is \$93 million. As a result, the overcollateralization is \$3 million (\$93 million – \$90 million).

Overcollateralization represents a form of internal credit enhancement because it can be used to absorb losses. For example, if the liability of the structure (i.e., par value of all the bond classes) is \$100 million and the collateral's value is \$105 million, then the first \$5 million of losses will not result in a loss to any of the bond classes in the structure.

c. Senior-Subordinate Structure Earlier in this section we explained a senior-subordinate structure in describing the bonds that can be issued in a securitization. We explained that there are senior bond classes and subordinate bond classes. The subordinate bond classes are also referred to as junior bond classes or non-senior bond classes.

As explained earlier, the creation of a senior-subordinate structure is done to provide credit tranching. More specifically, the senior-subordinate structure is a form of internal credit enhancement because the subordinate bond classes provide credit support for the senior bond classes. To understand why, the hypothetical HAT structure with one subordinate bond class that was described earlier is reproduced below:

Bond class	Par value (million)
A (senior)	\$90
B (subordinate)	10
Total	\$100

The senior bond class, A, is credit enhanced because the first \$10 million in losses is absorbed by the subordinate bond class, B. Consequently, if defaults do not exceed \$10 million, then the senior bond will receive the entire par value of \$90 million.

Note that one subordinate bond class can provide credit enhancement for another subordinate bond class. To see this, consider the hypothetical HAT structure with two subordinate bond classes presented earlier:

Bond class	Par value (million)
A (senior)	\$90
B (subordinate)	7
C (subordinate)	3
Total	\$100

Bond Class C, the first loss piece, provides credit enhancement for not only the senior bond class, but also the subordinate bond class B.

The basic concern in the senior-subordinate structure is that while the subordinate bond classes provide a certain level of credit protection for the senior bond class at the closing of the deal, the level of protection changes over time due to prepayments. Faster prepayments can remove the desired credit protection. Thus, the objective after the deal closes is to distribute any prepayments such that the credit protection for the senior bond class does not deteriorate over time.

In real-estate related asset-backed securities, as well as nonagency mortgage-backed securities, the solution to the credit protection problem is a well developed mechanism called the **shifting interest mechanism**. Here is how it works. The percentage of the mortgage balance of the subordinate bond class to that of the mortgage balance for the entire deal is called the **level of subordination** or the **subordinate interest**. The higher the percentage, the greater the level of protection for the senior bond classes. The subordinate interest changes after the deal is closed due to prepayments. That is, the subordinate interest shifts (hence the term “shifting interest”). The purpose of a shifting interest mechanism is to allocate prepayments so that the subordinate interest is maintained at an acceptable level to protect the senior bond class. In effect, by paying down the senior bond class more quickly, the amount of subordination is maintained at the desired level.

The prospectus will provide the shifting interest percentage schedule for calculating the **senior prepayment percentage** (the percentage of prepayments paid to the senior bond class). For mortgage loans, a commonly used shifting interest percentage schedule is as follows:

Year after issuance	Senior prepayment percentage
1–5	100%
6	70
7	60
8	40
9	20
after year 9	0

So, for example, if prepayments in month 20 are \$1 million, the amount paid to the senior bond class is \$1 million and no prepayments are made to the subordinated bond classes. If prepayments in month 90 (in the seventh year after issuance) are \$1 million, the senior bond class is paid \$600,000 ($60\% \times \$1$ million).

The shifting interest percentage schedule given in the prospectus is the “base” schedule. The set of shifting interest percentages can change over time depending on the performance of the collateral. If the performance is such that the credit protection for the senior bond class has deteriorated because credit losses have reduced the subordinate bond classes, the base shifting interest percentages are overridden and a higher allocation of prepayments is made to the senior bond class.

Performance analysis of the collateral is undertaken by the trustee for determining whether or not to override the base schedule. The performance analysis is in terms of tests and if the collateral fails any of the tests, this will trigger an override of the base schedule.

It is important to understand that the presence of a shifting interest mechanism results in a trade-off between credit risk and contraction risk for the senior bond class. The shifting interest mechanism reduces the credit risk to the senior bond class. However, because the senior bond class receives a larger share of any prepayments, contraction risk increases.

G. Call Provisions

Corporate, federal agency, and municipal bonds may contain a call provision. This provision gives the issuer the right to retire the bond issue prior to the stated maturity date. The issuer motivation for having the provision is to benefit from a decline in interest rates after the bond is issued. Asset-backed securities typically have call provisions. The motivation is twofold. As with other bonds, the issuer (the SPV) will want to take advantage of a decline in interest rates. In

addition, to reduce administrative fees, the trustee may want to call in the issue because the par value of a bond class is small and it is more cost effective to payoff the one or more bond classes.

Typically, for a corporate, federal agency, and municipal bond the trigger event for a call provision is that a specified amount of time has passed.⁵ In the case of asset-backed securities, it is not simply the passage of time whereby the trustee is permitted to exercise any call option. There are trigger events for exercising the call option based on the amount of the issue outstanding.

There are two call provisions where the trigger that grants the trustee to call in the issue is based on a date being reached: (1) call on or after specified date and (2) auction call. A **call on or after specified date** operates just like a standard call provision for corporate, federal agency, and municipal securities: once a specified date is reached, the trustee has the option to call all the outstanding bonds. In an **auction call**, at a certain date a call will be exercised if an auction results in the outstanding collateral being sold at a price greater than its par value. The premium over par value received from the auctioned collateral is retained by the trustee and is eventually distributed to the seller of the assets.

Provisions that allow the trustee to call an issue or a tranche based on the par value outstanding are referred to as **optional clean-up call provisions**. Two examples are (1) percent of collateral call and (2) percent of bond call. In a **percent of collateral call**, the outstanding bonds can be called at par value if the outstanding *collateral's* balance falls below a predetermined percent of the original collateral's balance. This is the most common type of clean-up call provision for amortizing assets and the predetermined level is typically 10%. For example, suppose that the value for the collateral is \$100 million. If there is a percent of collateral call provision with a trigger of 10%, then the trustee can call the entire issue if the value of the call is \$10 million or less. In a **percent of bond call**, the outstanding bonds can be called at par value if the outstanding *bond's* par value relative to the original par value of bonds issued falls below a specified amount.

There is a call option that combines two triggers based on the amount outstanding and date. In a **latter of percent or date call**, the outstanding bonds can be called if either (1) the collateral's outstanding balance reaches a predetermined level before the specified call date or (2) the call date has been reached even if the collateral outstanding is above the predetermined level.

In addition to the above call provisions which permit the trustee to call the bonds, there may be an **insurer call**. Such a call permits the insurer to call the bonds if the collateral's cumulative loss history reaches a predetermined level.

III. HOME EQUITY LOANS

A **home equity loan** (HEL) is a loan backed by residential property. At one time, the loan was typically a second lien on property that was already pledged to secure a first lien. In some cases, the lien was a third lien. In recent years, the character of a home equity loan has changed. Today, a home equity loan is often a first lien on property where the borrower has either an *impaired credit history* and/or the payment-to-income ratio is too high for the loan to qualify as a conforming loan for securitization by Ginnie Mae, Fannie Mae, or Freddie Mac. Typically, the borrower used a home equity loan to consolidate consumer debt using the current home as collateral rather than to obtain funds to purchase a new home.

⁵As explained earlier, the calling of a portion of the issue is permitted to satisfy any sinking fund requirement.

Home equity loans can be either closed end or open end. A **closed-end HEL** is structured the same way as a fully amortizing residential mortgage loan. That is, it has a fixed maturity and the payments are structured to fully amortize the loan by the maturity date. With an **open-end HEL**, the homeowner is given a credit line and can write checks or use a credit card for up to the amount of the credit line. The amount of the credit line depends on the amount of the equity the borrower has in the property. Because home equity loan securitizations are predominately closed-end HELs, our focus in this section is securities backed by them.

There are both fixed-rate and variable-rate closed-end HELs. Typically, variable-rate loans have a reference rate of 6-month LIBOR and have periodic caps and lifetime caps. (A periodic cap limits the change in the mortgage rate from the previous time the mortgage rate was reset; a lifetime cap sets a maximum that the mortgage rate can ever be for the loan.) The cash flow of a pool of closed-end HELs is comprised of interest, regularly scheduled principal repayments, and prepayments, just as with mortgage-backed securities. Thus, it is necessary to have a prepayment model and a default model to forecast cash flows. The prepayment speed is measured in terms of a conditional prepayment rate (CPR).

A. Prepayments

As explained in the previous chapter, in the agency MBS market the PSA prepayment benchmark is used as the base case prepayment assumption in the prospectus. This benchmark assumes that the conditional prepayment rate (CPR) begins at 0.2% in the first month and increases linearly for 30 months to 6% CPR. From month 36 to the last month that the security is expected to be outstanding, the CPR is assumed to be constant at 6%. At the time that the prepayment speed is assumed to be constant, the security is said to be **seasoned**. For the PSA benchmark, a security is assumed to be seasoned in month 36. When the prepayment speed is depicted graphically, the linear increase in the CPR from month 1 to the month when the security is assumed to be seasoned is called the **prepayment ramp**. For the PSA benchmark, the prepayment ramp begins at month 1 and extends to month 30. Speeds that are assumed to be faster or slower than the PSA prepayment benchmark are quoted as a multiple of the base case prepayment speed.

There are differences in the prepayment behavior for home equity loans and agency MBS. Wall Street firms involved in the underwriting and market making of securities backed by HELs have developed prepayment models for these deals. Several firms have found that the key difference between the prepayment behavior of HELs and agency residential mortgages is the important role played by the credit characteristics of the borrower.⁶

Borrower characteristics and the amount of seasoning (i.e., how long the loans have been outstanding) must be kept in mind when trying to assess prepayments for a particular deal. In the prospectus of a HEL, a base case prepayment assumption is made. Rather than use the PSA prepayment benchmark as the base case prepayment speed, issuer's now use a base case prepayment benchmark that is specific to that issuer. The benchmark prepayment speed in the prospectus is called the **prospectus prepayment curve** or PPC. As with the PSA benchmark, faster or slower prepayments speeds are a quoted as a multiple of the PPC. Having an issuer-specific prepayment benchmark is preferred to a generic benchmark such as the PSA

⁶Dale Westhoff and Mark Feldman, "Prepayment Modeling and Valuation of Home Equity Loan Securities," Chapter 18 in Frank J. Fabozzi, Chuck Ramsey, and Michael Marz (eds.), *The Handbook of Nonagency Mortgage-Backed Securities: Second Edition* (New Hope, PA: Frank J. Fabozzi Associates, 2000).

benchmark. The drawback for this improved description of the prepayment characteristics of a pool of mortgage loans is that it makes comparing the prepayment characteristics and investment characteristics of the collateral between issuers and issues (newly issued and seasoned issues) difficult.

Since HEL deals are backed by both fixed-rate and variable-rate loans, a separate PPC is provided for each type of loan. For example, in the prospectus for the Contimortgage Home Equity Loan Trust 1998–2, the base case prepayment assumption for the fixed-rate collateral begins at 4% CPR in month 1 and increases 1.45455% CPR per month until month 12, at which time it is 20% CPR. Thus, the collateral is assumed to be seasoned in 12 months. The prepayment ramp begins in month 1 and ends in month 12. If an investor analyzed the deal based on 200% PPC, this means doubling the CPRs cited and using 12 months for when the collateral seasons. For the variable-rate collateral in the ContiMortgage deal, 100% PPC assumes the collateral is seasoned after 18 months with the CPR in month 1 being 4% and increasing 1.82353% CPR each month. From month 18 on, the CPR is 35%. Thus, the prepayment ramp starts at month 1 and ends at month 18. Notice that for this issuer, the variable-rate collateral is assumed to season slower than the fixed-rate collateral (18 versus 12 months), but has a faster CPR when the pool is seasoned (35% versus 20%).

B. Payment Structure

As with nonagency mortgage-backed securities discussed in the previous chapter, there are passthrough and paythrough home equity loan-backed structures.

Typically, home equity loan-backed securities are securitized by both closed-end fixed-rate and adjustable-rate (or variable-rate) HELs. The securities backed by the latter are called **HEL floaters**. The reference rate of the underlying loans typically is 6-month LIBOR. The cash flow of these loans is affected by periodic and lifetime caps on the loan rate.

Institutional investors that seek securities that better match their floating-rate funding costs are attracted to securities that offer a floating-rate coupon. To increase the attractiveness of home equity loan-backed securities to such investors, the securities typically have been created in which the reference rate is 1-month LIBOR. Because of (1) the mismatch between the reference rate on the underlying loans (6-month LIBOR) and that of the HEL floater and (2) the periodic and life caps of the underlying loans, there is a cap on the coupon rate for the HEL floater. Unlike a typical floater, which has a cap that is fixed throughout the security's life, the effective periodic and lifetime cap of a HEL floater is variable. The effective cap, referred to as the **available funds cap**, will depend on the amount of funds generated by the net coupon on the principal, less any fees.

Let's look at one issue, Advanta Mortgage Loan Trust 1995–2 issued in June 1995. At the offering, this issue had approximately \$122 million closed-end HELs. There were 1,192 HELs consisting of 727 fixed-rate loans and 465 variable-rate loans. There were five classes (A-1, A-2, A-3, A-4, and A-5) and a residual. The five classes are summarized below:

Class	Par amount (\$)	Passthrough coupon rate (%)
A-1	9,229,000	7.30
A-2	30,330,000	6.60
A-3	16,455,000	6.85
A-4	9,081,000	floating rate
A-5	56,917,000	floating rate

The collateral is divided into group I and group II. The 727 fixed-rate loans are included in group I and support Classes A-1, A-2, A-3, and A-4 certificates. The 465 variable-rate loans are in group II and support Class A-5.

Tranches have been structured in home equity loan deals so as to give some senior tranches greater prepayment protection than other senior tranches. The two types of structures that do this are the non-accelerating senior tranche and the planned amortization class tranche.

1. Non-Accelerating Senior Tranches A **non-accelerating senior tranche** (NAS tranche) receives principal payments according to a schedule. The schedule is not a dollar amount. Rather, it is a principal schedule that shows for a given month the share of pro rata principal that must be distributed to the NAS tranche. A typical principal schedule for a NAS tranche is as follows:⁷

Months	Share of pro rata principal
1 through 36	0%
37 through 60	45%
61 through 72	80%
73 through 84	100%
After month 84	300%

The average life for the NAS tranche is stable for a large range of prepayments because for the first three years all prepayments are made to the other senior tranches. This reduces the risk of the NAS tranche contracting (i.e., shortening) due to fast prepayments. After month 84, 300% of its pro rata share is paid to the NAS tranche thereby reducing its extension risk.

The average life stability over a wide range of prepayments is illustrated in Exhibit 2. The deal analyzed is the ContiMortgage Home Equity Loan Trust 1997–2.⁸ Class A-9 is the NAS tranche. The analysis was performed on Bloomberg shortly after the deal was issued using the issue's PPC. As can be seen, the average life is fairly stable between 75% to 200% PPC. In fact, the difference in the average life between 75% PPC and 200% PPC is slightly greater than 1 year.

In contrast, Exhibit 2 also shows the average life over the same prepayment scenarios for a non-NAS sequential-pay tranche in the same deal—Class A-7. Notice the substantial average life variability. While the average life difference between 75% and 200% PPC for the NAS tranche is just over 1 year, it is more than 9 years for the non-NAS tranche. Of course, the non-NAS in the same deal will be less stable than a regular sequential tranche because the non-NAS gets a greater share of principal than it would otherwise.

2. Planned Amortization Class Tranche In our discussion of collateralized mortgage obligations issued by the agencies in the previous chapter we explained how a planned amortization class tranche can be created. These tranches are also created in HEL structures. Unlike agency CMO PAC tranches that are backed by fixed-rate loans, the collateral for HEL deals is both fixed rate and adjustable rate.

An example of a HEL PAC tranche in a HEL-backed deal is tranche A-6 in ContiMortgage 1998–2. We described the PPC for this deal in Section III.A.1 above. There is a separate PAC

⁷Charles Schorin, Steven Weinreich, and Oliver Hsiang, "Home Equity Loan Transaction Structures," Chapter 6 in Frank J. Fabozzi, Chuck Ramsey, and Michael Marz, *Handbook of Nonagency Mortgage-Backed Securities: Second Edition* (New Hope, PA: Frank J. Fabozzi Associates, 2000).

⁸This illustration is from Schorin, Weinreich, and Hsiang, "Home Equity Loan Transaction Structures."

EXHIBIT 2 Average Life for NAS Tranche (Class A-9) and Non-Nas Tranche (Class A-7) for ContiMortgage Home Equity Loan Trust 1997–2 for a Range of Prepayments

	% PPC												Avg life difference 75% to 200% PPC
	0	50	75	100	120	150	200	250	300	350	400	500	
Plateau CPR	0	10	15	20	24	30	40	50	60	70	80	100	
Avg Life													
NAS Bond	11.71	7.81	7.06	6.58	6.30	6.06	5.97	3.98	2.17	1.73	1.38	0.67	1.09
Non-NAS Bond	21.93	14.54	11.94	8.82	6.73	4.71	2.59	1.96	1.55	1.25	1.03	0.58	9.35

Calculation: Bloomberg Financial Markets. Reported in Charles Schorin, Steven Weinreich, and Oliver Hsiang, “Home Equity Loan Transaction Structures,” Chapter 6 in Frank J. Fabozzi, Chuck Ramsey, and Michael Marz, *Handbook of Nonagency Mortgage-Backed Securities: Second Edition* (New Hope, PA: Frank J. Fabozzi Associates, 2000).

collar for both the fixed-rate and adjustable-rate collateral. For the fixed-rate collateral the PAC collar is 125%-175% PPC; for the adjustable-rate collateral the PAC collar is 95%-130% PPC. The average life for tranche A-6 (a tranche backed by the fixed-rate collateral) is 5.1 years. As explained in Chapter 3, the effective collar for shorter tranches can be greater than the upper collar specified in the prospectus. The effective upper collar for tranche A-6 is actually 180% PPC (assuming that the adjustable-rate collateral pays at 100% PPC).⁹

For shorter PACs, the effective upper collar is greater. For example, for tranche A-3 in the same deal, the initial PAC collar is 125% to 175% PPC with an average life of 2.02 years. However, the effective upper collar is 190% PPC (assuming the adjustable-rate collateral pays at 100% PPC).

The effective collar for PAC tranches changes over time based on actual prepayments and therefore based on when the support tranches depart from the initial PAC collar. For example, if for the next 36 months after the issuance of the ContiMortgage 1998–2 actual prepayments are a constant 150% PPC, then the effective collar would be 135% PPC to 210% PPC.¹⁰ That is, the lower and upper collar will increase. If the actual PPC is 200% PPC for the 10 months after issuance, the support bonds will be fully paid off and there will be no PAC collateral. In this situation the PAC is said to be a **broken PAC**.

IV. MANUFACTURED HOUSING-BACKED SECURITIES

Manufactured housing-backed securities are backed by loans for manufactured homes. In contrast to site-built homes, manufactured homes are built at a factory and then transported to a site. The loan may be either a mortgage loan (for both the land and the home) or a consumer retail installment loan.

Manufactured housing-backed securities are issued by Ginnie Mae and private entities. The former securities are guaranteed by the full faith and credit of the U.S. government. The manufactured home loans that are collateral for the securities issued and guaranteed by Ginnie Mae are loans guaranteed by the Federal Housing Administration (FHA) or Veterans Administration (VA).

⁹For a more detailed analysis of this tranche, see Schorin, Weinreich, and Hsiang, “Home Equity Loan Transaction Structures.”

¹⁰Schorin, Weinreich, and Hsiang, “Home Equity Loan Transaction Structures.”

Loans not backed by the FHA or VA are called **conventional loans**. Manufactured housing-backed securities that are backed by such loans are called **conventional manufactured housing-backed securities**. These securities are issued by private entities.

The typical loan for a manufactured home is 15 to 20 years. The loan repayment is structured to fully amortize the amount borrowed. Therefore, as with residential mortgage loans and HELs, the cash flow consists of net interest, regularly scheduled principal, and prepayments. However, prepayments are more stable for manufactured housing-backed securities because they are not sensitive to refinancing.

There are several reasons for this. First, the loan balances are typically small so that there is no significant dollar savings from refinancing. Second, the rate of depreciation of mobile homes may be such that in the earlier years depreciation is greater than the amount of the loan paid off. This makes it difficult to refinance the loan. Finally, typically borrowers are of lower credit quality and therefore find it difficult to obtain funds to refinance.

As with residential mortgage loans and HELs, prepayments on manufactured housing-backed securities are measured in terms of CPR and each issue contains a PPC.

The payment structure is the same as with nonagency mortgage-backed securities and home equity loan-backed securities.

V. RESIDENTIAL MBS OUTSIDE THE UNITED STATES

Throughout the world where the market for securitized assets has developed, the largest sector is the residential mortgage-backed sector. It is not possible to provide a discussion of the residential mortgage-backed securities market in every country. Instead, to provide a flavor for this market sector and the similarities with the U.S. nonagency mortgage-backed securities market, we will discuss just the market in the United Kingdom and Australia.

A. U.K. Residential Mortgage-Backed Securities

In Europe, the country in which there has been the largest amount of issuance of asset-backed securities is the United Kingdom.¹¹ The largest component of that market is the residential mortgage-backed security market which includes “prime” residential mortgage-backed securities and “nonconforming” residential mortgage-backed securities. In the U.S. mortgage market, a nonconforming mortgage loan is one that does not meet the underwriting standards of Ginnie Mae, Fannie Mae, or Freddie Mac. However, this does not mean that the loan has greater credit risk. In contrast, in the U.K. mortgage market, nonconforming mortgage loans are made to borrowers that are viewed as having greater credit risk—those that do not have a credit history and those with a history of failing to meet their obligations.

¹¹Information about the U.K. residential mortgage-backed securities market draws from the following sources: Phil Adams, “UK Residential Mortgage-Backed Securities,” and “UK Non-Conforming Residential Mortgage-Backed Securities,” in *Building Blocks*, Asset-Backed Securities Research, Barclays Capital, January 2001; Christopher Flanagan and Edward Reardon, *European Structured Products: 2001 Review and 2002 Outlook*, Global Structured Finance Research, J.P. Morgan Securities Inc., January 11, 2002; and, “UK Mortgages—MBS Products for U.S. Investors,” *Mortgage Strategist*, UBS Warburg, February 27, 2001, pp. 15–21.

The standard mortgage loan is a variable rate, fully amortizing loan. Typically, the term of the loan is 25 years. As in the U.S. mortgage market, borrowers seeking a loan with a high loan-to-value ratio are required to obtain mortgage insurance, called a “mortgage indemnity guarantee” (MIG).

The deals are more akin to the nonagency market since there is no guarantee by a federally related agency or a government sponsored enterprise as in the United States. Thus, there is credit enhancement as explained below.

Because the underlying mortgage loans are floating rate, the securities issued are floating rate (typically, LIBOR is the reference rate). The cash flow depends on the timing of the principal payments. The deals are typically set up as a sequential-pay structure. For example, consider the Granite Mortgage 00–2 transaction, a typical structure in the United Kingdom.¹² The mortgage pool consists of prime mortgages. There are four bond classes. The two class A tranches, Class A-1 and Class A-2, are rated AAA. One is a dollar denominated tranche and the other a pound sterling tranche. Class B is rated single A and tranche C is rated triple BBB. The sequence of principal payments is as follows: Class A-1 and Class A-2 are paid off on a pro rata basis, then Class B is paid off, and then Class C is paid off.

The issuer has the option to call the outstanding notes under the following circumstances:

- A withholding tax is imposed on the interest payments to noteholders
- a clean up call (if the mortgage pool falls to 10% or less of the original pool amount)
- on a specified date (called the “step up date”) or dates in the future

For example, for the Granite Mortgage 00–02 transaction, the step up date is September 2007. The issuer is likely to call the issue because the coupon rate on the notes increases at that time. In this deal, as with most, the margin over LIBOR doubles.

Credit enhancement can consist of excess spread, reserve fund, and subordination. For the Granite Mortgage 00–2, there was subordination: Class B and C tranches for the two Class A tranche and Class C tranche for the Class B tranche. The reserve was fully funded at the time of issuance and the excess spread was used to build up the reserve fund. In addition, there is a “principal shortfall provision.” This provision requires that if the realized losses for a period are such that the excess reserve for that period is not sufficient to cover the losses, as excess spread becomes available in future periods they are used to cover these losses. Also there are performance triggers that under certain conditions will provide further credit protection to the senior bonds by modifying the payment of principal. When the underlying mortgage pool consists of nonconforming mortgage loans, additional protections are provided for investors.

Since prepayments will reduce the average life of the senior notes in a transaction, typical deals have provision that permit the purchase of substitute mortgages if the prepayment rate exceeds a certain rate. For example, in the Granite Mortgage 00–02 deal, this rate is 20% per annum.

B. Australian Mortgage-Backed Securities

In Australia, lending is dominated by mortgage banks, the larger ones being ANZ, Commonwealth Bank of Australia, National Australia Bank, Westpac, and St. George Bank.¹³

¹²For a more detailed discussion of this structure, see Adams, “UK Residential Mortgage-Backed Securities,” pp. 31–37.

¹³Information about the Australian residential mortgage-backed securities market draws from the following sources: Phil Adams, “Australian Residential Mortgage-Backed Securities,” in *Building Blocks*;

Non-mortgage bank competitors who have entered the market have used securitization as a financing vehicle. The majority of the properties are concentrated in New South Wales, particularly the city of Sydney. Rating agencies have found that the risk of default is considerably less than in the U.S. and the U.K.

Loan maturities are typically between 20 and 30 years. As in the United States, there is a wide range of mortgage designs with respect to interest rates. There are fixed-rate, variable-rate (both capped and uncapped), and rates tied to a benchmark.

There is mortgage insurance for loans to protect lenders, called “lenders mortgage insurance” (LMI). Loans typically have LMI covering 20% to 100% of the loan. The companies that provide this insurance are private corporations.¹⁴ When mortgages loans that do not have LMI are securitized, typically the issuer will purchase insurance for those loans.

LMI is important for securitized transactions since it is the first layer of credit enhancement in a deal structure. The rating agencies recognize this in rating the tranches in a structure. The amount that a rating agency will count toward credit enhancement for LMI depends on the rating agency’s assessment of the mortgage insurance company.

When securitized, the tranches have a floating rate. There is an initial revolving period—which means that no principal payments are made to the tranche holders but instead reinvested in new collateral. As with the U.K. Granite Mortgage 00–02 deal, the issuer has the right to call the issue if there is an imposition of a withholding tax on note holders’ interest payments, after a certain date, or if the balance falls below a certain level (typically, 10%).

Australian mortgage-backed securities have tranches that are U.S. dollar denominated and some that are denominated in Euros.¹⁵ These global deals typically have two or three AAA senior tranches and one AA or AA– junior tranche.

For credit enhancement, there is excess spread (which in most deals is typically small), subordination, and, as noted earlier, LMI. To illustrate this, consider the Interstar Millennium Series 2000–3E Trust—a typical Australian MBS transaction. There are two tranches: a senior tranche (Class A) that was rated AAA and a subordinated tranche (Class B) that was AA–. The protection afforded the senior tranche is the subordinated tranche, LMI (all the properties were covered up to 100% and were insured by all five major mortgage insurance companies), and the excess spread.

VI. AUTO LOAN-BACKED SECURITIES

Auto loan-backed securities represents one of the oldest and most familiar sectors of the asset-backed securities market. Auto loan-backed securities are issued by:

1. the financial subsidiaries of auto manufacturers

Karen Weaver, Eugene Xu, Nicholas Bakalar, and Trudy Weibel, “Mortgage-Backed Securities in Australia,” Chapter 41 in *The Handbook of Mortgage-Backed Securities: Fifth Edition* (New York, NY: McGraw Hill, 2001); and, “Australian Value Down Under,” *Mortgage Strategist*, UBS Warburg, February 6, 2001, pp. 14–22.

¹⁴The five major ones are Royal and Sun Alliance Lenders Mortgage Insurance Limited, CGU Lenders Mortgage Insurance Corporation Ltd., PMI mortgage insurance limited, GE Mortgage Insurance Property Ltd., and GE Mortgage Insurance Corporation.

¹⁵The foreign exchange risk for these deals is typically hedged using various types of swaps (fixed/floating, floating/floating, and currency swaps).

2. commercial banks
3. independent finance companies and small financial institutions specializing in auto loans

Historically, auto loan-backed securities have represented between 18% to 25% of the asset-backed securities market. The auto loan market is tiered based on the credit quality of the borrowers. “Prime auto loans” are of fundamentally high credit quality and originated by the financial subsidiaries of major auto manufacturers. The loans are of high credit quality for the following reasons. First, they are a secured form of lending. Second, they begin to repay principal immediately through amortization. Third, they are short-term in nature. Finally, for the most part, major issuers of auto loans have tended to follow reasonably prudent underwriting standards.

Unlike the sub-prime mortgage industry, there is less consistency on what actually constitutes various categories of prime and sub-prime auto loans. Moody’s assumes the *prime* market is composed of issuers typically having cumulative losses of less than 3%; *near-prime* issuers have cumulative losses of 3–7%; and sub-prime issuers have losses exceeding 7%.

The auto sector was a small part of the European asset-backed securities market in 2002, about 5% of total securitization. There are two reasons for this. First, there is lower per capita car ownership in Europe. Second, there is considerable variance of tax and regulations dealing with borrower privacy rules in Europe thereby making securitization difficult.¹⁶ Auto deals have been done in Italy, the U.K., Germany, Portugal, and Belgium.

A. Cash Flow and Prepayments

The cash flow for auto loan-backed securities consists of regularly scheduled monthly loan payments (interest and scheduled principal repayments) and any prepayments. For securities backed by auto loans, prepayments result from (1) sales and trade-ins requiring full payoff of the loan, (2) repossession and subsequent resale of the automobile, (3) loss or destruction of the vehicle, (4) payoff of the loan with cash to save on the interest cost, and (5) refinancing of the loan at a lower interest cost.

Prepayments due to repossession and subsequent resale are sensitive to the economic cycle. In recessionary economic periods, prepayments due to this factor increase. While refinancings may be a major reason for prepayments of mortgage loans, they are of minor importance for automobile loans. Moreover, the interest rates for the automobile loans underlying some deals are substantially below market rates since they are offered by manufacturers as part of a sales promotion.

B. Measuring Prepayments

For most asset-backed securities where there are prepayments, prepayments are measured in term of the conditional prepayment rate, CPR. As explained in the previous chapter, monthly prepayments are quoted in terms of the single monthly mortality (SMM) rate. The *convention* for calculating and reporting prepayment rates for auto-loan backed securities is different. Prepayments for auto loan-backed securities are measured in terms of the **absolute**

¹⁶Flanagan and Reardon, *European Structures Products: 2001 Review and 2002 Outlook*, p. 9.

prepayment speed, denoted by ABS.¹⁷ The ABS is the monthly prepayment expressed as a percentage of the original collateral amount. As explained in the previous chapter, the SMM (monthly CPR) expresses prepayments based on the prior month's balance.

There is a mathematical relationship between the SMM and the ABS measures. Letting M denote the number of months after loan origination, the SMM rate can be calculated from the ABS rate using the following formula:

$$\text{SMM} = \frac{\text{ABS}}{1 - [\text{ABS} \times (M - 1)]}$$

where the ABS and SMM rates are expressed in decimal form.

For example, if the ABS rate is 1.5% (i.e., 0.015) at month 14 after origination, then the SMM rate is 1.86%, as shown below:

$$\text{SMM} = \frac{0.015}{1 - [0.015 \times (14 - 1)]} = 0.0186 = 1.86\%$$

The ABS rate can be calculated from the SMM rate using the following formula:

$$\text{ABS} = \frac{\text{SMM}}{1 + [\text{SMM} \times (M - 1)]}$$

For example, if the SMM rate at month 9 after origination is 1.3%, then the ABS rate is:

$$\text{ABS} = \frac{0.013}{1 + [0.013 \times (9 - 1)]} = 0.0118 = 1.18\%$$

Historically, when measured in terms of SMM rate, auto loans have experienced SMMs that increase as the loans season.

VII. STUDENT LOAN-BACKED SECURITIES

Student loans are made to cover college cost (undergraduate, graduate, and professional programs such as medical and law school) and tuition for a wide range of vocational and trade schools. Securities backed by student loans, popularly referred to as SLABS (student loan asset-backed securities), have similar structural features as the other asset-backed securities we discussed above.

The student loans that have been most commonly securitized are those that are made under the Federal Family Education Loan Program (FFELP). Under this program, the

¹⁷The only reason for the use of ABS rather than SMM/CPR in this sector is historical. Auto-loan backed securities (which were popularly referred to at one time as CARS (Certificates of Automobile Receivables)) were the first non-mortgage assets to be developed in the market. (The first non-mortgage asset-backed security was actually backed by computer lease receivables.) The major dealer in this market at the time, First Boston (now Credit Suisse First Boston) elected to use ABS for measuring prepayments. You may wonder how one obtains "ABS" from "absolute prepayment rate." Again, it is historical. When the market first started, the ABS measure probably meant "asset-backed security" but over time to avoid confusion evolved to absolute prepayment rate.

government makes loans to students via private lenders. The decision by private lenders to extend a loan to a student is not based on the applicant's ability to repay the loan. If a default of a loan occurs and the loan has been properly serviced, then the government will guarantee up to 98% of the principal plus accrued interest.¹⁸

Loans that are not part of a government guarantee program are called **alternative loans**. These loans are basically consumer loans and the lender's decision to extend an alternative loan will be based on the ability of the applicant to repay the loan. Alternative loans have been securitized.

A. Issuers

Congress created Fannie Mae and Freddie Mac to provide liquidity in the mortgage market by allowing these government sponsored enterprises to buy mortgage loans in the secondary market. Congress created the Student Loan Marketing Association (nicknamed "Sallie Mae") as a government sponsored enterprise to purchase student loans in the secondary market and to securitize pools of student loans. Since its first issuance in 1995, Sallie Mae is now the major issuer of SLABS and its issues are viewed as the benchmark issues.¹⁹ Other entities that issue SLABS are either traditional corporate entities (e.g., the Money Store and PNC Bank) or non-profit organizations (Michigan Higher Education Loan Authority and the California Educational Facilities Authority). The SLABS of the latter typically are issued as tax-exempt securities and therefore trade in the municipal market. In recent years, several not-for-profit entities have changed their charter and applied for "for profit" treatment.

B. Cash Flow

Let's first look at the cash flow for the student loans themselves. There are different types of student loans under the FFELP including subsidized and unsubsidized Stafford loans, Parental Loans for Undergraduate Students (PLUS), and Supplemental Loans to Students (SLS). These loans involve three periods with respect to the borrower's payments—deferment period, grace period, and loan repayment period. Typically, student loans work as follows. While a student is in school, no payments are made by the student on the loan. This is the **deferment period**. Upon leaving school, the student is extended a **grace period** of usually six months when no payments on the loan must be made. After this period, payments are made on the loan by the borrower.

Student loans are floating-rate loans, exclusively indexed to the 3-month Treasury bill rate. As a result, some issuers of SLABS issue securities whose coupon rate is indexed to the 3-month Treasury bill rate. However, a large percentage of SLABS issued are indexed to LIBOR floaters.²⁰

Prepayments typically occur due to defaults or loan consolidation. Even if there is no loss of principal faced by the investor when defaults occur, the investor is still exposed to

¹⁸Actually, depending on the origination date, the guarantee can be up to 100%.

¹⁹In 1997 Sallie Mae began the process of unwinding its status as a GSE; until this multi-year process is completed, all debt issued by Sallie Mae under its GSE status will be "grandfathered" as GSE debt until maturity.

²⁰This creates a mismatch between the collateral and the securities. Issuers have dealt with this by hedging with the risk by using derivative instruments such as interest rate swaps (floating-to-floating rate swaps described in Chapter 14) or interest rate caps (described in Chapter 14).

contraction risk. This is the risk that the investor must reinvest the proceeds at a lower spread and in the case of a bond purchased at a premium, the premium will be lost. Studies have shown student loan prepayments are insensitive to the level of interest rates. Consolidations of a loan occur when the student who has loans over several years combines them into a single loan. The proceeds from the consolidation are distributed to the original lender and, in turn, distributed to the bondholders.

VIII. SBA LOAN-BACKED SECURITIES

The Small Business Administration (SBA) is an agency of the U.S. government empowered to guarantee loans made by approved SBA lenders to qualified borrowers. The loans are backed by the full faith and credit of the government. Most SBA loans are variable-rate loans where the reference rate is the prime rate. The rate on the loan is reset monthly on the first of the month or quarterly on the first of January, April, July, and October. SBA regulations specify the maximum coupon allowable in the secondary market. Newly originated loans have maturities between 5 and 25 years.

The Small Business Secondary Market Improvement Act passed in 1984 permitted the pooling of SBA loans. When pooled, the underlying loans must have similar terms and features. The maturities typically used for pooling loans are 7, 10, 15, 20, and 25 years. Loans without caps are not pooled with loans that have caps.

Most variable-rate SBA loans make monthly payments consisting of interest and principal repayment. The amount of the monthly payment for an individual loan is determined as follows. Given the coupon formula of the prime rate plus the loan's quoted margin, the interest rate is determined for each loan. Given the interest rate, a level payment amortization schedule is determined. It is this level payment that is paid for the next month until the coupon rate is reset.

The monthly cash flow that the investor in an SBA-backed security receives consists of

- the coupon interest based on the coupon rate set for the period
- the scheduled principal repayment (i.e., scheduled amortization)
- prepayments

Prepayments for SBA-backed securities are measured in terms of CPR. Voluntary prepayments can be made by the borrower without any penalty. There are several factors contributing to the prepayment speed of a pool of SBA loans. A factor affecting prepayments is the maturity date of the loan. It has been found that the fastest speeds on SBA loans and pools occur for shorter maturities.²¹ The purpose of the loan also affects prepayments. There are loans for working capital purposes and loans to finance real estate construction or acquisition. It has been observed that SBA pools with maturities of 10 years or less made for working capital purposes tend to prepay at the fastest speed. In contrast, loans backed by real estate that are long maturities tend to prepay at a slow speed.

²¹ Donna Faulk, "SBA Loan-Backed Securities," Chapter 10 in *Asset-Backed Securities*.

IX. CREDIT CARD RECEIVABLE-BACKED SECURITIES

When a purchase is made on a credit card, the issuer of the credit card (the lender) extends credit to the cardholder (the borrower). Credit cards are issued by banks (e.g., Visa and MasterCard), retailers (e.g., Sears and Target Corporation), and travel and entertainment companies (e.g., American Express). At the time of purchase, the cardholder is agreeing to repay the amount borrowed (i.e., the cost of the item purchased) plus any applicable finance charges. The amount that the cardholder has agreed to pay the issuer of the credit card is a receivable from the perspective of the issuer of the credit card. Credit card receivables are used as collateral for the issuance of an asset-backed security.

A. Cash Flow

For a pool of credit card receivables, the cash flow consists of finance charges collected, fees, and principal. Finance charges collected represent the periodic interest the credit card borrower is charged based on the unpaid balance after the grace period. Fees include late payment fees and any annual membership fees.

Interest to security holders is paid periodically (e.g., monthly, quarterly, or semiannually). The interest rate may be fixed or floating—roughly half of the securities are floaters. The floating rate is uncapped.

A credit card receivable-backed security is a nonamortizing security. For a specified period of time, the lockout period or revolving period, the principal payments made by credit card borrowers comprising the pool are retained by the trustee and reinvested in additional receivables to maintain the size of the pool. The lockout period can vary from 18 months to 10 years. So, during the lockout period, the cash flow that is paid out to security holders is based on finance charges collected and fees. After the lockout period, the principal is no longer reinvested but paid to investors, the principal-amortization period and the various types of structures are described next.

B. Payment Structure

There are three different amortization structures that have been used in credit card receivable-backed security deals: (1) passthrough structure, (2) controlled-amortization structure, and (3) bullet-payment structure. The latter two are the more common. One source reports that 80% of the deals are bullet structures and the balance are controlled amortization structures.²²

In a **passthrough structure**, the principal cash flows from the credit card accounts are paid to the security holders on a pro rata basis. In a **controlled-amortization structure**, a scheduled principal amount is established, similar to the principal window for a PAC bond. The scheduled principal amount is sufficiently low so that the obligation can be satisfied even under certain stress scenarios, where cash flow is decreased due to defaults or slower repayment by borrowers. The security holder is paid the lesser of the scheduled principal amount and the pro rata amount. In a **bullet-payment structure**, the security holder receives the entire amount in one distribution. Since there is no assurance that the entire amount can be paid in one lump sum, the procedure is for the trustee to place principal monthly into an

²²Thompson, “MBNA Tests the Waters.”

account that generates sufficient interest to make periodic interest payments and accumulate the principal to be repaid. These deposits are made in the months shortly before the scheduled bullet payment. This type of structure is also often called a **soft bullet** because the maturity is technically not guaranteed, but is almost always satisfied. The time period over which the principal is accumulated is called the **accumulation period**.

C. Performance of the Portfolio of Receivables

There are several concepts that must be understood in order to assess the performance of the portfolio of receivables and the ability of the issuer to meet its interest obligation and repay principal as scheduled.

We begin with the concept of the **gross portfolio yield**. This yield includes finance charges collected and fees. **Charge-offs** represent the accounts charged off as uncollectible. **Net portfolio yield** is equal to gross portfolio yield minus charge-offs. The net portfolio yield is important because it is from this yield that the bondholders will be paid. So, for example, if the average yield (WAC) that must be paid to the various tranches in the structure is 5% and the net portfolio yield for the month is only 4.5%, there is the risk that the bondholder obligations will not be satisfied.

Delinquencies are the percentages of receivables that are past due for a specified number of months, usually 30, 60, and 90 days. They are considered an indicator of potential future charge-offs.

The **monthly payment rate** (MPR) expresses the monthly payment (which includes finance charges, fees, and any principal repayment) of a credit card receivable portfolio as a percentage of credit card debt outstanding in the previous month. For example, suppose a \$500 million credit card receivable portfolio in January realized \$50 million of payments in February. The MPR would then be 10% (\$50 million divided by \$500 million).

There are two reasons why the MPR is important. First, if the MPR reaches an extremely low level, there is a chance that there will be extension risk with respect to the principal payments on the bonds. Second, if the MPR is very low, then there is a chance that there will not be sufficient cash flows to pay off principal. This is one of the events that could trigger early amortization of the principal (described below).

At issuance, portfolio yield, charge-offs, delinquency, and MPR information are provided in the prospectus. Information about portfolio performance is then available from Bloomberg, the rating agencies, and dealers.

D. Early Amortization Triggers

There are provisions in credit card receivable-backed securities that require early amortization of the principal if certain events occur. Such provisions, which as mentioned earlier in this chapter are referred to as early amortization or rapid amortization provisions, are included to safeguard the credit quality of the issue. The only way that the principal cash flows can be altered is by the triggering of the early amortization provision.

Typically, early amortization allows for the rapid return of principal in the event that the 3-month average excess spread earned on the receivables falls to zero or less. When early amortization occurs, the credit card tranches are retired sequentially (i.e., first the AAA bond then the AA rated bond, etc.). This is accomplished by paying the principal payments made by the credit card borrowers to the investors instead of using them to purchase more receivables. The length of time until the return of principal is largely a function of the monthly payment

rate. For example, suppose that a AAA tranche is 82% of the overall deal. If the monthly payment rate is 11% then the AAA tranche would return principal over a 7.5-month period (82%/11%). An 18% monthly payment rate would return principal over a 4.5-month period (82%/18%).

X. COLLATERALIZED DEBT OBLIGATIONS

A **collateralized debt obligation** (CDO) is a security backed by a diversified pool of one or more of the following types of debt obligations:

- U.S. domestic high-yield corporate bonds
- structured financial products (i.e., mortgage-backed and asset-backed securities)
- emerging market bonds
- bank loans
- special situation loans and distressed debt

When the underlying pool of debt obligations are bond-type instruments (high-yield corporate, structured financial products, and emerging market bonds), a CDO is referred to as a **collateralized bond obligation** (CBO). When the underlying pool of debt obligations are bank loans, a CDO is referred to as a **collateralized loan obligation** (CLO).

A. Structure of a CDO

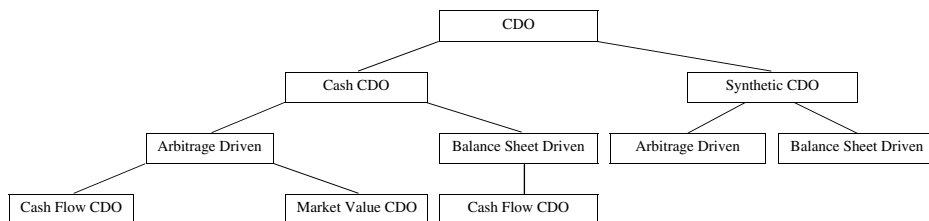
In a CDO structure, there is an asset manager responsible for managing the portfolio of debt obligations. There are restrictions imposed (i.e., restrictive covenants) as to what the asset manager may do and certain tests that must be satisfied for the tranches in the CDO to maintain the credit rating assigned at the time of issuance and determine how and when tranches are repaid principal.

The funds to purchase the underlying assets (i.e., the bonds and loans) are obtained from the issuance of debt obligations (i.e., tranches) and include one or more senior tranches, one or more mezzanine tranches, and a subordinate/equity tranche. There will be a rating sought for all but the subordinate/equity tranche. For the senior tranches, at least an A rating is typically sought. For the mezzanine tranches, a rating of BBB but no less than B is sought. As explained below, since the subordinate/equity tranche receives the residual cash flow, no rating is sought for this tranche.

The ability of the asset manager to make the interest payments to the tranches and payoff the tranches as they mature depends on the performance of the underlying assets. The proceeds to meet the obligations to the CDO tranches (interest and principal repayment) can come from (1) coupon interest payments of the underlying assets, (2) maturing assets in the underlying pool, and (3) sale of assets in the underlying pool.

In a typical structure, one or more of the tranches is a floating-rate security. With the exception of deals backed by bank loans which pay a floating rate, the asset manager invests in fixed-rate bonds. Now that presents a problem—paying tranche investors a floating rate and investing in assets with a fixed rate. To deal with this problem, the asset manager uses derivative instruments to be able to convert fixed-rate payments from the assets into floating-rate payments. In particular, interest rate swaps are used. This derivative instrument allows a market participant to swap fixed-rate payments for floating-rate payments or vice

EXHIBIT 3 CDO Family Tree



versa. Because of the mismatch between the nature of the cash flows of the debt obligations in which the asset manager invests and the floating-rate liability of any of the tranches, the asset manager must use an interest rate swap. A rating agency will require the use of swaps to eliminate this mismatch.

B. Family of CDOs

The family of CDOs is shown in Exhibit 3. While each CDO shown in the exhibit will be discussed in more detail below, we will provide an overview here.

The first breakdown in the CDO family is between cash CDOs and synthetic CDOs. A **cash CDO** is backed by a pool of cash market debt instruments. We described the range of debt obligations earlier. These were the original types of CDOs issued. A **synthetic CDO** is a CDO where the investor has the economic exposure to a pool of debt instrument but this exposure is realized via a credit derivative instrument rather than the purchase of the cash market instruments. We will discuss the basic elements of a synthetic CDO later.

Both a cash CDO and a synthetic CDO are further divided based on the motivation of the sponsor. The motivation leads to balance sheet and arbitrage CDOs. As explained below, in a **balance sheet CDO**, the motivation of the sponsor is to remove assets from its balance sheet. In an **arbitrage CDO**, the motivation of the sponsor is to capture a spread between the return that it is possible to realize on the collateral backing the CDO and the cost of borrowing funds to purchase the collateral (i.e., the interest rate paid on the obligations issued).

Cash CDOs that are arbitrage transactions are further divided in cash flow and market value CDOs depending on the primary source of the proceeds from the underlying asset used to satisfy the obligation to the tranches. In a **cash flow CDO**, the primary source is the interest and maturing principal from the underlying assets. In a **market value CDO**, the proceeds to meet the obligations depends heavily on the total return generated from the portfolio. While cash CDOs that are balance sheet motivated transactions can also be cash flow or market value CDOs, only cash flow CDOs have been issued.

C. Cash CDOs

In this section, we take a closer look at cash CDOs. Before we look at cash flow and market value CDOs, we will look at the type of cash CDO based on the sponsor motivation: arbitrage and balance sheet transactions. As can be seen in Exhibit 3, cash CDOs are categorized based on the motivation of the sponsor of the transaction. In an arbitrage transaction, the motivation of the sponsor is to earn the spread between the yield offered on the debt obligations in the underlying pool and the payments made to the various tranches in the structure. In a balance sheet transaction, the motivation of the sponsor is to remove debt instruments (primarily

loans) from its balance sheet. Sponsors of balance sheet transactions are typically financial institutions such as banks seeking to reduce their capital requirements by removing loans due to their higher risk-based capital requirements. Our focus in this section is on arbitrage transactions because such transactions are the largest part of the cash CDO sector.

1. Cash CDO Arbitrage Transactions The key as to whether or not it is economic to create an arbitrage CDO is whether or not a structure can be created that offers a competitive return for the subordinate/equity tranche.

To understand how the subordinate/equity tranche generates cash flows, consider the following basic \$100 million CDO structure with the coupon rate to be offered at the time of issuance as shown below:

Tranche	Par Value	Coupon rate
Senior	\$80,000,000	LIBOR + 70 basis points
Mezzanine	10,000,000	10-year Treasury rate plus 200 basis points
Subordinate/Equity	10,000,000	

Suppose that the collateral consists of bonds that all mature in 10 years and the coupon rate for every bond is the 10-year Treasury rate plus 400 basis points. The asset manager enters into an interest rate swap agreement with another party with a notional amount of \$80 million in which it agrees to do the following:

- pay a fixed rate each year equal to the 10-year Treasury rate plus 100 basis points
- receive LIBOR

The interest rate agreement is simply an agreement to periodically exchange interest payments. The payments are benchmarked off of a notional amount. This amount is not exchanged between the two parties. Rather it is used simply to determine the dollar interest payment of each party. This is all we need to know about an interest rate swap in order to understand the economics of an arbitrage transaction. Keep in mind, the goal is to show how the subordinate/equity tranche can be expected to generate a return.

Let's assume that the 10-year Treasury rate at the time the CDO is issued is 7%. Now we can walk through the cash flows for each year. Look first at the collateral. The collateral will pay interest each year (assuming no defaults) equal to the 10-year Treasury rate of 7% plus 400 basis points. So the interest will be:

$$\text{Interest from collateral: } 11\% \times \$100,000,000 = \$11,000,000$$

Now let's determine the interest that must be paid to the senior and mezzanine tranches. For the senior tranche, the interest payment will be:

$$\text{Interest to senior tranche: } \$80,000,000 \times (\text{LIBOR} + 70 \text{ bp})$$

The coupon rate for the mezzanine tranche is 7% plus 200 basis points. So, the coupon rate is 9% and the interest is:

$$\text{Interest to mezzanine tranche: } 9\% \times \$10,000,000 = \$900,000$$

Finally, let's look at the interest rate swap. In this agreement, the asset manager is agreeing to pay some party (we'll call this party the "swap counterparty") each year 7% (the 10-year Treasury rate) plus 100 basis points, or 8%. But 8% of what? As explained above, in an interest rate swap payments are based on a notional amount. In our illustration, the notional amount is \$80 million. The reason the asset manager selected the \$80 million was because this is the amount of principal for the senior tranche which receives a floating rate. So, the asset manager pays to the swap counterparty:

$$\text{Interest to swap counterparty: } 8\% \times \$80,000,000 = \$6,400,000$$

The interest payment received from the swap counterparty is LIBOR based on a notional amount of \$80 million. That is,

$$\text{Interest from swap counterparty: } \$80,000,000 \times \text{LIBOR}$$

Now we can put this all together. Let's look at the interest coming into the CDO:

Interest from collateral	\$11,000,000
Interest from swap counterparty	$\$80,000,000 \times \text{LIBOR}$
Total interest received	$\$11,000,000 + \$80,000,000 \times \text{LIBOR}$

The interest to be paid out to the senior and mezzanine tranches and to the swap counterparty include:

Interest to senior tranche	$\$80,000,000 \times (\text{LIBOR} + 70 \text{ bp})$
Interest to mezzanine tranche	\$900,000
Interest to swap counterparty	\$6,400,000
Total interest paid	$\$7,300,000 + \$80,000,000 \times (\text{LIBOR} + 70 \text{ bp})$

Netting the interest payments coming in and going out we have:

Total interest received	$\$11,000,000 + \$80,000,000 \times \text{LIBOR}$
Total interest paid	$\$7,300,000 + \$80,000,000 \times (\text{LIBOR} + 70 \text{ bp})$
Net interest	$\$3,700,000 - \$80,000,000 \times (70 \text{ bp})$

Since 70 bp times \$80 million is \$560,000, the net interest remaining is \$3,140,000 (= \$3,700,000 - \$560,000). From this amount any fees (including the asset management fee) must be paid. The balance is then the amount available to pay the subordinate/equity tranche. Suppose that these fees are \$634,000. Then the cash flow available to the subordinate/equity tranche for the year is \$2.5 million. Since the tranche has a par value of \$10 million and is assumed to be sold at par, this means that the annual return is 25%.

Obviously, some simplifying assumptions have been made. For example, it is assumed that there are no defaults. It is assumed that all of the issues purchased by the asset manager are noncallable and therefore the coupon rate would not decline because issues are called. Moreover, as explained below, after some period the asset manager must begin repaying principal to the senior and mezzanine tranches. Consequently, the interest rate swap must be structured to take this into account since the entire amount of the senior tranche is not outstanding for the life of the collateral. Despite the simplifying assumptions, the illustration does demonstrate the basic economics of an arbitrage transaction, the need for the use of an interest rate swap, and how the subordinate/equity tranche will realize a return.

2. Cash Flow CDO Structure In a cash flow CDO, the objective of the asset manager is to generate cash flow (primarily from interest earned and proceeds from bonds that have matured, have been called, or have amortized) to repay investors in the senior and mezzanine tranches. Because the cash flows from the structure are designed to accomplish the objective for each tranche, restrictions are imposed on the asset managers. The conditions for disposing of issues held are specified and are usually driven by credit risk considerations. Also, in assembling the portfolio, the asset manager must meet certain requirements set forth by the rating agency or agencies that rate the deal.

There are three relevant periods. The first is the *ramp up period*. This is the period that follows the closing date of the transaction where the manager begins investing the proceeds from the sale of the debt obligations issued. This period is usually less than one year. The *reinvestment period* or *revolving period* is where principal proceeds are reinvested and is usually for five or more years. In the final period, the portfolio assets are sold and the debt holders are paid off as described below.

a. Distribution of Income Income is derived from interest income from the underlying assets and capital appreciation. The income is then used as follows. Payments are first made to the trustee and administrators and then to the asset manager.²³ Once these fees are paid, then the senior tranches are paid their interest. At this point, before any other payments are made, there are certain tests that must be passed.

These tests are called coverage tests and will be discussed later. If the coverage tests are passed then interest is paid to the mezzanine tranches. Once the mezzanine tranches are paid, interest is paid to the subordinate/equity tranche.

In contrast, if the coverage tests are not passed then there are payments that are made so as to protect the senior tranches. The remaining income after paying the fees and senior tranche interest is used to redeem the senior tranches (i.e., pay off principal) until the coverage tests are brought into compliance. If the senior tranches are paid off fully because the coverage tests are not brought into compliance, then any remaining income is used to redeem the mezzanine tranches. Any remaining income is then used to redeem the subordinate/equity tranche.

b. Distribution of Principal Cash Flow The principal cash flow is distributed as follows after the payment of the fees to the trustees, administrators, and asset manager. If there is a shortfall in interest paid to the senior tranches, principal proceeds are used to make up the shortfall. Assuming that the coverage tests are satisfied, during the reinvestment period the principal is reinvested. After the reinvestment period or if the coverage tests are failed, the principal cash flow is used to pay down the senior tranches until the coverage tests are satisfied. If all the senior tranches are paid down, then the mezzanine tranches are paid off and then the subordinate/equity tranche is paid off.

c. Restrictions on Management The asset manager in both a cash flow CDO and a market value CDO (discussed next) actively manage the portfolio. The difference is in the degree of active management. In a cash flow CDO, the asset manager initially structures and then rebalances the portfolio so that interest from the pool of assets plus repaid principal is sufficient to meet the obligations of the tranches. In contrast, the asset manager in a market value CDO seeks to generate trading profits to satisfy a portion of the obligations to the tranches.

²³There are other management fees that are usually made based on performance. But these payments are made after payments to the mezzanine tranches.

The asset manager for both types of CDOs must monitor the collateral to ensure that certain tests imposed by the rating agencies are being met. For cash flow CDOs there are two types of tests: quality tests and coverage tests.

In rating a transaction, the rating agencies are concerned with the diversity of the assets. There are tests that relate to the diversity of the assets. These tests are called **quality tests**. An asset manager may not undertake a trade that will result in the violation of any of the quality tests. Quality tests include (1) a minimum asset diversity score,²⁴ (2) a minimum weighted average rating, and (3) maturity restrictions.

There are tests to ensure that the performance of the collateral is sufficient to make payments to the various tranches. These tests are called **coverage tests**. There are two types of quality tests: par value tests and interest coverage ratio. Recall that if the coverage tests are violated, then income from the collateral is diverted to pay down the senior tranches.

3. Market Value CDO As with a cash flow CDO, in a market value CDO there are debt tranches and a subordinate/equity tranche. However, because in a market value CDO the asset manager must sell assets in the underlying pool in order to generate proceeds for interest and repayment of maturing tranches, there is a careful monitoring of the assets and their price volatility. This is done by the frequent marking to market of the assets.

Because a market value CDO relies on the activities of the asset manager to generate capital appreciation and enhanced return to meet the obligations of the tranches in the structure, greater flexibility is granted to the asset manager with respect to some activities compared to a cash flow CDO. For example, while in a cash flow CDO the capital structure is fixed, in a market value CDO the asset manager is permitted to utilize additional leverage after the closing of the transaction.

D. Synthetic CDOs

A synthetic CDO is so named because the CDO does not actually own the underlying assets on which it has risk exposure. That is, in a synthetic CDO the CDO debt holders absorb the economic risks, but not the legal ownership, of a pool of assets.

1. Key Elements of a Synthetic CDO In a synthetic CDO there is credit risk exposure to a portfolio of assets, call the **reference asset**. The reference asset serves as the basis for a contingent payment as will be explained later. The reference asset can be a bond market index such as a high-yield bond index or a mortgage index. Or, the reference asset can be a portfolio of corporate loans that is owned by a bank.

The credit risk associated with the reference asset is divided into two sections: (1) senior section and (2) junior section. In a typical synthetic CDO structure, the senior section is about 90% and the junior section is about 10%. (We'll see what we mean by 90% and 10% shortly.) The losses that are realized from the reference are first realized by the junior section up to a notional amount and then after that full loss is realized, the senior section begins realizing losses.

For example, let's suppose that the reference asset is a high-yield corporate bond index. An amount of credit risk exposure in terms of market value must be determined. Suppose that it is \$500 million. The \$500 million is referred to as the notional amount. Suppose further that

²⁴Rating agencies have developed measures that quantify the diversity of a portfolio. These measures are referred to as "diversity scores."

the credit risk associated with the \$500 million credit exposure is divided into a \$450 million senior section and \$50 million junior section. The \$450 million is the notional amount for the senior section and the \$50 million is the notional amount for the junior section. The first \$50 million of loss to the reference asset due to a credit event (explained later) is absorbed by the junior section. Only after the junior section absorbs the first \$50 million in loss will the senior section realize any loss.

You may wonder why we refer to senior and junior “sections” rather than senior and junior “note holders.” The reason is that in a synthetic CDO structure, no debt obligations are issued to fund the senior section. However, for the junior section, debt obligations are issued. In our illustration, \$50 million of junior notes are issued. They are issued in the same way as in a cash CDO structure. That is, there is typically several tranches of junior notes issued by the special purpose vehicle (SPV). There will be the most senior tranche of the junior notes and there will be the subordinate/equity tranche.

The proceeds received from the issuance of the junior notes are then invested by the asset manager. However, the investments are restricted to high quality debt instruments. This includes government securities, federal agency debentures, and corporate, mortgage-backed, and asset-backed securities rated triple A.

Now we introduce the key to a synthetic CDO—a credit derivative instrument. An interest rate derivative is used by an investor to protect against interest rate risk. (We actually illustrated the use of one type of interest rate derivative, an interest rate swap, earlier when we demonstrated the economics of an arbitrage CDO transaction.) A credit derivative, as the name indicates, is used to protect against credit risk. The type of credit derivative used in a synthetic CDO is a **credit default swap**. Here we discuss the essential elements of a credit default swap, just enough to understand its role in a synthetic CDO.

A credit default swap is conceptually similar to an insurance policy. There is a “protection buyer” who purchases protection against credit risk on the reference asset. In a synthetic CDO, the insurance buyer is the asset manager. The protection buyer (the asset manager in a synthetic CDO) pays a periodic fee (like an insurance premium) and receives, in return, payment from the protection seller in the event of a “credit event” affecting any asset included in the reference asset. Who is the seller of the protection seller? It is the SPV on behalf of the junior note holders.

As with an interest rate swap, a credit default swap has a notional amount. The notional amount will be equal to the senior section, \$450 million in our example.

Let’s clarify this by continuing with our earlier illustration and look at the return to the junior note holder in the structure. The junior note holders are getting payments that come from two sources:

- (1) the income from the high quality securities purchased with the funds from the issuance of the junior debt obligations and
- (2) the insurance premium (premium from the credit default swap) paid by the asset manager to the SPV

Effectively, the junior note holders are receiving the return on a portfolio of high-quality assets subsidized by the insurance premium (i.e., the payment from the credit default swap). However, this ignores the obligation of the junior note holders with respect to the credit default swap. If there is a credit event (discussed below) that requires the junior note holders to make a payment to the protection buyer, then this reduces the return to the junior note holders. As noted earlier, the effect on a particular tranche of the junior section depends on its

priority. That is, the subordinate/equity tranche is affected first before the most senior tranche and the other tranches superior to the subordinate/equity tranche is affected.

So what becomes critical for the junior note holders' return is when it must make a payment. In credit derivatives, a payoff by the protection seller occurs when there is a **credit event**. Credit events are defined in the credit derivative documentation. On a debt instrument a credit event generally includes: bankruptcy, failure to pay when due, cross default/cross acceleration, repudiation, and restructuring. This credit event applies to any of the assets within the reference asset. For example, if a high-yield corporate bond index is the reference asset and Company X is in the index, a credit event with respect to Company X results in a payment to the protection buyer. If a designated portfolio of bank loans to corporations is the reference asset and Corporation Y's loan is included, then a credit event with respect to Corporation Y results in a payment to the protection buyer.

How much must be paid by the protection seller (the junior tranches in our illustration) to the protection buyer (the asset manager)? Should a credit event occur, there is an intent that the protection buyer be made whole: The protection buyer should be paid the difference between par and the "fair value" of the securities. How this is determined is set forth in the credit derivative agreement.

What is the motivation for the creation of synthetic CDOs? There exist two types: synthetic balance sheet CDOs and synthetic arbitrage CDOs. In the case of a synthetic balance sheet CDO, by embedding a credit default swap within a CDO structure, a bank can shed the credit risk of a portfolio of bank loans without having to notify any borrowers that they are selling the loans to another party, a requirement in some countries. No consent is needed from borrowers to transfer the credit risk of the loans, as is effectively done in a credit default swap. This is the reason synthetic balance sheet CDOs were initially set up to accommodate European bank balance sheet deals.

For a synthetic arbitrage CDO, there are several economic advantages of using a synthetic CDO structure rather than a cash CDO structure. First, it is not necessary to obtain funding for the senior section, thus making it easier to do a CDO transaction.²⁵ Second, the ramp-up period is shorter than for a cash CDO structure since only the high-quality assets need be assembled, not all of the assets contained in the reference asset. Finally, there are opportunities in the market to be able to effectively acquire the assets included in the reference asset via a credit default swap at a cheaper cost than buying the assets directly.²⁶ It because of these three advantages that issuance of synthetic CDO structures has increased dramatically since 2001 and is expected to continue to increase relative to cash CDO structures.

²⁵It is for this reason that a nonsynthetic CDO structure is referred to as a "cash" CDO structure because cash is required to be raised to purchase all the collateral assets.

²⁶For a more detailed discussion of these advantages and how it impacts the economics of a CDO, see Chapter 8 in Laurie S. Goodman and Frank J. Fabozzi, *Collateralized Debt Obligations: Structures and Analysis* (New York, NY: John Wiley & Sons, 2002).

CHAPTER 12

VALUING MORTGAGE-BACKED AND ASSET-BACKED SECURITIES

I. INTRODUCTION

In the two previous chapters, we looked at mortgage-backed and asset-backed securities. Our focus was on understanding the risks associated with investing in these securities, how they are created (i.e., how they are structured), and why the products are created. Specifically, in the case of agency mortgage-backed securities we saw how prepayment risk can be redistributed among different tranches to create securities with a prepayment risk profile that is different from the underlying pool of mortgages. For asset-backed securities and nonagency mortgage-backed security, we saw how to create tranches with different degrees of credit risk.

What we did not discuss in describing these securities is how to value them and how to quantify their exposure to interest rate risk. That is, we know, for example, that a support tranche in a CMO structure has greater prepayment risk than a planned amortization class (PAC) tranche. However, how do we determine whether or not the price at which a support tranche is offered in the market adequately compensates for the greater prepayment risk? In this chapter, we will describe and then apply a methodology for valuing mortgage-backed securities and some types of asset-backed securities—Monte Carlo simulation. As we stressed in Chapter 9, a byproduct of a valuation model is the option-adjusted spread. We will see how the option-adjusted spread for a mortgage-backed or an asset-backed security is computed and applied. From a valuation model the effective duration and effective convexity of any security can be computed. We will explain how to compute effective duration and effective convexity using the Monte Carlo simulation model. However, in the case of mortgage-backed securities, there have been several alternative measures of duration used by practitioners. These measures will be identified along with their advantages and disadvantages.

Admittedly, the majority of this chapter is devoted to the valuation of mortgage-backed securities and by extension to all real estate-related asset-backed securities. They are the most difficult asset-backed products to value and to quantify in terms of interest rate exposure. At the end of this chapter, we provide a framework for determining which analytical measures discussed in this chapter are appropriate for valuing any asset-backed security. In fact, the principles apply to all fixed income products.

II. CASH FLOW YIELD ANALYSIS

Let's begin with the traditional analysis of mortgage-backed and asset-backed securities—**cash flow yield analysis**. As explained in Chapter 6, the yield on any financial instrument is the interest rate that makes the present value of the expected cash flow equal to its market price plus accrued interest. When applied to mortgage-backed and asset-backed securities, this yield is called a **cash flow yield**. The problem in calculating the cash flow yield of a mortgage-backed and asset-backed securities is that the cash flow is unknown because of prepayments. Consequently, to determine a cash flow yield, some assumption about the prepayment rate must be made. And, in the case of all but agency mortgage-backed securities, an assumption about default rates and recovery rates must be made.

The cash flow for mortgage-backed and asset-backed securities is typically monthly. The convention is to compare the yield on mortgage-backed and asset-backed securities to that of a Treasury coupon security by calculating the security's *bond-equivalent yield*. As explained in Chapter 6, the bond-equivalent yield for a Treasury coupon security is found by doubling the semiannual yield to maturity. However, it is incorrect to do this for a mortgage-backed or an asset-backed security because the investor has the opportunity to generate greater reinvestment income by reinvesting the more frequent (i.e., monthly) cash flows. The **market convention** is to calculate a yield so as to make it comparable to the yield to maturity on a bond-equivalent basis. The formula for annualizing the monthly cash flow yield for a monthly-pay product is therefore:

$$\text{bond-equivalent yield} = 2[(1 + i_M)^6 - 1]$$

where i_M is the monthly interest rate that will equate the present value of the projected monthly cash flow equal to the market price (plus accrued interest) of the security.

To illustrate the calculation of the bond-equivalent yield, suppose that the monthly yield is 0.6%. That is, i_M is 0.006. Then

$$\text{bond-equivalent yield} = 2[(1.006)^6 - 1] = 0.0731 = 7.31\%$$

A. Limitations of Cash Flow Yield Measure

All yield measures suffer from problems that limit their use in assessing a security's potential return. The yield to maturity has two major shortcomings as a measure of a bond's potential return. To realize the stated yield to maturity, the investor must:

1. reinvest the coupon payments at a rate equal to the yield to maturity, and
2. hold the bond to the maturity date

As explained in Chapter 6, the reinvestment of the coupon payments is critical and for long-term bonds can be as much as 80% of the bond's return. Reinvestment risk is the risk of having to reinvest the interest payments at less than the computed yield. Interest rate risk is the risk associated with having to sell the security before its maturity date at a price less than the purchase price.

These shortcomings are equally applicable to the cash flow yield measure:

1. the projected cash flows are assumed to be reinvested at the cash flow yield, and

2. the mortgage-backed or asset-backed security is assumed to be held until the final payout based on some prepayment assumption.

The importance of reinvestment risk, the risk that the cash flow will have to be reinvested at a rate less than the cash flow yield, is particularly important for many mortgage-backed and asset-backed securities because payments are monthly and both interest and principal must be reinvested. Moreover, an additional assumption is that the projected cash flow is actually realized. If the prepayment, default, and recovery experience is different from that assumed, the cash flow yield will not be realized.

B. Nominal Spread

Given the computed cash flow yield and the average life for a mortgage-backed or asset-backed security based on some prepayment, default, and recovery assumption, the next step is to compare the yield to the yield for a comparable Treasury security. “Comparable” is typically defined as a Treasury security with the same maturity as the security’s average life. The difference between the cash flow yield and the yield on a comparable Treasury security is called the **nominal spread**.

Unfortunately, it is the nominal spread that some managers will use as a measure of relative value. However, this spread masks the fact that a portion of the nominal spread is compensation for accepting prepayment risk. For example, CMO support tranches have been offered at large nominal spreads. However, the nominal spread embodies the substantial prepayment risk associated with support tranches. The manager who buys solely on the basis of nominal spread fails to determine whether or not that nominal spread offered adequate compensation given the substantial prepayment risk faced by the holder of a support tranche.

Instead of nominal spread, managers need a measure that indicates the potential compensation after adjusting for prepayment risk. This measure is called the **option-adjusted spread**. We discussed this measure in Chapter 9 where we covered the valuation of corporate and agency bonds with embedded options. Before discussing this measure for structured products, we describe another spread measure commonly quoted for structured products called the **zero-volatility spread**, a measure described in Chapter 9.

III. ZERO-VOLATILITY SPREAD

As explained earlier, the proper procedure to compare any security to a U.S. Treasury security is to compare it to a portfolio of Treasury securities that have the same cash flow. The value of the security is then equal to the present value of all of the cash flows. The security’s value, assuming the cash flows are default-free, will equal the present value of the replicating portfolio of Treasury securities. In turn, these cash flows are valued at the Treasury spot rates.

The **zero-volatility spread** is a measure of the spread that the investor would realize over the entire Treasury spot rate curve if the mortgage-backed or asset-backed security is held to maturity. It is not a spread off one point on the Treasury yield curve, as is the nominal spread. The zero-volatility spread (also called the **Z-spread** and the **static spread**) is the spread that will make the present value of the cash flows from the mortgage-backed or asset-backed security when discounted at the Treasury spot rate plus the spread equal to the price of the security.

A trial-and-error procedure (or search algorithm) is required to determine the zero-volatility spread. We illustrated this in Chapter 6.¹

Also as explained at Level I, in general, the shorter the maturity or average life of a structured product, the less the zero-volatility spread will differ from the nominal spread. The magnitude of the difference between the nominal spread and the zero-volatility spread also depends on the shape of the yield curve. The steeper the yield curve, the greater the difference.

One of the objectives of this chapter is to explain when it is appropriate to use the Z-spread instead of the OAS. What will be seen is that if a structured product has an option and the borrower tends to take advantage of that option when interest rates decline, then the OAS should be used. If the borrower has an option but tends not to take advantage of it when interest rates decline, then the Z-spread should be used.

IV. MONTE CARLO SIMULATION MODEL AND OAS

In Chapter 9, we discussed one model that is used to value callable agency debentures and corporate bonds, the binomial model. This valuation model accommodates securities in which the decision to exercise a call option is not dependent on how interest rates evolved over time. That is, the decision of an issuer to call a bond will depend on the level of the rate at which the issue can be refunded relative to the issue's coupon rate, and not the path interest rates took to get to that rate. In contrast, there are fixed income securities and derivative instruments for which the periodic cash flows are "interest rate path-dependent." This means that the cash flow received in one period is determined not only by the current interest rate level, but also by the path that interest rates took to get to the current level.

For example, in the case of passthrough securities, prepayments are interest rate path-dependent because this month's prepayment rate depends on whether there have been prior opportunities to refinance since the underlying mortgages were originated. This phenomenon is referred to as "prepayment burnout." Pools of passthroughs are used as collateral for the creation of CMOs. Consequently, there are typically two sources of path dependency in a CMO tranche's cash flows. First, the collateral prepayments are path-dependent as discussed above. Second, the cash flows to be received in the current month by a CMO tranche depend on the outstanding balances of the other tranches in the deal. Thus, we need the history of prepayments to calculate these balances.

Conceptually, the valuation of agency passthroughs using the Monte Carlo model is simple. In practice, however, it is very complex. The simulation involves generating a set of cash flows based on simulated future mortgage refinancing rates, which in turn imply simulated prepayment rates.

Valuation modeling for agency CMOs is similar to valuation modeling for passthroughs, although the difficulties are amplified because the issuer has distributed both the prepayment risk and the interest rate risk into different tranches. The sensitivity of the passthroughs comprising the collateral to these two risks is not transmitted equally to every tranche. Some of the tranches wind up more sensitive to prepayment risk and interest rate risk than the collateral, while some of them are much less sensitive.

The objective is to figure out how the value of the collateral gets transmitted to the tranches in a deal. More specifically, the objective is to find out where the value goes and

¹Most common spreadsheet programs offer this type of algorithm.

where the risk goes so that one can identify the tranches with low risk and high value: the tranches a manager wants to consider for purchase. The good news is that this combination usually exists in every deal. The bad news is that in every deal there are usually tranches with low value and high risk that managers want to avoid purchasing.

A. Simulating Interest Rate Paths and Cash Flows

To generate these random interest rate paths, the typical model used by Wall Street firms and commercial vendors takes as input today's term structure of interest rates and a volatility assumption. (We discussed these topics in Chapter 9.) The term structure of interest rates is the theoretical spot rate (or zero coupon) curve implied by today's Treasury securities. The simulations should be calibrated so that the average simulated price of a zero-coupon Treasury bond equals today's actual price.

On-the-run Treasury issues are often used in the calibration process. Some dealers and vendors of analytical systems use the LIBOR curve instead of the Treasury curve—or give the user a choice to use either the Treasury curve or the LIBOR curve. The reason is that some investors are interested in spreads that they can earn relative to their funding costs and LIBOR for many investors is a better proxy for that cost than Treasury rates. (We will discuss the different types of investors later.)

As explained in Chapter 9, every dealer and vendor of analytical systems employs an interest rate model. This is a model that assumes how interest rates will change over time. The interest rate models employed by most dealers and vendors of analytical systems are similar. However, one input to all interest rate models is the interest rate volatility assumption. It is that assumption that varies by dealer and vendor. As will be illustrated later in this chapter, it is a critical input.

The volatility assumption determines the dispersion of future interest rates in the simulation. Today, many dealers and vendors do not use one volatility number for the yield of all maturities of the yield curve. Instead, they use either a short/long yield volatility or a term structure of yield volatility. A short/long yield volatility means that volatility is specified for maturities up to a certain number of years (short yield volatility) and a different yield volatility for longer maturities (long yield volatility). The short yield volatility is assumed to be greater than the long yield volatility. A term structure of yield volatilities means that a yield volatility is assumed for each maturity.

Based on the interest rate model and the assumed volatility, a series of interest rate paths will be generated. We will see shortly how a security is valued on each interest rate path. However, there is nothing that we have explained thus far that assures us that the values produced by the model will be arbitrage free. Recall from Chapter 9 that the binomial interest rate tree by design is constructed to be arbitrage free. That is, if any of the on-the-run issues that were used to construct the binomial interest rate tree are valued using the tree, the model would produce a value for that on-the-run issue equal to its market value. There is nothing we described so far about the Monte Carlo simulation to assure this.

More specifically, in the case of Monte Carlo simulation for valuing mortgage-backed and asset-backed securities, the on-the-run Treasury issues are typically used. What assurance is there that if an on-the-run Treasury issue is valued using the Monte Carlo simulation model it will be arbitrage free? That is, what assurance is there that the value produced by the model will equal the market price? Nothing. That's right, nothing. What the model builder must do is "adjust" the interest rate paths so that the model produces the correct values for the on-the-run Treasury issues. A discussion of this adjustment process is not important to us. In

fact, there are very few published sources that describe how this is done. The key point here is that no such adjustment is necessary in a binomial model for valuing corporate and agency bonds with embedded options because the tree is built to be arbitrage free. In the case of the Monte Carlo simulation model, the builder must make an arbitrary adjustment to the interest rate paths to get the model to be arbitrage free.

The simulation works by generating many scenarios of future interest rate paths. As just explained, the “raw” interest rate paths that are simulated must be “adjusted” so as to make the model generate arbitrage-free values for whatever benchmark interest rates are used—typically, the on-the-run Treasury issues. *So, in the remainder of this chapter, when we refer to interest rate paths it is understood that it is the “adjusted” interest rate paths where “adjusted” means that each interest rate path is adjusted so that the model will produce arbitrage-free values.*

In each month of the scenario (i.e., path), a monthly interest rate and a mortgage refinancing rate are generated. The monthly interest rates are used to discount the projected cash flows in the scenario. The mortgage refinancing rate is needed to determine the cash flows because it represents the opportunity cost the borrower (i.e., mortgagor) is facing at that time.

If the refinancing rates are high relative to the borrower’s original coupon rate (i.e., the rate on the borrower’s loan), the borrower will have less incentive to refinance, or even a disincentive (i.e., the homeowner will avoid moving in order to avoid refinancing). If the refinancing rate is low relative to the borrower’s original coupon rate, the borrower has an incentive to refinance.

Prepayments are projected by feeding the refinancing rate and loan characteristics into a prepayment model. Given the projected prepayments, the cash flows along an interest rate path can be determined.

To make this more concrete, consider a newly issued mortgage passthrough security with a maturity of 360 months. Exhibit 1 shows N “adjusted” simulated interest rate path scenarios—adjusted to be arbitrage free. Each scenario consists of a path of 360 simulated 1-month future interest rates. (The number of paths generated is based on a well known principle in simulation which will not be discussed here.) So, our first assumption that we make to get Exhibit 1 is the volatility of interest rates.

Exhibit 2 shows the paths of simulated mortgage refinancing rates corresponding to the scenarios shown in Exhibit 1. In going from Exhibit 1 to Exhibit 2, an assumption must be made about the relationship between the Treasury rates and refinancing rates. The assumption is that there is a constant spread relationship between the rate that the borrower will use to determine whether or not to refinance (i.e., the refinancing rate) and the 1-month interest rates shown in Exhibit 1. For example, for 30-year mortgage loans, model builders use the 10-year Treasury rate as a proxy for the refinancing rate.

Given the mortgage refinancing rates, the cash flows on each interest rate path can be generated. For agency mortgage-backed securities, this requires a prepayment model. For asset-backed securities and nonagency mortgage-backed securities, this requires both a prepayment model and a model of defaults and recoveries. So, our next assumption is that the output of these models (prepayments, defaults, and recoveries) are correct. The resulting cash flows are depicted in Exhibit 3.

B. Calculating the Present Value for an Interest Rate Path

Given the cash flows on an interest rate path, the path’s present value can be calculated. The discount rate for determining the present value is the simulated spot rate for each month on the interest rate path plus an appropriate spread. The spot rate on a path can be determined

from the simulated future monthly rates. The relationship that holds between the simulated spot rate for month T on path n and the simulated future 1-month rates is:

$$z_T(n) = \{[1 + f_1(n)][1 + f_2(n)] \cdots [1 + f_T(n)]\}^{1/T} - 1$$

where

$$\begin{aligned} z_T(n) &= \text{simulated spot rate for month } T \text{ on path } n \\ f_t(n) &= \text{simulated future 1-month rate for month } t \text{ on path } n \end{aligned}$$

In Chapter 6 we explained the relationship between spot rates and forward rates.

Consequently, the interest rate path for the simulated future 1-month rates can be converted to the interest rate path for the simulated monthly spot rates as shown in Exhibit 4. Therefore, the present value of the cash flows for month T on interest rate path n discounted at the simulated spot rate for month T plus some spread is:

$$PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

where

$$\begin{aligned} PV[C_T(n)] &= \text{present value of cash flows for month } T \text{ on path } n \\ C_T(n) &= \text{cash flow for month } T \text{ on path } n \\ z_T(n) &= \text{spot rate for month } T \text{ on path } n \\ K &= \text{spread} \end{aligned}$$

The spread, K , reflects the risks that the investor feels are associated with realizing the cash flows.

The present value for path n is the sum of the present value of the cash flows for each month on path n . That is,

$$PV[\text{Path}(n)] = PV[C_1(n)] + PV[C_2(n)] + \cdots + PV[C_{360}(n)]$$

where $PV[\text{Path}(n)]$ is the present value of interest rate path n .

C. Determining the Theoretical Value

The present value of a given interest rate path can be thought of as the theoretical value of a passthrough if that path was actually realized. The theoretical value of the passthrough can be determined by calculating the average of the theoretical values of all the interest rate paths. That is, the theoretical value is equal to

$$\text{theoretical value} = \frac{PV[\text{Path}(1)] + PV[\text{Path}(2)] + \cdots + PV[\text{Path}(N)]}{N}$$

where N is the number of interest rate paths. The theoretical value derived from the above equation is based on some spread, K . It follows the usual practice of discounting cash flows at spot rates plus a spread—in this case the spread is K .

This procedure for valuing a passthrough is also followed for a CMO tranche. The cash flow for each month on each interest rate path is found according to the principal repayment and interest distribution rules of the deal.

EXHIBIT 4 “Adjusted” Simulated Paths of Monthly Arbitrage-Free Spot Rates

Month	Interest rate path number						
	1	2	3	...	n	...	N
1	$z_1(1)$	$z_1(2)$	$z_1(3)$	...	$z_1(n)$	...	$z_1(N)$
2	$z_2(1)$	$z_2(2)$	$z_2(3)$	...	$z_2(n)$	...	$z_2(N)$
3	$z_3(1)$	$z_3(2)$	$z_3(3)$	...	$z_3(n)$	...	$z_3(N)$
...	...	...	...	...	...	...	...
t	$z_t(1)$	$z_t(2)$	$z_t(3)$	...	$z_t(n)$	...	$z_t(N)$
...	...	...	...	...	...	...	...
358	$z_{358}(1)$	$z_{358}(2)$	$z_{358}(3)$	...	$z_{358}(n)$	...	$z_{358}(N)$
359	$z_{359}(1)$	$z_{359}(2)$	$z_{359}(3)$	...	$z_{359}(n)$	...	$z_{359}(N)$
360	$z_{360}(1)$	$z_{360}(2)$	$z_{360}(3)$	...	$z_{360}(n)$	...	$z_{360}(N)$
Notation:							
$z_t(n)$ = spot rate for month t on path n							
N = total number of interest rate paths							

D. Selecting the Number of Interest Rate Paths

Let’s now address the question of the number of scenario paths, N , needed to value a security. The number of interest rate paths determines how “good” the estimate is, not relative to the truth but relative to the model used. The more paths, the more the average value produced tends to converge. It is simply a statistical sampling problem.

Most models employ some form of variance reduction to cut down on the number of sample paths necessary to get a good statistical sample.² Several vendor firms have developed computational procedures that reduce the number of paths required but still provide the accuracy of a full Monte Carlo analysis. The procedure is to use statistical techniques to reduce the number of interest rate paths to sets of similar paths. These paths are called **representative paths**. For example, suppose that 2,000 sample paths are generated. Using a certain statistical technique, these 2,000 sample paths can be collapsed to, say, 16 representative paths. The security is then valued on each of these 16 representative paths. The theoretical value of the security is then the *weighted* average of the 16 representative paths. The weight for a path is the percentage of that representative path relative to the total sample paths. Vendors often give the investor or portfolio manager the choice of whether to use the “full Monte Carlo simulation” or to specify a number of representative paths.

E. Option-Adjusted Spread

In Chapter 9, we explained the option-adjusted spread (OAS). Specifically, we explained (1) how to compute the OAS for corporate and agency bonds with embedded options and (2) how to interpret the OAS and apply it in relative analysis. Below we cover the same issues for the OAS computed for mortgage-backed securities.

²The variance reduction technique is described in books on management science and Monte Carlo simulation.

1. **Computing the OAS** In the Monte Carlo model, the OAS is the spread that when added to all the spot rates on all interest rate paths will make the average present value of the paths equal to the observed market price (plus accrued interest). Mathematically, OAS is the value for K (the spread) that will satisfy the following condition:

$$\frac{PV[\text{Path}(1)] + PV[\text{Path}(2)] + \dots + PV[\text{Path}(N)]}{N} = \text{market price}$$

where N is the number of interest rate paths. The left-hand side of the above equation looks identical to that of the equation for the theoretical value. The difference is that the objective is to determine what spread, K , will make the model produce a theoretical value equal to the market price.

The procedure for determining the OAS is straightforward and involves the same search algorithm explained for the zero-volatility spread. The next question, then, is how to interpret the OAS. Basically, the OAS is used to reconcile value with market price. On the *right*-hand side of the previous equation is the market's statement: the price of a structured product. The average present value over all the paths on the *left*-hand side of the equation is the model's output, which we refer to as the theoretical value.

2. **Interpreting the OAS and Relative Value Application** What an investor or a portfolio manager seeks to do is to buy a mortgage-backed security where value is greater than price. By using a valuation model such as the Monte Carlo model, a portfolio manager could estimate the value of a security, which at this point would be sufficient in determining whether to buy a security. That is, the portfolio manager can say that this security is 1 point cheap or 2 points cheap, and so on. The model does not stop here. Instead, it converts the divergence between price and value into some type of spread measure since most market participants find it more convenient to think about spreads than price differences.

The OAS was developed as a measure of the spread that can be used to convert dollar differences between value and price. As we explained in Chapter 9, in the binomial model a spread is measured relative to the benchmark interest rates used to generate the interest rate tree and which is therefore used to make the tree arbitrage free. The same is true in the case of the Monte Carlo simulation model. The spread is measured relative to the benchmark interest rates that were used to generate the interest rate paths and to adjust the interest rate paths to make them arbitrage free. Typically, for a mortgage-backed security the benchmark interest rates are the on-the-run Treasury rates. The OAS is then measuring the average spread over the Treasury spot rate curve, not the Treasury yield as explained in Chapter 9. It is an average spread since the OAS is found by averaging over the interest rate paths for the possible Treasury spot rate curves. Of course, if the LIBOR curve is used, the OAS is the spread over that curve.

This spread measure is superior to the nominal spread which gives no recognition to the prepayment risk. As explained in Chapter 9, the OAS is "option adjusted" because the cash flows on the interest rate paths are adjusted for the option of the borrowers to prepay. While we may understand the mechanics of how to compute the OAS and why it is appropriate to use the OAS rather than the nominal spread or Z-spread, the question is what does the spread represent? In Chapter 9, a discussion of what compensation the OAS reflects for corporate and agency bonds with embedded options was presented. The compensation is for a combination of credit risk and liquidity risk. The compensation depends on the benchmark interest rates used in the analysis. For example, consider the use of Treasury interest rates and more specifically the Treasury spot rate curve since this is

the benchmark typically used in the calculation of an OAS for a mortgage-backed security. If there is an OAS computed using the Treasury benchmark, then what is that OAS compensating?

Consider first Ginnie Mae mortgage passthrough securities. As explained earlier (Chapter 3), Ginnie Mae is an arm of the U.S. government. The securities it issues are backed by the full faith and credit of the U.S. government. Effectively, a Ginnie Mae mortgage-backed security is a Treasury security with prepayment risk. The OAS removes the prepayment risk (i.e., the option risk). Additionally, if the benchmark is Treasury rates, the OAS should not be compensation for credit risk. That leaves liquidity risk. While Ginnie Mae mortgage passthrough securities may not be as liquid as on-the-run Treasury issues, they are fairly liquid as measured by the bid-ask spread. Nevertheless, part of the OAS should reflect compensation for liquidity risk. There is one more risk that was not the focus of Chapter 9, modeling risk. In our explanation of the Monte Carlo model, there were several critical assumptions and parameters required. If those assumptions prove incorrect or if the parameters are misestimated, the prepayment model will not calculate the true level of risk. Thus, probably a good part of the compensation for a Ginnie Mae mortgage passthrough security reflects payment for this model uncertainty.

If we consider CMOs issued by Ginnie Mae rather than the mortgage passthrough securities, the OAS would reflect the complexity associated with a particular tranche. For example, a planned amortization class (PAC) tranche would be less exposed to modeling risk than a support tranche in the same CMO structure. Hence, compensation for modeling risk would be greater for a support tranche than a PAC tranche in the same structure. Moreover, PAC tranches have greater liquidity than support tranches, so compensation is less for the former relative to the latter.

As we move from Ginnie Mae issued mortgage products to those issued by Freddie Mac and Fannie Mae, we introduce credit risk. As explained in Chapter 3, Freddie Mac and Fannie Mae are government sponsored enterprises (GSEs). As such, there is no requirement that a GSE be bailed out by the U.S. government. The GSEs are viewed as triple A rated. Consequently, in addition to modeling risk and liquidity risk, a portion of the OAS reflects credit risk relative to Treasury securities.

Moving on to nonagency mortgage-backed securities and real estate backed asset-backed securities, the OAS is compensating for: (1) credit risk (which varies depending on the credit rating for the tranche under consideration), (2) liquidity risk (which is greater than for Ginnie Mae, Fannie Mae, and Freddie Mac mortgage products), and (3) modeling risk.

F. Option Cost

The implied cost of the option embedded for a mortgage-backed or asset-backed security can be obtained by calculating the difference between the option-adjusted spread at the assumed volatility of interest rates and the zero-volatility spread. That is,

$$\text{option cost} = \text{zero-volatility spread} - \text{option-adjusted spread}$$

The Option cost, option cost measures the prepayment (or option) risk embedded in the security. Note that the cost of the option is a byproduct of the option-adjusted spread analysis, not valued explicitly with some option pricing model.

G. Illustrations

We will use two deals to show how CMOs can be analyzed using the Monte Carlo model/OAS procedure discussed above—a simple structure and a PAC/support structure.³

1. Simple Structure The simple structure analyzed is Freddie Mac (FHLMC) 1915. It is a simple sequential-pay CMO bond structure. The structure includes eight tranches, A, B, C, D, E, F, G, and S. The focus of our analysis is on tranches A, B, and C. All three tranches were priced at a premium.

The top panel of Exhibit 5 shows the OAS, the option cost, and effective duration⁴ for the collateral and the three tranches in the CMO structure. However, tranche A had the smallest effective duration and tranche C had the largest effective duration. The OAS for the collateral is 51 basis points. Since the option cost is 67 basis points, the zero-volatility spread is 118 basis points (51 basis points plus 67 basis points).

At the time this analysis was performed, March 10, 1998, the Treasury yield curve was not steep. We explained that when the yield curve is relatively flat the zero-volatility spread will not differ significantly from the nominal spread. Thus, for the three tranches shown in Exhibit 5, the zero-volatility spread is 83 basis points for A, 115 basis points for B, and 116 basis points for C.

Notice that the tranches did not share the OAS equally. The same is true for the option cost. Both the Z-spread and the option cost increase as the effective duration increases. Whether or not any of these tranches were attractive investments requires a comparison to other tranches in the market with the same effective duration. While not presented here, all three tranches offered an OAS similar to other sequential-pay tranches with the same effective duration available in the market. On a relative basis (i.e., relative to the other tranches analyzed in the deal), the only tranche where there appears to be a bit of a bargain is tranche C. A portfolio manager contemplating the purchase of this last cash flow tranche can see that C offers a higher OAS than B and appears to bear less of the risk (i.e., has lower option cost), as measured by the option cost. The problem portfolio managers may face is that they might not be able to go out as long on the yield curve as tranche C because of effective duration, maturity, and average life constraints relative to their liabilities, for example.

Now let's look at modeling risk. Examination of the sensitivity of the tranches to changes in prepayments and interest rate volatility will help us to understand the interaction of the tranches in the structure and who is bearing the risk. How the deal behaves under various scenarios should reinforce and be consistent with the valuation (i.e., a tranche may look "cheap" for a reason).

We begin with prepayments. Specifically, we keep the same interest rate paths as those used to get the OAS in the base case (the top panel of Exhibit 5), but reduce the prepayment rate on each interest rate path to 80% of the projected rate. As can be seen in the second panel of Exhibit 5, slowing down prepayments increases the OAS and price for the collateral. The exhibit reports two results of the sensitivity analysis. First, it indicates the change in the OAS. Second, it indicates the change in the price, holding the OAS constant at the base case.

³These illustrations are from Frank J. Fabozzi, Scott F. Richard, and David S. Horowitz, "Valuation of CMOs," Chapter 6 in Frank J. Fabozzi (ed.), *Advances in the Valuation and Management of Mortgage-Backed Securities* (New Hope, PA: Frank J. Fabozzi Associates, 1998).

⁴We will explain how to compute the effective duration using the Monte Carlo methodology in Section V. A.

EXHIBIT 5 OAS Analysis of FHLMC 1915 Classes A, B, and C (As of 3/10/98)

All three tranches were trading at a premium as of the date of the analysis.

Base Case (Assumes 13% Interest Rate Volatility)

	OAS (in basis points)	Option cost (in basis points)	Z-Spread (in basis points)	Effective duration (in years)
Collateral	51	67	118	1.2
Tranche				
A	32	51	83	0.9
B	33	82	115	2.9
C	46	70	116	6.7

Prepayments at 80% and 120% of Prepayment Model (Assumes 13% Interest Rate Volatility)

	New OAS (in basis points)		Change in price per \$100 par (holding OAS constant)	
	80%	120%	80%	120%
Collateral	63	40	\$0.45	−\$0.32
Tranche				
A	40	23	0.17	−0.13
B	43	22	0.54	−0.43
C	58	36	0.97	−0.63

Interest Rate Volatility of 9% and 17%

	New OAS (in basis points)		Change in price per \$100 par (holding OAS constant)	
	9%	17%	9%	17%
Collateral	79	21	\$1.03	−\$0.94
Tranche				
A	52	10	0.37	−0.37
B	66	−3	1.63	−1.50
C	77	15	2.44	−2.08

To see how a portfolio manager can use the information in the second panel, consider tranche A. At 80% of the prepayment speed, the OAS for this tranche increases from 32 basis points to 40 basis points. If the OAS is held constant, the panel indicates that the buyer of tranche A would gain \$0.17 per \$100 par value.

Notice that for all of the tranches reported in Exhibit 5 there is a gain from a slowdown in prepayments. This is because all of the sequential tranches in this deal are priced over par. (An investor in a tranche priced at a premium benefits from a slowdown in prepayments because the investor receives the higher coupon for a longer period and postpones the capital loss from a prepayment.) Also notice that while the changes in OAS are about the same for the different tranches, the changes in price are quite different. This arises because the shorter tranches have less duration. Therefore, their prices do not move as much from a change in OAS as a longer average life tranche. A portfolio manager who is willing to go to the long end of the yield curve, such as tranche C, would realize the most benefit from the slowdown in prepayments.

Also shown in the second panel of the exhibit is the second part of our experiment to test the sensitivity of prepayments: the prepayment rate is assumed to be 120% of the base case. The collateral loses money in this scenario because it is trading above par. This is reflected in the OAS of the collateral which declines from 51 basis points to 40 basis points. Now look at the three tranches. They all lost money because the tranches were all at a premium and the speeding of prepayments adversely affects the tranche.

Before looking at the last panel that shows the effect of a change in interest rate volatility on the OAS, let's review the relationship between expected interest rate volatility and the value of a mortgage-backed security. Recall that the investor in a mortgage-backed security has sold an option to homeowners (borrowers). Thus, the investor is short an option. As will be explained in Chapter 14, the value of an option depends on expected interest rate volatility. When expected interest rate volatility decreases, the value of the option embedded in a mortgage-backed security decreases and therefore the value of a mortgage-backed security increases. The opposite is true when expected interest rate volatility increases—the value of the embedded option increases and the value of a mortgage-backed security decreases.

Now let's look at the sensitivity to the interest rate volatility assumption, 13% in the base case. Two experiments are performed: reducing the volatility assumption to 9% and increasing it to 17%. These results are reported in the third panel of Exhibit 5.

Reducing the volatility to 9% increases the dollar price of the collateral by \$1.03 and increases the OAS from 51 in the base case to 79 basis points. However, this \$1.03 increase in the price of the collateral is not equally distributed among the three tranches. Most of the increase in value is realized by the longer tranches. The OAS gain for each of the tranches follows more or less the effective durations of those tranches. This makes sense, because the longer the duration, the greater the risk, and when volatility declines, the reward is greater for the accepted risk. At the higher level of assumed interest rate volatility of 17%, the collateral is severely affected. The longer the duration, the greater the loss. These results for a decrease and an increase in interest rate volatility are consistent with what we explained earlier.

Using the Monte Carlo simulation/OAS analysis, a fair conclusion that can be made about this simple structure is: what you see is what you get. The only surprise in this structure is the lower option cost in tranche C. In general, however, a portfolio manager willing to extend duration gets paid for that risk in this structure.

2. PAC/Support Tranche Structure Now let's look at how to apply the methodology to a more complicated CMO structure, FHLMC Series 1706. The collateral (i.e., pool of passthroughs) for this structure is Freddie Mac 7s (7% coupon rate). A partial summary of the deal is provided in Exhibit 6. That is, only the tranches we will be discussing in this section are shown in the exhibit.⁵

While this deal looks complicated, it is relatively simple compared to many deals that have been issued. Nonetheless, it brings out all the key points about application of OAS analysis, specifically, the fact that most deals include cheap bonds, expensive bonds, and fairly priced bonds. The OAS analysis helps identify how a tranche should be classified. A more proper analysis would compare the OAS for each tranche to a similar duration tranche available in the market.

All of the tranches in Exhibit 6 were discussed in Chapter 10. At issuance, there were 10 PAC tranches, three scheduled tranches, a floating-rate support tranche, and an inverse floating-rate support. Recall that the “scheduled tranches” are support tranches with a schedule, referred to in Chapter 10 as “PAC II tranches.”

The first two PAC tranches in the deal, tranche A and tranche B, were paid off at the time of the analysis. The other PAC tranches were still available at the time of the analysis. The prepayment protection for the PAC tranches is provided by the support tranches. The support tranches in this deal that are shown in Exhibit 6 are tranches LA, LB, and M. There were other support tranches not shown in Exhibit 6. LA is the shortest average life support tranche (a scheduled (SCH) bond).

⁵This deal was described in Chapter 10.

EXHIBIT 6 Summary of Federal Home Loan Mortgage Corporation—Multiclass Mortgage Participation Certificates (Guaranteed), Series 1706

Total Issue: \$300,000,000		Issue Date: 2/18/94		Original issue pricing (225% PSA assumed)	
Tranche	Original balance (\$)	Coupon (%)	Stated maturity	Average life (yrs)	Expected maturity
PAC Tranches					
C (PAC Bond)	25,500,000	5.25	4/15/14	3.5	6/15/98
D (PAC Bond)	9,150,000	5.65	8/15/15	4.5	1/15/99
E (PAC Bond)	31,650,000	6.00	1/15/19	5.8	1/15/01
G (PAC Bond)	30,750,000	6.25	8/15/21	7.9	5/15/03
H (PAC Bond)	27,450,000	6.50	6/15/23	10.9	10/15/07
J (PAC Bond)	5,220,000	6.50	10/15/23	14.4	9/15/09
K (PAC Bond)	7,612,000	7.00	3/15/24	18.8	5/15/19
Support Tranches					
LA (SCH Bond)	26,673,000	7.00	11/15/21	3.5	3/15/02
LB (SCH Bond)	36,087,000	7.00	6/15/23	3.5	9/15/02
M (SCH Bond)	18,738,000	7.00	3/15/24	11.2	10/15/08

The collateral for this deal was trading at a premium. That is, the homeowners (borrowers) were paying a higher mortgage rate than available in the market at the time of the analysis. This meant that the value of the collateral would increase if prepayments slow down but would decrease if prepayments increase. What is important to note, however, is that a tranche could be trading at a discount, par, or premium even though the collateral is priced at a premium. For example, PAC C had a low coupon rate at the time of the analysis and therefore was trading at a discount. Thus, while the collateral (which was selling at a premium) loses value from an increase in prepayments, a discount tranche such as tranche C would increase in value if prepayments increase. (Recall that in the simple structure analyzed earlier, the collateral and all the tranches were trading at a premium.)

The top panel of Exhibit 7 shows the base case OAS, the option cost, and the effective duration for the collateral and tranches in Exhibit 6. The collateral OAS is 60 basis points, and the option cost is 44 basis points. The Z-spread of the collateral to the Treasury spot curve is 104 basis points.

The 60 basis points of OAS did not get equally distributed among the tranches—as was the case with the simple structure analyzed earlier. Tranche LB, the scheduled support, did not realize a good OAS allocation, only 29 basis points, and had an extremely high option cost. Given the prepayment uncertainty associated with this tranche, its OAS would be expected to be higher. The reason for the low OAS is that this tranche was priced so that its cash flow yield is high. Using the Z-spread as a proxy for the nominal spread (i.e., spread over the Treasury yield curve), the 103 basis point spread for tranche LB is high given that this appears to be a short average life tranche. Consequently, “yield buyers” (i.e., investors with a preference for high nominal yield, who may not be attentive to compensation for prepayment risk) probably bid aggressively for this tranche and thereby drove down its OAS, trading off “yield” for OAS. From a total return perspective, however, tranche LB should be avoided. It is a rich, or expensive, tranche. The other support tranche analyzed, tranche M, had an OAS of 72 basis points and at the time of this analysis was similar to that offered on comparable duration tranches available in the market.

EXHIBIT 7 OAS Analysis of FHLMC 1706 (As of 3/10/98)

Base Case (Assumes 13% Interest Rate Volatility)

	OAS (in basis points)	Option cost (in basis points)	Z-Spread (in basis points)	Effective duration (in years)
Collateral	60	44	104	2.6
PAC Tranches				
C(PAC)	15	0	15	0.2
D(PAC)	16	4	20	0.6
E(PAC)	26	4	30	1.7
G(PAC)	42	8	50	3.3
H(PAC)	50	12	62	4.9
J(PAC)	56	14	70	6.8
K(PAC)	57	11	68	8.6
Support Tranches				
LA (SCH)	39	12	51	1.4
LB (SCH)	29	74	103	1.2
M(SCH)	72	53	125	4.9

**Prepayments at 80% and 120% of Prepayment Model
(Assumes 13% Interest Rate Volatility)**

	Base case OAS	New OAS (in basis points)		Change in price per \$100 par (holding OAS constant)	
		80%	120%	80%	120%
Collateral	60	63	57	\$0.17	−0.11
PAC Tranches					
C(PAC)	15	15	15	0.00	0.00
D(PAC)	16	16	16	0.00	0.00
E(PAC)	26	27	26	0.01	−0.01
G(PAC)	42	44	40	0.08	−0.08
H(PAC)	50	55	44	0.29	−0.27
J(PAC)	56	63	50	0.50	−0.47
K(PAC)	57	65	49	0.77	−0.76
Support Tranches					
LA(SCH)	39	31	39	−0.12	0.00
LB(SCH)	29	39	18	0.38	−0.19
M(SCH)	72	71	76	−0.07	0.18

Interest Rate Volatility of 9% and 17%

	Base case OAS	New OAS (in basis points)		Change in price per \$100 par (holding OAS constant)	
		9%	17%	9%	17%
Collateral	60	81	35	\$0.96	−0.94
PAC Tranches					
C(PAC)	15	15	15	0.00	0.00
D(PAC)	16	16	16	0.00	0.00
E(PAC)	26	27	24	0.02	−0.04
G(PAC)	42	48	34	0.21	−0.27
H(PAC)	50	58	35	0.48	−0.72
J(PAC)	56	66	41	0.70	−1.05
K(PAC)	57	66	44	0.82	−1.19
Support Tranches					
LA(SCH)	39	47	24	0.09	−0.18
LB(SCH)	29	58	−4	0.80	−0.82
M(SCH)	72	100	41	1.80	−1.72

The analysis reported in the top panel of Exhibit 7 help us identify where the cheap tranches are in the deal. The long average life and effective duration tranches in the deal are the PAC tranches G, H, J, and K. These tranches have high OAS relative to the other tranches and low option cost. They appear to be the cheap tranches in the deal. These PAC tranches had well protected cash flows and exhibited positive convexity (i.e., these tranches lose less in an adverse scenario than they gain in a positive scenario).

The next two panels in Exhibit 7 show the sensitivity of the OAS and the price (holding OAS constant at the base case) to changes in the prepayment speed (80% and 120% of the base case) and to changes in volatility (9% and 17%). This analysis shows that the change in the prepayment speed does not affect the collateral significantly, while the change in the OAS (holding the price constant) and price (holding OAS constant) for each tranche can be significant.

Tranches C and D at the time of the analysis were priced at a discount with short average lives. The OAS and price of these two tranches were not affected by a slowing down or a speeding up of the prepayment model. Tranche H was a premium tranche with a medium-term average life at the time of the analysis. Because tranche H was trading at a premium, it benefits from a slowing in prepayments, as the bondholder will receive the coupon for a longer time. Faster prepayments represent an adverse scenario. The PAC tranches are quite well-protected. The longer average life PACs will actually benefit from a reduced prepayment rate because they will be earning the higher coupon interest longer. So, on an OAS basis, the earlier conclusion that the long PACs were allocated a good part of the deal's value holds up under our first stress test (i.e., changing prepayments).

The sensitivity of the collateral and the tranches to changes in volatility are shown in the third panel of Exhibit 7. A lower volatility increases the value of the collateral, while a higher volatility reduces its value (This is consistent with our option cost equation in Section IV.E.) The long average life PACs continue to be fairly well-protected, whether the volatility is lower or higher. In the two volatility scenarios they continue to get a good OAS on a relative value basis, although not as much as in the base case if volatility is higher (but the OAS still looks like a reasonable value in this scenario). This reinforces the earlier conclusion concerning the investment merit of the long PACs in this deal. Note, however, that PAC tranches H, J, and K are more sensitive to the volatility assumption than tranches C, D, E, and G and therefore the investor is accepting greater volatility risk (i.e., the risk that volatility will change) with tranches H, J, and K relative to tranches C, D, E, and G.

V. MEASURING INTEREST RATE RISK

As explained in Chapter 9, duration and convexity can be used to estimate the interest rate exposure to parallel shifts in the yield curve (i.e., a measure of level risk). In this section we will discuss duration measures for mortgage-backed securities. There are several duration measures that are used in practice. Two researchers who have done extensive work in measuring duration, Lakhbir Hayre and Hubert Chang, conclude that "No single duration measure will consistently work well for mortgage securities."⁶ To that conclusion should be added that there are some measures that do not work at all.

⁶Lakhbir Hayre and Hubert Chang, "Effective and Empirical Duration of Mortgage Securities," *The Journal of Fixed Income* (March 1997), pp. 17–33.

A. Duration Measures

Duration is a measure of the price sensitivity to changes in interest rates. We have seen how to compute the duration of a security by shocking rates up and down and determining how the price of the security changes. Duration is then computed as follows:

$$\text{duration} = \frac{V_- - V_+}{2V_0(\Delta y)}$$

where

Δy = change in rate used to calculate new values (i.e., the interest rate shock)

V_+ = estimated value if yield is increased by Δy

V_- = estimated value if yield is decreased by Δy

V_0 = initial price (per \$100 of par value)

For bonds with embedded options such as mortgage-backed securities, the appropriate measure is effective duration and to capture the negative convexity of a bond with an embedded option, effective convexity should be computed. We will see how to calculate the effective duration for a mortgage-backed security using the Monte Carlo simulation model. Then we will see how the assumptions of the model impact the duration estimate. Dealers and vendors use other measures of duration that will be described later.

1. Effective Duration To calculate effective duration, the value of the security must be estimated when rates are shocked up and down a given number of basis points. In terms of the Monte Carlo model, the yield curve used (either the Treasury yield curve or LIBOR curve) is shocked up and down and the new curve is used to generate the values to be used in the effective duration and effective convexity formulas. This is analogous to the process we used to compute effective duration and effective convexity using the binomial model in Chapter 9.

In generating the prices when rates are shocked up and down, there is an assumption that the relationships assumed in generating the initial price do not change when rates are shocked up and down. Specifically, the yield volatility is assumed to be unchanged to derive the new interest rate paths for a given shock (i.e., the new Exhibit 1), the spread between the mortgage rate and the 10-year Treasury rate is assumed to be unchanged in constructing the new Exhibit 2 from the newly constructed Exhibit 1, and the OAS is assumed to be constant. The constancy of the OAS comes into play because when discounting the new cash flows (i.e., the cash flows in the new Exhibit 3), the current OAS that was computed is assumed to be the same and is added to the new rates in the new Exhibit 1.

We'll use an illustration by Lakhbir Hayre and Hubert Chang to explain the calculation of effective duration for a mortgage-backed security, a FNMA 7.5% TBA passthrough on May 1, 1996.⁷ On that day, the base mortgage rate was 7.64%. The price of the issue at the time was 98.781 (i.e., 98-25). The OAS was 65 basis points. Based on a shock of 25 basis points, the estimated prices holding the OAS constant at 65 basis points were as follows:

V_- = 99.949 for a decrease in the yield curve of 25 basis points

V_+ = 97.542 for an increase in the yield curve of 25 basis points

⁷Hayre and Chang, "Effective and Empirical Duration of Mortgage Securities."

The effective duration based on a Δy of 0.0025 is then

$$\frac{99.949 - 97.542}{2 \times 98.781 \times 0.0025} = 4.87$$

There are differences in the effective durations for a given mortgage-backed security reported by dealers and vendors of analytical systems. Several practitioners have explained and illustrated why there are differences in effective duration estimates reported by dealers and vendors. The differences result from, among others:⁸

1. differences in the amount of the rate shock used
2. differences in prepayment models
3. differences in option-adjusted spread
4. differences in the relationship between short-term interest rates and refinancing rates

Earlier, we discussed the first reason. As explained, the rate shock is the amount interest rates are increased and decreased to obtain the two values that are inserted into the effective duration formula. If the change is too large, there is the problem with picking up the effect of convexity.

Prepayment models differ across dealers and vendors. Some dealer models consistently forecast slower prepayments relative to other dealer models and others the reverse.

The effective duration is dependent on the OAS computed. Recall that the calculation of the OAS is a byproduct of the Monte Carlo model. Therefore, the computed value for the OAS depends on all of the assumptions in the Monte Carlo model. Specifically, it depends on the yield volatility assumed and the prepayment model employed. Dealers and vendors make different assumptions regarding yield volatility and use proprietary prepayment models. These can result in differences in OAS. Since the OAS is added to the new simulated short-term rates to compute the new values for V_- and V_+ , a different OAS will result in different effective durations.

Finally, recall that in explaining the Monte Carlo simulation model that we stated that in moving from Exhibit 1 (the simulated short-term rates) to Exhibit 2 (the refinancing rates), an assumption must be made about the relationship between short-term rates and the 10-year Treasury rate (i.e., the rate used as a proxy for refinancing). Differences in models about how large the spread between these rates will be affects the value of a mortgage-backed security and therefore the values used in the duration equation when rates are shocked.

2. Other Duration Measures There have been other measures proposed for estimating the duration of a mortgage-backed security. These measures include **cash flow duration**, **coupon curve duration**, and **empirical duration**. The first two duration measures are forms of effective duration in that they do recognize that the values that should be used in the duration formula should take into account how the cash flows may change due to changes in prepayments when interest rates change. In contrast, empirical duration is a duration that is computed statistically using observed market prices. Below we describe how each of these duration measures is calculated, as well as the advantages and limitations of each.

⁸Sam Choi, "Effective Durations for Mortgage-Backed Securities: Recipes for Improvement," *The Journal of Fixed Income* (March 1996), pp. 24–30; and, Hayre and Chang, "Effective and Empirical Duration of Mortgage Securities."

a. Cash Flow Duration Recall from the general duration formula that there are two values that must be substituted into the numerator of the formula—the value if rates are decreased (V_-) and the value if rates are increased (V_+). With effective duration, these two values consider how changes in interest rates change the cash flow due to prepayments. This is done through the Monte Carlo simulation by allowing for the cash flows to change on the interest rate paths.

For **cash flow duration**, there is recognition that the cash flow can change but the analysis to obtain the cash flow is done following a static methodology. Specifically, the cash flow duration is calculated as follows:

- Step 1:* Calculate the cash flow based on some prepayment assumption.
- Step 2:* From the cash flow in Step 1 and the market price (V_0), compute the cash flow yield.
- Step 3:* Increase the cash flow yield by Δy and from a prepayment model determine the new prepayment rate at that higher cash flow yield. Typically, the prepayment rate will be lower than in Step 1 because of the higher yield level.
- Step 4:* Using the lower prepayment rate in Step 3 determine the cash flow and then value the cash flow using the higher cash flow yield as the discount rate. This gives the value (V_+).
- Step 5:* Decrease the cash flow yield by Δy and from a prepayment model determine the new prepayment rate at that lower cash flow yield. Typically, the prepayment rate will be higher than in Step 1 because of the lower yield level.
- Step 6:* Using the higher prepayment rate in Step 5 determine the cash flow and then value the cash flow using the lower cash flow yield as the discount rate. This gives the value (V_-).

From the change in basis points (Δy), the values for V_+ and V_- found in Steps 4 and 6, and the initial value V_0 , the duration can be computed.

We can use the hypothetical CMO structure in Chapter 10 to illustrate how to calculate cash flow duration. Specifically, in FJF-2, there were four tranches, A, B, C, and Z. Let's focus on tranche C. Suppose that the price for this tranche is 100.2813. Then the cash flow duration is computed as follows:

- Step 1:* Suppose that the assumed prepayment rate for this tranche is 165 PSA.
- Step 2:* Based on the assumed prepayment rate of 165 PSA and the price of 100.2813, it can be demonstrated that the cash flow yield is 7%.
- Step 3:* Suppose that the cash flow yield is increased (i.e., shocked) by 25 basis points (from 7% to 7.25%) and suppose that some prepayment model projects a prepayment rate of 150 PSA. (Note that this is a slower prepayment rate than at 7%.)
- Step 4:* Based on 150 PSA, a new cash flow can be generated. The cash flow is then discounted at 7.25% (the new cash flow yield). It can be demonstrated that the value of this tranche based on these assumptions would be 98.3438. This is the value V_+ .
- Step 5:* Suppose that the cash flow yield is decreased (i.e., shocked) by 25 basis points (from 7% to 6.75%) and suppose that some prepayment model projects a prepayment rate of 200 PSA. (Note that this is a faster prepayment rate than at 7%.)

Step 6: Based on 200 PSA, the new cash flow can be generated. The cash flow is then discounted at 6.75% (the new cash flow yield). It can be demonstrated that the value of this tranche based on these assumptions would be 101.9063. This is the value V_- .

Now we have the following information:

$$\begin{aligned} V_0 &= 100.2813 \\ V_+ &= 98.3438 \\ V_- &= 101.9063 \\ \Delta y &= 0.0025 \end{aligned}$$

Then using the general form of duration, we obtain:

$$\text{duration} = \frac{101.9063 - 98.3438}{2(100.2813)(0.0025)} = 7.11$$

What type of duration measure is the cash flow duration—effective duration or modified duration? Technically, it is a form of effective duration because notice that in Steps 3 and 5 when the rate is changed, the cash flow is allowed to change. However, as has been stressed throughout, the valuation model that is used to get the new values to substitute into the duration formula is critical. The valuation model in the case of the cash flow duration is based on the naive assumption that there is a single prepayment rate over the life of the mortgage-backed security for any given interest rate shock. This is in contrast to the values produced by the Monte Carlo simulation model that does more sophisticated analyses of how the cash flow can change when interest rates change.

Why bother discussing the cash flow duration if it is an inferior form of effective duration? The reason is that it is a commonly cited duration measure and practitioners should be aware of how it is computed. Likewise, we did discuss in detail yield calculations despite their limitations.

An interesting question is how does this form of duration compare to modified duration. Recall from Chapter 7 and Chapter 9 that modified duration does not assume that cash flows will change when rates are shocked. That is, in the steps discussed above to obtain cash flow duration, in Steps 3 and 5 it is assumed that the prepayment rate is the same as in Step 1.

To illustrate this, we'll once again use tranche C in FJF-2. In Step 3, the prepayment rate assumed is still 165 PSA despite the fact that rates are assumed to increase. That is, the cash flow is not assumed to change. Based on a cash flow yield of 7.25% and a prepayment rate of 165 PSA, the value of this tranche would decline to 98.4063. When the cash flow yield is assumed to decline to 6.75%, the prepayment rate is still assumed to be 165 PSA and the value of this tranche would be 102.1875. Then to calculate the modified duration we know:

$$\begin{aligned} V_0 &= 100.2813 \\ V_+ &= 98.4063 \\ V_- &= 102.1875 \\ \Delta y &= 0.0025 \end{aligned}$$

The modified duration is then:

$$\text{duration} = \frac{102.1875 - 98.4063}{2(100.2813)(0.0025)} = 7.54$$

Thus, the modified duration is greater than the cash flow duration for this tranche.

It is important to reiterate that the modified duration is inferior to the cash flow duration because the former gives absolutely no recognition to how prepayments may change when interest rates change. While cash flow duration is commonly cited in practice, it is a form of effective duration that does give some recognition that prepayments and therefore cash flow may change when interest rates change, but it is based on a naive assumption about how prepayments may change. The effective duration as computed using Monte Carlo simulation is superior to cash flow duration.

b. Coupon Curve Duration The **coupon curve duration** uses market prices to estimate the duration of a mortgage-backed security. This approach, first suggested by Douglas Breeden,⁹ starts with the coupon curve of prices for similar mortgage-backed securities. The coupon curve represents generic passthrough securities of a particular issuer with different coupon rates. By rolling up and down the coupon curve of prices, the duration can be obtained. Because of the way it is estimated, this approach to duration estimation was referred to by Breeden as the “roll-up, roll-down approach.” The prices obtained from rolling up and rolling down the coupon curve of prices are substituted into the duration formula.

To illustrate this approach, suppose that the coupon curve of prices for a passthrough security for some month is as follows:

Coupon	Price
6%	85.19
7%	92.06
8%	98.38
9%	103.34
10%	107.28
11%	111.19

Suppose that the coupon curve duration for the 8% coupon passthrough is sought. If the yield declines by 100 basis points, the assumption is that the price of the 8% coupon passthrough will increase to the price of the current 9% coupon passthrough. Thus, the price will increase from 98.38 to 103.34. Similarly, if the yield increases by 100 basis points, the assumption is that the price of the 8% coupon passthrough will decline to the price of the 7% coupon passthrough (92.06). Using the duration formula, the corresponding values are:

$$V_0 = 98.38$$

$$V_+ = 92.06$$

$$V_- = 103.34$$

$$\Delta y = 0.01$$

⁹Douglas Breeden, “Risk, Return, and Hedging of Fixed-Rate Mortgages,” *The Journal of Fixed Income* (September 1991), pp. 85–107.

The estimated duration based on the coupon curve is then:

$$\text{duration} = \frac{103.34 - 92.06}{2(98.38)(0.01)} = 5.73$$

Breeden tested the coupon curve durations and found them to be relatively accurate in estimating the interest rate risk of generic passthrough securities.¹⁰ Bennett Golub reports a similar finding.¹¹

While the advantages of the coupon curve duration are the simplicity of its calculation and the fact that current prices embody market expectations, there are disadvantages. The approach is limited to generic mortgage-backed securities and difficult to use for mortgage derivatives such as CMOs.

c. Empirical Duration When computing effective duration and cash flow duration, the values to be substituted into the duration formula are those based on some valuation model. For coupon curve duration, the observed market prices are used in the duration formula. In contrast, **empirical duration** is estimated statistically using historical market prices and market yields.¹² Regression analysis is used to estimate the relationship. Some firms such as PaineWebber use empirical duration, also called **implied duration**, as their primary measure of the duration of an MBS.

There are three advantages to the empirical duration approach.¹³ First, the duration estimate does not rely on any theoretical formulas or analytical assumptions. Second, the estimation of the required parameters is easy to compute using regression analysis. Finally, the only inputs that are needed are a reliable price series and Treasury yield series.

There are disadvantages.¹⁴ First, a reliable price series for the mortgage security may not be available. For example, there may be no price series available for a thinly traded mortgage derivative security or the prices may be matrix priced (i.e., priced by a pricing service based on issues with similar characteristics) or model priced rather than actual transaction prices. Second, an empirical relationship does not impose a structure for the options embedded in a mortgage-backed security and this can distort the empirical duration. This may occur after a sharp and sustained shock to interest rates have been realized. Finally, the volatility of the spread to Treasury yields can distort how the price of a mortgage-backed security reacts to yield changes.

¹⁰Breeden, "Risk, Return, and Hedging of Fixed-Rate Mortgages."

¹¹See Bennett W. Golub, "Towards a New Approach to Measuring Mortgage Duration," Chapter 32 in Frank J. Fabozzi (ed.), *The Handbook of Mortgage-Backed Securities* (Chicago: Probus Publishing, 1995), p. 673.

¹²This approach was first suggested in 1986 in Scott M. Pinkus and Marie A. Chandoha, "The Relative Price Volatility of Mortgage Securities," *Journal of Portfolio Management* (Summer 1986), pp. 9–22 and then in 1990 by Paul DeRossa, Laurie Goodman, and Mike Zazzarino, "Duration Estimates on Mortgage-Backed Securities," *Journal of Portfolio Management* (Winter 1993), pp. 32–37, and more recently in Laurie S. Goodman and Jeffrey Ho, "Mortgage Hedge Ratios: Which One Works Best?" *The Journal of Fixed Income* (December 1997), pp. 23–33, and Laurie S. Goodman and Jeffrey Ho, "An Integrated Approach to Hedging and Relative Value Analysis," Chapter 15 in Frank J. Fabozzi (ed.), *Advances in the Valuation and Management of Mortgage-Backed Securities* (New Hope, PA: Frank J. Fabozzi Associates, 1999).

¹³Golub, "Towards a New Approach to Measuring Mortgage Duration," p. 672.

¹⁴Golub, "Towards a New Approach to Measuring Mortgage Duration."

VI. VALUING ASSET-BACKED SECURITIES

From the description of the Monte Carlo model, it can be seen that the valuation process is complex. Rather than build their own valuation model, portfolio managers typically use a model of a third-party vendor of analytical systems or a model of a dealer firm to value mortgage-backed securities. But mortgage-backed securities are only one type of structured product. Asset-backed securities are also structured products. Is it necessary to use the Monte Carlo model for all asset-backed securities? Below we will explain the circumstances as to when the Monte Carlo model must be used and when it is sufficient to use the Z-spread.

The model that should be used for valuing an asset-backed security (ABS) depends on the characteristic of the loans or receivables backing the deal. An ABS can have one of the following three characteristics:

Characteristic 1: The ABS does not have a prepayment option.

Characteristic 2: The ABS has a prepayment option but borrowers do not exhibit a tendency to prepay when refinancing rates fall below the loan rate.

Characteristic 3: The ABS has a prepayment option and borrowers do exhibit a tendency to prepay when refinancing rates fall below the loan rate.

An example of a Characteristic 1 type ABS is a security backed by credit card receivables. An example of a Characteristic 2 type ABS is a security backed by automobile loans. A security backed by closed-end home equity loans where the borrowers are high quality borrowers (i.e., prime borrowers) is an example of a Characteristic 3 type ABS. There are some real-estate backed ABS we discussed in Chapter 11 where the verdict is still out as to the degree to which borrowers take advantage of refinancing opportunities. Specifically, these include securities backed by manufactured housing loans and securities backed by closed-end home equity loans to borrowers classified as low quality borrowers.

There are two possible approaches to valuing an ABS. They are the

1. zero-volatility spread (Z-spread) approach
2. option-adjusted spread (OAS) approach

For the Z-spread approach the interest rates used to discount the cash flows are the spot rates plus the zero-volatility spread. The value of an ABS is then the present value of the cash flows based on these discount rates. The Z-spread approach does not consider the prepayment option. Consequently, the Z-spread approach should be used to value Characteristic 1 type ABS. (In terms of the relationship between the Z-spread, OAS, and option cost discussed earlier in this chapter, this means that the value of the option is zero and therefore the Z-spread is equal to the OAS.) Since the Z-spread is equal to the OAS, the Z-spread approach to valuation can be used.

The Z-spread approach can also be used to value Characteristic 2 type ABS because while the borrowers do have a prepayment option, the option is not typically exercised. Thus, as with Characteristic 1 type ABS, the Z-spread is equal to the OAS.

The OAS approach—which is considerably more computationally extensive than the Z-spread approach—is used to value securities where there is an embedded option and there is an expectation that the option is expected to be exercised if it makes economic sense for the borrower to do so. Consequently, the OAS approach is used to value Characteristic 3

type ABS. The choice is then whether to use the binomial model (or a comparable model) or the Monte Carlo simulation model. Since typically the cash flow for an ABS with a prepayment option is interest rate path dependent—as with a mortgage-backed security—the Monte Carlo simulation model is used.

VII. VALUING ANY SECURITY

We conclude this chapter with a summary of the approaches to valuing any fixed income security using the two approaches that we discussed in the previous section—the Z-spread approach and the OAS approach.

Below we match the valuation approach with the type of security.

1. For an *option-free bond* the correct approach is the Z-spread approach.
2. For a *bond with an embedded option where the cash flow is not interest rate path dependent* (such as a callable corporate or agency debenture bond or a puttable bond) the correct approach is the OAS approach. Since the backward induction method can be used for such bonds, the binomial model or its equivalent should be used.
3. For a bond with an embedded option where the cash flow is interest rate path dependent (such as a mortgage-backed security or certain real estate-backed ABS) the correct approach is the OAS approach. However, because of the interest rate path dependency of the cash flow, the Monte Carlo simulation model should be used.

CHAPTER 13

INTEREST RATE DERIVATIVE INSTRUMENTS

I. INTRODUCTION

In this chapter we turn our attention to financial contracts that are popularly referred to as **interest rate derivative instruments** because they derive their value from some cash market instrument or reference interest rate. These instruments include futures, forwards, options, swaps, caps, and floors. In this chapter we will discuss the basic features of these instruments and in the next we will see how they are valued.

Why would a portfolio manager be motivated to use interest rate derivatives rather than the corresponding cash market instruments. There are three principal reasons for doing this when there is a well-developed interest rate derivatives market for a particular cash market instrument. First, typically it costs less to execute a transaction or a strategy in the interest rate derivatives market in order to alter the interest rate risk exposure of a portfolio than to make the adjustment in the corresponding cash market. Second, portfolio adjustments typically can be accomplished faster in the interest rate derivatives market than in the corresponding cash market. Finally, interest rate derivative may be able to absorb a greater dollar transaction amount without an adverse effect on the price of the derivative instrument compared to the price effect on the cash market instrument; that is, the interest rate derivative may be more liquid than the cash market. To summarize: There are three potential advantages that motivate the use of interest rate derivatives: cost, speed, and liquidity.

II. INTEREST RATE FUTURES

A **futures contract** is an agreement that requires a party to the agreement either to buy or sell something at a designated future date at a predetermined price. Futures contracts are products created by exchanges. Futures contracts based on a financial instrument or a financial index are known as **financial futures**. Financial futures can be classified as (1) stock index futures, (2) interest rate futures, and (3) currency futures. Our focus in this chapter is on interest rate futures.

A. Mechanics of Futures Trading

A futures contract is an agreement between a buyer (seller) and an established exchange or its clearinghouse in which the buyer (seller) agrees to take (make) delivery of something (the **underlying**) at a specified price at the end of a designated period of time. The price at which

the parties agree to transact in the future is called the **futures price**. The designated date at which the parties must transact is called the **settlement date** or **delivery date**.

1. Liquidating a Position Most financial futures contracts have settlement dates in the months of March, June, September, and December. This means that at a predetermined time in the contract settlement month the contract stops trading, and a price is determined by the exchange for settlement of the contract. The contract with the closest settlement date is called the **nearby futures contract**. The next futures contract is the one that settles just after the nearby futures contract. The contract farthest away in time from settlement is called the **most distant futures contract**.

A party to a futures contract has two choices on liquidation of the position. First, the position can be liquidated prior to the settlement date. For this purpose, the party must take an offsetting position in the same contract. For the buyer of a futures contract, this means selling the same number of the identical futures contracts; for the seller of a futures contract, this means buying the same number of identical futures contracts.

The alternative is to wait until the settlement date. At that time the party purchasing a futures contract accepts delivery of the underlying at the agreed-upon price; the party that sells a futures contract liquidates the position by delivering the underlying at the agreed-upon price. For some interest rate futures contracts, settlement is made in cash only. Such contracts are referred to as **cash settlement contracts**.

2. The Role of the Clearinghouse Associated with every futures exchange is a clearinghouse, which performs several functions. One of these functions is to guarantee that the two parties to the transaction will perform.

When an investor takes a position in the futures market, the clearinghouse takes the opposite position and agrees to satisfy the terms set forth in the contract. Because of the clearinghouse, the investor need not worry about the financial strength and integrity of the party taking the opposite side of the contract. After initial execution of an order, the relationship between the two parties ends. The clearinghouse interposes itself as the buyer for every sale and the seller for every purchase. Thus investors are free to liquidate their positions without involving the other party in the original contract, and without worrying that the other party may default. This is the reason that we define a futures contract as an agreement between a party and a clearinghouse associated with an exchange. Besides its guarantee function, the clearinghouse makes it simple for parties to a futures contract to unwind their positions prior to the settlement date.

3. Margin Requirements When a position is first taken in a futures contract, the investor must deposit a minimum dollar amount per contract as specified by the exchange. This amount is called **initial margin** and is required as deposit for the contract. The initial margin may be in the form of an interest-bearing security such as a Treasury bill. As the price of the futures contract fluctuates, the value of the margin account changes. Marking to market means effectively replacing the initiation price with a current settlement price. The contract thus has a new settlement price. At the end of each trading day, the exchange determines the current settlement price for the futures contract. This price is used to mark to market the investor's position, so that any gain or loss from the position is reflected in the margin account.¹

¹For a further discussion of margin requirements and illustrations of how the margin account changes as the futures price changes, see Don M. Chance, *Analysis of Derivatives for the CFA*

Maintenance margin is the minimum level (specified by the exchange) to which the margin account may fall to as a result of an unfavorable price movement before the investor is required to deposit additional margin. The additional margin deposited is called **variation margin**, and it is an amount necessary to bring the account back to its initial margin level. This amount is determined from the process of marking the position to market. Unlike initial margin, variation margin must be in cash, not interest-bearing instruments. Any excess margin in the account may be withdrawn by the investor. If a party to a futures contract who is required to deposit variation margin fails to do so within 24 hours, the futures position is closed out.

Although there are initial and maintenance margin requirements for buying securities on margin, the concept of margin differs for securities and futures. When securities are acquired on margin, the difference between the price of the security and the initial margin is borrowed from the broker. The security purchased serves as collateral for the loan, and the investor pays interest. For futures contracts, the initial margin, in effect, serves as “good faith” money, an indication that the investor will satisfy the obligation of the contract.

B. Forward Contracts

A **forward contract**, just like a futures contract, is an agreement for the future delivery of something at a specified price at the end of a designated period of time. Futures contracts are standardized agreements as to the delivery date (or month) and quality of the deliverable, and are traded on organized exchanges. A forward contract differs in that it is usually non-standardized (that is, the terms of each contract are negotiated individually between buyer and seller), there is no clearinghouse, and secondary markets are often non-existent or extremely thin. Unlike a futures contract, which is an exchange-traded product, a forward contract is an over-the-counter instrument.

Futures contracts are marked to market at the end of each trading day. Consequently, futures contracts are subject to interim cash flows as additional margin may be required in the case of adverse price movements, or as cash is withdrawn in the case of favorable price movements. A forward contract *may or may not be marked to market*, depending on the wishes of the two parties. For a forward contract that is *not* marked to market, there are no interim cash flow effects because no additional margin is required.

Finally, the parties in a forward contract are exposed to credit risk because either party may default on its obligation. This risk is called **counterparty risk**. This risk is minimal in the case of futures contracts because the clearinghouse associated with the exchange guarantees the other side of the transaction. In the case of a forward contract, both parties face counterparty risk. Thus, there exists **bilateral counterparty risk**.

Other than these differences, most of what we say about futures contracts applies equally to forward contracts.

C. Risk and Return Characteristics of Futures Contracts

When an investor takes a position in the market by buying a futures contract, the investor is said to be in **along position** or to **belong futures**. The buyer of the futures contract is also

referred to as the “long.” If, instead, the investor’s opening position is the sale of a futures contract, the investor is said to be in a **short position** or to be **short futures**. The seller of the futures contract is also referred to as the “short.” The buyer of a futures contract will realize a profit if the futures price increases; the seller of a futures contract will realize a profit if the futures price decreases.

When a position is taken in a futures contract, the party need not put up the entire amount of the investment. Instead, only initial margin must be put up. Consequently, an investor can effectively create a leveraged position by using futures. At first, the leverage available in the futures market may suggest that the market benefits only those who want to speculate on price movements. This is not true. As we shall see, futures markets can be used to control interest rate risk. Without the effective leverage possible in futures transactions, the cost of reducing price risk using futures would be too high for many market participants.

D. Exchange-Traded Interest Rate Futures Contracts

Interest rate futures contracts can be classified by the maturity of their underlying security. Short-term interest rate futures contracts have an underlying security that matures in less than one year. Examples of these are futures contracts in which the underlying is a 3-month Treasury bill and a 3-month Eurodollar certificate of deposit. The maturity of the underlying security of long-term futures contracts exceeds one year. Examples of these are futures contracts in which the underlying is a Treasury coupon security, a 10-year agency note, and a municipal bond index. Our focus will be on futures contracts in which the underlying is a Treasury coupon security (a Treasury bond or a Treasury note). These contracts are the most widely used by managers of bond portfolios and we begin with the specifications of the Treasury bond futures contract. We will also discuss the agency note futures contracts.

There are futures contracts on non-U.S. government securities traded throughout the world. Many of them are modeled after the U.S. Treasury futures contracts and consequently, the concepts discussed below apply directly to those futures contracts.

1. Treasury Bond Futures The Treasury bond futures contract is traded on the Chicago Board of Trade (CBOT). The underlying instrument for a Treasury bond futures contract is \$100,000 par value of a hypothetical 20-year coupon bond. The coupon rate on the hypothetical bond is called the **notional coupon**.

The futures price is quoted in terms of par being 100. Quotes are in 32nds of 1%. Thus a quote for a Treasury bond futures contract of 97–16 means 97 and $\frac{16}{32}$ or 97.50. So, if a buyer and seller agree on a futures price of 97–16, this means that the buyer agrees to accept delivery of the hypothetical underlying Treasury bond and pay 97.50% of par value and the seller agrees to accept 97.50% of par value. Since the par value is \$100,000, the futures price that the buyer and seller agree to for this hypothetical Treasury bond is \$97,500.

The minimum price fluctuation for the Treasury bond futures contract is $\frac{1}{32}$ of 1%, which is referred to as “a 32nd.” The dollar value of a 32nd for \$100,000 par value (the par value for the underlying Treasury bond) is \$31.25. Thus, the minimum price fluctuation is \$31.25 for this contract.

EXHIBIT 1 U.S. Treasury Bond Issues Acceptable for Delivery and Conversion Factors

Eligible for Delivery as of May 29, 2002.

Issue		Conversion factors											
		Mar. 2005	Jun. 2005	Sep. 2005	Dec. 2005	Mar. 2006	Jun. 2006	Sep. 2006	Dec. 2006	Mar. 2007	Jun. 2007		
5 ¹ / ₄	11/15/28	0.9062	0.9065	0.9071	0.9075	0.9081	0.9084	0.9090	0.9095	0.9101	0.9105		
5 ¹ / ₄	02/15/29	0.9056	0.9062	0.9065	0.9071	0.9075	0.9081	0.9084	0.9090	0.9095	0.9101		
5 ³ / ₈	02/15/31	0.9185	0.9189	0.9191	0.9196	0.9198	0.9203	0.9206	0.9210	0.9213	0.9218		
5	08/15/28	0.9376	0.9381	0.9383	0.9387	0.9389	0.9394	0.9396	0.9400	0.9403	0.9407		
6	02/15/26	0.9999	1.0000	0.9999	1.0000	0.9999	1.0000	0.9999	1.0000	0.9999	1.0000		
6 ¹ / ₈	11/15/27	1.0153	1.0151	1.0152	1.0150	1.0150	1.0148	1.0148	1.0146	1.0146	1.0144		
6 ¹ / ₈	08/15/29	1.0158	1.0158	1.0156	1.0156	1.0154	1.0155	1.0153	1.0153	1.0151	1.0152		
6 ¹ / ₄	08/15/23	1.0274	1.0273	1.0270	1.0269	1.0265	1.0264	1.0261	1.0260	1.0256	1.0255		
6 ¹ / ₄	05/15/30	1.0322	1.0319	1.0319	1.0316	1.0316	1.0313	1.0313	1.0310	1.0310	1.0307		
6 ³ / ₈	08/15/27	1.0456	1.0455	1.0451	1.0450	1.0446	1.0444	1.0441	1.0439	1.0435	1.0433		
6	11/15/26	1.0600	1.0595	1.0593	1.0588	1.0585	1.0580	1.0578	1.0573	1.0570	1.0565		
6 ⁵ / ₈	02/15/27	1.0752	1.0749	1.0744	1.0741	1.0735	1.0732	1.0726	1.0722	1.0716	1.0713		
6 ³ / ₄	08/15/26	1.0893	1.0889	1.0882	1.0878	1.0871	1.0867	1.0860	1.0855	1.0848	1.0843		
6 ⁷ / ₈	08/15/25	1.1017	1.1011	1.1003	1.0998	1.0990	1.0984	1.0976	1.0970	1.0961	1.0955		
7 ¹ / ₈	02/15/23	1.1217	1.1209	1.1197	1.1189	1.1177	1.1168	1.1156	1.1147	1.1135	1.1125		
7 ¹ / ₄	08/15/22	1.1331	1.1321	1.1308	1.1298	1.1285	1.1274	1.1261	1.1250	1.1236	1.1225		
7	11/15/24	1.1711	1.1697	1.1687	1.1673	1.1663	1.1649	1.1637	1.1623	1.1612	1.1597		
7 ⁵ / ₈	11/15/22	1.1746	1.1730	1.1717	1.1701	1.1687	1.1671	1.1657	1.1640	1.1625	1.1607		
7 ⁵ / ₈	02/15/25	1.1864	1.1853	1.1839	1.1828	1.1813	1.1801	1.1786	1.1774	1.1759	1.1746		
7 ⁷ / ₈	02/15/21	1.1892	1.1875	1.1855	1.1838	—	—	—	—	—	—		
8	11/15/21	1.2077	1.2056	1.2039	1.2018	1.2000	1.1979	1.1960	—	—	—		
8 ¹ / ₈	05/15/21	1.2166	1.2144	1.2125	1.2102	1.2083	—	—	—	—	—		
8 ¹ / ₈	08/15/21	1.2185	1.2166	1.2144	1.2125	1.2102	1.2083	—	—	—	—		
8 ³ / ₄	05/15/20	1.2695	—	—	—	—	—	—	—	—	—		
8 ³ / ₄	08/15/20	1.2721	1.2695	—	—	—	—	—	—	—	—		
No. of Eligible issues		25	24	23	23	22	21	20	19	19	19		

Source: Chicago Board of Trade

We have been referring to the underlying as a hypothetical Treasury bond. The seller of a Treasury bond futures contract who decides to make delivery rather than liquidate the position by buying back the contract prior to the settlement date must deliver some Treasury bond issue. But what Treasury bond issue? The CBOT allows the seller to deliver one of several Treasury bonds that the CBOT designates as acceptable for delivery. The specific issues that the seller may deliver are published by the CBOT for all contracts by settlement date. The CBOT makes its determination of the Treasury bond issues that are acceptable for delivery from all outstanding Treasury bond issues that have at least 15 years to maturity from the date of delivery.

Exhibit 1 shows the Treasury bond issues that the seller could have selected to deliver to the buyer of the CBOT Treasury bond futures contract as of May 29, 2002. Should the U.S. Department of the Treasury issue any Treasury bonds that meet the CBOT criteria for eligible delivery, those issues would be added to the list. Notice that for the Treasury bond futures contract settling (i.e., maturing) in March 2005, notice that there are 25 eligible issues. For contracts settling after March 2005, there are fewer than 25 eligible issues due to the shorter maturity of each previous eligible issue that results in a maturity of less than 15 years.

Although the underlying Treasury bond for this contract is a hypothetical issue and therefore cannot itself be delivered into the futures contract, the contract is not a cash settlement contract. The only way to close out a Treasury bond futures contract is to either initiate an offsetting futures position, or to deliver a Treasury bond issue satisfying the above-mentioned criteria into the futures contract.

a. Conversion Factors The delivery process for the Treasury bond futures contract makes the contract interesting. At the settlement date, the seller of a futures contract (the short) is now required to deliver to the buyer (the long) \$100,000 par value of a 6% 20-year Treasury bond. Since no such bond exists, the seller must choose from one of the acceptable deliverable Treasury bonds that the CBOT has specified. Suppose the seller is entitled to deliver \$100,000 of a 5% 20-year Treasury bond to settle the futures contract. The value of this bond is less than the value of a 6% 20-year bond. If the seller delivers the 5% 20-year bond, this would be unfair to the buyer of the futures contract who contracted to receive \$100,000 of a 6% 20-year Treasury bond. Alternatively, suppose the seller delivers \$100,000 of a 7% 20-year Treasury bond. The value of a 7% 20-year Treasury bond is greater than that of a 6% 20-year bond, so this would be a disadvantage to the seller.

How can this problem be resolved? To make delivery equitable to both parties, the CBOT has introduced conversion factors for adjusting the price of each Treasury issue that can be delivered to satisfy the Treasury bond futures contract. The conversion factor is determined by the CBOT before a contract with a specific settlement date begins trading.² The adjusted price is found by multiplying the conversion factor by the futures price. The adjusted price is called the **converted price**.

Exhibit 1 shows conversion factors as of May 29, 2002. The conversion factors are shown by contract settlement date. Note that the conversion factor depends not only on the issue delivered but also on the settlement date of the contract. For example, look at the first issue in Exhibit 1, the 5 1/4% coupon bond maturing 11/15/28. For the Treasury bond futures contract settling (i.e., maturing) in March 2005, the conversion factor is 0.9062. For the December 2005 contract, the conversion factor is 0.9075.

²The conversion factor is based on the price that a deliverable bond would sell for at the beginning of the delivery month if it were to yield 6%.

The price that the buyer must pay the seller when a Treasury bond is delivered is called the **invoice price**. The invoice price is the futures settlement price plus accrued interest. However, as just noted, the seller can deliver one of several acceptable Treasury issues and to make delivery fair to both parties, the invoice price must be adjusted based on the actual Treasury issue delivered. It is the conversion factors that are used to adjust the invoice price. The invoice price is:

$$\begin{aligned} \text{invoice price} &= \text{contract size} \times \text{futures settlement price} \times \text{conversion factor} \\ &+ \text{accrued interest} \end{aligned}$$

Suppose the Treasury March 2006 futures contract settles at 105–16 and that the issue delivered is the 8% of 11/15/21. The futures contract settlement price of 105–16 means 105.5% of par value or 1.055 times par value. As indicated in Exhibit 1, the conversion factor for this issue for the March 2006 contract is 1.2000. Since the contract size is \$100,000, the invoice price the buyer pays the seller is:

$$\$100,000 \times 1.055 \times 1.2000 + \text{accrued interest} = \$126,600 + \text{accrued interest}$$

b. Cheapest-to-Deliver Issue As can be seen in Exhibit 1, there can be more than one issue that is permitted to be delivered to satisfy a futures contract. In fact, for the March 2005 contract, there are 25 deliverable or eligible bond issues. It is the short that has the option of selecting which one of the deliverable bond issues if he decides to deliver.³ The decision of which one of the bond issues a short will elect to deliver is *not* made arbitrarily. There is an economic analysis that a short will undertake in order to determine the best bond issue to deliver. In fact, as we will see, all of the elements that go into the economic analysis will be the same for all participants in the market who are either electing to deliver or who are anticipating delivery of one of the eligible bond issues. In this section, how the best bond issue to deliver is determined will be explained.

The economic analysis is not complicated. The basic principle is as follows. Suppose that an investor enters into the following two transactions *simultaneously*:

1. buys one of the deliverable bond issues today with borrowed money and
2. sells a futures contract

The two positions (i.e., the long position in the deliverable bond issue purchased and the short position in the futures contract) will be held to the delivery date. At the delivery date, the bond issue purchased will be used to satisfy the short's obligation to deliver an eligible bond issue. The simultaneous transactions above and the delivery of the acceptable bond issue purchased to satisfy the short position in the futures contract is called a *cash and carry trade*. We will discuss this in more detail in the next chapter where the importance of selecting the best bond issue to deliver for the pricing of a futures contract is explained.

Let's look at the economics of this cash and carry trade. The investor (who by virtue of the fact that he sold a futures contract is the short), has synthetically created a short-term investment vehicle. The reason is that the investor has purchased a bond issue (one of the

³Remember that the short can always unwind his position by buying the same futures contract before the settlement date.

deliverable bond issues) and at the delivery date delivers that bond issue and receives the futures price. So, the investor knows the cost of buying the bond issue and knows how much will be received from the investment. The amount received is the coupon interest until the delivery date, any reinvestment income from reinvesting coupon payments, and the futures price at the delivery date. (Remember that the futures price at the delivery date for a given deliverable bond issue will be its converted price.) Thus, the investor can calculate the rate of return that will be earned on the investment. In the futures market, this rate of return is called the **implied repo rate**.

An implied repo rate can be calculated for *every* deliverable bond issue. For example, suppose that there are N deliverable bond issues that can be delivered to satisfy a bond futures contract. Market participants who want to know either the best issue to deliver or what issue is likely to be delivered will calculate an implied repo rate for all N eligible bond issues. Which would be the best issue to deliver by a short? Since the implied repo rate is the rate of return on an investment, the best bond issue is the one that has the *highest* implied repo rate (i.e., the highest rate of return). The bond issue with the highest implied repo rate is called the **cheapest-to-deliver issue**.

Now that we understand the economic principle for determining the best bond issue to deliver (i.e., the cheapest-to-deliver issue), let's look more closely at how one calculates the implied repo rate for each deliverable bond issue. This rate is computed using the following information for a given deliverable bond issue:

1. the price plus accrued interest at which the Treasury issue could be purchased
2. the converted price plus the accrued interest that will be received upon delivery of that Treasury bond issue to satisfy the short futures position
3. the coupon payments that will be received between today and the date the issue is delivered to satisfy the futures contract.
4. the reinvestment income that will be realized on the coupon payments between the time the interim coupon payment is received and the date that the issue is delivered to satisfy the Treasury bond futures contract.

The first three elements are known. The last element will depend on the reinvestment rate that can be earned. While the reinvestment rate is unknown, typically this is a small part of the rate of return and not much is lost by assuming that the implied repo rate can be predicted with certainty.

The general formula for the implied repo rate is as follows:

$$\text{implied repo rate} = \frac{\text{dollar return}}{\text{cost of the investment}} \times \frac{360}{\text{days}_1}$$

where days_1 is equal to the number of days until settlement of the futures contract. Below we will explain the other components in the formula for the implied repo rate.

Let's begin with the dollar return. The **dollar return** for an issue is the difference between the **proceeds received** and the **cost of the investment**. The proceeds received are equal to the proceeds received at the settlement date of the futures contract and any interim coupon payment plus interest from reinvesting the interim coupon payment. The proceeds received at the settlement date include the converted price (i.e., futures settlement price multiplied by the conversion factor for the issue) and the accrued interest received from delivery of the issue. That is,

$$\begin{aligned} \text{proceeds received} &= \text{converted price} + \text{accrued interest received} + \text{interim coupon payment} \\ &\quad + \text{interest from reinvesting the interim coupon payment} \end{aligned}$$

As noted earlier, all of the elements are known except the interest from reinvesting the interim coupon payment. This amount is estimated by assuming that the coupon payment can be reinvested at the term repo rate. Later, we describe the repo market and the term repo rate. The **term repo rate** is not only a borrowing rate for an investor who wants to borrow in the repo market but also the rate at which an investor can invest proceeds on a short-term basis. For how long is the reinvestment of the interim coupon payment? It is the number of days from when the interim coupon payment is received and the actual delivery date to satisfy the futures contract. The reinvestment income is then computed as follows:

$$\begin{aligned} &\text{interest from reinvesting the interim coupon payment} \\ &= \text{interim coupon} \times \text{term repo rate} \times (\text{days}_2/360) \end{aligned}$$

where

$$\begin{aligned} \text{days}_2 &= \text{number of days between when the interim coupon payment is received and} \\ &\quad \text{the actual delivery date of the futures contract} \end{aligned}$$

The reason for dividing days_2 by 360 is that the ratio represents the number of days the interim coupon is reinvested as a percentage of the number of days in a year as measured in the money market.

The cost of the investment is the amount paid to purchase the issue. This cost is equal to the purchase price plus accrued interest paid. That is,

$$\text{cost of the investment} = \text{purchase price} + \text{accrued interest paid}$$

Thus, the dollar return for the numerator of the formula for the implied repo rate is equal to

$$\text{dollar return} = \text{proceeds received} - \text{cost of the investment}$$

The dollar return is then divided by the cost of the investment.⁴

So, now we know how to compute the numerator and the denominator in the formula for the implied repo rate. The second ratio in the formula for the implied repo rate simply involves annualizing the return using a convention in the money market for the number of days. (Recall that in the money market the convention is to use a 360 day year.) Since the investment resulting from the cash and carry trade is a synthetic money market instrument, 360 days are used.

Let's compute the implied repo rate for a hypothetical issue that may be delivered to satisfy a hypothetical Treasury bond futures contract. Assume the following for the deliverable issue and the futures contract:

⁴Actually, the cost of the investment should be adjusted because the amount that the investor ties up in the investment is reduced if there is an interim coupon payment. We will ignore this adjustment here.

Futures contract

futures price = 96

days to futures delivery date (days_1) = 82 days

Deliverable issue

price of issue = 107

accrued interest paid = 3.8904

coupon rate = 10%

days remaining before interim coupon paid = 40 days

interim coupon = \$5

number of days between when the interim coupon payment is received and the actual delivery date of the futures contract (days_2) = 42

conversion factor = 1.1111

accrued interest received at futures settlement date = 1.1507

Other information:

42-day term repo rate = 3.8%

Let's begin with the proceeds received. We need to compute the converted price and the interest from reinvesting the interim coupon payment. The converted price is:

$$\begin{aligned}\text{converted price} &= \text{futures price} \times \text{conversion factor} \\ &= 96 \times 1.1111 = 106.6656\end{aligned}$$

The interest from reinvesting the interim coupon payment depends on the term repo rate. The term repo rate is assumed to be 3.8%. Therefore,

$$\begin{aligned}\text{interest from reinvesting the interim coupon payment} &= \$5 \times 0.038 \times \left(\frac{42}{360} \right) \\ &= 0.0222\end{aligned}$$

To summarize:

converted price	= 106.6656
accrued interest received at futures settlement date	= 1.1507
interim coupon payment	= 5.0000
interest from reinvesting the interim coupon payment	= 0.0222
proceeds received	= 112.8385

The cost of the investment is the purchase price for the issue plus the accrued interest paid, as shown below:

$$\text{cost of the investment} = 107 + 3.8904 = 110.8904$$

EXHIBIT 2 Delivery Options Granted to the Short (Seller) of a CBOT Treasury Bond Futures Contract

Delivery option	Description
Quality or swap option	Choice of which acceptable Treasury issue to deliver
Timing option	Choice of when in delivery month to deliver
Wild card option	Choice to deliver after the closing price of the futures contract is determined

The implied repo rate is then:

$$\text{implied repo rate} = \frac{112.8385 - 110.8904}{110.8904} \times \frac{360}{82} = 0.0771 = 7.71\%$$

Once the implied repo rate is calculated for each deliverable issue, the cheapest-to-deliver issue will be the one that has the highest implied repo rate (i.e., the issue that gives the maximum return in a cash-and-carry trade). As explained in the next chapter, this issue plays a key role in the pricing of a Treasury bond futures contract.

While an eligible bond issue may be the cheapest to deliver today, changes in factors may cause some other eligible bond issue to be the cheapest to deliver at a future date. A sensitivity analysis can be performed to determine how a change in yield affects the cheapest to deliver.

c. Other Delivery Options In addition to the choice of which acceptable Treasury issue to deliver—sometimes referred to as the **quality option** or **swap option**—the short has at least two more options granted under CBOT delivery guidelines. The short is permitted to decide when in the delivery month delivery actually will take place. This is called the **timing option**. The other option is the right of the short to give notice of intent to deliver up to 8:00 p.m. Chicago time after the closing of the exchange (3:15 p.m. Chicago time) on the date when the futures settlement price has been fixed. This option is referred to as the **wild card option**. The quality option, the timing option, and the wild card option (in sum referred to as the **delivery options**), mean that the long position can never be sure which Treasury bond will be delivered or when it will be delivered. These three delivery options are summarized in Exhibit 2.

d. Delivery Procedure For a short who wants to deliver, the delivery procedure involves three days. The first day is the **position day**. On this day, the short notifies the CBOT that it intends to deliver. The short has until 8:00 p.m. central standard time to do so. The second day is the **notice day**. On this day, the short specifies which particular issue will be delivered. The short has until 2:00 p.m. central standard time to make this declaration. (On the last possible notice day in the delivery month, the short has until 3:00 p.m.) The CBOT then selects the long to whom delivery will be made. This is the long position that has been outstanding for the greatest period of time. The long is then notified by 4:00 p.m. that delivery will be made. The third day is the **delivery day**. By 10:00 a.m. on this day the short must have in its account the Treasury issue that it specified on the notice day and by 1:00 p.m. must deliver that bond to the long that was assigned by the CBOT to accept delivery. The long pays the short the invoice price upon receipt of the bond.

2. Treasury Note Futures The three Treasury note futures contracts are 10-year, 5-year, and 2-year note contracts. All three contracts are modeled after the Treasury bond futures contract and are traded on the CBOT.

The underlying instrument for the 10-year Treasury note futures contract is \$100,000 par value of a hypothetical 10-year, 6% Treasury note. Several acceptable Treasury issues may be delivered by the short. An issue is acceptable if the maturity is not less than 6.5 years and not greater than 10 years from the first day of the delivery month. Delivery options are granted to the short position.

For the 5-year Treasury note futures contract, the underlying is \$100,000 par value of a 6% notional coupon U.S. Treasury note that satisfies the following conditions: (1) an original maturity of not more than 5 years and 3 months, (2) a remaining maturity no greater than 5 years and 3 months, and (3) a remaining maturity not less than 4 years and 2 months.

The underlying for the 2-year Treasury note futures contract is \$200,000 par value of a 6% notional coupon U.S. Treasury note with a remaining maturity of not more than 2 years and not less than 1 year and 9 months. Moreover, the original maturity of the note delivered to satisfy the 2-year futures cannot be more than 5 years and 3 months.

3. Agency Note Futures Contract In 2000, the CBOT and the Chicago Mercantile Exchange (CME) began trading in futures contracts in which the underlying is a Fannie Mae or Freddie Mac agency debenture security. (Agency debentures are explained in Chapter 3.) The underlying for the CBOT 10-year agency note futures contract is a Fannie Mae benchmark note or Freddie Mac reference note having a par value of \$100,000 and a notional coupon of 6%. The 10-year agency note futures contract of the CME is similar to that of the CBOT, but has a notional coupon of 6.5% instead of 6%.

As with the Treasury futures contract, more than one issue is deliverable for both the CBOT and CME agency note futures contract. The contract delivery months are March, June, September, and December. As with the Treasury futures contract a conversion factor applies to each eligible issue for each contract settlement date. Because many issues are deliverable, one issue is the cheapest-to-deliver issue. This issue is found in exactly the same way as with the Treasury futures contract.

III. INTEREST RATE OPTIONS

An **option** is a contract in which the writer of the option grants the buyer of the option the right, but not the obligation, to purchase from or sell to the writer something at a specified price within a specified period of time (or at a specified date). The **writer**, also referred to as the **seller**, grants this right to the buyer in exchange for a certain sum of money, called the **option price** or **option premium**. The price at which the underlying for the contract may be bought or sold is called the **exercise price** or **strike price**. The date after which an option is void is called the **expiration date**. Our focus is on options where the “something” underlying the option is an interest rate instrument or an interest rate.

When an option grants the buyer the right to purchase the designated instrument from the writer (seller), it is referred to as a **call option**, or **call**. When the option buyer has the right to sell the designated instrument to the writer, the option is called a **put option**, or **put**.

An option is also categorized according to when the option buyer may exercise the option. There are options that may be exercised at any time up to and including the expiration date. Such an option is referred to as an **American option**. There are options that may be exercised only at the expiration date. An option with this feature is called a **European option**. An option that can be exercised prior to maturity but only on designated dates is called a **modified American, Bermuda, or Atlantic option**.

A. Risk and Return Characteristics of Options

The maximum amount that an option buyer can lose is the option price. The maximum profit that the option writer can realize is the option price at the time of sale. The option buyer has substantial upside return potential, while the option writer has substantial downside risk.

It is assumed in this chapter that the reader has an understanding of the basic positions that can be created with options. These positions include:

1. long call position (buying a call option)
2. short call position (selling a call option)
3. long put position (buying a put option)
4. short put position (selling a put option)

Exhibit 3 shows the payoff profile for these four option positions *assuming that each option position is held to the expiration date and not exercised early*.

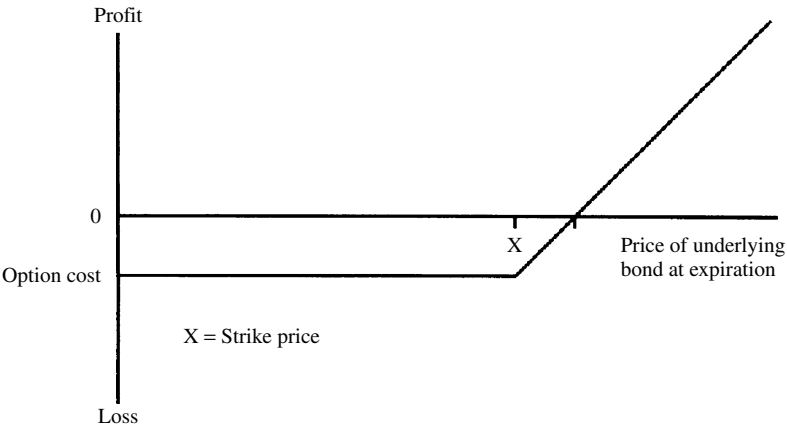
B. Differences Between Options and Futures Contracts

Unlike a futures contract, one party to an option contract is not obligated to transact. Specifically, the option buyer has the right, but not the obligation, to transact. The option writer does have the obligation to perform. In the case of a futures contract, both buyer and seller are obligated to perform. Of course, a futures buyer does not pay the seller to accept the obligation, while an option buyer pays the option seller an option price.

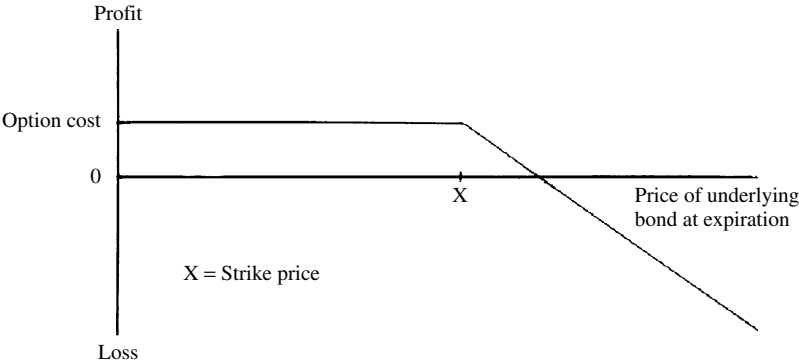
Consequently, the risk/reward characteristics of the two contracts are also different. In the case of a futures contract, the buyer of the contract realizes a dollar-for-dollar gain when the price of the futures contract increases and suffers a dollar-for-dollar loss when the price of the futures contract drops. The opposite occurs for the seller of a futures contract. Options do not provide this symmetric risk/reward relationship. The most that the buyer of an option can lose is the option price. While the buyer of an option retains all the potential benefits, the gain is always reduced by the amount of the option price. The maximum profit that the writer may realize is the option price; this is compensation for accepting substantial downside risk.

Both parties to a futures contract are required to post margin. There are no margin requirements for the buyer of an option once the option price has been paid in full. Because the option price is the maximum amount that the investor can lose, no matter how adverse the price movement of the underlying, there is no need for margin. Because the writer of an option has agreed to accept all of the risk (and none of the reward) of the position in the underlying, the writer is generally required to put up the option price received as margin. In addition, as price changes occur that adversely affect the writer's position, the writer is required to deposit additional margin (with some exceptions) as the position is marked to market.

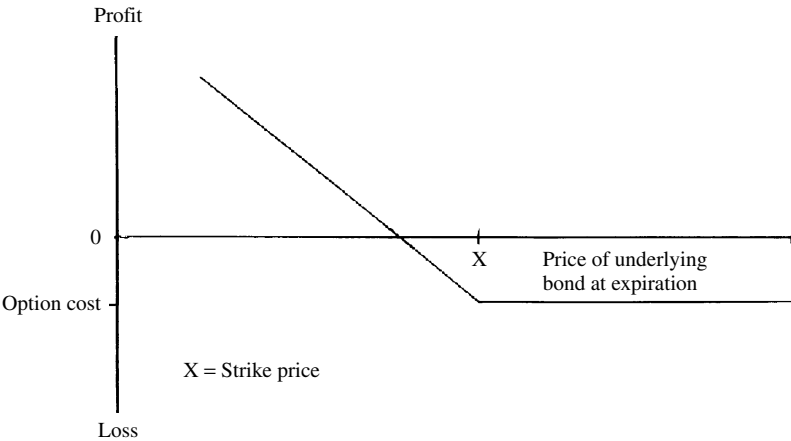
EXHIBIT 3 Payoff of Basic Option Positions if Held to Expiration Date



(a) Long Call Position

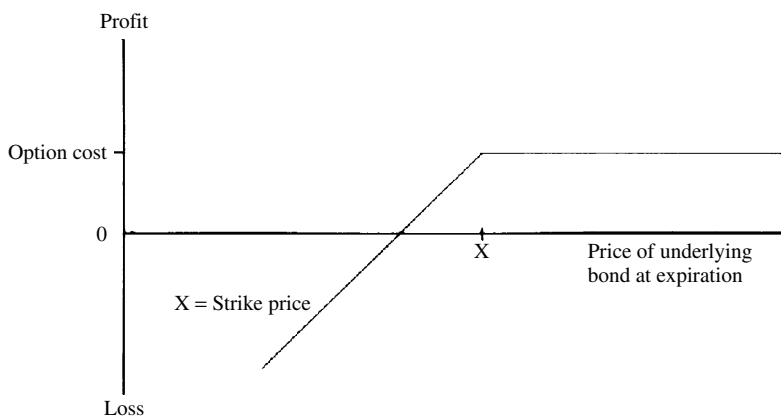


(b) Short Call Position



(c) Long Put Position

EXHIBIT 3 (Continued)



(d) Short Put Position

C. Exchange-Traded Versus OTC Options

Options, like other financial instruments, may be traded either on an organized exchange or in the over-the-counter (OTC) market. An exchange that wants to create an options contract must obtain approval from regulators. Exchange-traded options have three advantages. First, the strike price and expiration date of the contract are standardized.⁵ Second, as in the case of futures contracts, the direct link between buyer and seller is severed after the order is executed because of the interchangeability of exchange-traded options. The clearinghouse performs the same guarantor function in the options market that it does in the futures market. Finally, transaction costs are lower for exchange-traded options than for OTC options.

The higher cost of an OTC option reflects the cost of customizing the option for the many situations where an institutional investor needs to have a tailor-made option because the standardized exchange-traded option does not satisfy its investment objectives. Investment banking firms and commercial banks act as principals as well as brokers in the OTC options market. While an OTC option is less liquid than an exchange-traded option, this is typically not of concern to an institutional investor—most institutional investors use OTC options as part of an asset/liability strategy and intend to hold them to expiration.

Exchange-traded interest rate options can be written on a fixed income security or an interest rate futures contract. The former options are called *options on physicals*. For reasons to be explained later, options on interest rate futures are more popular than options on physicals. However, portfolio managers have made increasingly greater use of OTC options.

1. Exchange-Traded Futures Options There are futures options on all the interest rate futures contracts mentioned earlier in this chapter. An option on a futures contract, commonly referred to as a **futures option**, gives the buyer the right to buy from or sell to the writer a designated futures contract at the strike price at any time during the life of the option. If the futures option is a call option, the buyer has the right to purchase one designated futures

⁵Exchanges have developed put and call options issued by their clearinghouse that are customized with respect to expiration date, exercise style, and strike price. These options are called *flexible exchange options* and are nicknamed “Flex” options.

contract at the strike price. That is, the buyer has the right to acquire a long futures position in the underlying futures contract. If the buyer exercises the call option, the writer acquires a corresponding short position in the same futures contract.

A put option on a futures contract grants the buyer the right to sell one designated futures contract to the writer at the strike price. That is, the option buyer has the right to acquire a short position in the designated futures contract. If the put option is exercised, the writer acquires a corresponding long position in the designated futures contract.

As the parties to the futures option will realize a position in a futures contract when the option is exercised, the question is: what will the futures price be? What futures price will the long be required to pay for the futures contract, and at what futures price will the short be required to sell the futures contract?

Upon exercise, the futures price for the futures contract will be set equal to the strike price. The position of the two parties is then immediately marked-to-market in terms of the then-current futures price. Thus, the futures position of the two parties will be at the prevailing futures price. At the same time, the option buyer will receive from the option seller the economic benefit from exercising. In the case of a call futures option, the option writer must pay the difference between the current futures price and the strike price to the buyer of the option. In the case of a put futures option, the option writer must pay the option buyer the difference between the strike price and the current futures price.

For example, suppose an investor buys a call option on some futures contract in which the strike price is 85. Assume also that the futures price is 95 and that the buyer exercises the call option. Upon exercise, the call buyer is given a long position in the futures contract at 85 and the call writer is assigned the corresponding short position in the futures contract at 85. The futures positions of the buyer and the writer are immediately marked-to-market by the exchange. Because the prevailing futures price is 95 and the strike price is 85, the long futures position (the position of the call buyer) realizes a gain of 10, while the short futures position (the position of the call writer) realizes a loss of 10. The call writer pays the exchange 10 and the call buyer receives from the exchange 10. The call buyer, who now has a long futures position at 95, can either liquidate the futures position at 95 or maintain a long futures position. If the former course of action is taken, the call buyer sells his futures contract at the prevailing futures price of 95. There is no gain or loss from liquidating the position. Overall, the call buyer realizes a gain of 10 (less the option purchase price). The call buyer who elects to hold the long futures position will face the same risk and reward of holding such a position, but still realizes a gain of 10 from the exercise of the call option.

Suppose instead that the futures option with a strike price of 85 is a put rather than a call, and the current futures price is 60 rather than 95. Then, if the buyer of this put option exercises it, the buyer would have a short position in the futures contract at 85; the option writer would have a long position in the futures contract at 85. The exchange then marks the position to market at the then-current futures price of 60, resulting in a gain to the put buyer of 25 and a loss to the put writer of the same amount. The put buyer now has a short futures position at 60 and can either liquidate the short futures position by buying a futures contract at the prevailing futures price of 60 or maintain the short futures position. In either case the put buyer realizes a gain of 25 (less the option purchase price) from exercising the put option.

There are no margin requirements for the buyer of a futures option once the option price has been paid in full. Because the option price is the maximum amount that the buyer can lose regardless of how adverse the price movement of the underlying instrument, there is no need for margin. Because the writer (seller) of a futures option has agreed to accept all of the risk (and none of the reward) of the position in the underlying instrument, the writer (seller) is

required to deposit not only the margin required on the interest rate futures contract position but also (with certain exceptions) the option price that is received from writing the option.

The price of a futures option is quoted in 64ths of 1% of par value. For example, a price of 24 means $24/64$ of 1% of par value. Since the par value of a Treasury bond futures contract is \$100,000, an option price of 24 means: $[(24/64)/100] \times \$100,000 = \$375$. In general, the price of a futures option quoted at Q is equal to:

$$\text{Option price} = \left[\frac{Q/64}{100} \right] \times \$100,000$$

There are three reasons that futures options have largely supplanted options on fixed income securities as the options vehicle of choice for institutional investors who want to use exchange-traded options. First, unlike options on fixed income securities, options on Treasury coupon futures do not require payments for accrued interest to be made. Consequently, when a futures option is exercised, the call buyer and the put writer need not compensate the other party for accrued interest. Second, futures options are believed to be “cleaner” instruments because of the reduced likelihood of delivery squeezes. Market participants who must deliver an instrument are concerned that at the time of delivery the instrument to be delivered will be in short supply, resulting in a higher price to acquire the instrument. As the deliverable supply of futures contracts is infinite for futures options currently traded, there is no concern about a delivery squeeze. Finally, in order to price any option, it is imperative to know at all times the price of the underlying instrument. In the bond market, current prices are not as easily available as price information on the futures contract. The reason is that as bonds trade in the OTC market there is no single reporting system with recent price information. Thus, an investor who wanted to purchase an option on a Treasury bond would have to call several dealer firms to obtain a price. In contrast, futures contracts are traded on an exchange and, as a result, price information is reported.

2. Over-the-Counter Options Institutional investors who want to purchase an option on a specific Treasury security or a Ginnie Mae passthrough security can do so on an over-the-counter basis. There are government and mortgage-backed securities dealers who make a market in options on specific securities. OTC options, also called **dealer options**, usually are purchased by institutional investors who want to hedge the risk associated with a specific security. For example, a thrift may be interested in hedging its position in a specific mortgage passthrough security. Typically, the maturity of the option coincides with the time period over which the buyer of the option wants to hedge, so the buyer is not concerned with the option's liquidity.

In the absence of a clearinghouse the parties to any over-the-counter contract are exposed to counterparty risk.⁶ In the case of forward contracts where both parties are obligated to perform, both parties face counterparty risk. In contrast, in the case of an option, once the option buyer pays the option price, it has satisfied its obligation. It is only the seller that must perform if the option is exercised. Thus, the option buyer is exposed to counterparty risk—the risk that the option seller will fail to perform.

⁶There are well-established institutional arrangements for mitigating counterparty risk in not only OTC options but also the other OTC derivatives described in this chapter (swaps, caps, and floors). These arrangement include limiting exposure to a specific counterparty, marking to market positions, collateralizing trades, and netting arrangement. For a discussion of these arrangements, see Chance, *Analysis of Derivatives for the CFA Program*, pp. 595–598.

OTC options can be customized in any manner sought by an institutional investor. Basically, if a dealer can reasonably hedge the risk associated with the opposite side of the option sought, it will create the option desired by a customer. OTC options are not limited to European or American type. Dealers also create modified American (Bermuda or Atlantic) type options.

IV. INTEREST RATE SWAPS

In an interest rate swap, two parties agree to exchange periodic interest payments. The dollar amount of the interest payments exchanged is based on some predetermined dollar principal, which is called the **notional principal** or **notional amount**. The dollar amount each counterparty pays to the other is the agreed-upon periodic interest rate times the notional principal. The only dollars that are exchanged between the parties are the interest payments, not the notional principal. In the most common type of swap, one party agrees to pay the other party fixed interest payments at designated dates for the life of the contract. This party is referred to as the **fixed-rate payer**. The fixed rate that the fixed-rate payer must make is called the **swap rate**. The other party, who agrees to make interest rate payments that float with some reference rate, is referred to as the **fixed-rate receiver**.

The reference rates that have been used for the floating rate in an interest rate swap are those on various money market instruments: Treasury bills, the London interbank offered rate, commercial paper, bankers acceptances, certificates of deposit, the federal funds rate, and the prime rate. The most common is the London interbank offered rate (LIBOR). LIBOR is the rate at which prime banks offer to pay on Eurodollar deposits available to other prime banks for a given maturity. Basically, it is viewed as the global cost of bank borrowing. There is not just one rate but a rate for different maturities. For example, there is a 1-month LIBOR, 3-month LIBOR, 6-month LIBOR, etc.

To illustrate an interest rate swap, suppose that for the next five years party X agrees to pay party Y 6% per year (the swap rate), while party Y agrees to pay party X 6-month LIBOR (the reference rate). Party X is the fixed-rate payer, while party Y is the fixed-rate receiver. Assume that the notional principal is \$50 million, and that payments are exchanged every six months for the next five years. This means that every six months, party X (the fixed-rate payer) will pay party Y \$1.5 million (6% times \$50 million divided by 2).⁷ The amount that party Y (the fixed-rate receiver) will pay party X will be 6-month LIBOR times \$50 million divided by 2. If 6-month LIBOR is 5% at the beginning of the 6-month period, party Y will pay party X \$1.25 million (5% times \$50 million divided by 2). Mechanically, the floating-rate is determined at the beginning of a period and paid in arrears—that is, it is paid at the end of the period. The two payments are actually netted out so that \$0.25 million will be paid from party X to party Y. Note that we divide by two because one-half year's interest is being paid. This is illustrated in panel a of Exhibit 4.

The convention that has evolved for quoting a swap rate is that a dealer sets the floating rate equal to the reference rate and then quotes the fixed rate that will apply. The fixed rate is the swap rate and reflects a “spread” above the Treasury yield curve with the same term to maturity as the swap. This spread is called the **swap spread**.

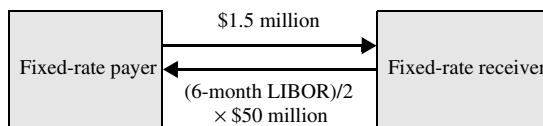
⁷In the next chapter we will fine tune our calculation to take into consideration day count conventions when computing swap payments.

EXHIBIT 4 Summary of How the Value of a Swap to Each Counterparty Changes when Interest Rates Change

a. Initial position

Swap rate	= 6%	Settlement	= semiannual
Reference rate	= 6-month LIBOR	Term of swap	= 5 years
Notional amount	= \$50 million	Payment by fixed-rate payer	= \$1.5 million

Every six months



b. Interest rates increase such that swap rate is 7% for new swaps

Fixed-rate payer pays initial swap rate of 6% to obtain 6-month LIBOR

Advantage to fixed-rate payer: pays only 6% not 7% to obtain 6-month LIBOR

Fixed-rate receiver pays 6-month LIBOR

Disadvantage to fixed-rate receiver: receives only 6% in exchange for 6-month LIBOR, not 7%

Results of a rise in interest rates:

Party	Value of swap
Fixed-rate payer	Increases
Fixed-rate receiver	Decreases

c. Interest rates decrease such that swap rate is 5% for new swaps

Fixed-rate payer pays initial swap rate of 6% to obtain 6-month LIBOR

Disadvantage to fixed-rate payer: must pay 6% not 5% to obtain 6-month LIBOR

Fixed-rate receiver pays 6-month LIBOR

Advantage to fixed-rate receiver: receives 6% in exchange for 6-month LIBOR, not 5%

Results of a decrease in interest rates:

Party	Value of swap
Fixed-rate payer	Decreases
Fixed-rate receiver	Increases

A. Entering Into a Swap and Counterparty Risk

Interest rate swaps are OTC instruments. This means that they are not traded on an exchange. An institutional investor wishing to enter into a swap transaction can do so through either a securities firm or a commercial bank that transacts in swaps.⁸ These entities can do one of the following. First, they can arrange or broker a swap between two parties that want to enter into an interest rate swap. In this case, the securities firm or commercial bank is acting in a brokerage capacity. The broker is not a party to the swap.

The second way in which a securities firm or commercial bank can get an institutional investor into a swap position is by taking the other side of the swap. This means that the

⁸Don't get confused here about the role of commercial banks. A bank can use a swap in its asset/liability management. Or, a bank can transact (buy and sell) swaps to clients to generate fee income. It is in the latter sense that we are discussing the role of a commercial bank in the swap market here.

securities firm or the commercial bank is a dealer rather than a broker in the transaction. Acting as a dealer, the securities firm or the commercial bank must hedge its swap position in the same way that it hedges its position in other securities that it holds. Also it means that the dealer (which we refer to as a **swap dealer**) is the counterparty to the transaction. If an institutional investor entered into a swap with a swap dealer, the institutional investor will look to the swap dealer to satisfy the obligations of the swap; similarly, that same swap dealer looks to the institutional investor to fulfill its obligations as set forth in the swap.

The risk that the two parties take on when they enter into a swap is that the other party will fail to fulfill its obligations as set forth in the swap agreement. That is, each party faces default risk and therefore there is bilateral counterparty risk.

B. Risk/Return Characteristics of an Interest Rate Swap

The value of an interest rate swap will fluctuate with market interest rates. As interest rates rise, the fixed-rate payer is receiving a higher 6-month LIBOR (in our illustration). He would need to pay more for a new swap. Let's consider our hypothetical swap. Suppose that interest rates change immediately after parties X and Y enter into the swap. Panel a in Exhibit 4 shows the transaction. First, consider what would happen if the market demanded that in any 5-year swap the fixed-rate payer must pay 7% in order to receive 6-month LIBOR. If party X (the fixed-rate payer) wants to sell its position to party A, then party A will benefit by having to pay only 6% (the original swap rate agreed upon) rather than 7% (the current swap rate) to receive 6-month LIBOR. Party X will want compensation for this benefit. Consequently, the value of party X's position has increased. Thus, if interest rates increase, the fixed-rate payer will realize a profit and the fixed-rate receiver will realize a loss. Panel b in Exhibit 4 summarizes the results of a rise in interest rates.

Next, consider what would happen if interest rates decline to, say, 5%. Now a 5-year swap would require a new fixed-rate payer to pay 5% rather than 6% to receive 6-month LIBOR. If party X wants to sell its position to party B, the latter would demand compensation to take over the position. In other words, if interest rates decline, the fixed-rate payer will realize a loss, while the fixed-rate receiver will realize a profit. Panel c in Exhibit 4 summarizes the results of a decline in interest rates.

While we know in what direction the change in the value of a swap will be for the counterparties when interest rates change, the question is how much will the value of the swap change. We show how to compute the change in the value of a swap in the next chapter.

C. Interpreting a Swap Position

There are two ways that a swap position can be interpreted: (1) a package of forward (futures) contracts and (2) a package of cash flows from buying and selling cash market instruments.

1. Package of Forward (Futures) Contracts Contrast the position of the counterparties in an interest rate swap summarized above to the position of the long and short interest rate futures (forward) contract. The long futures position gains if interest rates decline and loses if interest rates rise—this is similar to the risk/return profile for a floating-rate payer. The risk/return profile for a fixed-rate payer is similar to that of the short futures position: a gain if interest rates increase and a loss if interest rates decrease. By taking a closer look at the interest rate swap we can understand why the risk/return relationships are similar.

Consider party X's position in our previous swap illustration. Party X has agreed to pay 6% and receive 6-month LIBOR. More specifically, assuming a \$50 million notional principal, X has agreed to buy a commodity called "6-month LIBOR" for \$1.5 million. This is effectively a 6-month forward contract where X agrees to pay \$1.5 million in exchange for delivery of 6-month LIBOR. If interest rates increase to 7%, the price of that commodity (6-month LIBOR) is higher, resulting in a gain for the fixed-rate payer, who is effectively long a 6-month forward contract on 6-month LIBOR. The floating-rate payer is effectively short a 6-month forward contract on 6-month LIBOR. There is therefore an implicit forward contract corresponding to each exchange date.

Now we can see why there is a similarity between the risk/return relationship for an interest rate swap and a forward contract. If interest rates increase to, say, 7%, the price of that commodity (6-month LIBOR) increases to \$1.75 million (7% times \$50 million divided by 2). The long forward position (the fixed-rate payer) gains, and the short forward position (the floating-rate payer) loses. If interest rates decline to, say, 5%, the price of our commodity decreases to \$1.25 million (5% times \$50 million divided by 2). The short forward position (the floating-rate payer) gains, and the long forward position (the fixed-rate payer) loses.

Consequently, interest rate swaps can be viewed as a package of more basic interest rate derivatives, such as forwards.⁹ The pricing of an interest rate swap will then depend on the price of a package of forward contracts with the same settlement dates in which the underlying for the forward contract is the same reference rate. We will make use of this principle in the next chapter when we explain how to value swaps.

While an interest rate swap may be nothing more than a package of forward contracts, it is not a redundant contract for several reasons. First, maturities for forward or futures contracts do not extend out as far as those of an interest rate swap; an interest rate swap with a term of 15 years or longer can be obtained. Second, an interest rate swap is a more transactionally efficient instrument. By this we mean that in one transaction an entity can effectively establish a payoff equivalent to a package of forward contracts. The forward contracts would each have to be negotiated separately. Third, the interest rate swap market has grown in liquidity since its introduction in 1981; interest rate swaps now provide more liquidity than forward contracts, particularly long-dated (i.e., long-term) forward contracts.

2. Package of Cash Market Instruments To understand why a swap can also be interpreted as a package of cash market instruments, consider an investor who enters into the transaction below:

- buy \$50 million par of a 5-year floating-rate bond that pays 6-month LIBOR every six months
- finance the purchase by borrowing \$50 million for five years on terms requiring a 6% annual interest rate payable every six months

⁹More specifically, an interest rate swap is equivalent to a package of **forward rate agreements**. A forward rate agreement (FRA) is the over-the-counter equivalent of the exchange-traded futures contracts on short-term rates. Typically, the short-term rate is LIBOR. The elements of an FRA are the contract rate, reference rate, settlement rate, notional amount, and settlement date.

As a result of this transaction, the investor

- receives a floating rate every six months for the next five years
- pays a fixed rate every six months for the next five years

The cash flows for this transaction are set forth in Exhibit 5. The second column of the exhibit shows the cash flow from purchasing the 5-year floating-rate bond. There is a \$50 million cash outlay and then ten cash inflows. The amount of the cash inflows is uncertain because they depend on future LIBOR. The next column shows the cash flow from borrowing \$50 million on a fixed-rate basis. The last column shows the net cash flow from the entire transaction. As the last column indicates, there is no initial cash flow (no cash inflow or cash outlay). In all ten 6-month periods, the net position results in a cash inflow of LIBOR and a cash outlay of \$1.5 million. This net position, however, is identical to the position of a fixed-rate payer/floating-rate receiver.

It can be seen from the net cash flow in Exhibit 5 that a fixed-rate payer has a cash market position that is equivalent to a long position in a floating-rate bond and a short position in a fixed-rate bond—the short position being the equivalent of borrowing by issuing a fixed-rate bond.

What about the position of a floating-rate payer? It can be easily demonstrated that the position of a floating-rate payer is equivalent to purchasing a fixed-rate bond and financing that purchase at a floating rate, where the floating rate is the reference rate for the swap. That is, the position of a floating-rate payer is equivalent to a long position in a fixed-rate bond and a short position in a floating-rate bond.

D. Describing the Counterparties to a Swap Agreement

The terminology used to describe the position of a party in the swap markets combines cash market jargon and futures market jargon, given that a swap position can be interpreted as a position in a package of cash market instruments or a package of futures/forward positions. As we have said, the counterparty to an interest rate swap is either a fixed-rate payer or floating-rate payer.

Exhibit 6 lists how the counterparties to an interest rate swap agreement are described.¹⁰ To understand why the fixed-rate payer is viewed as “short the bond market,” and the floating-rate payer is viewed as “long the bond market,” consider what happens when interest rates change. Those who borrow on a fixed-rate basis will benefit if interest rates rise because they have locked in a lower interest rate. But those who have a short bond position will also benefit if interest rates rise. Thus, a fixed-rate payer can be said to be short the bond market. A floating-rate payer benefits if interest rates fall. A long position in a bond also benefits if interest rates fall, so terminology describing a floating-rate payer as long the bond market is not surprising. From our discussion of the interpretation of a swap as a package of cash market instruments, describing a swap in terms of the sensitivities of long and short cash positions follows naturally.¹¹

¹⁰Robert F. Kopprasch, John Macfarlane, Daniel R. Ross, and Janet Showers, “The Interest Rate Swap Market: Yield Mathematics, Terminology, and Conventions,” Chapter 58 in Frank J. Fabozzi and Irving M. Pollack (eds.), *The Handbook of Fixed Income Securities* (Homewood, IL: Dow Jones-Irwin, 1987).

¹¹It is common for market participants to refer to one leg of a swap as the “funding leg” and the other as the “asset leg.” This jargon is the result of the interpretation of a swap as a leveraged position in the asset.

EXHIBIT 5 Cash Flow for the Purchase of a 5-Year Floating-Rate Bond Financed by Borrowing on a Fixed-Rate Basis

Transaction:

- Purchase for \$50 million a 5-year floating-rate bond: floating rate = LIBOR, semiannual payments
- Borrow \$50 million for five years: fixed rate = 6%, semiannual payments

Six month period	Cash flow (In millions of dollars) from:		
	Floating-rate bond*	Borrowing at 5%	Net = Same as swap
0	−\$50	+\$50.0	\$0
1	$+(\text{LIBOR}_1/2) \times 50$	−1.5	$+(\text{LIBOR}_1/2) \times 50 - 1.5$
2	$+(\text{LIBOR}_2/2) \times 50$	−1.5	$+(\text{LIBOR}_2/2) \times 50 - 1.5$
3	$+(\text{LIBOR}_3/2) \times 50$	−1.5	$+(\text{LIBOR}_3/2) \times 50 - 1.5$
4	$+(\text{LIBOR}_4/2) \times 50$	−1.5	$+(\text{LIBOR}_4/2) \times 50 - 1.5$
5	$+(\text{LIBOR}_5/2) \times 50$	−1.5	$+(\text{LIBOR}_5/2) \times 50 - 1.5$
6	$+(\text{LIBOR}_6/2) \times 50$	−1.5	$+(\text{LIBOR}_6/2) \times 50 - 1.5$
7	$+(\text{LIBOR}_7/2) \times 50$	−1.5	$+(\text{LIBOR}_7/2) \times 50 - 1.5$
8	$+(\text{LIBOR}_8/2) \times 50$	−1.5	$+(\text{LIBOR}_8/2) \times 50 - 1.5$
9	$+(\text{LIBOR}_9/2) \times 50$	−1.5	$+(\text{LIBOR}_9/2) \times 50 - 1.5$
10	$+(\text{LIBOR}_{10}/2) \times 50 + 50$	−51.5	$+(\text{LIBOR}_{10}/2) \times 50 - 1.5$

*The subscript for LIBOR indicates the 6-month LIBOR as per the terms of the floating-rate bond at time t .

EXHIBIT 6 Describing the Parties to a Swap Agreement

Fixed-rate payer	Fixed-rate receiver
• pays fixed rate in the swap • receives floating in the swap • is short the bond market • has bought a swap • is long a swap • has established the price sensitivities of a longer-term fixed-rate liability and a floating-rate asset	• pays floating rate in the swap • receives fixed in the swap • is long the bond market • has sold a swap • is short a swap • has established the price sensitivities of a longer-term fixed-rate asset and a floating-rate liability

V. INTEREST RATE CAPS AND FLOORS

There are agreements between two parties whereby one party for an upfront premium agrees to compensate the other at specific time periods if the reference rate is different from a predetermined level. If one party agrees to pay the other when the reference rate exceeds a predetermined level, the agreement is referred to as an **interest rate cap** or **ceiling**. The agreement is referred to as an **interest rate floor** if one party agrees to pay the other when the reference rate falls below a predetermined level. The predetermined level is called the **strike**

The payment of the floating-rate is referred to as the “funding leg” and the fixed-rate side is referred to as the “asset side.”

rate. The strike rate for a cap is called the **cap rate**; the strike rate for a floor is called the **floor rate**.

The terms of a cap and floor agreement include:

1. the reference rate
2. the strike rate (cap rate or floor rate) that sets the ceiling or floor
3. the length of the agreement
4. the frequency of settlement
5. the notional principal

For example, suppose that C buys an interest rate cap from D with the following terms:

1. the reference rate is 3-month LIBOR.
2. the strike rate is 6%.
3. the agreement is for four years.
4. settlement is every three months.
5. the notional principal is \$20 million.

Under this agreement, every three months for the next four years, D will pay C whenever 3-month LIBOR exceeds 6% at a settlement date. The payment will equal the dollar value of the difference between 3-month LIBOR and 6% times the notional principal divided by 4. For example, if three months from now 3-month LIBOR on a settlement date is 8%, then D will pay C 2% (8% minus 6%) times \$20 million divided by 4, or \$100,000. If 3-month LIBOR is 6% or less, D does not have to pay anything to C.

In the case of an interest rate floor, assume the same terms as the interest rate cap we just illustrated. In this case, if 3-month LIBOR is 8%, C receives nothing from D, but if 3-month LIBOR is less than 6%, D compensates C for the difference. For example, if 3-month LIBOR is 5%, D will pay C \$50,000 (6% minus 5% times \$20 million divided by 4).¹²

A. Risk/Return Characteristics

In an interest rate agreement, the buyer pays an upfront fee which represents the maximum amount that the buyer can lose and the maximum amount that the seller (writer) can gain. The only party that is required to perform is the seller of the interest rate agreement. The buyer of an interest rate cap benefits if the reference rate rises above the strike rate because the seller must compensate the buyer. The buyer of an interest rate floor benefits if the reference rate falls below the strike rate, because the seller must compensate the buyer.

The seller of an interest rate cap or floor does not face counterparty risk once the buyer pays the fee. In contrast, the buyer faces counterparty risk. Thus, as with options, there is unilateral counterparty risk.

¹²Interest rate caps and floors can be combined to create an *interest rate collar*. This is done by buying an interest rate cap and selling an interest rate floor. The purchase of the cap sets a maximum rate; the sale of the floor sets a minimum rate. The range between the maximum and minimum rate is called the collar.

B. Interpretation of a Cap and Floor Position

In an interest rate cap and floor, the buyer pays an upfront fee, which represents the maximum amount that the buyer can lose and the maximum amount that the seller of the agreement can gain. The only party that is required to perform is the seller of the interest rate agreement. The buyer of an interest rate cap benefits if the reference rate rises above the strike rate because the seller must compensate the buyer. The buyer of an interest rate floor benefits if the reference rate falls below the strike rate because the seller must compensate the buyer.

How can we better understand interest rate caps and interest rate floors? In essence these contracts are equivalent to a *package of interest rate options* at different time periods. As with a swap, a complex contract can be seen to be a package of basic contracts—options in the case of caps and floors. Each of the interest rate options comprising a cap are called **caplets**; similarly, each of the interest rate options comprising a floor are called **floorlets**.

The question is what type of package of options is a cap and a floor. Note the following very carefully! It depends on whether the underlying is a rate or a fixed-income instrument. If the underlying is considered a fixed-income instrument, its value changes inversely with interest rates. Therefore:

- for a call option on a fixed-income instrument:
 - (1) interest rates increase → fixed-income instrument's price decreases
→ call option value decreases
 - and
 - (2) interest rates decrease → fixed-income instrument's price increases
→ call option value increases
- for a put option on a fixed-income instrument
 - (1) interest rates increase → fixed-income instrument's price decreases
→ put option value increases
 - and
 - (2) interest rates decrease → fixed-income instrument's price increases
→ put option value decreases

To summarize the situation for call and put options on a fixed-income instrument:

Value of:	When interest rates	
	increase	decrease
long call	decrease	increase
short call	increase	decrease
long put	increase	decrease
short put	decrease	increase

For a cap and floor, the situation is as follows

Value of:	When interest rates	
	increase	decrease
short cap	decrease	increase
long cap	increase	decrease
short floor	increase	decrease
long floor	decrease	increase

Therefore, buying a cap (long cap) is equivalent to buying a package of puts on a fixed-income instrument and buying a floor (long floor) is equivalent to buying a package of calls on a fixed-income instrument.

Caps and floors can also be seen as packages of options on interest rates. In the over-the-counter market one can purchase an option on an interest rate. These options work as follows in terms of their payoff. There is a strike rate. For a call option on an interest rate, there is a payoff if the reference rate is greater than the strike rate. This means that when interest rates increase, the call option's value increases and when interest rates decrease, the call option's value decreases. As can be seen from the payoff for a cap and a floor summarized above, this is the payoff of a long cap. Consequently, a cap is equivalent to a package of call options on an interest rate. For a put option on an interest rate, there is a payoff when the reference rate is less than the strike rate. When interest rates increase, the value of the put option on an interest rate decreases, as does the value of a long floor position (see the summary above); when interest rates decrease, the value of the put on an interest rate increases, as does the value of a long floor position (again, see the summary above). Thus, a floor is equivalent to a package of put options on an interest rate.

When market participants talk about the equivalency of caps and floors in terms of put and call options, they must specify the underlying. For example, a long cap is equivalent to a package of call options on interest rates or a package of put options on a fixed-income instrument.

C. Creation of an Interest Rate Collar

Interest rate caps and floors can be combined by borrowers to create an **interest rate collar**. This is done by buying an interest rate cap and selling an interest rate floor. The purchase of the cap sets a maximum interest rate that a borrower would have to pay if the reference rate rises. The sale of a floor sets the minimum interest rate that a borrower can benefit from if the reference rate declines. Therefore, there is a range for the interest rate that the borrower must pay if the reference rate changes. The net premium that a borrower who wants to create a collar must pay is the difference between the premium paid to purchase the cap and the premium received to sell the floor.

For example, consider the following collar created by a borrower: a cap purchased with a strike rate of 7% and a floor sold with a strike rate of 4%. If the reference rate exceeds 7%, the borrower receives a payment; if the reference rate is less than 4%, the borrower makes a payment. Thus, the borrower's cost will have a range from 4% to 7%. Note, however, that the borrower's effective interest cost is adjusted by the net premium that the borrower must pay.

CHAPTER 14

VALUATION OF INTEREST RATE DERIVATIVE INSTRUMENTS

I. INTRODUCTION

In the previous chapter, we described interest rate derivative instruments—futures, forwards, options, swaps, caps, and floors. In this chapter, we focus on the valuation of these instruments. Later, we will see how they can be used by portfolio managers to control interest rate risk.

II. INTEREST RATE FUTURES CONTRACTS

In this section we will use an illustration to show how a futures contract is valued. Suppose that a 20-year, \$100 par value bond with a coupon rate of 8% is selling at par and that the next coupon payment is six months from now. Also suppose that this bond is the deliverable for a futures contract that settles in three months. If the current 3-month interest rate at which funds can be loaned or borrowed is 4% per year, what should be the price of this futures contract?

Suppose the price of the futures contract is 105. Consider the following strategy:

Sell the futures contract that settles in three months at \$105.

Borrow \$100 for three months at 4% per year.

With the borrowed funds, purchase the underlying bond for the futures contract.

This strategy is shown in Exhibit 1.

Notice that, ignoring initial margin and other transaction costs, there is no cash outlay for this strategy because the borrowed funds are used to purchase the bond. Three months from now, the following must be done:

Deliver the purchased bond to settle the futures contract.

Repay the loan.

When the bond is delivered to settle the futures contract three months from now, the amount received is the futures price of \$105 plus the accrued interest. Since the coupon rate is 8% for the bond delivered and the bond is held for three months, the accrued interest is

EXHIBIT 1 Cash and Carry Trade

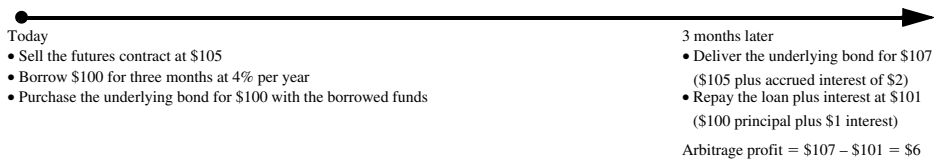
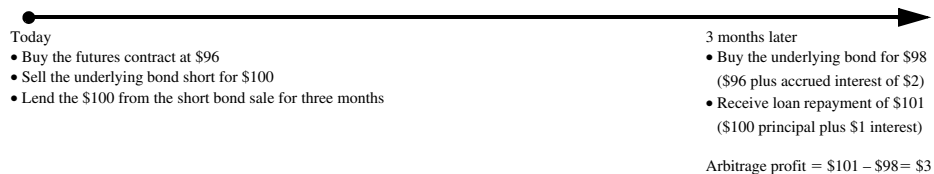


EXHIBIT 2 Reverse Cash and Carry Trade



$\$2[(8\% \times \$100)/4]$. Thus, the amount received is \$107 (\$105 + \$2). The amount that must be paid to repay the loan is the \$100 principal plus the interest. Since the interest rate for the loan is 4% per year and the loan is for three months, the interest cost is \$1. Thus, the amount paid is \$101 (\$100 + \$1).¹ To summarize, at the end of three months the cash flow will be:

Cash inflow from delivery of the bond	=	\$107
Cash outflow from repayment of the loan	=	<u>-\$101</u>
Profit	=	\$6

This strategy guarantees a profit of \$6. Moreover, the profit is generated with no initial outlay because the funds used to purchase the bond are borrowed. The profit will be realized *regardless of the futures price at the settlement date*. Obviously, in a well-functioning market, arbitrageurs would buy the bond and sell the futures, forcing the futures price down and bidding up the bond price so as to eliminate this profit.

This strategy of purchasing a bond with borrowed funds and simultaneously selling a futures contract is called a **cash and carry trade**.

In contrast, suppose that the futures price is \$96 instead of \$105. Consider the strategy below and which is depicted in Exhibit 2:

- Buy the futures contract that settles in three months at \$96.
- Sell (short) the bond underlying the futures contract for \$100.
- Invest (lend) the \$100 proceeds from the short sale for three months at 4% per year.

Once again, there is no cash outlay if we ignore the initial margin for the futures contract and other transaction costs. Three months from now when the futures contract must be settled, the following must be done:

- Purchase the underlying bond to settle the futures contract.
- Receive proceeds from repayment of the loan.

¹Note that there are no interim coupon payments to be considered for potential reinvestment income because we assume that the next coupon payment is six months from the time the strategy is implemented.

When the bond is delivered to settle the futures contract three months from now, the amount paid is the futures price of \$96 plus the accrued interest of \$2, or \$98. The amount that will be received from the proceeds invested (lent) for three months is \$101, or \$100 principal plus interest of \$1.² To summarize, at the end of three months the cash flow will be:

Cash inflow from the amount invested (lent)	=	\$101
Cash outflow to purchase the bond	=	-\$98
Profit	=	\$3

A profit of \$3 will be realized. This is an arbitrage profit because it requires no initial cash outlay and will be realized regardless of the futures price at the settlement date.

Because this strategy involves initially selling the underlying bond, it is called a **reverse cash and carry trade**.

There is a futures price that eliminates any arbitrage profit. There will be no arbitrage profit if the futures price is \$99. Let's look at what would happen if each of the two previous strategies is followed when the futures price is \$99. First, consider the cash and carry trade:

Sell the futures contract that settles in three months at \$99.

Borrow \$100 for three months at 4% per year.

With the borrowed funds purchase the underlying bond for the futures contract.

When the bond is delivered to settle the futures contract three months from now, the amount received is the futures price of \$99 plus the accrued interest of \$2, or \$101. The amount required to repay the loan is the \$100 principal plus the interest of \$1. Thus, the amount paid is \$101. To summarize, at the end of three months the cash flow will be:

Cash inflow from delivery of the bond	=	\$101
Cash outflow from repayment of the loan	=	-\$101
Profit	=	\$0

Thus, there is no arbitrage profit if the futures price is \$99.

Next, consider the reverse cash and carry trade. In this trade the following is done today:

Buy the futures contract that settles in three months at \$99.

Sell the bond underlying the futures contract for \$100.

Invest (lend) the \$100 proceeds from the short sale for three months at 4% per year.

Three months from now when the futures contract must be settled, the amount to be paid is the futures price of \$99 plus the accrued interest of \$2, or \$101. The amount that will be

²Note that the short seller must pay the party from whom the bond was borrowed any coupon payments that were made. In our illustration, we assumed that the next coupon payment would be in six months so there are no coupon payments. However, the short seller must pay any accrued interest. In our illustration, since the investor purchases the underlying bond for \$96 plus accrued interest, the investor has paid the accrued interest. When the bond is delivered to cover the short position, the bond includes the accrued interest. So, no adjustment to the arbitrage profit is needed in our illustration to take accrued interest into account.

received from the proceeds of the three month loan is \$101, \$100 plus interest of \$1. At the end of three months the cash flow will be:

Cash inflow from the amount invested (lent)	=	\$101
Cash outflow to purchase the bond	=	<u>-\$101</u>
Profit	=	\$0

Thus, neither strategy results in a profit or loss. Hence, the futures price of \$99 is the equilibrium or theoretical price, because any higher or lower futures price will permit arbitrage profits.

A. Theoretical Futures Price Based on Arbitrage Model

Considering the arbitrage arguments (based on the cash and carry trade) just presented, the theoretical futures price can be determined from the following information:

1. The price of the underlying bond in the cash market. (In our example, the price of the bond is \$100.)
2. The coupon rate on the bond. (In our example, the coupon rate is 8% per year.)
3. The interest rate for borrowing and lending until the settlement date. The borrowing and lending rate is referred to as the **financing rate**. (In our example, the financing rate is 4% per year.)

We will let

- r = financing rate (in decimal)
 c = current yield, or annual dollar coupon divided by the cash market price (in decimal)
 P = cash market price
 F = futures price
 t = time, in years, to the futures delivery date

Given an assumption of no interim cash flows and no transaction costs, the equation below gives the theoretical futures price that produces a zero profit (i.e., no arbitrage profit) using either the cash and carry trade or the reverse cash and carry trade:

$$F = P + Pt(r - c) \quad (1)$$

Let's apply equation (1) to our previous example in which

$$r = 0.04 \quad c = 0.08 \quad P = 100 \quad t = 0.25$$

Then the theoretical futures price is

$$F = 100 + 100 \times 0.25 \times (0.04 - 0.08) = 100 - 1 = 99$$

This agrees with the theoretical futures price we derived earlier.

It is important to note that c is the current yield, found by dividing the coupon interest payment by the cash market price. In our illustration above, since the cash market price of the bond is 100, the coupon rate is equal to the current yield. If the cash market price is not the par value, the coupon rate is not equal to the current yield.

The theoretical futures price may be at a premium to the cash market price (higher than the cash market price) or at a discount from the cash market price (lower than the cash

market price), depending on $(r - c)$. The term $(r - c)$ is called the **net financing cost** because it adjusts the financing rate for the coupon interest earned. The net financing cost is more commonly called the **cost of carry**, or simply **carry**. **Positive carry** means that the current yield earned is greater than the financing cost; **negative carry** means that the financing cost exceeds the current yield. The relationships can be expressed as follows:

Carry	Futures price
Positive ($c > r$)	At a discount to cash price ($F < P$)
Negative ($c < r$)	At a premium to cash price ($F > P$)
Zero ($c = r$)	Equal to cash price ($F = P$)

In the case of interest rate futures, carry depends on the shape of the yield curve. When the yield curve is upward sloping, the short-term financing rate is lower than the current yield on the bond, resulting in positive carry. The futures contract then sells at a discount to the cash price for the bond. The opposite is true when the yield curve is inverted.

Earlier we explained how the cash and carry trade or the reverse cash and carry trade can be used to exploit any mispricing of the futures contract. Let's review when each trade is implemented based on the actual futures price relative to the theoretical futures price. In our illustration when the theoretical futures price was 99 but the actual futures price was 105, the arbitrage profit due to the futures contract being overpriced was captured using the cash and carry trade. Alternatively, when the cash market price was assumed to be 96, the arbitrage profit resulting from the cheapness of the futures contract was captured by the reverse cash and carry trade. To summarize:

Relationship between theoretical futures price and cash market price	Implement the following trade to capture the arbitrage profit
theoretical futures price > cash market price	cash and carry trade
theoretical futures price < cash market price	reverse cash and carry trade

B. A Closer Look at the Theoretical Futures Price

To derive the theoretical futures price using the arbitrage argument, we made several assumptions. Below we look at the implications of these assumptions.

1. Interim Cash Flows In the model we assumed no interim cash flows due to variation margin or coupon interest payments. However, we know that interim cash flows can occur for both of these reasons. Because we assumed no initial margin or variation margin, the price derived is technically the theoretical price for a forward contract that is not marked to market. Incorporating interim coupon payments into the pricing model is not difficult. However, the value of the coupon payments at the settlement date will depend on the interest rate at which they can be reinvested. The shorter the maturity of the futures contract and the lower the coupon rate, the less important the reinvestment income is in determining the futures price.

2. The Short-Term Interest Rate (Financing Rate) In presenting the theoretical futures price in equation (1), we assumed that the borrowing and lending rates are equal. Typically, however, the borrowing rate is higher than the lending rate. If we will let

$$r_B = \text{borrowing rate and } r_L = \text{lending rate}$$

and continue with our assumption of no interim cash flows and no transaction costs, then the futures price that would produce no cash and carry arbitrage profit is

$$F = P + Pt (r_B - c) \quad (2)$$

and the futures price that would produce no reverse cash and carry arbitrage profit is

$$F = P + Pt (r_L - c) \quad (3)$$

Equations (2) and (3) together provide boundaries for the theoretical futures price. Equation (2) provides the upper boundary and equation (3) the lower boundary. For example, assume that the borrowing rate is 4% per year, while the lending rate is 3.2% per year. Then using equation (2) and the previous example, the upper boundary is

$$F(\text{upper boundary}) = 100 + 100 \times 0.25 \times (0.04 - 0.08) = 99$$

The lower boundary, using equation (3), is

$$F(\text{lower boundary}) = 100 + 100 \times 0.25 \times (0.032 - 0.08) = 98.8$$

In calculating these boundaries, we assume no transaction costs are involved in taking the position. In actuality, the transaction costs of entering into and closing the cash position as well as the round-trip transaction costs for the futures contract must be considered because these transaction costs affect the boundaries for the futures contract.

3. Deliverable Bond Is Not Known The arbitrage arguments used to derive equation (1) assumed that only one instrument is deliverable. But as explained in the previous chapter, the futures contracts on Treasury bonds and Treasury notes are designed to allow the short the choice of delivering any one of a number of deliverable issues (the quality or swap option³). Because there may be more than one deliverable, market participants track the price of each deliverable bond and determine which bond is the cheapest to deliver. The futures price will then trade in relation to the cheapest-to-deliver issue.

There is the risk that while an issue may be the cheapest to deliver at the time a position in the futures contract is taken, it may not be the cheapest to deliver after that time. A change in the cheapest-to-deliver can dramatically alter the futures price. What are the implications of the quality (swap) option on the futures price? Because the swap option is an option granted by the long to the short, the long will want to pay *less* for the futures contract than indicated by equation (1). Therefore, as a result of the quality option, the theoretical futures price as given by equation (1) must be adjusted as follows:

$$F = P + Pt (r - c) - \text{value of quality option} \quad (4)$$

Market participants have employed theoretical models to estimate the fair value of the quality option. A discussion of these models is beyond the scope of this chapter.

³As explained in the previous chapter, this is the option granted to the short in the futures contract to select from among the eligible issues the one to deliver.

4. **Delivery Date Is Not Known** In the pricing model based on arbitrage arguments, a known delivery date is assumed. For Treasury bond and note futures contracts, the short has a timing option and a wild card option, so the long does not know when the security will be delivered. The effect of the timing and wild card options⁴ on the theoretical futures price is the same as with the quality option. These delivery options result in a theoretical futures price that is lower than the one suggested by equation (1), as shown below:

$$F = P + P(r - c) - \text{value of quality option} - \text{value of timing option} \\ - \text{value of wildcard option} \quad (5)$$

or alternatively,

$$F = P + P(r - c) - \text{delivery options} \quad (6)$$

Market participants attempt to value the delivery options in order to apply equation (6). A discussion of these models is a specialist topic.

5. **Putting It Altogether** To summarize, there is not one theoretical futures price that would eliminate any arbitrage profit, but a range for the theoretical futures prices based on borrowing and lending rates. Consequently, the futures price can fluctuate within this range and there will be no arbitrage profit. Once recognition is given to the delivery options granted to the short in the futures contract, the theoretical futures price is lower. Specifically, it is reduced by the value of the delivery options. This means that the lower boundary for the theoretical futures price shifts down by an amount equal to the value of the delivery options and the upper boundary for the theoretical futures price shifts down by the same amount.

III. INTEREST RATE SWAPS

In an interest rate swap, the counterparties agree to exchange periodic interest payments. The dollar amount of the interest payments exchanged is based on the notional principal. In the most common type of swap, there is a fixed-rate payer and a fixed-rate receiver. The convention for quoting swap rates is that a swap dealer sets the floating rate equal to the reference rate (i.e., the interest rate used to determine the floating-rate in a swap) and then quotes the fixed rate that will apply.

A. Computing the Payments for a Swap

In the previous chapter on interest rate derivative instruments, we described the basic features of an interest rate swap using rough calculations for the payments and explained how the parties to a swap either gain or lose when interest rates change. For valuation, however, we need more details. To value a swap it is necessary to determine the present value of the fixed-rate payments and the present value of the floating-rate payments. The difference between these

⁴As explained in the previous chapter, the timing option is the option granted to the short to select the delivery date in the delivery month. The wild card option is the option granted to the short to give notice of intent to deliver after the closing of the exchange on the date when the futures settlement price has been fixed.

two present values is the value of a swap. As will be explained below, whether the value is positive (i.e., an asset) or negative (i.e., a liability) depends on whether the party is the fixed-rate payer or the fixed-rate receiver.

We are interested in how the swap rate is determined at the inception of the swap. At the inception of the swap, the terms of the swap are such that the present value of the floating-rate payments is equal to the present value of the fixed-rate payments. At inception the value of the swap is equal to zero. This is the fundamental principle in determining the swap rate (i.e., the fixed rate that the fixed-rate payer will pay).

Here is a roadmap of the presentation. First we will look at how to compute the floating-rate payments. We will see how the future values of the reference rate are determined to obtain the floating rate for the period. From the future values of the reference rate we will then see how to compute the floating-rate payments, taking into account the number of days in the payment period. Next we will see how to calculate the fixed-rate payments given the swap rate. Before we look at how to calculate the value of a swap, we will see how to calculate the swap rate. This will require an explanation of how the present value of any cash flow in an interest rate swap is computed. Given the floating-rate payments and the present value of the floating-rate payments, the swap rate can be determined by using the principle that the swap rate is the fixed rate that makes the present value of the fixed-rate payments equal to the present value of the floating-rate payments. Finally, we will see how the value of a swap is determined after the inception of a swap.

1. Calculating the Floating-Rate Payments For the first floating-rate payment, the amount is known because the floating-rate is known at the beginning of the period even though it is paid at the end of the period (i.e., payment is made in arrears). For all subsequent payments, the floating-rate payment depends on the value of the reference rate when the floating rate is determined. To illustrate the issues associated with calculating the floating-rate payment, we assume that:

- swap starts today, January 1 of year 1
- the floating-rate payments are made quarterly based on “actual/360” (“actual” means the actual number of days in the quarter)
- the reference rate is 3-month LIBOR (London interbank offered rate)
- the notional amount of the swap is \$100 million
- the term of the swap is three years

The quarterly floating-rate payments are based on an “actual/360” day count convention. This convention means that we assume 360 days in a year and that, in computing the interest for the quarter, the actual number of days in the quarter is used. The floating-rate payment is set at the beginning of the quarter but paid at the end of the quarter—that is, the floating-rate payments are made in arrears.

Suppose that today 3-month LIBOR is 4.05%. Let’s look at what the fixed-rate payer will receive on March 31 of year 1—the date when the first quarterly swap payment is made. There is no uncertainty about what the floating-rate payment will be. In general, the floating-rate payment is determined as follows:

$$\text{notional amount} \times (3\text{-month LIBOR}) \times \frac{\text{no. of days in period}}{360}$$

In our illustration, assuming a non-leap year, the number of days from January 1 of year 1 to March 31 of year 1 (the first quarter) is 90. If 3-month LIBOR is 4.05%, then the fixed-rate payer will receive a floating-rate payment on March 31 of year 1 equal to:

$$\$100,000,000 \times 0.0405 \times \frac{90}{360} = \$1,012,500$$

Now the difficulty is in determining the floating-rate payments after the first quarterly payment. While the first quarterly payment is known, the next 11 are not. However, there is a way to hedge the next 11 floating-rate payments by using a futures contract. The futures contract equivalent to the future floating-rate payments in a swap whose reference rate is 3-month LIBOR is the Eurodollar CD futures contract. In effect then, the remaining swap payments are equivalent to a package of futures contracts. We will digress to discuss this contract.

a. The Eurodollar CD Futures Contract As explained in the previous chapter, a swap position can be interpreted as a package of forward/futures contracts or a package of cash flows from buying and selling cash market instruments. It is the former interpretation that will be used as the basis for valuing a swap.

Eurodollar certificates of deposit (CDs) are denominated in dollars but represent the liabilities of banks outside the United States. The contracts are traded on both the International Monetary Market of the Chicago Mercantile Exchange and the London International Financial Futures Exchange. The rate paid on Eurodollar CDs is LIBOR.

The 3-month Eurodollar CD is the underlying instrument for the Eurodollar CD futures contract. The contract is for \$1 million of face value and is traded on an index price basis. The index price basis is equal to 100 minus the product of the annualized LIBOR futures rate in decimal and 100. For example, a Eurodollar CD futures price of 94.00 means a 3-month LIBOR futures rate of 6% [100 minus (0.06 × 100)].

The Eurodollar CD futures contract is a cash settlement contract. That is, the parties settle in cash for the value of a Eurodollar CD based on LIBOR at the settlement date.

The Eurodollar CD futures contract allows the buyer of the contract to lock in the rate on 3-month LIBOR today for a future 3-month period. For example, suppose that on February 1 in Year 1 an investor purchases a Eurodollar CD futures contract that settles in March of Year 1. Assume that the LIBOR futures rate for this contract is 5%. This means that the investor has agreed to invest in a 3-month Eurodollar CD that pays a rate of 5%. Specifically, the investor has locked in a 3-month rate of 5% beginning March of Year 1. If on February 1 of Year 1 this investor purchased a contract that settles in September of Year 2 and the LIBOR futures rate is 5.4%, the investor has locked in the rate on a 3-month investment beginning September of Year 2.

The seller of a Eurodollar CD futures contract is agreeing to lend funds for three months at some future date at the LIBOR futures rate. For example, suppose that on February 1 of Year 1 a bank sells a Eurodollar CD futures contract that settles in March of Year 1 and the LIBOR futures rate is 5%. The bank locks in a borrowing rate of 5% for three months beginning in March of Year 1. If the settlement date is September of Year 2 and the LIBOR futures rate is 5.4%, the bank is locking in a borrowing rate of 5.4% for the 3-month period beginning September of Year 2.

The key point here is that the Eurodollar CD futures contract allows a participant in the financial market to lock in a 3-month rate on an investment or a 3-month borrowing rate. The 3-month period begins in the month that the contract settles.

b. Determining Future Floating-Rate Payments Now let's return to our objective of determining the future floating-rate payments. These payments can be locked in over the life of the swap using the Eurodollar CD futures contract. We will show how these floating-rate payments are computed using this contract.

We will begin with the next quarterly payment—for the quarter that runs from April 1 of year 1 to June 30 of year 1. This quarter has 91 days. The floating-rate payment will be determined by 3-month LIBOR on April 1 of year 1 and paid on June 30 of year 1. There is a 3-month Eurodollar CD futures contract for settlement on March 31 of year 1. The price of that futures contract will reflect the market's expectation of 3-month LIBOR on April 1 of year 1. For example, if the futures price for the 3-month Eurodollar CD futures contract that settles on March 31 of year 1 is 95.85, then as explained above, the 3-month Eurodollar futures rate is 4.15%. We will refer to that rate for 3-month LIBOR as the “forward rate.”⁵ Therefore, if the fixed-rate payer bought 100 of these 3-month Eurodollar CD futures contracts on January 1 of year 1 (the inception of the swap) that settle on March 31 of year 1, then the payment that will be locked in for the quarter (April 1 to June 30 of year 1) is

$$\$100,000,000 \times 0.0415 \times \frac{91}{360} = \$1,049,028$$

(Note that each futures contract is for \$1 million and hence 100 contracts have a notional amount of \$100 million.) Similarly, the Eurodollar CD futures contract can be used to lock in a floating-rate payment for each of the next 10 quarters. Once again, it is important to emphasize that the reference rate at the beginning of period t determines the floating-rate that will be paid for the period. However, the floating-rate payment is not made until the end of period t .

Exhibit 3 shows this for the 3-year swap. Shown in Column (1) is when the quarter begins and in Column (2) when the quarter ends. The payment of \$1,012,500 will be received at the end of the first quarter (March 31 of year 1). That is the known floating-rate payment as explained earlier. It is the only payment that is known. The information used to compute the first payment is in Column (4) which shows the current 3-month LIBOR (4.05%). The payment is shown in the last column, Column (8).

Notice that Column (7) numbers the quarters from 1 through 12. Look at the heading for Column (7). It identifies each quarter in terms of the end of the quarter. This is important because we will eventually be discounting the payments (cash flows). We must take care to understand when the payments are to be exchanged in order to discount properly. So, the first payment of \$1,012,500 is going to be received at the end of quarter 1. When we refer to the time period for any payment, the reference is to the end of quarter. So, the fifth payment of \$1,225,000 would be identified as the payment for period 5, where period 5 means that it will be exchanged at the end of the fifth quarter.

2. Calculating the Fixed-Rate Payments The swap specifies the frequency of settlement for the fixed-rate payments. The frequency need not be the same for the floating-rate payments. For example, in the 3-year swap we have been using to illustrate the calculation of the floating-rate payments, the frequency is quarterly. The frequency of the fixed-rate payments could be semiannual rather than quarterly.

⁵We discussed forward rates in earlier chapters. The reason that we refer to “forward rates” rather than “3-month Eurodollar futures rates” is because we will be developing generic formulas that can be used regardless of the reference rate for the swap. The formulas we present later in this chapter will be in terms of “forward rate for the period” and “period forward rate.”

EXHIBIT 3 Floating-Rate Payments Based on Initial LIBOR and Eurodollar CD Futures

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Quarter starts	Quarter ends	Number of days in quarter	Current 3-month LIBOR	3-month Eurodollar CD futures price	Forward rate*	Period = End of quarter	Floating-rate payment at end of quarter
Jan 1 year 1	Mar 31 year 1	90	4.05%		—	1	1,012,500
Apr 1 year 1	June 30 year 1	91		95.85	4.15%	2	1,049,028
July 1 year 1	Sept 30 year 1	92		95.45	4.55%	3	1,162,778
Oct 1 year 1	Dec 31 year 1	92		95.28	4.72%	4	1,206,222
Jan 1 year 2	Mar 31 year 2	90		95.10	4.90%	5	1,225,000
Apr 1 year 2	June 30 year 2	91		94.97	5.03%	6	1,271,472
July 1 year 2	Sept 30 year 2	92		94.85	5.15%	7	1,316,111
Oct 1 year 2	Dec 31 year 2	92		94.75	5.25%	8	1,341,667
Jan 1 year 3	Mar 31 year 3	90		94.60	5.40%	9	1,350,000
Apr 1 year 3	June 30 year 3	91		94.50	5.50%	10	1,390,278
July 1 year 3	Sept 30 year 3	92		94.35	5.65%	11	1,443,889
Oct 1 year 3	Dec 31 year 3	92		94.24	5.76%	12	1,472,000

*The forward rate is the 3-month Eurodollar futures rate.

In our illustration we will assume that the frequency of settlement is quarterly for the fixed-rate payments, the same as for the floating-rate payments. The day count convention is the same as for the floating-rate payment, “actual/360”. The equation for determining the dollar amount of the fixed-rate payment for the period is:

$$\text{notional amount} \times \text{swap rate} \times \frac{\text{no. of days in period}}{360}$$

This is the same equation used for determining the floating-rate payment except that the swap rate is used instead of the reference rate (3-month LIBOR in our illustration).

For example, suppose that the swap rate is 4.98% and that the quarter has 90 days. Then the fixed-rate payment for the quarter is:

$$\$100,000,000 \times 0.0498 \times \frac{90}{360} = \$1,245,000$$

If there are 92 days in a quarter, the fixed-rate payment for the quarter is:

$$\$100,000,000 \times 0.0498 \times \frac{92}{360} = \$1,272,667$$

Note that the rate is fixed for each quarter but the dollar amount of the payment depends on the number of days in the period.

Exhibit 4 shows the fixed-rate payments based on an *assumed* swap rate of 4.9875%. (Later we will see how the swap rate is determined.) The first three columns of the exhibit show the same information as in Exhibit 3—the beginning and end of the quarter and the number of days in the quarter. Column (4) simply uses the notation for the period. That is, period 1 means the end of the first quarter, period 2 means the end of the second quarter, and so on. Column (5) shows the fixed value payments for each period based on a swap rate of 4.9875%.

EXHIBIT 4 Fixed-Rate Payments Assuming a Swap Rate of 4.9875%

(1)	(2)	(3)	(4)	(5)
Quarter starts	Quarter starts	Number days in quarter	Period = end of quarter	Fixed-rate payment if swap rate is assumed to be 4.9875%
Jan 1 year 1	Mar 31 year 1	90	1	1,246,875
Apr 1 year 1	June 30 year 1	91	2	1,260,729
July 1 year 1	Sept 30 year 1	92	3	1,274,583
Oct 1 year 1	Dec 31 year 1	92	4	1,274,583
Jan 1 year 2	Mar 31 year 2	90	5	1,246,875
Apr 1 year 2	June 30 year 2	91	6	1,260,729
July 1 year 2	Sept 30 year 2	92	7	1,274,583
Oct 1 year 2	Dec 31 year 2	92	8	1,274,583
Jan 1 year 3	Mar 31 year 3	90	9	1,246,875
Apr 1 year 3	June 30 year 3	91	10	1,260,729
July 1 year 3	Sept 30 year 3	92	11	1,274,583
Oct 1 year 3	Dec 31 year 3	92	12	1,274,583

B. Calculation of the Swap Rate

Now that we know how to calculate the payments for the fixed-rate and floating-rate sides of a swap where the reference rate is 3-month LIBOR given (1) the current value for 3-month LIBOR, (2) a series for 3-month LIBOR in the future from the Eurodollar CD futures contract, and (3) the assumed swap rate, we can demonstrate how to compute the swap rate.

At the initiation of an interest rate swap, the counterparties are agreeing to exchange future payments. No upfront payments are made by either party. This means that the swap terms must be such that the present value of the payments to be made by the counterparties must be at least equal to the present value of the payments that will be received. In fact, to eliminate arbitrage opportunities, the present value of the payments made by a party will be equal to the present value of the payments received by that same party. *The equivalence of the present value of the payments (or no arbitrage) is the key principle in calculating the swap rate.*

Since we will have to calculate the present value of the payments, let's show how this is done.

1. Calculating the Present Value of the Floating-Rate Payments As explained earlier, we must be careful about how we compute the present value of payments. In particular, we must carefully specify (1) the timing of the payments and (2) the interest rates used to discount the payments. We already addressed the first issue. In constructing the exhibit for the payments, we indicated that the payments are made at the end of the quarter. So, we denoted the timing of the payments with respect to the end of the quarter.

Now let's turn to the interest rates that should be used for discounting. Earlier we emphasized two points. First, every cash flow should be discounted at its own discount rate using the relevant spot rate. So, if we discounted a cash flow of \$1 using the spot rate for period t , the present value would be:

$$\text{present value of \$1 to be received in period } t = \frac{\$1}{(1 + \text{spot rate for period } t)^t}$$

The second point we emphasized is that forward rates are derived from spot rates so that if we discount a cash flow using forward rates rather than a spot rate, we would arrive at the

same value. That is, the present value of \$1 to be received in period t can be rewritten as:

$$\text{present value of \$1 to be received in period } t = \frac{\$1}{(1 + \text{forward rate for period 1})(1 + \text{forward rate for period 2}) \dots (1 + \text{forward rate for period } t)}$$

We will refer to the present value of \$1 to be received in period t as the **forward discount factor**. In our calculations involving swaps, we will compute the forward discount factor for a period using the forward rates. These are the same forward rates that are used to compute the floating-rate payments—those obtained from the Eurodollar CD futures contract. We must make just one more adjustment. We must adjust the forward rates used in the formula for the number of days in the period (i.e., the quarter in our illustrations) in the same way that we made this adjustment to compute the payments. Specifically, the forward rate for a period, which we will refer to as the **period forward rate**, is computed using the following equation:

$$\text{period forward rate} = \text{annual forward rate} \times \left(\frac{\text{days in period}}{360} \right)$$

For example, look at Exhibit 3. The annual forward rate for period 4 is 4.72%. The period forward rate for period 4 is:

$$\text{period forward rate} = 4.72\% \times \left(\frac{92}{360} \right) = 1.2062\%$$

Column (5) in Exhibit 5 shows the annual forward rate for each of the 12 periods (reproduced from Exhibit 3) and Column (6) shows the period forward rate for each of the 12 periods. Note that the period forward rate for period 1 is $^{90}/_{360}$ 4.05%, which is $^{90}/_{360}$ of the known rate for 3-month LIBOR.

EXHIBIT 5 Calculating the Forward Discount Factor

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter starts	Quarter ends	Number of days in quarter	Period = end of quarter	Forward rate	Period forward rate	Forward discount factor
Jan 1 year 1	Mar 31 year 1	90	1	4.05%	1.0125%	0.98997649
Apr 1 year 1	June 30 year 1	91	2	4.15%	1.0490%	0.97969917
July 1 year 1	Sept 30 year 1	92	3	4.55%	1.1628%	0.96843839
Oct 1 year 1	Dec 31 year 1	92	4	4.72%	1.2062%	0.95689609
Jan 1 year 2	Mar 31 year 2	90	5	4.90%	1.2250%	0.94531597
Apr 1 year 2	June 30 year 2	91	6	5.03%	1.2715%	0.93344745
July 1 year 2	Sept 30 year 2	92	7	5.15%	1.3161%	0.92132183
Oct 1 year 2	Dec 31 year 2	92	8	5.25%	1.3417%	0.90912441
Jan 1 year 3	Mar 31 year 3	90	9	5.40%	1.3500%	0.89701471
Apr 1 year 3	June 30 year 3	91	10	5.50%	1.3903%	0.88471472
July 1 year 3	Sept 30 year 3	92	11	5.65%	1.4439%	0.87212224
Oct 1 year 3	Dec 31 year 3	92	12	5.76%	1.4720%	0.85947083

Also shown in Exhibit 5 is the forward discount factor for each of the 12 periods. These values are shown in the last column. Let's show how the forward discount factor is computed for periods 1, 2, and 3. For period 1, the forward discount factor is:

$$\begin{aligned}\text{forward discount factor} &= \frac{\$1}{1 + \text{period forward rate}_1} \\ &= \frac{\$1}{1.010125} = 0.98997649\end{aligned}$$

For period 2,

$$\text{forward discount factor} = \frac{\$1}{(1.010125)(1.010490)} = 0.97969917$$

For period 3,

$$\text{forward discount factor} = \frac{\$1}{(1.010125)(1.010490)(1.011628)} = 0.96843839$$

Given the floating-rate payment for a period and the forward discount factor for the period, the present value of the payment can be computed. For example, from Exhibit 3 we see that the floating-rate payment for period 4 is \$1,206,222. From Exhibit 5, the forward discount factor for period 4 is 0.95689609. Therefore, the present value of the payment is:

$$\text{present value of period 4 payment} = \$1,206,222 \times 0.95689609 = \$1,154,229$$

Exhibit 6 shows the present value for each payment. The total present value of the 12 floating-rate payments is \$14,052,917. Thus, the present value of the payments that the fixed-rate payer will receive is \$14,052,917 and the present value of the payments that the fixed-rate receiver will pay is \$14,052,917.

EXHIBIT 6 Present Value of the Floating-Rate Payments

(1)	(2)	(3)	(4)	(5)	(6)
Quarter starts	Quarter ends	Period = end of quarter	Forward discount factor	Floating-rate payment at end of quarter	PV of floating-rate payment
Jan 1 year 1	Mar 31 year 1	1	0.98997649	1,012,500	1,002,351
Apr 1 year 1	June 30 year 1	2	0.97969917	1,049,028	1,027,732
July 1 year 1	Sept 30 year 1	3	0.96843839	1,162,778	1,126,079
Oct 1 year 1	Dec 31 year 1	4	0.95689609	1,206,222	1,154,229
Jan 1 year 2	Mar 31 year 2	5	0.94531597	1,225,000	1,158,012
Apr 1 year 2	June 30 year 2	6	0.93344745	1,271,472	1,186,852
July 1 year 2	Sept 30 year 2	7	0.92132183	1,316,111	1,212,562
Oct 1 year 2	Dec 31 year 2	8	0.90912441	1,341,667	1,219,742
Jan 1 year 3	Mar 31 year 3	9	0.89701471	1,350,000	1,210,970
Apr 1 year 3	June 30 year 3	10	0.88471472	1,390,278	1,229,999
July 1 year 3	Sept 30 year 3	11	0.87212224	1,443,889	1,259,248
Oct 1 year 3	Dec 31 year 3	12	0.85947083	1,472,000	1,265,141
				Total	14,052,917

2. Determination of the Swap Rate The fixed-rate payer will require that the present value of the fixed-rate payments that must be made based on the swap rate not exceed the \$14,052,917 present value of the floating-rate payments to be received. The fixed-rate receiver will require that the present value of the fixed-rate payments to received be at least as great as the \$14,052,917 that must be paid. This means that both parties will require the present value of the fixed-rate payments to be \$14,052,917. If that is the case, the present value of the fixed-rate payments is equal to the present value of the floating-rate payments and therefore the value of the swap is zero for both parties at the inception of the swap. The interest rates used to compute the present value of the fixed-rate payments are the same as those used to discount the floating-rate payments.

Beginning with the basic relationship for no arbitrage to exist:

$$PV \text{ of floating-rate payments} = PV \text{ of fixed-rate payments}$$

it can be shown that the formula for the swap rate is:⁶

$$SR = \frac{PV \text{ of floating-rate payments}}{\sum_{t=1}^N \text{notional amount} \times \frac{\text{Days}_t}{360} \times FDF_t}$$

where

SR = swap rate

Days_t = number of days in period t

FDF_t = forward discount factor for period t

Note that all the values needed to compute the swap rate are known.

⁶The formula is derived as follows. The fixed-rate payment for period t is equal to:

$$\text{notional amount} \times SR \times \frac{\text{Days}_t}{360}$$

The present value of the fixed-rate payment for period t is found by multiplying the previous expression by the forward discount factor for period t (FDF_t). That is, the present value of the fixed-rate payment for period t is equal to:

$$\text{notional amount} \times SR \times \frac{\text{Days}_t}{360} \times FDF_t$$

Summing the present values of the fixed-rate payment for all periods gives the present value of the fixed-rate payments. Letting N equal the number of periods in the swap, then the present value of the fixed-rate payments can be expressed as:

$$SR \sum_{t=1}^N \text{notional amount} \times \frac{\text{Days}_t}{360} \times FDF_t$$

The condition for no arbitrage is that the present value of the fixed-rate payments as given by the expression above is equal to the present value of the floating-rate payments. That is,

$$SR \sum_{t=1}^N \text{notional amount} \times \frac{\text{Days}_t}{360} \times FDF_t = PV \text{ of floating-rate payments}$$

Solving for the swap rate gives the formula in the text.

EXHIBIT 7 Calculating the Denominator for the Swap Rate Formula

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter starts	Quarter ends	Number of days in quarter	Period = end of quarter	Forward discount factor	Days/360	Forward discount factor $\times$ days/ $360 \times$ notional
Jan 1 year 1	Mar 31 year 1	90	1	0.98997649	0.25000000	24,749,412
Apr 1 year 1	June 30 year 1	91	2	0.97969917	0.25277778	24,764,618
July 1 year 1	Sept 30 year 1	92	3	0.96843839	0.25555556	24,748,981
Oct 1 year 1	Dec 31 year 1	92	4	0.95689609	0.25555556	24,454,011
Jan 1 year 2	Mar 31 year 2	90	5	0.94531597	0.25000000	23,632,899
Apr 1 year 2	June 30 year 2	91	6	0.93344745	0.25277778	23,595,477
July 1 year 2	Sept 30 year 2	92	7	0.92132183	0.25555556	23,544,891
Oct 1 year 2	Dec 31 year 2	92	8	0.90912441	0.25555556	23,233,179
Jan 1 year 3	Mar 31 year 3	90	9	0.89701471	0.25000000	22,425,368
Apr 1 year 3	June 30 year 3	91	10	0.88471472	0.25277778	22,363,622
July 1 year 3	Sept 30 year 3	92	11	0.87212224	0.25555556	22,287,568
Oct 1 year 3	Dec 31 year 3	92	12	0.85947083	0.25555556	21,964,255
Total						281,764,281

Let's apply the formula to determine the swap rate for our 3-year swap. Exhibit 7 shows the calculation of the denominator of the formula. The forward discount factor for each period shown in Column (5) is obtained from Column (4) of Exhibit 6. The sum of the last column in Exhibit 7 shows that the denominator of the swap rate formula is \$281,764,281. We know from Exhibit 6 that the present value of the floating-rate payments is \$14,052,917. Therefore, the swap rate is

$$SR = \frac{\$14,052,917}{\$281,764,281} = 0.049875 = 4.9875\%$$

Given the swap rate, the **swap spread** can be determined. For example, since this is a 3-year swap, the convention is to use the 3-year on-the-run Treasury rate as the benchmark. If the yield on that issue is 4.5875%, the swap spread is 40 basis points (4.9875% – 4.5875%).

The calculation of the swap rate for all swaps follows the same principle: equating the present value of the fixed-rate payments to that of the floating-rate payments.

C. Valuing a Swap

Once the swap transaction is completed, changes in market interest rates will change the payments for the floating-rate side of the swap. The value of an interest rate swap is the difference between the present values of the payments for the two sides of the swap. The 3-month LIBOR forward rates from the current Eurodollar CD futures contracts are used to (1) calculate the floating-rate payments and (2) determine the discount factors used to calculate the present value of the payments.

To illustrate this, consider the 3-year swap used to demonstrate how to calculate the swap rate. Suppose that one year later, interest rates change as shown in Columns (4) and (6) in Exhibit 8. Column (4) shows the current 3-month LIBOR. In Column (5) are the Eurodollar CD futures prices for each period. These rates are used to compute the forward rates in Column (6). Note that interest rates have increased one year later since the rates in Exhibit 8 are greater than those in Exhibit 3. As in Exhibit 3, the current 3-month LIBOR and the

EXHIBIT 8 Rates and Floating-Rate Payments One Year Later if Rates Increase

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Quarter starts	Quarter ends	Number of days in quarter	Current 3-month LIBOR	3-month Eurodollar futures price	Forward rate*	Period = end of quarter	Floating-rate payments at end of quarter
Jan 1 year 2	Mar 31 year 2	90	5.25%			1	1,312,500
Apr 1 year 2	June 30 year 2	91		94.27	5.73%	2	1,448,417
July 1 year 2	Sept 30 year 2	92		94.22	5.78%	3	1,477,111
Oct 1 year 2	Dec 31 year 2	92		94.00	6.00%	4	1,533,333
Jan 1 year 3	Mar 31 year 3	90		93.85	6.15%	5	1,537,500
Apr 1 year 3	June 30 year 3	91		93.75	6.25%	6	1,579,861
July 1 year 3	Sept 30 year 3	92		93.54	6.46%	7	1,650,889
Oct 1 year 3	Dec 31 year 3	92		93.25	6.75%	8	1,725,000

*The forward rate is the 3-month Eurodollar futures rate.

EXHIBIT 9 Period Forward Rates and Forward Discount Factors One Year Later if Rates Increase

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter starts	Quarter ends	Number of days in quarter	Period = end of quarter	Forward rate	Period forward rate	Forward discount factor
Jan 1 year 2	Mar 31 year 2	90	1	5.25%	1.3125%	0.98704503
Apr 1 year 2	June 30 year 2	91	2	5.73%	1.4484%	0.97295263
July 1 year 2	Sept 30 year 2	92	3	5.78%	1.4771%	0.95879023
Oct 1 year 2	Dec 31 year 2	92	4	6.00%	1.5333%	0.94431080
Jan 1 year 3	Mar 31 year 3	90	5	6.15%	1.5375%	0.93001186
Apr 1 year 3	June 30 year 3	91	6	6.25%	1.5799%	0.91554749
July 1 year 3	Sept 30 year 3	92	7	6.46%	1.6509%	0.90067829
Oct 1 year 3	Dec 31 year 3	92	8	6.75%	1.7250%	0.88540505

forward rates are used to compute the floating-rate payments. These payments are shown in Column (8) of Exhibit 8.

In Exhibit 9, the forward discount factor is computed for each period, in the same way as it was calculated in Exhibit 5. The forward discount factor for each period is shown in the last column of Exhibit 9.

In Exhibit 10 the forward discount factor (from Exhibit 9) and the floating-rate payments (from Exhibit 8) are shown. The fixed-rate payments need not be recomputed. They are the payments shown in Column (8) of Exhibit 4. These are the fixed-rate payments for the swap rate of 4.9875% and they are reproduced in Exhibit 10. Now the two payment streams must be discounted using the new forward discount factors. As shown at the bottom of Exhibit 10, the two present values are as follows:

Present value of floating rate payments \$11,459,496
Present value of fixed rate payments \$9,473,390

The two present values are not equal and therefore for one party the value of the swap increased, and for the other party the value of the swap decreased. Let's look at which party gained and which party lost.

EXHIBIT 10 Valuing the Swap One Year Later if Rates Increase

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Quarter starts	Quarter ends	Forward discount factor	Floating cash flow at end of quarter	PV of floating cash flow	Fixed cash flow at end of quarter	PV of fixed cash flow
Jan 1 year 2	Mar 31 year 2	0.98704503	1,312,500	1,295,497	1,246,875	1,230,722
Apr 1 year 2	June 30 year 2	0.97295263	1,448,417	1,409,241	1,260,729	1,226,630
July 1 year 2	Sept 30 year 2	0.95879023	1,477,111	1,416,240	1,274,583	1,222,058
Oct 1 year 2	Dec 31 year 2	0.94431080	1,533,333	1,447,943	1,274,583	1,203,603
Jan 1 year 3	Mar 31 year 3	0.93001186	1,537,500	1,429,893	1,246,875	1,159,609
Apr 1 year 3	June 30 year 3	0.91554749	1,579,861	1,446,438	1,260,729	1,154,257
July 1 year 3	Sept 30 year 3	0.90067829	1,650,889	1,486,920	1,274,583	1,147,990
Oct 1 year 3	Dec 31 year 3	0.88540505	1,725,000	1,527,324	1,274,583	1,128,523
			Total	11,459,496		9,473,390

Summary	Fixed-rate payer	Fixed-rate receiver
PV of payments received	11,459,496	9,473,390
PV of payments made	9,473,390	11,459,496
Value of swap	1,986,106	−1,986,104

The fixed-rate payer will receive the floating-rate payments, which have a present value of \$11,459,496. The present value of the payments that must be made by the fixed-rate payer is \$9,473,390. Thus, the swap has a positive value for the fixed-rate payer equal to the difference in the two present values of \$1,986,106. This is the value of the swap to the fixed-rate payer. Notice that, consistent with what we said in the previous chapter, when interest rates increase (as they did in the illustration analyzed), the fixed-rate payer benefits because the value of the swap increases.

In contrast, the fixed-rate receiver must make payments with a present value of \$11,459,496 but will only receive fixed-rate payments with a present value equal to \$9,473,390. Thus, the value of the swap for the fixed-rate receiver is −\$1,986,106. Again, as explained in the previous chapter, the fixed-rate receiver is adversely affected by a rise in interest rates because of the resulting decline in the value of a swap.

The same valuation principle applies to more complicated swaps. For example, there are swaps whose notional amount changes in a predetermined way over the life of the swap. These include amortizing swaps, accreting swaps, and roller coaster swaps. Once the payments are specified, the present value is calculated as described above by simply adjusting the payment amounts for the changing notional amounts—the methodology does *not* change.⁷

IV. OPTIONS

An option grants the buyer of the option the right, but not the obligation, to purchase from or sell to the contract writer an asset (the underlying) at a specified price (the strike price) within a specified period of time (or at a specified date). The compensation that the option buyer pays to acquire the option from the option writer is the **option price**. (The option price is also referred to as the **option premium**.) A call option grants the buyer the right to purchase the

⁷These include amortizing swaps (swaps where the notional amount declines over time) and accreting swaps (swaps where the notional amount increases over time).

underlying from the writer (seller); a put option gives the buyer the right to sell the underlying to the writer. An American option allows the buyer to exercise the option at any time up to and including the expiration date. A European option allows the buyer to exercise the option only on the expiration date.

The maximum amount that an option buyer can lose is the option price. The maximum profit that the option writer can realize is the option price. The option buyer has substantial upside return potential, while the option writer has substantial downside risk.

A. Components of the Option Price

The option price can be decomposed into two parts: the intrinsic value and the time value. We describe each below.

1. Intrinsic Value The option value is a reflection of the option's intrinsic value and its time value. The **intrinsic value** of an option is its economic value if it is exercised immediately. If no positive economic value would result from exercising the option immediately, then the intrinsic value is zero.

For a call option, the intrinsic value is positive if the current market price of the underlying security is greater than the strike price. The intrinsic value is then the difference between the current market price of the underlying security and the strike price. If the strike price of a call option is greater than or equal to the current market price of the security, the intrinsic value is zero. For example, if the strike price for a call option is \$100 and the current market price of the security is \$105, the intrinsic value is \$5. That is, an option buyer exercising the option and simultaneously selling the underlying security would realize \$105 from the sale of the security, which would be covered by acquiring the security from the option writer for \$100, thereby netting a \$5 gain.

When an option has intrinsic value, it is said to be **in the money**. When the strike price of a call option exceeds the current price of the security, the call option is said to be **out of the money**; it has no intrinsic value. An option for which the strike price is equal to the current price of the security is said to be **at the money**. Both at-the-money and out-of-the-money options have an intrinsic value of zero because they are not profitable to exercise.

For a put option, the intrinsic value is equal to the amount by which the current price of the security is below the strike price. For example, if the strike price of a put option is \$100 and the current price of the security is \$92, the intrinsic value is \$8. The owner of the put option who simultaneously buys the underlying security and exercises the put will net \$8 since by exercising the option the security will be sold to the writer for \$100, while the security is purchased in the market for \$92. The intrinsic value is zero if the strike price is less than or equal to the current market price.

Our put option with a strike price of \$100 would be: (1) in the money when the security's price is less than \$100, (2) out of the money when the security's price exceeds \$100, and (3) at the money when the security's price is equal to \$100.

The relationships above are summarized in Exhibit 11.

2. Time Value The **time value** of an option is the amount by which the option price exceeds its intrinsic value. The option buyer hopes that at some time up to the expiration date, changes in the market price of the underlying security will increase the value of the rights conveyed by the option. For this prospect, the option buyer is willing to pay a premium above the intrinsic value.

EXHIBIT 11 Relationship Between Security Price, Strike Price, and Intrinsic Value

If Security price > Strike price	Call option	Put option
Intrinsic value	Security price – Strike price	Zero
Jargon	In-the-money	Out-of-the-money
If Security price < Strike price	Call option	Put option
Intrinsic value	Zero	Strike price – Security price
Jargon	Out-of-the-money	In-the-money
If Security price = Strike price	Call option	Put option
Intrinsic value	Zero	Zero
Jargon	At-the-money	At-the-money

EXHIBIT 12 Summary of Factors that Affect the Price of an American Option on a Fixed Income Instrument

Factor	General effect of an increase of factor on:	
	Call price	Put price
Price of underlying security	increase	decrease
Strike price	decrease	increase
Time to expiration	increase	increase
Expected interest rate volatility	increase	increase
Short-term risk-free rate	increase	decrease
Coupon payments	decrease	increase

For example, if the price of a call option with a strike price of \$100 is \$9 when the current price of the security is \$105, the time value of this option is \$4 (\$9 minus its intrinsic value of \$5). Had the current price of the security been \$90 instead of \$105, then the time value of this option would be \$9 because the option has no intrinsic value.

B. Factors that Influence the Value of an Option on a Fixed Income Instrument

There are six factors that influence the value of an option when the *underlying security is a fixed income instrument*:

1. current market price of the underlying security
2. strike price of the option
3. time to expiration of the option
4. expected interest rate volatility over the life of the option
5. short-term risk-free interest rate over the life of the option
6. coupon interest payment over the life of the option

The impact of each of these factors may depend on whether (1) the option is a call or a put, and (2) the option is an American option or a European option. A summary of the effect of each factor on American put and call option prices is presented in Exhibit 12.

1. Price of the Underlying Security The option price changes as the price of the underlying security changes. For a call option, as the price of the underlying security increases (holding all other factors constant), the option price increases. This is because the intrinsic

value of a call option increases when the price of the underlying security increases. The opposite holds for a put option: as the price of the underlying security increases, the price of a put option decreases. This is because the intrinsic value of a put option decreases when the price of the underlying security increases.

2. **Strike Price** All other factors equal, the lower the strike price, the higher the price of a call option. For put options, the higher the strike price, the higher the option price.

3. **Time to Expiration of the Option** An option is a “wasting asset.” That is, after the expiration date passes the option has no value. Holding all other factors equal, the longer the time to expiration of the option, the greater the option price. As the time to expiration decreases, less time remains for the underlying security’s price to rise (for a call buyer) or to fall (for a put buyer)—to compensate the option buyer for any time value paid—and, therefore, the probability of a favorable price movement decreases. Consequently, for American options, as the time remaining until expiration decreases, the option price approaches its intrinsic value.

4. **Expected Interest Rate Volatility Over the Life of the Option** All other factors equal, the greater the expected interest rate volatility or yield volatility, the more an investor would be willing to pay for the option, and the more an option writer would demand for it. This is because the greater the volatility, the greater the probability that the price of the underlying security will move in favor of the option buyer at some time before expiration. The procedure for estimating interest rate volatility is explained in Chapter 8.

5. **Short-Term Risk-Free Rate Over the Life of the Option** Buying the underlying security ties up one’s money. Buying an option on the dollar amount of the underlying security makes available for investment the difference between the security price and the option price at the risk-free rate. All other factors constant, the higher the short-term risk-free rate, the greater the cost of buying the underlying security and carrying it to the expiration date of the call option. Hence, the higher the short-term risk-free rate, the more attractive the call option is relative to the direct purchase of the underlying security. As a result, the higher the short-term risk-free rate, the greater the price of a call option. In the case of a put option, the alternative to buying a put is shorting the security. When the security is shorted, the proceeds received can be invested at the short-term risk-free rate. When the short-term risk-free rate increases, this makes it more attractive to short the security relative to buying a put option. Consequently, the value of a put option declines when the short-term risk-free rate increases.

6. **Coupon Payments Over the Life of the Option** Coupon interest payments on the underlying security tend to decrease the price of a call option because they make it more attractive to hold the underlying security than to hold the option. That is, the owner of the security receives the coupon payments but the buyer of the call option does not. The higher the coupon payment received by the owner of the security, the more attractive it is to own the security and the less attractive it is to own the call option. So, the value of a call option declines the higher the coupon payment.

The opposite is true for the put option. Coupon interest payments on the underlying security tend to increase the price of a put option. The buyer of a put option compares the position to a short position in the security. When shorting the security, the coupon payment must be paid by the short seller. So, the higher the coupon payment the less attractive it is to short the security and the more attractive it is to buy the put option. As a result, the value of a put option increases the higher the coupon payment.

C. Factors that Influence the Value of a Futures Option

There are five factors that influence the value of an option in which the underlying is a futures contract:

1. current futures price
2. strike price of the option
3. time to expiration of the option
4. expected interest rate volatility over the life of the option
5. short-term risk-free rate over the life of the option

These are the same factors that affect the value of an option on a fixed income instrument. Notice that the coupon payment is not a factor since the underlying is a futures contract.

D. Pricing Models for Options and Options on Futures

At any time, the intrinsic value of an option can be determined. The question is, what is an approximate time value for an option? To answer this question, option pricing models have been developed. Two models used to value options on a fixed income instrument are the Black-Scholes model and the arbitrage-free binomial model. For options on futures, the most common model is the Black model, a version of the Black-Scholes model. We will discuss the two common models for options on fixed income instruments before discussing the Black model for options on futures for fixed income instruments.

1. The Black-Scholes Model The most common model for the pricing of options is the Black-Scholes option pricing model. Although this model was developed for options on stocks, it has been applied to options on bonds as well. Such applications are fraught with problems. We will discuss the model for stocks first and then discuss the limitations of applying the model to bonds.

The model was developed for valuing European style call options on a non-dividend-paying common stock.⁸ To derive the option pricing model based on arbitrage arguments,

⁸The Black-Scholes option pricing formula is:

$$C = SN(d_1) - Xe^{-rt}N(d_2)$$

where

$$d_1 = \frac{\ln(S/X) + (r + 0.5s^2)t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

ln = natural logarithm

C = call option price

S = current stock price

X = strike price

r = short-term risk-free interest rate

e = 2.718 (natural antilog of 1)

t = time remaining to the expiration date (measured as a fraction of a year)

s = standard deviation of the stock returns

N(.) = the cumulative probability density. The value for N(.) is obtained from a normal distribution function.

certain assumptions are imposed. There have been a good number of extensions of the Black-Scholes option pricing that have been formulated by relaxing the assumptions. While the model was developed for common stock, it has been applied to value options on fixed income instruments. With the exception of the coupon payment, the factors that we explained earlier that determine the value of an option of a fixed income instrument are included in the Black-Scholes formula. As we will see, because of the assumptions imposed the Black-Scholes option pricing model does not necessarily produce reasonable values for options on bonds.

The option price derived from the Black-Scholes option pricing model is “fair” in the sense that if any other price existed, it would be possible to earn riskless arbitrage profits by taking an offsetting position in the underlying stock. If the price of the call option in the market is higher than that derived from the Black-Scholes option pricing model, an investor could sell the call option and buy a certain number of shares in the underlying stock. If the reverse is true and the market price of the call option is less than the “fair” price derived from the model, the investor could buy the call option and sell short a certain number of shares in the underlying stock. This process of hedging by taking a position in the underlying stock allows the investor to lock in the riskless arbitrage profit. The number of shares necessary to hedge the position changes as the factors that affect the option price change, so the hedged position must be changed constantly.

To understand the limitations of applying the model to bonds, let’s look at the values that would be derived in a couple of examples. We know that there are coupon-paying bonds and zero-coupon bonds. In our illustration we will use a zero-coupon bond. The reason is that the original Black-Scholes model was for common stock that did not pay a dividend and so a zero-coupon bond would be the equivalent type of instrument. Specifically, we will look at how the Black-Scholes option pricing model would value a zero-coupon bond with three years to maturity assuming the following:

Strike price = \$88.00

Time remaining to expiration = 2 years

Current bond price = \$83.96 (assuming for simplicity annual compounding)

Expected return volatility = standard deviation = 10%

Risk-free rate = 6%

The Black-Scholes formula gives a value of \$ 8.116.⁹ There is no reason to suspect that this estimated value is incorrect. However, let’s change the problem slightly. Instead of a strike

⁹The value is derived as follows. The current price of \$83.96 is the present value of the maturity value of \$100 discounted at 6% (assuming a flat yield curve). We know

$$S = 83.96 \quad X = 88.00 \quad t = 2 \quad s = 0.10 \quad r = 0.06$$

Substituting these values into the formula in the previous footnote:

$$d_1 = \frac{\ln(83.96/88) + (0.06 + 0.5(0.10)^2)2}{0.10\sqrt{2}} = 0.5869$$

$$d_2 = 0.5869 - 0.10\sqrt{2} = 0.4455$$

price of \$88, let's make the strike price \$100.25.¹⁰ The Black-Scholes option pricing model would give a fair value of \$2.79. Is there any reason to believe this is incorrect? Well, consider that this is a call option on a zero-coupon bond that will never have a value greater than its maturity value of \$100. Consequently, a call option with a strike price of \$100.25 must have a value of zero. Yet, the Black-Scholes option pricing model tells us that the value is \$2.79! In fact, if we assume a higher volatility, the model would give an even greater value for the call option.

Why is the Black-Scholes model off by so much in our illustration? The answer is that there are three assumptions underlying the Black-Scholes model that limit its use in pricing options on fixed income instruments.

The first assumption is that the probability distribution for the underlying security's prices assumed by the Black-Scholes model permits some probability—no matter how small—that the price can take on any positive value. But in the case of a zero-coupon bond, the price cannot take on a value above \$100. In the case of a coupon bond, we know that the price cannot exceed the sum of the coupon payments plus the maturity value. For example, for a 5-year 10% coupon bond with a maturity value of \$100, the price cannot be greater than \$150 (five coupon payments of \$10 plus the maturity value of \$100). Thus, unlike stock prices, bond prices have a maximum value. The only way that a bond's price can exceed the maximum value is if negative interest rates are permitted. While there have been instances where negative interest rates have occurred outside the United States, users of option pricing models assume that this is outcome cannot occur. Consequently, any probability distribution for prices assumed by an option pricing model that permits bond prices to be higher than the maximum bond value could generate nonsensical option prices. The Black-Scholes model does allow bond prices to exceed the maximum bond value (or, equivalently, assumes that interest rates can be negative).

The second assumption of the Black-Scholes model is that the short-term interest rate is constant over the life of the option. Yet the price of an interest rate option will change as interest rates change. A change in the short-term interest rate changes the rates along the yield curve. Therefore, for interest rate options it is inappropriate to assume that the short-term rate will be constant. The third assumption is that the variance of returns is constant over the life of the option. As a bond moves closer to maturity its price volatility declines and therefore its

From a normal distribution table it can be determined that $N(0.5869) = 0.7214$ and $N(0.4455) = 0.6720$. Then

$$C = 83.96(0.7214) - 88[e^{-(0.06)(2)}(0.6720)] = \$8.116.$$

¹⁰Substituting the new strike price, we get

$$d_1 = \frac{\ln(83.96/100.25) + (0.06 + 0.5(0.10)^2)}{0.10\sqrt{2}} = -0.3346$$

$$d_2 = -0.3346 - 0.10\sqrt{2} = -0.4761$$

From a normal distribution table $N(-0.3346) = 0.3689$ and $N(-0.4761) = 0.3170$. Then

$$C = 83.96(0.3689) - 100.25[e^{-(0.06)(2)}(0.3170)] = \$2.79.$$

return volatility declines. (We discussed this in Chapter 5 where we demonstrated the “pull to par” characteristic of a bond.) Therefore, the assumption that variance of returns is constant over the life of the option is inappropriate.¹¹

The limitations of the Black-Scholes model in pricing options on bonds are summarized below:

	Assumptions	Bond characteristics
1.	The price of the underlying has some possibility of rising to any price.	There is a maximum price for a bond and any higher price implies that a negative interest rate is possible.
2.	Short-term rates remain constant.	Changes in short-term rates occur which cause bond price to change.
3.	Volatility (variance) of returns is constant over the life of the option.	Bond return volatility decreases as the bond approaches maturity.

2. Arbitrage-Free Binomial Model The proper way to value options on bonds is to use an arbitrage-free model that takes into account the yield curve. This model can incorporate different volatility assumptions along the yield curve. The most common model employed by dealer firms is the Black-Derman-Toy model.¹²

We have already developed the basic principles for employing this model. In the chapter on valuing bonds with embedded options, Chapter 9, we explained how to construct a binomial interest rate tree such that the tree would be arbitrage free. We used the interest rate tree to value bonds (both option-free bonds and bonds with embedded options). The same tree can be used to value a stand-alone option on a bond.

To illustrate how this is done, let’s consider a 2-year European call option on a 4-year Treasury bond with a 6.5% coupon rate and a strike price of 100.25. That is, if the call option is exercised at the option expiration date, the option buyer has the right to purchase a 6.5% coupon Treasury bond with two years remaining to maturity at a price of 100.25. We will assume that the estimated par Treasury yield curve for maturities up to four years is as follows:

Maturity	Yield to maturity	Market price
1 year	3.5%	100
2 years	4.2%	100
3 years	4.7%	100
4 years	5.2%	100

For purposes of this illustration, we will assume annual-pay bonds as we did in our illustrations in Chapter 9.

The next step is to construct an arbitrage-free binomial interest rate tree. We explained the general principles of constructing the tree in Chapter 9. Exhibit 13a shows the binomial

¹¹While we have illustrated the problem of using the Black-Scholes model to price interest rate options, it can also be shown that the binomial option pricing model based on the price distribution of the underlying bond suffers from the same problems.

¹²Fischer Black, Emanuel Derman, and William Toy, “A One-Factor Model of Interest Rates and Its Application to Treasury Bond Options,” *Financial Analysts Journal* (January-February 1990), pp. 24–32.

interest rate tree assuming interest rate volatility of 10%;¹³ as noted earlier in our discussion of the factors that affect the value of an option, the volatility assumption is critical.

Exhibit 13b uses the interest rate tree in Exhibit 13a to value the underlying for our option: the 4-year Treasury bond with a 6.5% coupon rate. The boxes at each node and the backward induction method for valuation using the interest rate tree were described in Chapter 9. The valuation of our 4-year Treasury bond using the binomial interest rate tree using the backward induction method is as follows. Begin in Year 4 (the bond's maturity). We know that regardless of the path taken by interest rates, at Year 4 our 4-year Treasury bond will have a value of \$100 and there will be a coupon payment of \$6.5. That is why each box at Year 4 shows a value of \$100 and \$6.5. Working backwards to Year 3, recall from Chapter 9 that the present value of the cash flow shown at each of the two nodes to the right (Year 4 in our illustration) are discounted at the interest rate shown in the box at Year 3 and the present values are then averaged because we are assuming the two cash flows have an equal probability of occurring. For example, look at the N_{HHH} . The situation is simple in this case. The two nodes to the right (N_{HHHH} and N_{HHHL}) both have a cash flow of \$106.50. Discounting at the interest rate of 9.1987% shown in the box at N_{HHH} , the present value is \$97.529 which is also shown in the box at N_{HHH} .

Let's do one more calculation to show the backward induction procedure. Look at N_{HH} . The two nodes to the right of N_{HH} are N_{HHH} and N_{HHL} . The value shown at N_{HH} is found by (a) calculating the present value of $(\$97.529 + \$6.5)$ at 7.0053%, (b) calculating the present value of $(\$99.041 + \$6.5)$ at 7.0053%, and (c) averaging the two present values. The result of this calculation is \$97.925 and it is this value that is shown in the box at N_{HH} .

Applying the backward induction method to "Today" in Exhibit 13, the value of \$104.643 is shown. This is the arbitrage-free value of the 4-year 6.5% Treasury bond.

Our objective is not to show how to value the arbitrage-free bond. We demonstrated how this is done in Chapter 9. Our objective is to show how to value a 2-year European call option on the 4-year Treasury bond with a coupon rate of 6.5%. To do so, we use a portion of Exhibit 13b. Specifically, Exhibit 14 shows the value of our hypothetical Treasury bond (excluding coupon interest) at each node at the end of Year 2.

The same backward induction procedure used for valuing a bond is used for valuing an option on a bond. Now we start at the end of the tree, Year 2 for our option. It is the end of the tree because our option is a 2-year European option so it can only be exercised at the end of Year 2. Hence, we will not be concerned with Year 3 and Year 4. The decision rule at a node for determining the value of an option on a bond depends on whether or not the option being valued is in the money. (The exercise decision is only applied at the option's expiration date because we are valuing a European option.) That is, a call option will be exercised at one of the nodes at the option's expiration date if the bond's price at the node is greater than the strike price (i.e., if the call option is in the money). In the case of a put option, the option will be exercised at one of the nodes at the option's expiration date if the strike price at the node is greater than the bond's price (i.e., if the put option is in the money).

Start at the end of the tree, Year 2 (the expiration date of the option). Three values are shown in Exhibit 14: 97.925, 100.418, and 102.534. Given these three values, the value of a call option with a strike price of 100.25 can be determined at each node. For example, if in Year 2 the price of this Treasury bond is 97.925, then since the strike price is 100.25, the

¹³Notice that the binomial interest rate tree shown in Exhibit 13a is the same as Exhibit 5 in Chapter 9. In that chapter, the yield curve used was assumed to be that of the issuer's yield curve. In the current chapter, we are assuming it is the Treasury yield curve.

value of the call option would be zero. In the other two cases, since the price in Year 2 is greater than the strike price, the value of the call option is the difference between the price of the bond at the node and 100.25.

Exhibit 14 shows the value of the call option two years from now (the option expiration date) at each of the three nodes. Given these values, the binomial interest rate tree is used to find the present value of the call option using the backward induction procedure. The discount rates are those from the binomial interest rate tree and are shown as the second number at each node. The first number at each node for Year 1 is the average present value found by discounting the call option value at the two nodes to the right using the discount rate at the node. (It is the average because it is assumed that each present value has an equal probability of occurring.) Now let's move back one year to "Today." The value of the option is the first number shown at the root (i.e., Today) of the tree, \$0.6056. The calculations to obtain this value are explained below.

To obtain the value for the option at N_H , the values of 0 and 0.168414 are discounted and then averaged. The discount rate is the rate shown at N_H of 5.4289%. That is,

$$0.5 \left(\frac{0}{1.054289} + \frac{0.168414}{1.054289} \right) = 0.079871$$

The multiplication by 0.5 in the above formula is because we are taking the average of the two present values.

To obtain the value for option at N_L , the values of 0.168414 and 2.283501 are discounted at 4.4448% and then averaged (i.e., multiplied by one half):

$$0.5 \left(\frac{0.168414}{1.044448} + \frac{2.283501}{1.044448} \right) = 1.173785$$

The value at the root, N , is found by discounting at 3.5% the value at N_H of 0.079871 and the value at N_L of 1.173785 and then averaging the values to get the value of 0.6056 as shown below:

$$0.5 \left(\frac{0.079871}{1.035} + \frac{1.173785}{1.035} \right) = 0.6056$$

The same procedure is used to value a European put option. This is illustrated in Exhibit 15 assuming that the buyer of the put option has the right to put the current 4-year 6.5% coupon Treasury bond in two years and the strike price is 100.25. The value of the put option two years from now is shown at each of the three nodes in Year 2.

3. Black Model The most commonly used model for futures options is the one developed by Black.¹⁴ The model was initially developed for valuing European options on forward contracts.

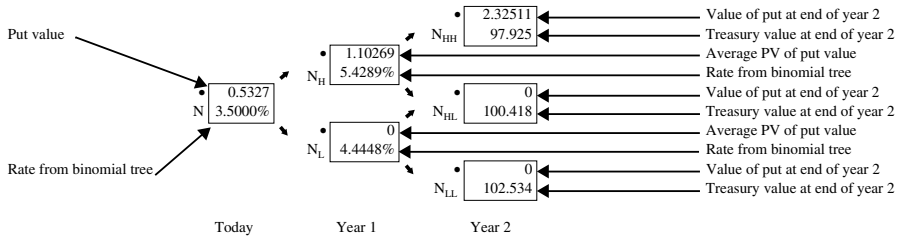
¹⁴Fischer Black, "The Pricing of Commodity Contracts," *Journal of Financial Economics* (March 1976), pp. 161-179. The value of a call and put based on the Black model is:

$$C = e^{-rt} [FN(d_1) - XN(d_2)]$$

$$P = e^{-rt} [XN(-d_2) - FN(-d_1)]$$

EXHIBIT 15 Valuing a European Put Option Using the Arbitrage-Free Binomial Method

Expiration: 2 years; Strike price: 100.25; Coupon: 6.5%; Current Price: 104.643; Volatility assumption: 10 %



There are two problems with this model. First, the Black model does not overcome the problems cited earlier for the Black-Scholes model. Failing to recognize the yield curve means that there will not be consistency between pricing Treasury futures and pricing options on Treasury futures. Second, the Black model was developed for pricing European options on futures contracts. Treasury futures options, however, are American options. Despite its limitations, the Black model is the most common model for pricing short-dated options on Treasury futures.

E. Sensitivity of Option Price to Change in Factors

In employing options in an investment strategy, a money manager would like to know how sensitive the price of an option is to a change in any one of the factors that affects its price. These measures are commonly referred to as the “Greeks.” Here we look at the sensitivity of a call option’s price to changes in the price of the underlying bond, the time to expiration, and expected interest rate volatility. The same measures apply when the underlying is a Treasury bond futures contract.

1. The Call Option Price and the Price of the Underlying Bond The sensitivity of an option to a change in the price of the underlying bond is called the **delta** of the option. Specifically,

$$\text{delta} = \frac{\text{change in option price}}{\text{change in price of underlying bond}}$$

where

$$d_1 = \frac{\ln(F/X) + 0.5s^2t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

ln = natural logarithm

C = call option price

P = put option price

F = futures price

X = strike price

r = short-term risk-free rate

e = 2.718 (natural antilog of 1)

t = time remaining to the expiration date (measured as a fraction of a year)

s = standard deviation of the return

N(.) = the cumulative probability density. The value for N(.) is obtained from a normal distribution function.

For a call option, the delta is positive since as we noted earlier, the higher the price of the underlying bond, the higher the option price. For a put option, delta is negative because the higher the price of the underlying bond, the lower the option price.

Let's interpret the delta. Suppose that the delta of a call option is 0.4. This means that if the price of the underlying bond increases by \$1, the price of the call option will increase by approximately \$0.40. Suppose that the delta of a put option is -0.2 . This means that if the price of the underlying bond increases by \$1 the price of the put option will decrease by approximately \$0.20. The delta of an option changes as the price of the underlying moves closer to or away from the strike price.

For an option where the intrinsic value is zero and the price of the underlying is very far from the strike price (i.e., an option that is said to be deep out of the money), the delta is close to 0. For example, consider a call option that expires in one year and has a strike price of \$100. Suppose that the current price of the underlying bond is \$45. Then an increase in the price of the underlying bond by \$1 (from \$45 to \$46) would not be expected to change the value of the call option.

For an option that is deep in the money, the delta of a call option is close to 1 and the delta of a put option is close to -1 . This is because the change in the option's price will closely mirror the change in the price of the underlying bond.

Delta plays the same role in approximating the sensitivity of the option's price to changes in the price of the underlying bond as duration does for measuring the sensitivity of the bond's price to changes in interest rates. In both cases, the changes are approximations. For bonds, the approximation can be improved by using the convexity measure. For an option, the approximation can be improved by calculating the **gamma** of an option. The gamma for an option is:

$$\text{gamma} = \frac{\text{change in delta}}{\text{change in price of underlying bond}}$$

2. The Call Option Price and Time to Expiration All other factors constant, the longer the time to expiration, the greater the option price. Since each day the option moves closer to the expiration date, the time to expiration decreases. The **theta** of an option measures the change in the option price as the time to expiration decreases, or equivalently, it is a measure of time decay. Theta is measured as follows:

$$\text{theta} = \frac{\text{change in price of option}}{\text{decrease in time to expiration}}$$

Assuming that the price of the underlying bond does not change (which means that the intrinsic value of the option does not change), theta measures how quickly the time value of the option changes as the option moves towards expiration.

Buyers of options prefer a low theta so that the option price does not decline quickly as it moves toward the expiration date. An option writer benefits from an option that has a high theta. This is because a high theta means that as the option moves closer to the expiration date, the option price falls faster than an option with a low theta. The option writer wants the option price to fall quickly as the option moves toward the expiration date because the option can then be bought back at a lower price.

3. The Call Option Price and Expected Interest Rate Volatility All other factors constant, a change in the expected interest rate volatility will change the option price.

The **kappa** of an option measures the change in the price of the option for a 1% change in expected interest rate volatility. (An option's kappa is also referred to as its **vega**.) That is,

$$\text{kappa} = \frac{\text{change in option price}}{1\% \text{ change in expected interest rate volatility}}$$

The kappa of an option is positive because, as explained earlier, the option price increases when expected interest rate volatility increases.

V. CAPS AND FLOORS

In applying the backward induction methodology to valuing caps and floors, the decision to exercise at a node will depend on whether or not the cap or the floor is in the money. Remember that a cap and a floor are nothing more than a package or strip of options. More specifically, they are a strip of European options on interest rates. Thus, to value a cap, the value of each period's cap, called a **caplet**, is found and the values of all the caplets are then summed. The same can be done for a floor.

To illustrate how this is done, we will use the binomial tree given in Exhibit 13a. We will simplify the analysis by ignoring the difference in the timing of the payments for caps and floors. Specifically, the settlement payment for caps and floors is in arrears. Allowing for that would require complicating the presentation and revising the binomial tree.¹⁵ Consider first a 5.2% 3-year cap with a notional amount of \$10 million. The reference rate is the 1-year rate in the binomial tree. The payoff for the cap is annual.

Exhibits 16a, 16b, and 16c show how this cap is valued by valuing the three caplets. The value for the caplet for any year, say Year X, is found as follows. First, calculate the payoff in Year X at each node as either:¹⁶

1. zero if the 1-year rate at the node is less than or equal to 5.2%, or
2. the notional principal amount of \$10 million times the difference between the 1-year rate at the node and 5.2% if the 1-year rate at the node is greater than 5.2%.

Then, the backward induction method is used to determine the value of the Year X caplet.

For example, consider the Year 3 caplet. At the top node in Year 3 of Exhibit 16c, the 1-year rate is 9.1987%. Since the 1-year rate at this node exceeds 5.2%, the payoff in Year 3 is:

$$\$10,000,000 \times (0.091987 - 0.052) = \$399,870$$

Let's show how the values shown at the nodes N_{HH} , N_H , and the root of the tree, N , are determined. For node N_{HH} we look at the value for the cap at the two nodes to its right, N_{HHH} and N_{HHL} . The backward induction method involves discounting the values at these

¹⁵Professor Donald Smith of Boston University provides a framework for the adjustment. A presentation using his framework is provided in Chapter 28 of Frank J. Fabozzi, *Bond Markets, Analysis, and Strategies* (Upper Saddle River, NJ: Pearson Prentice Hall, 2006).

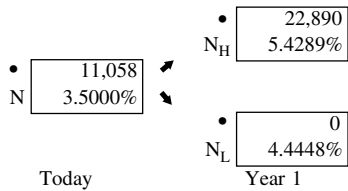
¹⁶Mathematically, the decision at a node is expressed as follows:

$$\$10,000,000 \times \text{Maximum}[(\text{Rate at node} - 5.2\%), 0]$$

EXHIBIT 16 Valuation of a 3-Year 5.2% Cap (10% Volatility Assumed)

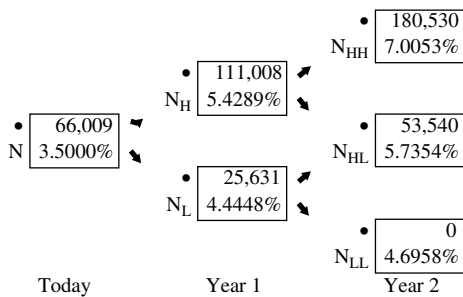
Assumptions
Cap rate: 5.2%
Notional amount: \$10,000,000
Payment frequency: Annual

Panel a: The Value of the Year 1 Caplet



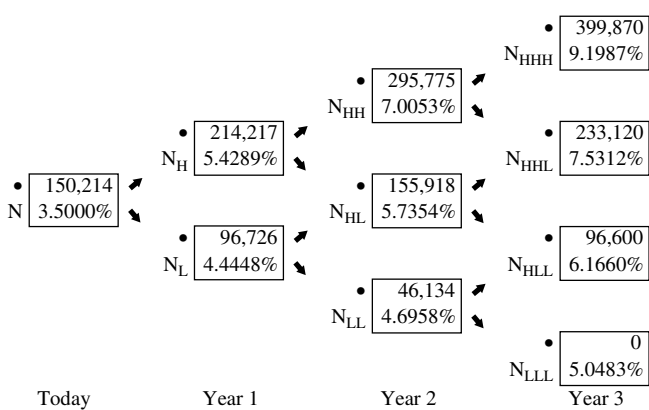
Value of Year 1 caplet = \$11,058

Panel b: The Value of the Year 2 Caplet



Value of Year 2 caplet = \$66,009

Panel c: The Value of the Year 3 Caplet



Value of Year 3 caplet = \$150,214

Summary: Value of 3-Year Cap = \$11,058 + \$66,009 + \$150,214 = \$227,281
Note on calculations: Payoff in last box of each exhibit is

$\$10,000,000 \times \text{Maximum} [(\text{Rate at node} - 5.2\%, 0)]$

nodes, \$399,870 and \$233,120, by the interest rate from the binomial tree at node N_{HH} , 7.0053%, and computing the average present value. That is,

$$\text{Value at } N_{HH} = [\$399,870/(1.070053) + \$233,120/(1.070053)]/2 = \$295,775$$

This is the value reported at N_{HH} .

Now let's see how the value at node N_H is determined. Using the backward induction method, the values at nodes N_{HH} and N_{HL} are discounted at the interest rate from the binomial tree at node N_H , 5.4289%, and then the present value is averaged. That is,

$$\text{Value at } N_H = [\$295,775/(1.054289) + \$155,918/(1.054289)]/2 = \$214,217$$

This is the value reported at N_H .

Finally, we get the value at the root, node N , which is the value of the Year 3 caplet found by discounting the value at N_H and N_L by 3.5% (the interest rate at node N) and then averaging the two present values. Doing so gives:

$$\text{Value at } N = [\$214,217/(1.035) + \$96,726/(1.035)]/2 = \$150,214$$

This is the value reported at N .

Following the same procedure, the value of the Year 2 caplet is found to be \$66,009 and the value of the Year 1 caplet is \$11,058. The value of the cap is then the sum of the values of the three caplets. That is,

$$\text{value of cap} = \text{value of Year 1 caplet} + \text{value of Year 2 caplet} + \text{value of Year 3 caplet}$$

Thus, the value of the cap is \$227,281, found by adding \$11,058, \$66,009, and \$150,214.

Similarly, an interest rate floor can be valued. Exhibit 17 shows how for a 4.8% 3-year floor with a notional amount of \$10 million. Again, the reference rate is the 1-year rate in the binomial tree and the payoff for the floor is annual. The value for the floor for any year, called a **floorlet**, say Year X , is found as follows. First, calculate the payoff in Year X at each node as either¹⁷

1. zero if the 1-year rate at the node is greater than or equal to 4.8%, or
2. the notional amount of \$10 million times the difference between 4.8% and the 1-year rate at the node if the 1-year rate at the node is less than 4.8%.

Let's see how the value of the Year 2 floorlet is determined using the backward induction method. Specifically, we will see how to compute the values at nodes N_{LL} , N_L , and N (the root of the tree and the value of the Year 2 floorlet). Look first at N_{LL} . Since the rate of 4.6958% is less than the floor rate of 4.8%, there is a payoff equal to

$$\$10,000,000 \times (0.048 - 0.046958) = \$10,420$$

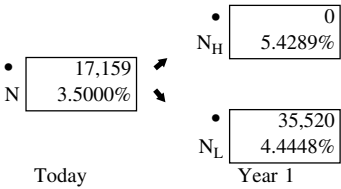
¹⁷Mathematically, the decision to exercise at a node is expressed as follows:

$$\$10,000,000 \times \text{Maximum} [(4.8\% - \text{Rate at node}), 0]$$

EXHIBIT 17 Valuation of a 3-Year 4.8% Floor (10% Volatility Assumed)

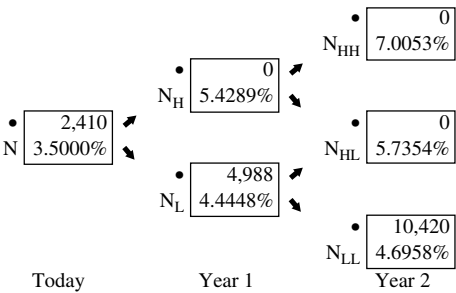
Assumptions
Floor rate: 4.8%
Notional amount: \$10,000,000
Payment frequency: Annual

Panel a: The Value of the Year 1 Floorlet



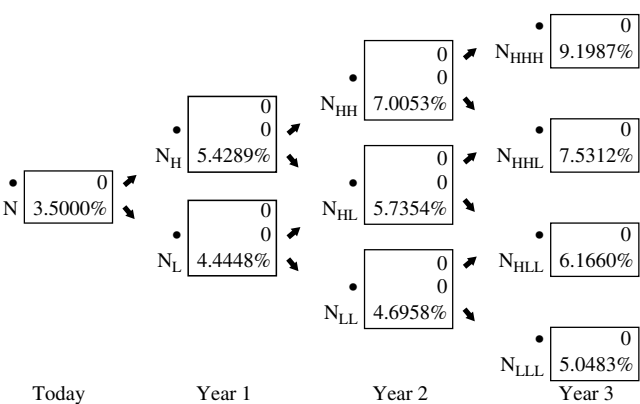
Value of Year 1 floorlet = \$17,159

Panel b: The Value of the Year 2 Floorlet



Value of Year 2 floorlet = \$2,410

Panel c: The Value of the Year 3 Floorlet



Value of Year 3 floorlet = \$0

Summary: Value of 3-Year Floor = \$17,159 + \$2,410 + \$0 = \$19,569

Note on calculations: Payoff in last box of each exhibit is

$\$10,000,000 \times \text{Maximum} [(4.8\% - \text{Rate at node}), 0]$

This is the value shown at node N_{LL} . Now we use the backward induction method to compute the value at N_L . We use the values at N_{LL} and N_{HL} to get the value at N_L . The two values are discounted at 4.4448% (the interest rate at node N_L) and then averaged. That is,

$$\text{Value at } N_L = [0/(1.044448) + \$10,420/(1.044448)]/2 = \$4,988$$

This is the value reported at N_L .

Finally, we compute the value for the Year 2 floorlet by discounting the values at N_H and N_L at 3.5% and then averaging, as shown below:

$$\text{Value at } N = [0/(1.035) + \$4,988/(1.035)]/2 = \$2,410$$

This is the value shown at the root of the tree and is the value of the Year 2 floorlet.

Adding the Year 1 floorlet, Year 2 floorlet, and Year 3 floorlet shown in Exhibit 17 gives the value of the 3-year floor: $\$17,159 + \$2,410 + \$0 = \$19,569$.

CHAPTER 15

GENERAL PRINCIPLES OF CREDIT ANALYSIS

I. INTRODUCTION

The credit risk of a bond includes:

1. the risk that the issuer will default on its obligation and
2. the risk that the bond's value will decline and/or the bond's price performance will be worse than that of other bonds against which the investor is compared because either (a) the market requires a higher spread due to a perceived increase in the risk that the issuer will default or (b) companies that assign ratings to bonds will lower a bond's rating.

The first risk is referred to as **default risk**. The second risk is labeled based on the reason for the adverse or inferior performance. The risk attributable to an increase in the spread, or more specifically the credit spread, is referred to as **credit spread risk**; the risk attributable to a lowering of the credit rating (i.e., a downgrading) is referred to as **downgrade risk**.¹

Credit analysis of any entity—a corporation, a municipality, or a sovereign government—involves the analysis of a multitude of quantitative and qualitative factors over the past, present, and future. There are four general approaches to gauging credit risk:

- credit ratings
- traditional credit analysis
- credit scoring models
- credit risk models

In this chapter, we discuss each approach. Our primary focus is on the credit analysis of corporate bonds.

II. CREDIT RATINGS

A **credit rating** is a formal opinion given by a specialized company of the default risk faced by investing in a particular issue of debt securities. The specialized companies that provide credit

¹These types of credit risk were discussed in detail earlier.

ratings are referred to as “rating agencies.” The three nationally recognized rating agencies in the United States are Moody’s Investors Service, Standard & Poor’s Corporation, and Fitch Ratings. The symbols used by these rating agencies and a summary description of each rating is provided in Chapter 2.

A. Rating Process, Surveillance, and Review

The rating process begins when a rating agency receives a formal request from an entity planning to issue a bond in which it seeks a rating for the bond issue (i.e., an “issue specific credit rating”). The cost associated with obtaining a credit rating is paid by the entity making the request for a rating. The request for a rating is made because without one, it would be difficult for the entity to issue a bond. The rating assigned applies to the specific bond to be issued, *not* to the entity requesting the rating. A rating agency may also be requested to provide a rating for a company that has no public debt outstanding (i.e., an “issuer credit rating”). This is done for companies that are parties in derivative transactions, such as swaps, so that market participants can assess counterparty risk.²

Once a credit rating is assigned to a corporate debt obligation, a rating agency monitors the credit quality of the issuer and can reassign a different credit rating to its bonds. An “upgrade” occurs when there is an improvement in the credit quality of an issue; a “downgrade” occurs when there is a deterioration in the credit quality of an issue. As noted earlier, downgrade risk is the risk that an issue will be downgraded.

Typically, before an issue’s rating is changed, the rating agency will announce in advance that it is reviewing the issue with the potential for upgrade or downgrade. The issue in such cases is said to be on “rating watch” or “credit watch.” In the announcement, the rating agency will state the direction of the potential change in rating—upgrade or downgrade. Typically, a decision will be made within three months.

In addition, rating agencies will issue rating outlooks. A **rating outlook** is a projection of whether an issue in the long term (from six months to two years) is likely to be upgraded, downgraded, or maintain its current rating. Rating agencies designate a rating outlook as either positive (i.e., likely to be upgraded), negative (i.e., likely to be downgraded), or stable (i.e., likely to be no change in the rating).

B. Gauging Default Risk and Downgrade Risk

The information available to investors from rating agencies about credit risk are: (1) ratings, (2) rating watches or credit watches, and (3) rating outlooks. Moreover, periodic studies by the rating agencies provide information to investors about credit risk. Below we describe how the information provided by rating agencies can be used to gauge two forms of credit risk: default risk and downgrade risk.

For long-term debt obligations, a **credit rating** is a forward-looking assessment of (1) the probability of default and (2) the relative magnitude of the loss should a default occur. For short-term debt obligations (i.e., obligations with initial maturities of one year or less), a credit rating is a forward-looking assessment of the probability of default. Consequently, credit ratings are the rating agencies assessment of the default risk associated with a bond issue.

Periodic studies by rating agencies provide information about two aspects of default risk—default rates and default loss rates. First, rating agencies study and make available to

²Counterparty risk is the risk that a party to a financial transaction will default on its obligation.

investors the percentage of bonds of a given rating at the beginning of a period that have defaulted at the end of the period. This percentage is referred to as the **default rate**.³ For example, a rating agency might report that the one-year default rate for triple B rated bonds is 1.8%. These studies have shown that the lower the credit rating, the higher the default rate. Rating agency studies also show default loss rates by rating and other characteristics of the issue (e.g., level of seniority and industry). A **default loss rate** is a measure of the magnitude of the potential of the loss should a default occur.⁴

A study by Moody's found that for a corporate bond, its ratings combined with its rating watches and rating outlook status provide a better gauge for default risk than using the ratings alone.⁵ The authors of the study looked at one-year and three-year default rates from 1996 through 2003 for senior unsecured rated bonds and within each rating by rating watch (watch upgrade and watch downgrade) and rating outlook status (positive, stable, and negative). The one-year default rate results for three selected ratings are shown below:

Rating	Watch up	Positive	Stable	Negative	Watch down
Baa3	NA	0.20%	0.60%	1.25%	2.26%
B1	NA	0.98%	2.53%	5.07%	12.03%
Caa1	3.7%	3.82%	8.43%	14.93%	42.21%

Notice that as one moves from left to right for a given credit rating in the above table that the default rate increases. Look at the Caa1 rating. For issues that were on rating watch for a potential upgrade at the beginning of the period, 3.7% defaulted in one year. However, for those on rating watch for a potential downgrade at the beginning of the period, 42.21% defaulted in one year. This suggests that rating watches contain useful information in gauging default risk. Look at the rating outlook status for Caa1. Issues that had a negative rating outlook at the beginning of the year had a one-year default rate that was almost four times greater than issues that had a positive rating outlook.

Moody's makes the following suggestion as to how an analyst can combine the information contained in rating watches and outlook rating status to adjust the senior unsecured rating of a corporate bond:

<i>For issues on:</i>	<i>Suggestion:</i>
downgrade watch	reduce current rating by two rating notches ⁶
upgrade watch	increase current rating by two rating notches
negative outlook	reduce current rating by one rating notch
stable outlook	keep current rating
positive outlook	increase current rating by one rating notch

Of course, portfolio managers may elect to develop their own system for adjusting the current rating of a bond based on their assessment of the findings of the study by Moody's.

³There are several ways that default rates can be measured. These are described in Chapter 3.

⁴The default loss rate is described in Chapter 3.

⁵David T. Hamilton and Richard Cantor, *Rating Transitions and Defaults Conditional on Watchlist, Outlook and Rating History*, Moody's Investors Service, February 2004.

⁶A rating "notch" is a rating based on the modified rating (i.e., in the case of Moody's with the "1", "2", and "3" modifiers). For example, if an issue is rated Baa2, then a reduction of one rating notch would be a rating of Baa3. A reduction of two rating notches would be a rating of Ba1.

What is essential, however, is that in assessing the default risk when using credit ratings, portfolio managers should take into consideration rating watches and rating outlook status.

While the discussion above has focused on default risk, other studies by rating agencies also provide information. In Chapter 2, the rating transition matrix published periodically by the rating agencies was explained. A rating transition table shows the percentage of issues of each rating at the beginning of a period that was downgraded or upgraded by the end of the time period. Consequently, by looking at the percentage of downgrades for a given rating, an estimate can be obtained of the probability of a downgrade and this can serve as a measure of downgrade risk.⁷

III. TRADITIONAL CREDIT ANALYSIS

In traditional credit analysis, the analyst considers the four C's of credit:

- capacity
- collateral
- covenants
- character

Capacity is the ability of an issuer to repay its obligations. **Collateral** is looked at not only in the traditional sense of assets pledged to secure the debt, but also to the quality and value of those unpledged assets controlled by the issuer. In both senses the collateral is capable of supplying additional aid, comfort, and support to the debt and the debtholder. Assets form the basis for the generation of cash flow which services the debt in good times as well as bad. **Covenants** are the terms and conditions of the lending agreement. They lay down restrictions on how management operates the company and conducts its financial affairs. Covenants can restrict management's discretion. A default or violation of any covenant may provide a meaningful early warning alarm enabling investors to take positive and corrective action before the situation deteriorates further. Covenants have value as they play an important part in minimizing risk to creditors. They help prevent the transfer of wealth from debt holders to equity holders. **Character** of management is the foundation of sound credit. This includes the ethical reputation as well as the business qualifications and operating record of the board of directors, management, and executives responsible for the use of the borrowed funds and repayment of those funds.

A. Analysis of the Capacity to Pay

A corporation will generate the funds to service its debt from its cash flow. The cash flow is generated from revenues and reduced by the costs of operations. Therefore, in assessing the ability of an issuer to pay, an analysis of the financial statements as discussed later in this chapter is undertaken. In addition to management quality, the factors examined by analysts at Moody's are:⁸

⁷An illustration of a rating transition matrix and the calculation of the probability of downgrade were provided.

⁸"Industrial Company Rating Methodology," Moody's Investors Services: Global Credit Research (July 1998), p. 3.

1. industry trends
2. the regulatory environment
3. basic operating and competitive position
4. financial position and sources of liquidity
5. company structure (including structural subordination and priority of claim)
6. parent company support agreements
7. special event risk

In considering industry trends, analysts look at the vulnerability of the company to economic cycles, the barriers to entry, and the exposure of the company to technological changes. For firms in regulated industries, proposed changes in regulations must be analyzed to assess their impact on future cash flows. At the company level, diversification of the product line and the cost structure are examined in assessing the basic operating position of the firm.

In addition to the measures described later in this chapter for assessing a company's financial position over the past three to five years, an analyst must look at the capacity of a firm to obtain additional financing and back-up credit facilities. There are various forms of back-up credit facilities. The strongest forms of back-up credit facilities are those that are contractually binding and do not include provisions that permit the lender to refuse to provide funds. An example of such a provision is one that allows the bank to refuse funding if the bank feels that the borrower's financial condition or operating position has deteriorated significantly. (Such a provision is called a **material adverse change clause**.) Non-contractual facilities such as lines of credit that make it easy for a bank to refuse funding should be of concern to the analyst. The analyst must also examine the quality of the bank providing the back-up facility.

Analysts should also assess whether the company can use securitization as a funding source for generating liquidity. Asset securitization involves using a pool of loans or receivables as collateral for a security. The decision of whether to securitize assets to borrow or use traditional borrowing sources is done on the basis of cost. However, if traditional sources dry up when a company faces a liquidity crisis, securitization may provide the needed liquidity. An analyst should investigate the extent to which management has considered securitization as a funding source.

Other sources of liquidity for a company may be third-party guarantees, the most common being a contractual agreement with its parent company. When such a financial guarantee exists, the analyst must undertake a credit analysis of the parent company.

In the analysis of an issuer's ability to pay, the analyst will analyze the issuer's financial statements (income statement, balance sheet, and statement of cash flows), project future financial statements based on certain assumptions, and compute various measures. These measures include traditional ratio measures and cash flow measures. Below we review these measures and explain how an additional analysis of cash flows provides a better early warning alarm of potential financial difficulties than traditional ratios.

1. Traditional Ratios Traditional ratios to evaluate the ability of an issuer to meet its obligations include:

- profitability ratios
- debt and coverage ratios

a. Profitability Ratios Equity analysts focus on the earnings of a firm, particularly the earnings per share. While a holder of the debt obligation of a firm does not have the opportunity to share in the economic growth of the firm, this does not mean that a credit analyst should

ignore a firm's profitability. It is from revenues that a firm will continue to grow in order to generate cash flow to meet obligations.

Profitability ratios are utilized to explore the underlying causes of a change in the company's earnings. They show the combined effects of liquidity and asset and debt management on the profitability of the firm. These ratios break earnings per share into its basic determinants for purposes of assessing the factors underlying the profitability of the firm. They help to assess the adequacy of historical profits, and to project future profitability through better understanding of its underlying causes.

Standards for a given ratio will vary according to operating characteristics of the company being analyzed and general business conditions; such standards cannot be stated as fixed and immutable. It is assumed that the analyst has made all adjustments deemed necessary to reflect comparable and true earning power of the corporation before calculating the ratios discussed below. It is important to stress that ratios are utilized to raise significant questions requiring further analysis, not to provide answers.

Equity analysts use the DuPont formula (explained in textbooks on equity analysis) to assess the determinants of a company's earnings per share. The profitability ratios analyzed to assess earnings per share are:

- return on stockholders' equity
- return on total assets
- profit margin
- asset turnover

Each of these measures and their limitations are explained in textbooks on financial statement analysis and equity analysis so they will not be repeated here.

b. Debt and Coverage Analysis There are three sets of ratios that are used by credit analysts as indicators to assess the ability of a firm to satisfy its debt obligations:

- short-term solvency ratios
- capitalization (or financial leverage) ratios
- coverage ratios

i. Short-Term Solvency Ratios Short-term solvency ratios are used to judge the adequacy of liquid assets for meeting short-term obligations as they come due. A complete analysis of the adequacy of working capital for meeting current liabilities as they come due and assessing management's efficiency in using working capital would require a thorough analysis of cash flows and forecasts of fund flows in future periods that will be discussed in the next section. However, ratios provide a crude but useful assessment of working capital. The following two ratios are calculated to assess the adequacy of working capital for a firm:

- the current ratio
- the acid-test ratio

The current ratio is calculated by dividing current assets by current liabilities:

$$\text{current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

The current ratio indicates the company's coverage of current liabilities by current assets. For example, if the ratio were 2:1, the firm could realize only half of the values stated in the balance sheet in liquidating current assets and still have adequate funds to pay all current liabilities.

A general standard for this ratio (such as 2:1) is *not* useful. Such a standard fails to recognize that an appropriate current ratio is a function of the nature of a company's business and would vary with differing operating cycles of different businesses. The **operating cycle** of a company is the duration from the time cash is invested in goods and services to the time that investment produces cash.⁹

A **current asset** is one that is expected to be converted into cash in the ordinary operating cycle of a business. Inventory, therefore, is a current asset. In a tobacco or liquor manufacturing company, inventory may be as much as 80% to 90% of current assets. However, for a liquor company that inventory may have to age four years or more before it can be converted into a salable asset. Such a company typically would require a much higher current ratio than average to have adequate liquidity to meet current liabilities maturing in one year. For a public utility company where there is no inventory or receivables collection problem, a current ratio of 1.1 or 1.2 to 1 has proved satisfactory. Industry averages are published by organizations such as Dun & Bradstreet and Robert Morris Associates. While industry averages have their faults, they are preferable to general standards that do not recognize operating differences among classes of companies.

The current ratio has a major weakness as an analytical tool. It ignores the composition of current assets, which may be as important as their relationship with current liabilities. Therefore, current ratio analysis must be supplemented by other working capital ratios.

Since the problem in meeting current liabilities may rest on slowness or even inability to convert inventories into cash to meet current obligations, the **acid-test ratio** (also called the **quick ratio**) is recommended. This is the ratio of current assets minus inventories to current liabilities; that is:

$$\text{acid-test ratio} = \frac{\text{current assets} - \text{inventories}}{\text{current liabilities}}$$

This ratio does assume that receivables are of good quality and will be converted into cash over the next year.

ii. Capitalization Ratios Credit analysts also calculate **capitalization ratios** to determine the extent to which the corporation is using financial leverage. These ratios, also called **financial leverage ratios**, can be interpreted only in the context of the stability of industry and company earnings and cash flow. The assumption is that the greater the stability of industry and company earnings and cash flow, the more the company is able to accept the risk associated with financial leverage, and the higher the allowable ratio of debt to total capitalization (the total dollar amount of all long-term sources of funds in the balance sheet).

⁹For example, a firm that produces and sells goods has an operating cycle comprising of four phases: (1) purchase of raw material and produce goods, investing in inventory; (2) sell goods, generating sales, which may or may not be for cash; (3) extend credit, creating accounts receivable; and, (4) collect accounts receivable, generating cash. The four phases make up the cycle of cash use and generation. The operating cycle would be somewhat different for companies that produce services rather than goods, but the idea is the same—the operating cycle is the length of time it takes to generate cash through the investment of cash.

There are many variations to be found within the industry to calculate capitalization ratios. Two such ratios are shown below:

$$\begin{aligned} \text{long-term debt to capitalization} &= \frac{\text{long-term debt}}{\text{long-term debt} + \text{shareholders' equity including minority interest}} \\ \text{total debt to capitalization} &= \frac{\text{current liabilities} + \text{long-term debt}}{\text{long-term debt} + \text{current liabilities} + \text{shareholders' equity including minority interest}} \end{aligned}$$

where shareholders' equity includes preferred stock.

For both ratios, the higher the ratio, the greater the financial leverage. The value used to measure debt in both ratios is book value. It is useful to calculate stockholders' equity at market as well as at book value for the purpose of determining these ratios. A market calculation for common equity may indicate considerably more or less financial leverage than a book calculation.

Commercial rating companies and most Wall Street analysts rely heavily upon the long-term debt to capitalization ratio, and this is often provided in research reports sent out to clients. While this ratio can be useful, it should be noted that in recent years, given the uncertain interest rate environment, many corporations have taken to financing a good deal of their business with short-term debt. Indeed, an imaginative treasurer with a keen insight into money market activities can earn as much for a company as a plant manager, simply by switching debt from long term to short term and vice versa, at the right time.

Other considerations in using the long-term debt to capitalization ratio involves leased assets. Many corporations rent buildings and equipment under long-term lease contracts. Required rental payments are contractual obligations similar to bond coupon and repayment obligations. However, assets acquired through leasing (i.e., those leases classified as operating leases) may not be capitalized and shown in the balance sheet. Two companies, therefore, might work with the same amount of fixed assets and produce the same profits before interest or rental payments, but the one leasing a high proportion of its productive equipment could show significantly lower financial leverage.

iii. Coverage Tests **Coverage ratios** are used to test the adequacy of cash flows generated through earnings for purposes of meeting debt and lease obligations. The four most commonly used coverage ratios are:

- EBIT interest coverage ratio
- EBITDA interest coverage ratio
- funds from operations/total debt ratio
- free operating cash flow/total debt ratio

EBIT stands for “earnings before interest and taxes.” The **EBIT interest coverage ratio** is simply EBIT divided by the annual interest expense. (Interest expense includes “capitalized interest.” This is effectively interest expense imputed for capitalized assets, the most important of which is leased assets.) Interest expense is tax deductible and, therefore, all earnings before taxes are available for paying such charges. Also, the interest should be added back to determine the amount available to meet annual interest expenses.

EBITDA stands for “earnings before interest, taxes, depreciation, and amortization.” The **EBITDA interest coverage ratio** is simply the ratio of EBITDA divided by the annual interest expense.

The last two ratios listed above indicate the amount of funds from operations relative to the amount of total debt. The funds from operations includes net income plus the following: depreciation, amortization, deferred income taxes, and other noncash items. The definition of free operating cash flows varies by rating agency. In the next section we describe one variant of free operating cash flow.

Suggested standards for coverage ratios are based on experience and empirical studies relating the incidence of defaults over a number of years to such ratios. Different standards are needed for a highly cyclical company than for a stable company. In the case study presented in the appendix to this chapter, benchmark ratios (as measured in terms of median ratios) by credit rating are presented for the coverage ratios described above, as well as for the capitalization ratios.

2. Cash Flow Analysis Will the ratios just described be sufficient to help an analyst identify companies that may encounter financial difficulties? Consider the study by Largay and Stickney who analyzed the financial statements of W.T. Grant during the 1966–1974 period preceding its bankruptcy in 1975 and ultimate liquidation.¹⁰ They noted that financial indicators such as profitability ratios, turnover ratios, and liquidity ratios showed some down trends, but provided no definite clues to the company’s impending bankruptcy. A study of cash flows from operations, however, revealed that company operations were causing an increasing drain on cash, rather than providing cash.¹¹ This necessitated an increased use of external financing, the required interest payments on which exacerbated the cash flow drain. Cash flow analysis clearly was a valuable tool in this case since W.T. Grant had been running a negative cash flow from operations for years. Yet none of the traditional ratios discussed above take into account the cash flow from operations.

The need to look at cash flow is emphasized by Standard & Poor’s:

*Cash flow analysis is the single most critical aspect of all credit rating decisions. It takes on added importance for speculative-grade issuers. While companies with investment-grade ratings generally have ready access to external cash to cover temporary shortfalls, junk-bond issuers lack this degree of flexibility and have fewer alternatives to internally generated cash for servicing debt.*¹²

S&P also notes that: “Discussions about cash flow often suffer from lack of uniform definition of terms.”¹³ Below we describe how S&P’s terminology with respect to four cash flow concepts: operating cash flow, free operating cash flow, discretionary cash flow, and prefinancing cash flow. In addition, we discuss the various ratios employing these cash flow measures.

¹⁰J.A. Largay III and C.P. Stickney, “Cash Flows, Ratio Analysis and the W.T. Grant Company Bankruptcy,” *Financial Analysts Journal* (July-August 1980), pp. 51-54.

¹¹For the period investigated, a statement of changes of financial position (on a working capital basis) was required prior to 1988.

¹²Standard & Poor’s, *Corporate Ratings Criteria*, undated, p. 26.

¹³*Corporate Ratings Criteria*, p. 27.

a. Cash Flow Measures Prior to the adoption of the statement of cash flows in 1987, the information regarding a firm's cash flows was quite limited. The **statement of cash flows** is a summary over a period of time of a firm's cash flows from operating, investing, and financing activities. The firm's statement of cash flows lists separately its

- cash flows from operating activities
- cash flows from investing
- cash flow financing activities

Typically the corresponding cash flows are referred to as:¹⁴

- Cash provided by operating activities
- Cash provided by/(used for) investing activities
- Cash used for financing activities

“Cash provided by operating activities” is also referred to as “cash flow from operations.”

By analyzing these individual statement of cash flows, creditors can examine such aspects of the business as:

- The source of financing for business operations, whether through internally generated funds or external sources of funds.
- The ability of the company to meet debt obligations (interest and principal payments).
- The ability of the company to finance expansion through cash flow from its operating activities.
- The ability of the company to pay dividends to shareholders.
- The flexibility the business has in financing its operations.

A firm that generates cash flows only by selling off its assets (obtaining cash flows from investing) or by issuing more securities (obtaining cash flows from financing) cannot keep that up for very long. For future prosperity and the ability to meet its obligations, the firm must be able to generate cash flows from its operations.

Analysts have reformatted the information from the firm's income statement and statement of cash flows to obtain what they view as a better description of the company's activities. S&P begins with what it refers to as funds from operations. The **funds from operations** is defined as net income adjusted for depreciation and other noncash debits and credits. Then from the funds from operation, the following cash flow measures are computed:¹⁵

Funds from operations

Decrease (increase) in noncash current assets

Increase (decrease) in nondebt current liabilities

Operating cash flow

Decrease by capital expenditures

¹⁴Some firms use different labels. For example, Microsoft refers to these cash flows as: Net cash from operations, Net cash used for financing, and Net cash used for investing.

¹⁵*Corporate Ratings Criteria*, p. 27.

Free operating cash flow

Decrease by cash dividends

Discretionary cash flow

Decrease by acquisitions

Increase by asset disposals

Net other sources (uses) of cash

Prefinancing cash flow

Operating cash flow is therefore funds from operations reduced by changes in the investment in working capital (current assets less current liabilities). Subtracting capital expenditures gives what S&P defines as **free operating cash flow**. It is this cash flow measure that can be used to pay dividends and make acquisitions.¹⁶ Reducing free operating cash flow by cash dividends gives **discretionary cash flow**. Adjusting discretionary cash flow for managerial discretionary decisions for acquisition of other companies, the disposal of assets (e.g., lines of business or subsidiaries), and others sources or uses of cash gives **prefinancing cash flow**. As stated by S&P, prefinancing cash flow “represents the extent to which company cash flow from all internal sources have been sufficient to cover all internal needs.”¹⁷

b. Cash Flow Ratios S&P uses the following cash flow ratios (based on its cash flow definitions described above) in analyzing a company in addition to the measures described earlier for coverage ratios:

$$\frac{\text{Funds from operations}}{\text{Total debt (adjusted for off-balance sheet liabilities)}}$$

$$\frac{\text{Free operating cash flow} + \text{Interest}}{\text{Interest}}$$

$$\frac{\text{Free operating cash flow} + \text{Interest}}{\text{Interest} + \text{Annual principal repayment obligation}}$$

(the above ratio is referred to as the “debt service coverage ratio”)

$$\frac{\text{Total debt}}{\text{Discretionary cash flow}}$$

¹⁶One of the most popular measures of cash flow in equity analysis is the “free cash flow.” This cash flow measure is defined as “the cash flow available to the company’s suppliers of capital after all operating expenses (including taxes) have been paid and necessary investments in working capital (e.g., inventory) and fixed capital (e.g., equipment) have been made.” (See, John D. Stowe, Thomas R. Robinson, Jerald E. Pinto, and Dennis W. McLeavey, *Analysis of Equity Investments: Valuation* (Charlottesville, VA: Association for Investment Management and Research, 2002), p. 115.) Analysts will make different adjustments to the statement of cash flows to obtain the free cash flow depending on the accounting information that is available. The procedure for calculating free cash flow starting with net income or statement of cash flows is explained in Stowe, Robinson, Pinto, and McLeavey, *Analysis of Equity Investments: Valuation*, pp. 119–124.

¹⁷*Corporate Ratings Criteria*, p. 27.

(the above ratio is called the “debt payback period”)

$$\frac{\text{Funds from operations}}{\text{Capital spending requirements}}$$

The particular cash flow ratios that S&P focuses on depends on the type of company being analyzed. According to S&P:

Where long-term viability is more assured (i.e., higher in the rating spectrum) there can be greater emphasis on the level of funds from operations and its relation to total debt burden. These measures clearly differentiate between levels of protection over time. Focusing on debt service coverage and free cash flow becomes more critical in the analysis of a weaker company. Speculative-grade issuers typically face near-term vulnerabilities, which are better measured by free cash flow ratios.¹⁸

B. Analysis of Collateral

A corporate debt obligation can be secured or unsecured. In our discussion of creditor rights in a bankruptcy, we explained that in the case of a liquidation, proceeds from a bankruptcy are distributed to creditors based on the absolute priority rule. However, in the case of a reorganization, the absolute priority rule rarely holds. That is, an unsecured creditor may receive distributions for the entire amount of his or her claim and common stockholders may receive something, while a secured creditor may receive only a portion of its claim. The reason is that a reorganization requires approval of all the parties. Consequently, secured creditors are willing to negotiate with both unsecured creditors and stockholders in order to obtain approval of the plan of reorganization.

The question is then, what does a secured position mean in the case of a reorganization if the absolute priority rule is not followed in a reorganization? The claim position of a secured creditor is important in terms of the negotiation process. However, because absolute priority is not followed and the final distribution in a reorganization depends on the bargaining ability of the parties, some analysts place less emphasis on collateral compared to the other factors discussed earlier and covenants discussed next.

We discussed the various types of collateral used for a corporate debt issue and features that analysts should be cognizant of in looking at an investor's secured position. Other important features are covered in our discussion of covenants below.

C. Analysis of Covenants

Covenants deal with limitations and restrictions on the borrower's activities. Some covenants are common to all indentures, such as

- to pay interest, principal, and premium, if any, on a timely basis
- to pay all taxes and other claims when due unless contested in good faith
- to maintain all properties used and useful in the borrower's business in good condition and working order
- to submit periodic certificates to the trustee stating whether the debtor is in compliance with the loan agreement

¹⁸ *Corporate Ratings Criteria*, p. 27.

These covenants are called **affirmative covenants** since they call upon the debtor to make promises to do certain things.

Negative covenants are those which require the borrower not to take certain actions. There are an infinite variety of restrictions that can be placed on borrowers, depending on the type of debt issue, the economics of the industry and the nature of the business, and the lenders' desires. Some of the more common restrictive covenants include various limitations on the company's ability to incur debt, since unrestricted borrowing can lead a company and its debtholders to ruin. Thus, debt restrictions may include limits on the absolute dollar amount of debt that may be outstanding or may require a ratio test—for example, debt may be limited to no more than 60% of total capitalization or that it cannot exceed a certain percentage of net tangible assets.

There may be an interest or fixed charge coverage test. The two common tests are:

- **maintenance test:** This test requires the borrower's ratio of earnings available for interest or fixed charges to be at least a certain minimum figure on each required reporting date (such as quarterly or annually) for a certain preceding period.
- **debt incurrence test:** only comes into play when the company wishes to do additional borrowing. In order to take on additional debt, the required interest or fixed charge coverage figure adjusted for the new debt must be at a certain minimum level for the required period prior to the financing. Debt incurrence tests are generally considered less stringent than maintenance provisions.

There could also be **cash flow tests** (or **cash flow requirements**) and **working capital maintenance provisions**.

Some indentures may prohibit subsidiaries from borrowing from all other companies except the parent. Indentures often classify subsidiaries as restricted or unrestricted. **Restricted subsidiaries** are those considered to be consolidated for financial test purposes; **unrestricted subsidiaries** (often foreign and certain special-purpose companies) are those excluded from the covenants governing the parent. Often, subsidiaries are classified as unrestricted in order to allow them to finance themselves through outside sources of funds.

Limitations on dividend payments and stock repurchases may be included in indentures. Often, cash dividend payments will be limited to a certain percentage of net income earned after a specific date (often the issuance date of the debt, called the "peg date") plus a fixed amount. Sometimes the dividend formula might allow the inclusion of the net proceeds from the sale of common stock sold after the peg date. In other cases, the dividend restriction might be so worded as to prohibit the declaration and payment of cash dividends if tangible net worth (or other measures, such as consolidated quick assets) declines below a certain amount.

D. Character of a Corporation

Character analysis involves the analysis of the quality of management. In discussing the factors it considers in assigning a credit rating, Moody's Investors Service notes the following regarding the quality of management:¹⁹

Although difficult to quantify, management quality is one of the most important factors supporting an issuer's credit strength. When the unexpected occurs, it is a management's ability to react appropriately that will sustain the company's performance.

¹⁹"Industrial Company Rating Methodology," p. 6.

In assessing management quality, the analysts at Moody's, for example, try to understand the business strategies and policies formulated by management. Following are factors that are considered: (1) strategic direction, (2) financial philosophy, (3) conservatism, (4) track record, (5) succession planning, and (6) control systems.

In recent years, focus has been on the corporate governance of the firm and the role of the board of directors.

1. Corporate Governance The bylaws are the rules of governance for the corporation. The bylaws define the rights and obligations of officers, members of the board of directors, and shareholders. In most large corporations, it is not possible for each owner to participate in monitoring of the management of the business. Therefore, the owners of a corporation elect a board of directors to represent them in the major business decisions and to monitor the activities of the corporation's management. The board of directors, in turn, appoints and oversees the officers of the corporation. Directors who are also employees of the corporation are called **inside directors**; those who have no other position within the corporation are **outside directors** or **independent directors**.

It is the board of directors that decides whether to hire, retain, or dismiss the chief executive officer, to establish the compensation system for senior management, and to ensure that the proper internal corporate control systems are in place to monitor management. Generally it is believed that the greater the proportion of outside directors, the greater the board independence from the management of the company. The proportion of outside directors on corporate boards varies significantly.

Recently, there have been a number of scandals and allegations regarding the financial information that is being reported to shareholders and the market. Financial results reported in the income statements and balance sheets of some companies indicated much better performance than the true performance or much better financial condition than actual.²⁰ Along with these financial reporting issues, the independence of the auditors and the role of financial analysts have been brought to the forefront.

The eagerness of managers to present favorable results to shareholders and the market appears to be a major factor in several of the scandals. Personal enrichment at the expense of shareholders seems to explain some of the scandals. Whatever the motivation, chief executive officers (CEOs), chief financial officers (CFOs), and board members are being held directly accountable for financial disclosures. For example, in 2002, the U.S. Securities and Exchange Commission ordered sworn statements attesting to the accuracy of financial statements. The first deadline for such statements resulted in several companies restating financial results.

The accounting scandals are creating an awareness of the importance of corporate governance, the importance of the independence of the public accounting auditing function, the role of financial analysts, and the responsibilities of CEOs and CFOs.

2. Agency Problem Corporate financial theory helps us understand how the abuses that diminish shareholder value arise and the potential for mitigating the abuse. In a publicly traded company, typically the managers of a corporation are not the major owners. The managers make decisions for owners. Thus, the managers act as agents. An **agent** is a person who acts for—and exerts powers on behalf of—another person or group of persons. The person (or

²⁰Examples include Xerox, which was forced to restate earnings for several years because it had inflated pre-tax profits by \$1.4 billion, Enron, which is accused of inflating earnings and hiding substantial debt, and Worldcom, which failed to properly account for \$3.8 billion of expenses.

group of persons) the agent represents is referred to as the **principal**. The relationship between the agent and his or her principal is an **agency relationship**. There is an agency relationship between the managers and the shareholders of corporations.²¹

In an agency relationship, the agent is charged with the responsibility of acting for the principal. As a result, it is possible that the agent may not act in the best interest of the principal, but instead act in his or her own self-interest. This is because the agent has his or her own objective of maximizing personal wealth. In a large corporation, for example, the managers may enjoy many fringe benefits, such as golf club memberships, access to private jets, and company cars. These benefits (also called perquisites, or “perks”) may be useful in conducting business and may help attract or retain management personnel, but there is room for abuse. The abuse of perquisites imposes costs on the firm—and ultimately on the owners of the firm. There is also a possibility that managers who feel secure in their positions may not bother to expend their best efforts toward the business. Finally, there is the possibility that managers will act in their own self-interest, rather than in the interest of the shareholders when those interests clash. For example, management may fight the acquisition of their firm by some other firm even if the acquisition would benefit shareholders. This is because in most takeovers, the management of the acquired firm generally lose their jobs. Consequently, a manager’s self-interest may be placed ahead of those of shareholders who may be offered an attractive price for their stock in the acquisition.

There are costs involved with any effort to minimize the potential for conflict between the principal’s interest and the agent’s interest. Such costs are called **agency costs**, and they are of three types: monitoring costs, bonding costs, and residual loss.

Monitoring costs are costs incurred by the principal to monitor or limit the actions of the agent. In a corporation, shareholders may require managers to periodically report on their activities via audited accounting statements, which are sent to shareholders. The accountants’ fees and the management time lost in preparing such statements are monitoring costs. Another example is the implicit cost incurred when shareholders limit the decision-making power of managers. By doing so, the owners may miss profitable investment opportunities; the foregone profit is a monitoring cost.

The board of directors of corporation has a fiduciary duty to shareholders; that is, the legal responsibility to make decisions (or to see that decisions are made) that are in the best interests of shareholders. Part of that responsibility is to ensure that managerial decisions are also in the best interests of the shareholders. Therefore, at least part of the cost of having directors is a monitoring cost.

Bonding costs are incurred by agents to assure principals that they will act in the principal’s best interest. The name comes from the agent’s promise or bond to take certain actions. A manager may enter into a contract that requires him or her to stay on with the firm even though another company acquires it; an implicit cost is then incurred by the manager, who foregoes other employment opportunities. Even when monitoring and bonding devices are used, there may be some divergence between the interests of principals and those of agents. The resulting cost, called the **residual loss**, is the implicit cost that results because the principal’s and the agent’s interests cannot be perfectly aligned even when monitoring and bonding costs are incurred.

²¹The seminal paper on the agency-principal relationship in corporate finance is Michael Jensen and William Meckling, “Theory and the Firm: Managerial Behavior, Agency Costs and Ownership Structure,” *Journal of Financial Economics* (October 1976), pp. 305-360.

3. Stakeholders and Corporate Governance When managers of a corporation assess a potential investment in a new product, they examine the risks and the potential benefits and costs. Similarly, managers assess current investments for the same purpose; if benefits do not continue to outweigh costs, they will not continue to invest in the product but will shift their investment elsewhere. This is consistent with the goal of shareholder wealth maximization and with the allocative efficiency of the market economy.

Discontinuing investment in an unprofitable business may mean closing down plants, laying off workers, and, perhaps destroying an entire town that depends on the business for income. So decisions to invest or disinvest may affect great numbers of people. All but the smallest business firms are linked in some way to groups of persons who are dependent to a degree on the business. These groups may include suppliers, customers, the community itself, and nearby businesses, as well as employees and shareholders. The various groups of persons that depend on a firm are referred to as its **stakeholders**; they all have some *stake* in the outcomes of the firm. For example, if the Boeing Company lays off workers or increases production, the effects are felt by Seattle and the surrounding communities.

Can a firm maximize the wealth of shareholders and stakeholders at the same time? Probably. If a firm invests in the production of goods and services that meet the demand of consumers in such a way that benefits exceed costs, then the firm will be allocating the resources of the community efficiently, employing assets in their most productive use. If later the firm must disinvest—perhaps close a plant—it has a responsibility to assist employees and other stakeholders who are affected. Failure to do so could tarnish its reputation, erode its ability to attract new stakeholder groups to new investments, and ultimately act to the detriment of shareholders.

The effects of a firm's actions on others are referred to as **externalities**. Pollution is an important example. Suppose the manufacture of a product creates air pollution. If the polluting firm acts to reduce this pollution, it incurs a cost that either increases the price of its product or decreases profit and the market value of its stock. If competitors do not likewise incur costs to reduce their pollution, the firm is at a disadvantage and may be driven out of business through competitive pressure.

The firm may try to use its efforts at pollution control to enhance its reputation in the hope that this will lead to a sales increase large enough to make up for the cost of reducing pollution. This is a market solution: the market places a value on the pollution control and rewards the firm (or an industry) for it. If society really believes that pollution is bad and that pollution control is good, the interests of owners and society can be aligned.

It is more likely, however, that pollution control costs will be viewed as reducing owners' wealth. Then firms must be forced to reduce pollution through laws or government regulations. But such laws and regulations also come with a cost—the cost of enforcement. Again, if the benefits of mandatory pollution control outweigh the cost of government action, society is better off. In such a case, if the government requires all firms to reduce pollution, then pollution control costs simply become one of the conditions under which owner wealth-maximizing decisions are to be made.

4. Mitigating the Agency Problem: Standard and Codes of Best Practices for Corporate Governance Let's focus on just shareholders and the agency problem as it relates to shareholders. There are three ways that shareholders can reduce the likelihood that management will act in its self interest. First, the compensation of the manager can be tied to the price performance of the firm. Second, managers can be granted a significant equity interest in the company. While an interesting solution, in practice most CEOs and boards

have an extremely small equity interest in their firms. For example, a study of 1,000 of the largest U.S. corporations found that the median holdings of CEOs was less than 0.2% of the outstanding equity.²²

Finally, the firm's internal corporate control systems can provide a means for effectively monitoring the performance and decision-making behavior of management. The timely removal of the CEO by the board of directors who believe that a CEO's performance is not in the best interest of the shareholders is one example of how an internal corporate control system can work. In general, there are several key elements of an internal corporate control system that are necessary for the effective monitoring of management. It is the breakdown of the internal control corporate control systems that lead to corporate difficulties and the destruction of shareholder wealth.

Because of the important role placed by the board of directors, the structure and composition of the board is critical for effective corporate governance. The key is to remove the influence of the CEO on board members. This can be done in several ways. First, while there is no optimal board size, the more members the less likely the influence of the CEO. With more board members, a larger number of committees can be formed to deal with important matters of the firm. At a minimum, there should be an auditing committee, a nominating committee (for board members), and a compensation committee. Second, the composition of the committee should have a majority of independent directors and the committees should include only independent directors. Third, the nominating committee should develop sound criteria for the selection of potential directors and the retention of current board members. The nominating committee should use the services of recruiting agencies to identify potential independent board members rather than rely on the CEO or management to put forth a slate of candidates. Fourth, the sole chairman of the board of directors should not be the CEO. This practice allows the CEO to exert too much influence over board members and set agenda items at board meetings. A compromise position is having the chair of the board being jointly held by the CEO and an independent board member.

The standards and codes of best practice for effective corporate governance are evolving. Unlike securities laws or regulatory requirements (such as exchange listing requirements) which set forth rules that affect corporate governance, standards and codes of best practice go beyond the applicable securities law of the country and are adopted voluntarily by a corporation. The expectation is that the adoption of best practice for corporate governance is a signal to investors about the character of management. The standards of best practice that have become widely accepted as a benchmark are those set forth by the Organisation of Economic Cooperation and Development (OECD) in 1999. The OECD Principles of Corporate Governance cover:

- the basic rights of shareholders
- equitable treatment of shareholders
- the role of stakeholders
- disclosure and transparency
- the role of the board of directors.

Other entities that have established standards and codes for corporate governance are the Commonwealth Association for Corporate Governance, the International Corporate

²²Michael Jensen, "The Modern Industrial Revolution, Exist, and the Failure of Internal Control Systems," in Donald H. Chew, Jr. (ed.), *The New Corporate Finance: Second Edition* (New York, NY: McGraw-Hill, 1999).

Governance Network, and the Business Roundtable. Countries have established their own code and standards using the OECD principles.²³

A survey of more than 200 institutional investors throughout the world conducted between April and May 2002 by McKinsey & Company found that investors “put corporate governance on a par with financial indicators when evaluating investment decisions.”²⁴ The investors surveyed indicated that they were prepared to pay a premium for the stock of companies that they felt exhibited high governance standards.

5. Corporate Governance and Bond Ratings Empirically, there have been several studies that have investigated the impact of corporate governance on stockholder returns.²⁵ Our interest in this chapter is the relationship between corporate governance and bond ratings (and hence bond yields). A study by Bhojraj and Sengupta investigates this relationship using a large sample of 1,001 industrial bonds for the period 1991–1996.²⁶

They note that a firm’s likelihood of default can be decomposed into two risks, information risk and agency risk. **Information risk** is the risk that the available information for evaluating default risk is not credible. There are two studies that support the position that corporate governance mechanisms reduce information risk. Beasley found that the greater the proportion of a board composed of outsiders, the lower the probability of financial statement fraud.²⁷ Sengupta found that the higher the quality of corporate disclosure, the higher the bond rating.²⁸ **Agency risk** is the risk that management will make decisions in its own self interest, thereby reducing firm value. There is mixed evidence on how the different types of corporate governance mechanism affect equity returns.

Bhojraj and Sengupta argue that if corporate governance mechanisms reduce agency risk and information risk, the result is that strong corporate governance should be associated with superior bond ratings and therefore lower yields. They find that companies that have greater institutional ownership and stronger outside control of the board benefitted from lower bond yields and higher ratings on their new bond issues. Bhojraj and Sengupta conclude that their findings “are consistent with the view that institutional owners and outside directors play an active role in reducing management opportunism and promoting firm value.”

They also investigate the effect of governance mechanisms on lower rated corporate bonds. Bhojraj and Sengupta argue that the monitoring role of governance mechanisms would be more important when dealing with such bonds because traditional measures for

²³The World Bank updates country progress on corporate governance on its web site: www.worldbank.org/html/fpd/privatesector/cg/codes.htm.

²⁴McKinsey & Company’s *Global Investor Opinion Survey*, May 2002.

²⁵For a review of the impact of various corporate governance mechanism on shareholder returns, as well as the experiences and perspective of the California Public Employee Pension Fund (CalPERS), see Chapter 22 in Mark J.P. Anson, *Handbook of Alternative Assets* (Hoboken, NJ: John Wiley & Sons, 2002).

²⁶Sanjeev Bhojraj and Partha Sengupta, “Effect of Corporate Governance on Bond Ratings and Yields: The Role of Institutional Investors and Outside Directors,” *Journal of Business*, Vol. 76, No. 3 (2003), pp. 455–476.

²⁷M. Beasley, “An Empirical Analysis of the Relation Between the Board of Director Composition and Financial Statement Fraud,” *Accounting Review* (October 1996), pp. 443–465.

²⁸Partha Sengupta, “Corporate Disclosure Quality and the Cost of Debt,” *Accounting Review* (October 1998), pp. 459–474.

assessing default risk described earlier in this chapter (profitability ratios, debt and coverage ratios, and cash flow measures) may not be informative about future prospects for satisfying debt obligations. Their results are consistent with a greater role for corporate governance mechanisms in reducing default risk for corporations that issue lower rated bonds.

6. Corporate Governance Ratings Several firms have developed services that assess corporate governance. One type of service provides confidential assessment of the relative strength of a firm's corporate governance practices. The customer for this service is a corporation seeking external evaluations of its current practice. The second is a service that rates (or scores) the corporate governance mechanisms of companies. Generally, these ratings are made public at the option of the company requesting an evaluation. The motivation for developing a corporate governance rating is described by one of the firms that provides this service, Governance Metrics International (GMI), as follows:

Why are we undertaking this challenge? Our premise is simple: companies that focus on corporate governance and transparency will, over time, generate superior returns and economic performance and lower their cost of capital. The opposite is also true: companies weak in corporate governance and transparency represent increased investment risks and result in a higher cost of capital. Our hope is that GMI research and ratings will help diligent investors and corporations focus on governance on an ongoing basis, identify companies and particular items that need improvement and, just as important, recognise companies that are clearly trying to set the best example with a positive rating.²⁹

Firms that provide corporate governance ratings for companies fall into two categories. The first are those that provide ratings for companies within a country. Examples of countries where firms have produced or plan to produce corporate governance ratings are Australia, Brazil, Greece, India, Malaysia, Philippines, Russia, South Korea, and Thailand.³⁰ The second category includes firms that rate across country borders. Examples of firms that fall into this category are Standard & Poor's, Governance Metrics International, The Corporate Library, and Deminor. We discuss each below.

Standard & Poor's produces a Corporate Governance Score which, at the option of the company, may be disclosed to the public. The score or rating is based on a review of both publicly available information, interviews with senior management and directors, and confidential information that S&P may have available from its credit rating of the corporation's debt.

S&P believes that its Corporate Governance Score helps companies in the following ways:

- Benchmark their current governance practices against global best practices
- Communicate both the substance and form of their governance practices to investors, insurers, creditors, customers, regulators, employees, and other stakeholders
- Enhance the investor relations process when used as part of a program designed to highlight governance effectiveness to both potential and current investors, thus differentiating the company from its competitors³¹

²⁹Howard Sherman, "Corporate Governance Ratings," *Corporate Governance* (January 2004), p. 6.

³⁰Sherman, "Corporate Governance Ratings," p. 5.

³¹Standard & Poor's, *Corporate Governance Evaluations & Scores*, undated, p. 2.

The score is based on four key elements evaluated by S&P:³²

1. *Ownership structure and external influences*
 - Transparency of ownership structure
 - Concentration and influence of ownership and external stakeholders
2. *Shareholder rights and stakeholder relations*
 - Shareholder meeting and voting procedures
 - Ownership rights and takeover defenses
 - Stakeholder relations
3. *Transparency, disclosure and audit*
 - Content of public disclosure
 - Timing of and access to public disclosure
 - Audit process
4. *Board structure and effectiveness*
 - Board structure and independence
 - Role and effectiveness of the board
 - Director and senior executive compensation

Based on the S&P's analysis of the four key elements listed above, its assessment of the company's corporate governance practices and policies and how its policies serve shareholders and other stakeholders is reflected in the Corporate Governance Score. The score ranges from 10 (the highest score) to 1 (the lowest score).

Governance Metrics International (GMI) provides two types of ratings. The first is what GMI refers to as a its "basic" rating; the information for this rating is based on publicly available information (regulatory filings, company websites, and news services); no fee is charged to a company receiving a basic rating. The second is a fee-based "comprehensive" rating obtained from interviews with outside directors and senior management. The seven categories analyzed by GMI are shareholder rights, compensation policies, accountability of the board, financial disclosure, market for control, shareholder base, and corporate reputation. There are more than 600 metrics that are used in generating the rating. GMI's scoring model calculates a value between 1 (lowest score) and 10 (highest score). The scores are relative to the other companies that are included in the universe researched by GMI. The ratings provided include a global rating (which allows a comparison to all the companies in the universe), a home market rating (which allows a comparison to all of the companies in the home country or region), and corresponding ratings for each of the seven categories analyzed by GMI.

The Corporate Library (TCL) has a rating service it calls "Board Effectiveness Rating." Rather than using best practice standards or codes in developing its corporate governance indicators, ratings are based on what this firm believes are "proven dynamics indicators of interest to shareholders and investors."³³ The indicators include compensation, outside director shareholdings, board structure and make-up, accounting and audit oversight, and board decision-making. The focus is on "which boards are most likely to enhance and preserve shareholder value, and which boards might actually increase investor risk."³⁴ The ratings are

³² *Corporate Governance Evaluations & Scores*, p. 2.

³³ <http://www.thecorporatelibrary.net/products/ratings2003.html>.

³⁴ Indicators that best practice might suggest such as a split chairman of the board/CEO role or a lead independent director are not considered by TCL because the firm does not believe they are significant in improving board effectiveness.

expressed on a scale ranging from A (highest effectiveness) to F (lowest effectiveness). TCL does not intend for its ratings to be used on a stand alone basis. Rather, the ratings are intended to improve current investment research methods employed by investors.

Finally, Deminor focuses on corporate governance ratings for Western European firms. The rating is based on 300 corporate governance indicators obtained from public and non-public information provided by the company being rated, as well as on interviews with members of the board of directors and executive committee. The ratings are based on standards that are internationally recognized such as the OECD Principles of Corporate Governance. The four categories analyzed are (1) rights and duties of shareholders, (2) commitment to shareholder value, (3) disclosure on corporate governance, and (4) board structure and functioning. The ratings range from 1 (lowest) to 10 (highest). There is an overall rating and a rating for each of the four categories.

E. Special Considerations for High-Yield Corporate Bonds

The discussion thus far has focused on credit analysis for any issuer regardless of credit rating. There are some unique factors that should be considered in the analysis of high-yield bonds. We will discuss the following:

- analysis of debt structure
- analysis of corporate structure
- analysis of covenants

In addition, we will discuss the reasons why an equity analysis approach to high-yield bond issuers is being used.

1. Analysis of Debt Structure In January 1990, the Association for Investment Management and Research held a conference on high-yield bonds. One of the presenters at the conference was William Cornish, then President of Duff & Phelps Credit Rating Company.³⁵ In his presentation he identified a unique factor in the credit analysis of high-yield issuers—the characteristics of the types of debt obligations comprising a high-yield issuer's debt structure.³⁶

Cornish explained why it was necessary for an analyst to examine a high-yield issuer's debt structure. At the time of his presentation, new types of bonds were being introduced into the high-yield market such as deferred coupon bonds. He noted that the typical debt structure of a high-yield issuer includes:

- bank debt
- brokers loans or "bridge loans"
- reset notes
- senior debt
- senior subordinated debt
- subordinated debt (payment in kind bonds)

³⁵Duff & Phelps was acquired by Fitch.

³⁶William A. Cornish, "Unique Factors in the Credit Analysis of High-Yield Bonds," in Frank K. Reilly (ed.), *High-Yield Bonds: Analysis and Risk Assessment* (Charlottesville, VA: Association for Investment Management and Research, 1990).

Cornish then went on to explain the importance of understanding the characteristics of the diverse debt obligations that are included in a typical high-yield debt structure.

Consider first bank loans. While investment-grade issuers also have bank debt in their capital structure, high-yield issuers rely to a greater extent on this form of debt because of a lack of alternative financing sources. Banks loans have three key characteristics. First, holders of bank debt have a priority over other debt holders on the firm's assets. Second, bank debt is typically short-term (usually it is not greater than two years). Finally, the rate on bank debt floats with the level of interest rates.

There are three implications of these characteristics of bank debt for the analysis of the credit worthiness of high-yield issuers. First, because the cost of this source of debt financing is affected by changes in short-term interest rates, the analyst must incorporate changing interest rate scenarios into cash flow projections. A rise in short-term interest rates can impose severe cash flow problems for an issuer heavily financed by bank debt.

Second, because the debt is short term, bank debt must be repaid in the near future. The challenge that the analyst faces is determining where the funds will be obtained to pay off maturing bank debt. There are three sources available:

1. repayment from operating cash flow
2. refinancing
3. sale of assets

Typically, it is a combination of the above three sources that a high-yield issuer will use. The implication is that the analyst must carefully examine the timing and amount of maturing bank debt and consider the sources for repayment.

If the repayment is to come from operations, the projections of cash flow from operations become even more critical than for a high-grade issuer which can rely on a wider range of funding sources such as commercial paper. When refinancing is the source of funds for loan repayment, there is the issue discussed earlier that future conditions in the financial market must be incorporated into the analyst's projections in order to assess future funding costs.

If the source of the loan repayment is the sale of assets, the analyst must consider which assets will be sold and how the sale of such assets will impact future cash flow from operations. If key assets must be sold to pay off maturing bank debt, management is adversely impacting the ability to repay other debt in the future from cash flow from operations. In leveraged buyouts, the new management will have a specific plan for the disposal of certain assets in order to pay off bank debt and other debt or related payments. One credit analyst, Jane Tripp Howe, suggests that the analyst ask the following questions regarding asset sales:³⁷

- Can the company meet its cash obligations if the sale of assets is delayed?
- How liquid are the assets that are scheduled for sale?
- Are the appraised values for these assets accurate?

Banks will not provide short-term funds where there are insufficient assets to cover a loan in the case of liquidation. If short-term to intermediate-term funds are needed, a high-yield issuer will turn to broker loans (or bridge loans) and/or reset notes. Earlier (Chapter 1), we covered reset notes. A reset note is a security where the coupon rate is reset periodically such

³⁷Jane Tripp Howe, "Credit Considerations in Evaluating High-Yield Bonds," Chapter 21 in Frank J. Fabozzi (ed.), *Handbook of Fixed Income Securities* (Burr Ridge, IL: Irwin Professional Publishing, 1997), p. 408.

that the security price will trade at some specified premium above par value. The presence of reset notes in the debt structure is of particular concern to the analyst for two reasons. First, there is the need to analyze the impact of future interest rates and spreads to assess the impact of higher borrowing costs. Second, to avoid a higher reset rate when interest rates rise due to rising interest rates in general and/or because of a higher spread demanded by the market for the particular issuer, the issuer may seek to dispose of assets. Again the assets sold may have an adverse impact on future cash flow from operations.

While there are typically longer term bonds referred to as “senior bonds” in a high-yield issuer’s debt structure, the term “senior bonds” is misleading in the presence of bank loans. Moreover, there are deferred coupon bonds. One such bond structure is a zero-coupon bond. Deferred coupon bonds permit the issuer to postpone interest payment to some future year. As a result, the interest burden is placed on future cash flow to meet the interest obligations. Because of this burden, the presence of deferred coupon bonds may impair the ability of the issuer to improve its credit quality in future periods. Moreover, if senior bonds have deferred coupon payments, the subordinated bonds will be adversely affected over time as the amount of senior bonds increases over time relative to the amount of subordinated bonds. For example, one type of deferred coupon bond that was commonly issued at one time was the payment-in-kind (PIK) bond. With this bond structure, a high-yield issuer has the option to either pay interest in cash or pay the equivalent of interest with another bond with the same coupon rate. If the issuer does not have the ability to pay the interest in cash, payment with another bond will increase future interest expense and thereby adversely impact the issuer’s future cash flow. If the PIK bonds are senior bonds, subordinated bonds are adversely affected over time as more senior bonds are added to the capital structure and future interest expense is increased further.

2. Analysis of Corporate Structure High-yield issuers usually have a holding company structure. The assets to pay creditors of the holding company will come from the operating subsidiaries. Cornish explains why it is critical to analyze the corporate structure for a high-yield issuer. Specifically, the analyst must understand the corporate structure in order to assess how cash will be passed between subsidiaries and the parent company and among the subsidiaries. The corporate structure may be so complex that the payment structure can be confusing.

Cornish provides an illustration of this. At the time of his presentation (January 1990), Farley Inc. had the following debt structure: senior subordinated debt, subordinated notes, and junior subordinated debt. The question raised by Cornish was where Farley Inc. was going to obtain cash flow to make payments to its creditors. One possibility was to obtain funds from its operating subsidiaries. At the time, Farley Inc. had three operating subsidiaries: Fruit of the Loom, Acme Boot, and West Point Pepperell. An examination of the debt structure of Fruit of the Loom (20% owned by Farley Inc.) indicated that there was bank debt and no intercompany loans were permitted. While there were restrictions on dividend payments, none were being paid at the time. An examination of the Acme Boot (100% owned by Farley Inc.) showed that there was bank debt and while there were restrictions but no prohibitions on intercompany loans, Farley Inc. had in fact put cash into this operating subsidiary. Finally, West Point Pepperell (95% owned by Farley Inc.) had bridge loans that restricted asset sales and dividend payments. Moreover, any payments that could be made to Farley Inc. from West Point Pepperell had to be such that they would not violate West Point Pepperell’s financial ratio requirements imposed by its bridge loan. The key point of the illustration is that an analyst evaluating the ability of Farley Inc. to meet its obligations to creditors would have to look very closely at the three operating subsidiaries. Just looking at financial ratios for the

entire holding company structure would not be adequate. At the time, it was not likely that the three operating subsidiaries would be able to make any contribution to assist the parent company in paying off its creditors.

3. **Analysis of Covenants** While an analyst should of course consider covenants when evaluating any bond issue (investment grade or high yield), it is particularly important for the analysis of high-yield issuers. The importance of understanding covenants was summarized by one high-yield portfolio manager, Robert Levine, as follows:³⁸

Covenants provide insight into a company's strategy. As part of the credit process, one must read covenants within the context of the corporate strategy. It is not sufficient to hire a lawyer to review the covenants because a lawyer might miss the critical factors necessary to make the appropriate decision. Also, loopholes in covenants often provide clues about the intentions of management teams.

4. **Equity Analysis Approach** Historically, the return on high-yield bonds has been greater than that of high-grade corporate bonds but less than that of common stocks. The risk (as measured in terms of the standard deviation of returns) has been greater than the risk of high-grade bonds but less than that of common stock. Moreover, high-yield bond returns have been found to be more highly correlated to equity returns than to investment grade bond returns. This is why, for example, managers hedging high-yield bond portfolios have found that a combination of stock index futures contracts and Treasury futures contracts has offered a better hedging alternative than just hedging with Treasury bond futures.³⁹

Consequently, some portfolio managers strongly believe that high-yield bond analysis should be viewed from an equity analyst's perspective. As Stephen Esser notes:⁴⁰

Using an equity approach, or at least considering the hybrid nature of high-yield debt, can either validate or contradict the results of traditional credit analysis, causing the analyst to dig further.

He further states:⁴¹

For those who work with investing in high-yield bonds, whether issued by public or private companies, dynamic, equity-oriented analysis is invaluable. If analysts think about whether they would want to buy a particular high-yield company's stock and what will happen to the future equity value of that company, they have a useful approach because, as equity values go up, so does the equity cushion beneath the company's debt. All else being equal, the bonds then become better credits and should go up in value relative to competing bond investments.

We will not review the equity analysis framework here. But, there has been strong sentiment growing in the investment community that an equity analysis approach will provide a better framework for high-yield bond analysis than a traditional credit approach.

³⁸Robert Levine, "Unique Factors in Managing High-Yield Bond Portfolios," in *High-Yield Bonds*, p. 35.

³⁹Kenneth S. Choie, "How to Hedge a High-Yield Bond Portfolio," Chapter 13 in Frank J. Fabozzi (ed.), *The New High-Yield Debt Market* (New York, NY: HarperBusiness, 1990).

⁴⁰Stephen F. Esser, "High-Yield Bond Analysis: The Equity Perspective," in Ashwinpaul C. Sondhi (ed.), *Credit Analysis of Nontraditional Debt Securities* (Charlottesville, VA: Association for Investment Management and Research, 1995), p. 47.

⁴¹Esser, "High-Yield Bond Analysis: The Equity Perspective," p. 54.

F. Credit Analysis of Non-Corporate Bonds

In this section we will look at the key factors analyzed in assessing the credit of the following non-corporate bonds:

- asset-backed securities and non-agency mortgage-backed securities
- municipal bonds
- sovereign bonds

1. Asset-Backed Securities and Non-Agency Mortgage-Backed Securities Asset-backed securities and non-agency mortgage-backed securities expose investors to credit risk. The three nationally recognized statistical rating organizations rate asset-backed securities. We begin with the factors considered by rating agencies in assigning ratings to asset-backed securities. Then we will discuss how the agencies differ with respect to rating asset-backed securities versus corporate bonds.

a. Factors Considered by Rating Agencies In analyzing credit risk, the rating companies focus on: (1) credit quality of the collateral, (2) quality of the seller/servicer, (3) cash flow stress and payment structure, and (4) legal structure.⁴² We discuss each below.

i. Credit Quality of the Collateral Analysis of the credit quality of the collateral depends on the asset type. The rating companies will look at the underlying borrower's ability to pay and the borrower's equity in the asset. The latter will be a key determinant as to whether the underlying borrower will default or sell the asset and pay off a loan. The rating companies will look at the experience of the originators of the underlying loans and will assess whether the loans underlying a specific transaction have the same characteristics as the experience reported by the issuer.

The concentration of loans is examined. The underlying principle of asset securitization is that the large number of borrowers in a pool will reduce the credit risk via diversification. If there are a few borrowers in the pool that are significant in size relative to the entire pool balance, this diversification benefit can be lost, resulting in a higher level of default risk. This risk is called **concentration risk**. In such instances, rating companies will set concentration limits on the amount or percentage of receivables from any one borrower. If the concentration limit at issuance is exceeded, the issue will receive a lower credit rating than if the concentration limit was not exceeded. If after issuance the concentration limit is exceeded, the issue may be downgraded.

Based on its analysis of the collateral and other factors described below, a rating company will determine the amount of credit enhancement necessary for an issue to receive a particular rating. Credit enhancement levels are determined relative to a specific rating desired for a security and can be either internal or external. External credit enhancement can be either insurance, corporate guarantees, letters of credit, or cash collateral reserves. Internal credit enhancements include reserve funds, overcollateralization, and senior/subordinated structures.

⁴²Suzanne Michaud, "A Rating Agency Perspective on Asset-Backed Securities," Chapter 16 in Anand K. Bhattacharya and Frank J. Fabozzi (eds.), *Asset-Backed Securities* (New Hope, PA: Frank J. Fabozzi Associates, 1997).

ii. Quality of the Seller/Servicer All loans must be serviced. Servicing involves collecting payments from borrowers, notifying borrowers who may be delinquent, and, when necessary, recovering and disposing of the collateral if the borrower does not make loan repayments by a specified time. These responsibilities are fulfilled by a third-party to an asset-backed securities transaction called a **servicer**. The servicer may be the originator of the loans used as the collateral.

In addition to the administration of the loan portfolio as just described, the servicer is responsible for distributing the proceeds collected from the borrowers to the different bondholders according to the payment priorities. Where there are floating-rate securities in the transaction, the servicer will determine the interest rate for the period. The servicer may also be responsible for advancing payments when there are delinquencies in payments (that are likely to be collected in the future) resulting in a temporary shortfall in the payments that must be made to the bondholders.

The role of the servicer is critical in a securitization transaction. Therefore, rating agencies look at the ability of a servicer to perform its duties before assigning a rating to the bonds in a transaction. For example, the following factors are reviewed when evaluating servicers: servicing history, experience, underwriting standard for loan originations, servicing capabilities, human resources, financial condition, and growth/competition/business environment.

As explained in the chapter on asset-backed securities, the issuer is the special purpose vehicle or trust. There are no employees. The trust simply has loans and receivables. The servicer therefore plays an important role in assuring that the payments are made to the bondholders. Shortly we will see how the characteristics of the servicer affect the way in which an issue is evaluated in terms of credit quality in comparison to the rating of a corporate bond issue.

iii. Cash Flow Stress and Payment Structure As explained in the chapter on asset-backed securities, the waterfall describes how the cash flow (i.e., interest and principal payments) from the collateral will be distributed to pay trustee fees, servicing fees, other administrative fees, and interest and principal to the bondholders in the structure. In determining a rating for bond class, the process begins with an analysis of the cash flow from the collateral under different assumptions about losses and delinquencies and economic scenarios established by the rating agency. (That is, the rating agencies perform a scenario analysis.) Then in each scenario, the cash flow is distributed to all bond classes in accordance with the structure's waterfall. Once a determination of what loss to the different bond's in the structure can occur, a rating can be assigned.

iv. Legal Structure A corporation using structured financing seeks a rating on the securities it issues that is higher than its own corporate bond rating. If that is not possible, the corporation seeking funds would be better off simply issuing a corporate bond.

As explained in the discussion on securitization in Chapter 11, the corporation seeking funds will sell collateral to a special purpose vehicle (SPV). The corporation selling the collateral to the SPV is called the "seller." It is the SPV that issues the securities and is therefore referred to as the "issuer." The SPV is used so that the collateral is no longer an asset of the corporation that sold it and therefore available to the seller's creditors. The key in a securitization is to protect the buyers of the asset-backed securities issued by the SPV from having a bankruptcy judge redirect the collateral to the

creditors of the selling corporation. Consequently, the rating agencies examine the legal structure and the underlying legal documents to assure that this will not happen in a bankruptcy.⁴³

b. Corporate Bond versus Asset-Backed Securities Credit Analysis Let's look at how the rating of an asset-backed security differs from that of a corporate bond issue. To understand the difference, it is important to appreciate how the cash flow that must be generated differs for a corporate bond issue and a securitization transaction from which the asset-backed securities are created.

In a corporate bond issue, management through its operations must undertake the necessary activities that will produce revenues and collect revenues. Management will incur costs in creating products and services. These costs include management compensation, employee salaries, the costs of raw materials, and financial costs. Consequently, in evaluating the credit risk of a corporate bond issue, an analyst will examine the factors discussed earlier in this chapter regarding the corporation's capacity to pay and the corporation's character.

In contrast, in a securitization transaction, there are assets (loans or receivables) that are to be collected and distributed to bondholders (i.e., investors in the asset-backed securities). There are no operating or business risks such as the competitive environment or existence of control systems that are needed to assess the cash flow. What is important is the quality of the collateral in generating the cash flow needed to make interest and principal payments. The assurance of cash flow based on different scenarios regarding defaults and delinquencies that the rating agencies will review. The rating agencies will review the likelihood of the cash flow based on different scenarios regarding defaults and delinquencies. The greater predictability of the cash flow in a securitization transaction that will be distributed to each bond class issue due to the absence of operational risks that distinguishes it from a corporate bond issue. It is the greater predictability of the cash flow in an asset-backed security transaction due to the absence of operational risks that distinguishes it from a corporate bond issue.

In a "true" securitization transaction, the role of the servicer is to simply collect the cash flow. There is no active management with respect to the collateral as is the case of the management necessary to operate a corporation to generate cash flow to pay bondholders. Standard & Poor's defines a "true securitization" as follows:

*In a true securitization, repayment is not dependent on the ability of the servicer to replenish the pool with new collateral or to perform more than routine administrative functions.*⁴⁴

There are securitization transactions where the role of the servicer is more than administrative. Where the role of the servicer is more than administrative, Standard & Poor's, for example, refers to such transactions as **hybrid transactions**. This is because such transactions have elements of an asset-backed security transaction and a corporation performing a service. According to Standard & Poor's:

*In a hybrid transaction, the role of the servicer is akin to that of a business manager. The hybrid servicer performs not only administrative duties, as in a true securitization, but also . . . [other] services that are needed to generate cash flow for debt service.*⁴⁵

⁴³In the chapter on asset-backed securities, the role of the attorneys in a transaction was described.

⁴⁴Standard & Poor's, "Rating Hybrid Securitizations," *Structured Finance* (October 1999), p. 2.

⁴⁵"Rating Hybrid Securitizations," p. 3.

Moreover, Standard & Poor's notes that:

*Unlike a true securitization, where the servicer is a fungible entity replaceable with few, if any, consequences to the transaction, bondholders depend on the expertise of the hybrid servicer for repayment. . . . Not coincidentally, these are the same attributes that form the basis of a corporate rating of the hybrid servicer. They also explain the rating linkage between the securitization and its hybrid servicer.*⁴⁶

Standard & Poor's provides an illustration of the distinction between a true asset-backed securitization transaction and one requiring a more active role for the servicer.⁴⁷ Consider a railcar company that has several hundred leases and the leases are with a pool of diversified highly rated companies. Suppose that each lease is for 10 years and it is the responsibility of the customers—not the railcar company—to perform the necessary maintenance on the leased railcars. If there is an asset-backed security transaction backed by these leases and the term of the transaction is 10 years, then the role of the servicer is minimal. Since the leases are for 10 years and the securities issued are for 10 years, the servicer is just collecting the lease payments and distributing them to the holders of the securities. In such a transaction, it *is* possible for this issue to obtain a high investment-grade rating as a true asset-backed security transaction.

Suppose we change the assumptions as follows. The securities issued are for 25 years, not 10 years. Also assume that the railcar company, not the customers, is responsible for the servicing. Now the role of the servicer changes. The servicer will be responsible for finding new companies to release the railcars to when the original leases terminate in 10 years. This is necessary because the securities issued have a maturity of 25 years but the original leases only cover payments to securityholders for the first 10 years. It is the releasing of the railcars that is required for the last 15 years. The servicer under this new set of assumptions is also responsible for the maintenance of the railcars leased. Thus, the servicer must be capable of maintaining the railcars or have on-going arrangements with one or more companies that have the ability to perform such maintenance.

How do rating agencies evaluate hybrid transactions? These transactions will be rated both in terms of a standard methodology for rating an asset-backed security transaction and using a “quasi-corporate approach” (in the words of Standard & Poor's) which involves an analysis of the servicer. The relative weight of the evaluations in assigning a rating to an asset-backed security transaction will depend on the involvement of the servicer. The more important the role of the servicer, the more weight will be assigned to the quasi-corporate approach analysis.

2. Municipal Bonds Earlier we discussed municipal bonds available in the United States—tax-backed debt and revenue bonds. However, municipal governments in other countries are making greater use of bonds with similar structures to raise funds. Below we discuss the factors that should be considered in assessing the credit risk of an issue.

a. Tax-Backed Debt In assessing the credit risk of tax-backed debt, there are four basic categories that should be considered. The first category includes information on the issuer's debt structure to determine the overall debt burden.⁴⁸ The second category relates to the

⁴⁶“Rating Hybrid Securitizations,” p. 3.

⁴⁷“Rating Hybrid Securitizations,” p. 3.

⁴⁸For municipalities, the debt burden usually is composed of the debt per capita as well as the debt as percentages of real estate valuations and personal incomes.

issuer's ability and political discipline to maintain sound budgetary policy. The focus of attention here usually is on the issuer's general operating funds and whether it has maintained at least balanced budgets over three to five years. The third category involves determining the specific local taxes and intergovernmental revenues available to the issuer, as well as obtaining historical information both on tax collection rates, which are important when looking at property tax levies, and on the dependence of local budgets on specific revenue sources. The final category of information necessary to the credit analysis is an assessment of the issuer's overall socioeconomic environment. The determinations that have to be made here include trends of local employment distribution and composition, population growth, real estate property valuation, and personal income, among other economic factors.

b. Revenue Bonds Revenue bonds are issued for either project or enterprise financings where the bond issuers pledge to the bondholders the revenues generated by the operating projects financed, or for general public-purpose financings in which the issuers pledge to the bondholders the tax and revenue resources that were previously part of the general fund.

While there are numerous security structures for revenue bonds, the underlying principle in assessing an issuer's credit worthiness is whether the project being financed will generate sufficient cash flows to satisfy the obligations due bondholders. Consequently, the analysis of revenue bonds is similar to the analysis of corporate bonds.

In assessing the credit risk of revenue bonds, the trust indenture and legal opinion should provide legal comfort in the following bond-security areas: (1) the limits of the basic security, (2) the flow-of-funds structure, (3) the rate, or user-charge, covenant, (4) the priority-of-revenue claims, (5) the additional-bonds tests, and (6) other relevant covenants.

i. Limits of the Basic Security The trust indenture and legal opinion should explain the nature of the revenues for the bonds and how they realistically may be limited by federal, state, and local laws and procedures. The importance of this is that while most revenue bonds are structured and appear to be supported by identifiable revenue streams, those revenues sometimes can be negatively affected directly by other levels of government.

ii. Flow of Funds Structure for Revenue Bonds For a revenue bond, the revenue of the enterprise is pledged to service the debt of the issue. The details of how revenue received by the enterprise will be disbursed are set forth in the trust indenture. Typically, the flow of funds for a revenue bond is as follows. First, all revenues from the enterprise are put into a **revenue fund**. It is from the revenue fund that disbursements for expenses are made to the following funds: **operation and maintenance fund, sinking fund, debt service reserve fund, renewal and replacement fund, reserve maintenance fund, and surplus fund**.

There are structures in which it is legally permissible for others to tap the revenues of the enterprise prior to the disbursement set forth in the flow of funds structure just described. For example, it is possible that the revenue bond could be structured such that the revenue is first applied to the general obligation of the municipality that has issued the bond.

Operations of the enterprise have priority over the servicing of the issue's debt, and cash needed to operate and maintain the enterprise is deposited from the revenue fund into the operation and maintenance fund. The pledge of revenue to the bondholders is a net revenue pledge, "net" meaning after operation expenses, so cash required to service the debt is deposited next into the sinking fund. Disbursements are then made to bondholders as specified in the trust indenture. Any remaining cash is then distributed to the reserve funds.

The purpose of the debt service reserve fund is to accumulate cash to cover any shortfall of future revenue to service the issue's debt. The specific amount that must be deposited is stated

in the trust indenture. The function of the renewal and replacement fund is to accumulate cash for regularly scheduled major repairs and equipment replacement. The function of the reserve maintenance fund is to accumulate cash for extraordinary maintenance or replacement costs that might arise. Finally, if any cash remains after disbursement for operations, debt servicing, and reserves, it is deposited in the surplus fund. The entity issuing the bond can use the cash in this fund in any way it deems appropriate.

iii. Rate, or User-Charge, Covenants There are various restrictive covenants included in the trust indenture for a revenue bond to protect the bondholders. A **rate covenant** (or **user charge covenant**) dictates how charges will be set on the product or service sold by the enterprise. The covenant could specify that the minimum charges be set so as to satisfy both expenses and debt servicing, or to yield a higher rate to provide for a certain amount of reserves.

iv. Priority-of-Revenue Claims The legal opinion as summarized in the official statement should clearly indicate whether or not others can legally tap the revenue of the issuer even before they start passing through the issuer's flow-of-funds structure.

v. Additional-Bonds Test An **additional-bonds test covenant** indicates whether additional bonds with the same lien (i.e., claim against property) may be issued. If additional bonds with the same lien may be issued, the conditions that must first be satisfied are specified. Other covenants specify that the facility may not be sold, the amount of insurance to be maintained, requirements for recordkeeping and for the auditing of the enterprise's financial statements by an independent accounting firm, and requirements for maintaining the facilities in good order.

vi. Other Relevant Covenants There are other relevant covenants for the bondholder's protection that the trust indenture and legal opinion should cover. These usually include pledges by the issuer of the bonds to have insurance on the project, to have accounting records of the issuer annually audited by an outside certified public accountant, to have outside engineers annually review the condition of the facility, and to keep the facility operating for the life of the bonds.

c. Corporate Versus Municipal Bond Credit Analysis The credit analysis of municipal bonds involves the same factors and quantitative measures as in corporate credit analysis. For tax-backed debt, the analysis of the character of the public officials is the same as that of the analysis of the character of management for a corporate bond. The analysis of the ability to pay in the case of tax-backed debt involves looking at the ability of the issuing entity to generate taxes and fees. As a corporate analyst would look at the composition of the revenues and profits by product line for a corporation, the municipal analyst will look at employment, industry, and real estate valuation trends needed to generate taxes and fees.

The credit analysis of municipal revenue bonds is identical to that of a corporate bond analysis. Effectively, the enterprise issuing a municipal revenue bond must generate cash flow from operations to satisfy the bond payments. For example, here are the types of questions that a municipal analyst evaluating a toll road, bridge, or tunnel revenue bond would ask. As you read these questions you will see that they are the same types of questions that a corporate analyst would ask in evaluating a corporate issuer if it could issue a bond for a toll road, bridge, or tunnel.⁴⁹

⁴⁹Sylvan G. Feldstein and Frank J. Fabozzi, *The Dow Jones-Irwin Guide to Municipal Bonds* (Homewood, IL: Dow Jones-Irwin, 1987), p. 72.

1. What is the traffic history and how sensitive is the demand to the toll charged? Equivalently, does the toll road, bridge, or tunnel provide a vital transportation link or does it face competition from interstate highways, toll-free bridges, or mass transportation?
2. How well is the facility maintained? Has the issuer established a maintenance reserve fund at a reasonable level to use for such repair work as road resurfacing and bridge painting?
3. What is the history of labor-management relations, and can public employee strikes substantially reduce toll collections?

The covenants that are unique to a municipal revenue bond and impact the credit analysis are the rate covenants and the priority-of-revenue covenants. The former dictates how the user charges will be set to meet the bond obligations. Also, just as in the case of a bond issue of a regulated corporate entity, restrictions on pricing must be recognized. In a municipal revenue bond the analyst must determine whether changes in the user charge require approval of other governmental entities such as the governor or state legislature. Priority-of-revenue covenants specify if other parties can legally tap the revenue of the enterprise before the revenue can be passed through to bondholders.

3. Sovereign Bonds The debt of other national governments is rated by nationally recognized statistical rating organizations. These ratings are referred to as **sovereign ratings**. Standard & Poor's and Moody's rate sovereign debt. We will first look at the factors considered by rating agencies in assigning sovereign ratings and then look at a structured approach that an analyst familiar with corporate credit analysis can use in assessing sovereign credits.

The categories used by S&P in deriving their ratings are listed in Exhibit 1. The two general categories are economic risk and political risk. The former category represents S&P's assessment of the ability of a government to satisfy its obligations. Both quantitative and qualitative analyses are used in assessing economic risk. Political risk is an assessment of the willingness of a government to satisfy its obligations. A government may have the ability to pay, but many be unwilling to pay. Political risk is assessed based on qualitative analysis of the economic and political factors that influence a government's economic policies.

There are two ratings assigned to each national government. One is a **local currency debt rating** and the other is a **foreign currency debt rating**. The reason for distinguishing between the two types of debt is that historically, the default frequency differs by the currency denomination of the debt. Specifically, defaults have been greater on foreign currency denominated debt.⁵⁰

The reason for the difference in default rates for local currency debt and foreign currency debt is that if a government is willing to raise taxes and control its domestic financial system, it can generate sufficient local currency to meet its local currency debt obligation. This is not the case with foreign currency denominated debt. A national government must purchase foreign currency to meet a debt obligation in that foreign currency and therefore has less control with respect to its exchange rate. Thus, a significant depreciation of the local currency relative to a foreign currency in which a debt obligation is denominated will impair a national government's ability to satisfy a foreign currency obligation.

⁵⁰David T. Beers and Marie Cavanaugh, "Sovereign Ratings: A Primer," Chapter 6 in Frank J. Fabozzi and Alberto Franco (eds.), *Handbook of Emerging Fixed Income & Currency Markets* (New Hope, PA: Frank J. Fabozzi Associates, 1997).

EXHIBIT 1 S&P Sovereign Ratings Methodology Profile

<i>Political Risk</i> • Form of government and adaptability of political institutions • Extent of popular participation • Orderliness of leadership succession • Degree of consensus on economic policy objectives • Integration in global trade and financial system • Internal and external security risks <i>Income and Economic Structure</i> • Living standards, income, and wealth distribution • Market, non-market economy • Resource endowments, degree of diversification <i>Economic Growth Prospects</i> • Size, composition of savings, and investment • Rate, pattern of economic growth <i>Fiscal Flexibility</i> • General government operating and total budget balances • Tax competitiveness and tax-raising flexibility • Spending pressures	<i>Public Debt Burden</i> • General government financial assets • Public debt and interest burden • Currency composition, structure of public debt • Pension liabilities • Contingent liabilities <i>Price Stability</i> • Trends in price inflation • Rates of money and credit growth • Exchange rate policy • Degree of central bank autonomy <i>Balance of Payments Flexibility</i> • Impact on external accounts of fiscal and monetary policies • Structure of the current account • Composition of capital flows <i>External Debt and Liquidity</i> • Size and currency composition of public external debt • Importance of banks and other public and private entities as contingent liabilities of the sovereign • Maturity structure and debt service burden • Debt service track record • Level, composition of reserves and other public external assets

Source: David T. Beers and Marie Cavanaugh, "Sovereign Ratings: A Primer," Chapter 6 in Frank J. Fabozzi and Alberto Franco (eds.), *Handbook of Emerging Fixed Income & Currency Markets* (New Hope, PA: Frank J. Fabozzi Associates, 1997), p. 67.

The implication of this is that the factors S&P analyzes in assessing the credit worthiness of a national government's local currency debt and foreign currency debt will differ to some extent. In assessing the credit quality of local currency debt, for example, S&P emphasizes domestic government policies that foster or impede timely debt service. The key factors looked at by S&P are:

- the stability of political institutions and degree of popular participation in the political process,
- income and economic structure,

- fiscal policy and budgetary flexibility,
- monetary policy and inflation pressures, and
- public debt burden and debt service track record.⁵¹

For foreign currency debt, credit analysis by S&P focuses on the interaction of domestic and foreign government policies. S&P analyzes a country's balance of payments and the structure of its external balance sheet. The area of analysis with respect to its external balance sheet are the net public debt, total net external debt, and net external liabilities.

IV. CREDIT SCORING MODELS

The previous section described the traditional ratios and other measures that credit analysts use in assessing default risk. Several researchers have used these measures as input to assess the default risk of issuers using the statistical technique of multiple discriminant analysis (MDA). This statistical technique is primarily a classification technique that is helpful in distinguishing between or among groups of objects and in identifying the characteristics of objects responsible for their inclusion in one or another group. One of the chief advantages of MDA is that it permits a simultaneous consideration of a large number of characteristics and does not restrict the investigator to a sequential evaluation of each individual attribute. For example, MDA permits a credit analyst studying ratings of corporate bonds to examine, at one time, the total and joint impact on ratings of multiple financial ratios, financial measures, and qualitative factors. Thus, the analyst is freed from the cumbersome and possibly misleading task of looking at each characteristic in isolation from the others. MDA seeks to form groups that are internally as similar as possible but that are as different from one another as possible.

From the above description of MDA it can be seen why it has been applied to problems of why bonds get the ratings they do and what variables seem best able to account for a bond's rating. Moreover, MDA has been used as a predictor of bankruptcy. While the steps involved in MDA for predicting bond ratings and corporate bankruptcies are a specialist topic, we will discuss the results of the work by Edward Altman, the primary innovator of MDA for predicting corporate bankruptcy.⁵² The models of Altman and others involved in this area are updated periodically. Our purpose here is only to show what an MDA model looks like.

In one of Altman's earlier models, referred to as the "Z-score model," he found that the following MDA could be used to predict corporate bankruptcy:⁵³

$$Z = 1.2 X_1 + 1.4 X_2 + 3.3 X_3 + 0.6 X_4 + 1.0 X_5$$

⁵¹Beers and Cavanaugh, "Sovereign Credit Ratings: A Primer," p. 68.

⁵²See Chapters 8 and 9 in Edward I. Altman, *Corporate Financial Distress and Bankruptcy: A Complete Guide to Predicting and Avoiding Distress and Profiting from Bankruptcy* (Hoboken, NJ: John Wiley & Sons, 1993). For discussion of MDA applied to predicting municipal bond ratings, see Michael G. Ferri and Frank J. Fabozzi, "Statistical Techniques for Predicting the Credit Worthiness of Municipal Bonds," Chapter 44 in Frank J. Fabozzi, Sylvan G. Feldstein, Irving M. Pollack, and Frank G. Zarb (eds.), *The Municipal Bond Handbook: Volume I* (Homewood, IL: Dow-Jones Irwin 1983).

⁵³Edward I. Altman, "Financial Bankruptcies, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance* (September 1968), pp. 589–699.

where

- X_1 = Working capital/Total assets (in decimal)
- X_2 = Retained earnings/Total assets (in decimal)
- X_3 = Earnings before interest and taxes/Total assets (in decimal)
- X_4 = Market value of equity/Total liabilities (in decimal)
- X_5 = Sales/Total assets (number of times)
- Z = Z-score

Given the value of the five variables for a given firm, a Z-score is computed. It is the Z-score that is used to classify firms with respect to whether or not there is potentially a serious credit problem that would lead to bankruptcy. Specifically, Altman found that Z-scores less than 1.81 indicated a firm with serious credit problems while a Z-score in excess of 3.0 indicated a healthy firm.

Subsequently, Altman and his colleagues revised the Z-score model based on more recent data. The resulting model, referred to as the “Zeta model,” found that the following seven variables were important in predicting corporate bankruptcies and were highly correlated with bond ratings:⁵⁴

- Earnings before interest and taxes (EBIT)/Total assets
- Standard error of estimate of EBIT/Total assets (normalized) for 10 years
- EBIT/Interest charges
- Retained earnings/Total assets
- Current assets/Current liabilities
- Five-year average market value of equity/Total capitalization
- Total tangible assets, normalized

While credit scoring models have been found to be helpful to analysts and bond portfolio managers, they do have limitations as a replacement for human judgment in credit analysis. Marty Fridson, for example, provides the following sage advice about using MDA models:

... quantitative models tend to classify as troubled credits not only most of the companies that eventually default, but also many that do not default. Often, firms that fall into financial peril bring in new management and are revitalized without ever failing in their debt service. If faced with a huge capital loss on the bonds of a financially distressed company, an institutional investor might wish to assess the probability of a turnaround—an inherently difficult-to-quantify prospect—instead of selling purely on the basis of a default model.⁵⁵

Fridson then goes on to explain that credit analyst must bear in mind that “companies can default for reasons that a model based on reported financial data cannot pick up” and provides several actual examples of companies that filed for bankruptcy for such reasons.

⁵⁴Edward I. Altman, Robert G. Haldeman, and Paul Narayann, “Zeta Analysis: A New Model to Identify Bankruptcy Risk of Corporations,” *Journal of Banking and Finance* (June 1977), pp. 29-54.

⁵⁵Martin S. Fridson, *Financial Statement Analysis: A Practitioner’s Guide, Second Edition* (Hoboken, NJ: John Wiley & Sons, 1995), p. 195.

V. CREDIT RISK MODELS

Historically, credit risk modeling has focused on credit ratings, default rates, and traditional credit analysis. In recent years, models for assessing credit risk to value corporate bonds have been introduced. The models can be divided into two groups: structural models and reduced form models. In this section we briefly describe these models.

A. Structural Models

Structural models are credit risk models developed on the basis of option pricing theory presented by Fisher Black and Myron Scholes⁵⁶ and Robert Merton.⁵⁷ The basic idea, common to all structural-type models, is that a company defaults on its debt if the value of the assets of the company falls below a certain default point. For this reason, these models are also known as “firm-value models.”⁵⁸ In these models it has been demonstrated that default can be modeled as an option to the stockholders granted by the bondholders and, as a result, an analyst can apply the same principles used for option pricing to the valuation of risky corporate securities.⁵⁹

The use of the option pricing theory set forth by Black-Scholes-Merton (BSM) provides a significant improvement over traditional methods for valuing default risky bonds. Subsequent to the work of BSM, there have been many extensions on both a theoretical and practical level. The BSM framework has been used by a number of credit software/consulting companies, including Moody’s KMV Corporation and JP Morgan’s Credit Metrics (co-developed with Reuters). Both systems use the BSM approach to model defaults and obtain the probability of default. (Moody’s KMV refers to this probability as the “expected default frequency.”) To make the BSM model operational, model developers define default occurring when the equity price falls below a certain barrier. This simplification is due to the fact that equity prices are much more available than asset values of the company.

B. Reduced Form Models

In structural models, the default process of a corporation is driven by the value of its assets. Since the value of any option depends on the volatility of the underlying (the volatility of the asset value in structural models), the probability of default is explicitly linked to the expected volatility of a corporation’s asset value. Thus, in structural models both the default process and recovery rates should a bankruptcy occur depend on the corporation’s structural characteristics.

⁵⁶Fischer Black and Myron Scholes, “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy* (May-June 1973), pp. 637–654.

⁵⁷Robert Merton, “Theory of Rational Option Pricing,” *Bell Journal of Economics and Management* (Spring 1973), pp. 141–183, and “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance*, Vol. 29, No. 2 (1974), pp. 449–470.

⁵⁸For a more detailed discussion of these models, see Chapter 8 in Mark J.P. Anson, Frank J. Fabozzi, Mooraad Choudhry, and Ren Raw Chen, *Credit Derivatives: Instruments, Applications, and Pricing* (Hoboken, NJ: John Wiley & Sons, 2004).

⁵⁹For the underlying theory and an illustration, see Don M. Chance, *Analysis of Derivatives for the CFA Program* (Charlottesville, VA: Association for Investment Management and Research, 2003), pp. 588–591.

In contrast, **reduced form models** do not look “inside the firm,” but instead model directly the probability of default or downgrade.⁶⁰ That is, the default process and the recovery process are (1) modeled independently of the corporation’s structural features and (2) are independent of each other.

The two most popular reduced form models are the Jarrow and Turnbull model⁶¹ and the Duffie and Singleton model.⁶² The statistical tools for modeling the default process and the recovery process are a specialized topic.⁶³

APPENDIX: CASE STUDY: BERGEN BRUNSWIG CORPORATION

The purpose of this case is to illustrate how the analysis of financial statements based on traditional ratios discussed in this chapter can be used to identify a corporate issuer that might be downgraded. The corporation used in the illustration is Bergen Brunswig Corporation.

I. BACKGROUND INFORMATION

Bergen Brunswig Corporation is a supply channel management company that provides pharmaceuticals, medical-surgical supplies, and specialty products. The company also provides information management solutions and outsourcing services, as well as develops disease-specific treatment protocols and pharmaco-economic initiatives to assist in the reduction of health care costs.

The original corporate bond rating was BBB+. On December 17, 1999, S&P lowered the company’s rating to BBB– citing “disappointing results at the company’s two recently acquired businesses, PharMerica and Statlander.” PharMerica, an institutional pharmacy, suffered from changes in Medicare reimbursement policies which reduced hospital patient occupancy and the use of high-margin drugs.

On February 2, 2000, S&P decided to downgrade the company’s corporate rating again to BB. S&P’s rationale was “deteriorating conditions in the company’s core drug distribution business as well as continued losses at Statlander, a specialty drug distributor acquired in 1999.”

⁶⁰The name “reduced form” was first given by Darrell Duffie to differentiate these models from the structural form models of the Black-Scholes-Merton type.

⁶¹Robert Jarrow and Stuart Turnbull, “Pricing Derivatives on Financial Securities Subject to Default Risk,” *Journal of Finance*, Vol. 50, No. 1 (1995), pp. 53–86.

⁶²Darrell Duffie and Kenneth Singleton, “Modeling Term Structures of Defaultable Bonds,” *Review of Financial Studies*, Vol. 12 (1999), pp. 687–720.

⁶³For a further discussion of reduced form models, see Chapter 9 in Anson, Fabozzi, Choudhry, and Chen, *Credit Derivatives: Instruments, Applications, and Pricing*; Darrell Duffie and Kenneth J. Singleton, *Credit Risk: Pricing Measurement, and Management* (Princeton, NJ: Princeton University Press, 2003), or Srichander Ramaswamy, *Managing Credit Risk in Corporate Bond Portfolios* (Hoboken, NJ: John Wiley & Sons, 2003).

EXHIBIT A1 Financial Data and Selected Ratios for Bergen Brunswig: Fiscal Years 1996–1999
Based on 10K Data

		1999	1998	1997	1996
1	Revenue	1	\$17,244,905	\$13,720,017	\$11,659,127
	COGS	1	\$(16,145,378)	\$(12,969,752)	\$(11,004,696)
	SG & A		\$(837,700)	\$(534,119)	\$(479,399)
	EBIT		\$261,827	\$216,146	\$175,032
	Interest Expense	2	\$74,143	\$39,996	\$30,793
	EBIT interest coverage		3.53	5.40	5.68
2	EBIT		\$261,827	\$216,146	\$175,032
	Depreciation & Amortization		\$66,031	\$37,465	\$40,756
	EBITDA		\$327,858	\$253,611	\$215,788
	Interest Expense		\$74,143	\$39,996	\$30,793
	EBITDA interest coverage		4.42	6.34	7.01
3	Net Income		\$70,573	\$3,102	\$81,679
	Depreciation & Amortization		\$66,031	\$37,465	\$40,756
	Current Deferred Income Taxes		\$10,840	\$41,955	\$10,577
	Other Noncash Items				
	Deferred Compensation		\$2,552	\$2,809	\$2,266
	Doubtful Receivables		\$85,881	\$11,934	\$11,899
	Writedown of goodwill			\$87,271	
	Abandonment of capitalized			\$5,307	
	Funds from operations		\$235,877	\$189,843	\$147,177
	Long Term Debt		\$1,041,983	\$464,778	\$437,956
	Lease Debt Equivalent		\$82	\$53	\$59
	Long Term Debt*		\$1,042,065	\$464,831	\$438,015
	Current Maturity of LTD		\$545,923	\$6,029	\$1,021
	Total Debt		\$1,587,988	\$470,860	\$439,036
	Funds from operations/Total debt		14.85%	40.32%	33.52%
4	Funds from operations		\$235,877	\$189,843	\$147,177
	Capital Expenditure		(\$305,535)	(\$52,361)	(\$23,806)
	Working Capital		\$1,199,527	\$518,443	\$474,910
	Change in WC		\$(681,084)	\$(43,533)	\$168,697
	Free operating cash flow	3	\$(750,742)	\$93,949	\$292,068
	Total Debt		\$1,587,988	\$470,860	\$439,036
	Free operating cash flow/Total debt		−47.28%	19.95%	66.52%
5	EBIT		\$261,827	\$216,146	\$175,032
	Total debt		\$1,587,988	\$470,860	\$439,036
	Equity		\$1,495,490	\$629,064	\$644,861
	Non-current deferred taxes			\$1,791	
	Total Capital		\$3,083,478	\$1,099,924	\$1,085,688
	Average Capital		2,091,701.10	1,092,806.15	1,086,503.89
	Pretax return on capital		12.52%	19.78%	16.11%
6	Operating Income		\$261,827	\$216,146	\$175,032
	Sales		\$17,244,905	\$13,720,017	\$11,659,127
	Operating Income/Sales		1.52%	1.58%	1.50%
7	Long Term Debt*		\$1,042,065	\$464,831	\$438,015
	Long-term debt		\$1,041,983	\$464,778	\$437,956
	Shareholders' equity		\$1,495,490	\$629,064	\$644,861
	Capitalization		\$2,537,473	\$1,093,842	\$1,082,817
	Long-term debt/Capitalization		41.07%	42.50%	40.45%

EXHIBIT A1 (Continued)

		1999	1998	1997	1996
8	Total Debt	\$1,587,988	\$470,860	\$439,036	
	Shareholders' equity	\$1,495,490	\$629,064	\$644,861	
	Capitalization	\$3,083,478	\$1,099,924	\$1,083,897	
	Total debt/Capitalization	51.50%	42.81%	40.51%	

*Bergen Brunswig fiscal year ends September 30th.

1. Revenues and Cost of Goods Sold excludes bulk shipment to customers' warehouse sites. The Company only serves as a intermediary and there is no material impact on the Company's operating earnings.
2. (a) Does not include pre-tax distributions on the Company's Preferred Securities.
(b) Although the S&P formulas call for "Gross Interest Expense" net interest is used because Gross Interest Expense was unavailable in the filings and could not be inferred.
3. Free operating cash flow = Funds from operations + Capital expenditure + Change in WC.

Note for the above formula for free operating cash flow:

- (a) Capital expenditure is shown as a negative in the exhibit. That is why it is added to obtain free operating cash flow. (This is consistent with S&P's formula for free operating cash flow as given in formula (4) in Exhibit A2.)
- (b) Increase in working capital for 1998 and 1999 is shown as a negative, so is added to free operating cash flow as per the S&P formula (4) in Exhibit A2.

II. ANALYSIS

Exhibit A1 shows the financial data for the 1996–1999 fiscal years and various financial ratios. (The financial ratios are shaded in the exhibit.) The company's fiscal year ends on September 30th. Since each rating agency uses slightly different inputs for the ratios it computes, we have included the ratio definitions used by S&P in Exhibit A2.

Exhibit A3 provides a summary of all the ratios that show a deteriorating trend of Bergen Brunswig's financial condition. The eight ratios shown in the exhibit strongly suggest that Bergen Brunswig was losing its financial strength and could possibly be a candidate for downgrade. To show the degree of deterioration, Exhibit A3 also displays for the eight ratios the median ratios for BBB and BB ratings.

Exhibit A4 highlights the trending of two key ratios—EBIT interest coverage and funds from operations/total debt—relative to the BBB benchmark. For 1999, the EBIT fell below 4 times, the median for BBB rated firms. For 1999, the ratio of funds from operations to total debt fell below the median for BB rated firms.

III. CONCLUSION

An analysis of the key ratios would have clearly signaled by December 1999 that Bergen Brunswig was a candidate for downgrading. As noted earlier, S&P lowered the company's corporate credit rating from BBB+ to BBB– on December 17, 1999 and then lowered it again on February 2, 2000 to BB. Among the reasons for the downgrade, S&P indicated that it expected EBITDA/interest to drop below 4 times in fiscal year 2000.

EXHIBIT A2 S&P's Formulas for Key Ratios

1. EBIT interest coverage =	$\frac{\text{Earnings from continuing operations}^{**} \text{ before interest and taxes}}{\text{Gross interest incurred before subtracting (1) capitalized interest and (2) interest income}}$
2. EBITDA interest coverage =	$\frac{\text{Earnings from continuing operations}^{**} \text{ before interest and taxes, depreciation, and amortization}}{\text{Gross interest incurred before subtracting (1) capitalized interest and (2) interest income}}$
3. Funds from operations/total debt =	$\frac{\text{Net income from continuing operations plus depreciation, amortization, deferred income taxes, and other noncash items}}{\text{Long-term debt}^{*} \text{ plus current maturities, commercial paper, and other short-term borrowings}}$
4. Free operating cash flow/total debt =	$\frac{\text{Funds from operations minus capital expenditures, minus (plus) the increase (decrease) in working capital (excluding changes in cash)}}{\text{Long-term debt}^{*} \text{ plus current maturities, commercial paper, and other short-term borrowings}}$
5. Pretax return on capital =	$\frac{\text{EBIT}}{\text{Average of beginning of year and end of year capital, including short-term debt, current maturities, long-term debt}^{*}, \text{ non-current deferred taxes, and equity}}$
6. Operating income/sales =	$\frac{\text{Sales minus cost of goods manufactured (before depreciation and amortization), selling, general and administrative, and research and development costs}}{\text{Sales}}$
7. Long-term debt/capitalization =	$\frac{\text{Long-term debt}^{*}}{\text{Long-term debt} + \text{shareholders' equity (including preferred stock)} \text{ plus minority interest}}$
8. Total debt/capitalization =	$\frac{\text{Long-term debt}^{*} \text{ plus current maturities, commercial paper, and other short-term borrowings}}{\text{Long-term debt}^{*} \text{ plus current maturities, commercial paper, and other short-term borrowings} + \text{shareholders' equity (including preferred stock)} \text{ plus minority interest}}$

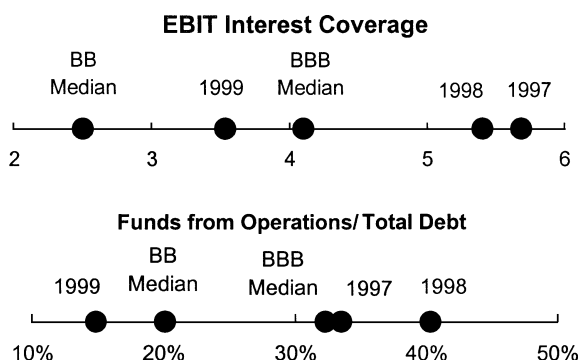
*Including amount for operating lease debt equivalent

**Including interest income and equity earnings; excluding nonrecurring items.

EXHIBIT A3 Summary of Ratios Showing Deteriorating Trend and Median Ratio for S&P BBB and BB Ratings

	1999	1998	1997	BBB Median	BB Median
EBIT interest coverage	3.53	5.40	5.68	4.10	2.5
EBITDA interest coverage	4.42	6.34	7.01	6.30	3.9
Funds from operations/total debt	14.85%	40.32%	33.52%	32.30%	20.10%
Free operating cash flow/total debt	-47.28%	19.95%	66.52%	6.30%	1.00%
Pretax return on capital	12.52%	19.78%	16.11%	15.40%	12.60%
Operating income/Sales	1.52%	1.58%	1.50%	15.80%	14.40%
Long-term debt/Capitalization	41.07%	42.50%	40.45%	40.80%	55.30%
Total debt/Capitalization	51.50%	42.81%	40.51%	46.40%	58.50%

EXHIBIT A4 Trending of EBIT Interest Leverage and Funds from Operating/Total Debt Ratios Relative to BBB and BB Benchmark



While we have demonstrated that traditional analysis could have identified a potential downgrade, what we did not address was the timing of information for preparing the analysis and reaching our conclusion. Specifically, S&P downgraded Bergen Brunswig the first time on December 17, 1999, but the company did not file its 10K (annual filing with the SEC) until December 29, 1999. Although we use the 10K numbers (since they are more accurate) for this project, an analyst would most probably estimate the ratios by looking at the 10Qs (the quarterly filings with the SEC).

Exhibit A5 shows how this would be done for the first two ratios (EBIT interest coverage and EBITDA interest coverage). An analyst would add the results from the first 9 months of 1999 (fiscal year) to the last quarter of 1998 in order to estimate the annual ratio and show the trend.

As we can see from Exhibit A5, the annualized EBITDA interest coverage ratio (5.65) is much lower than the 1998 ratio. If we take a closer look, the ratio for the first nine months of 1999 is even lower. This strongly suggests a deteriorating trend. Bergen Brunswig filed its 1999 third quarter 10Q on August 17, 1999, so the information was available to the analyst before December 1999.

EXHIBIT A5 Using Quarterly Financial Data from Bergen Brunswig to Compute EBIT and EBITDA Interest Coverage Ratios

	1999 9 months	1998 4Q	Annualized estimate
Revenue	\$12,716,939	\$3,666,166	\$16,383,105
COGS	\$(11,938,185)	\$(3,466,355)	\$(15,404,540)
SG & A	\$(559,142)	\$(144,725)	\$(703,867)
EBIT	\$219,612	\$55,086	\$274,698
Interest Expense	47,906	\$9,657	57,563
EBIT interest coverage	4.58	5.70	4.77
EBIT	\$219,612	\$55,086	\$274,698
Depreciation & Amortization	\$40,497	\$9,847	\$50,344
EBITDA	\$260,109	\$64,933	\$325,042
Interest Expense	\$47,906	\$9,657	\$57,563
EBITDA interest coverage	5.43	6.72	5.65

We believe S&P further lowered Bergen Brunswig's rating because of the first quarter result. Again, S&P's action date (February 2) is slightly earlier than the date that the company filed with the SEC (February 17). However, the weakening of Bergen Brunswig's financial position is apparent by the fact that the full-year 1999 ratios are even weaker than those for the first nine months of 1999.

CHAPTER 16

INTRODUCTION TO BOND PORTFOLIO MANAGEMENT

I. INTRODUCTION

The products of the fixed-income market, the risks associated with investing in fixed-income securities, and the fundamentals of valuation and interest rate risk measurement were covered. In depth coverage of the valuation of fixed-income securities with embedded options, features of structured products (mortgage-backed securities and asset-backed securities), and the principles of credit analysis were covered. Now, we will put all these tools together to demonstrate how to construct portfolios in such a way as to increase the likelihood of meeting an investment objective.

In this chapter, we set forth the framework for the investment management process. Regardless of the asset class being managed (i.e., stocks, bonds, or real estate), the investment management process follows the same integrated activities. John Maginn and Donald Tuttle define these activities as follows:

1. An investor's objectives, preferences, and constraints are identified and specified to develop explicit investment policies;
2. Strategies are developed and implemented through the choice of optimal combinations of financial and real assets in the marketplace;
3. Market conditions, relative asset values, and the investor's circumstances are monitored; and
4. Portfolio adjustments are made as appropriate to reflect significant change in any or all of the relevant variables.¹

In this chapter, we will use the Maginn-Tuttle framework to describe the investment management process for fixed-income portfolios. We refer to these four activities in the investment management process as:

1. setting the investment objectives
2. developing and implementing a portfolio strategy

¹John L. Maginn and Donald L. Tuttle, "The Portfolio Management Process and Its Dynamics," Chapter 1 in John L. Maginn and Donald L. Tuttle (editors), *Managing Investment Portfolios: A Dynamic Process* (New York: Warren, Gorham & Lamont, sponsored by the Institute of Chartered Financial Analysts, Second Edition, 1990), pp. 1–3 and 1–5.

3. monitoring the portfolio
4. adjusting the portfolio

A discussion of the investment management process provides a context in which to appreciate the significance of the chapters to follow in this book. Our focus, of course, is on the management of fixed-income portfolios.

II. SETTING INVESTMENT OBJECTIVES FOR FIXED-INCOME INVESTORS

The investment objectives of a fixed-income investor are often specified in terms of return and risk. The investment objectives should be expressed quantitatively in terms of some benchmark. The benchmark varies by the type of investor.

In general, we can divide fixed-income investors into two categories based on the characteristic of the benchmark. The first category of investor specifies the benchmark in terms of the investor's liability structure. The investment objective is to generate a cash flow from the fixed-income portfolio that, at a minimum, satisfies the liability structure. The second category of fixed-income investor specifies the benchmark as a particular bond market index. The investment objective may be to match the performance of the bond market index after management fees, or to outperform the bond market index after management fees by at least a predetermined number of basis points. Below we discuss each category of investor further in terms of its investment objectives.

A. Return Objectives

An investor identifies the benchmark to the portfolio manager. The manager may be employed by the investor or may be an external manager. Once the investment objective is specified in terms of a benchmark, the performance of a manager will be evaluated relative to that benchmark. It is important to note that, even if a manager is successful in terms of outperforming the benchmark, the client may not realize its investment objective. This can occur if the client does not correctly specify a benchmark to reflect its investment needs from a risk, return, and cash flow perspective. A good example is a defined benefit pension fund that specifies the investment objective in terms of a particular bond market index when, in fact, a more appropriate benchmark would be the liability structure of the fund. If the manager outperforms the benchmark, then the manager is successful. However, if sufficient cash flow is not generated to satisfy the pension liabilities, then the failure lies not with the manager but with the client's inability to properly specify an appropriate benchmark.

1. Liabilities as the Investment Objective In general, investors who specify the benchmark in terms of a liability structure that must be satisfied fall into two categories. The first category consists of investors who borrow funds and then invest those funds. Here, the objective is to earn a return on the funds borrowed that is greater than the cost of borrowing. The difference between the return on the funds invested and the cost of borrowing is called the **spread**. These investors are referred to as **funded investors**.

Depository institutions (banks, savings and loan associations, and credit unions) are clearly funded investors. Insurance companies have a wide range of products. For some products, the insurance company is a funded investor. For example, an insurance company that issues

a guaranteed investment contract (i.e., a policy where the insurance company guarantees a specified interest rate to policyholders for a specific time period) has basically borrowed money from policyholders and created a liability. Another example of a funded investor is a hedge fund, which is a highly leveraged entity that borrows on a short-term basis, typically using a repurchase agreement. (In Chapter 18 we will explain how a manager can use a repurchase agreement to borrow funds using the securities purchased as collateral.) The objective is to earn a return on the funds obtained through the repurchase agreement that is greater than the borrowing cost (i.e., repo rate).

The second category consists of institutional investors who must satisfy a liability structure but who did not borrow the funds that created the liability structure. One example is a pension sponsor that faces a liability structure based on defined benefits. A second example is a state that invests proceeds from a lottery so as to meet the state's obligation to make payments to a lottery winner.

Earlier, the focus was on fixed-income products. Now we are concerned with bond portfolio management, so we must understand how the liability structure affects the selection of a portfolio strategy. In this section we take a closer look at the nature of liabilities.

a. Liabilities Defined A liability is a potential cash outlay to be made at a future date to satisfy the contractual terms of an obligation. An institutional investor is concerned with both the **amount of the liability** and the **timing of the liability**, because the investor's assets must produce sufficient cash flow to meet promised payments.

b. Classification of Liabilities Liabilities are classified according to the degree of uncertainty with regard to amount and timing, as shown in Exhibit 1. This exhibit assumes that the holder of the obligation will not elect to cancel the obligation prior to any actual or projected payout date.

The classification of cash outlays as either "known" or "uncertain" is undoubtedly broad. When we refer to a cash outlay as being uncertain, we do not mean that it cannot be predicted. For some liabilities, the "law of large numbers" makes it easier to predict the timing and/or the amount of the cash outlays. This work is typically done by actuaries. Below we illustrate each type of liability.

A Type-I liability is one for which both the amount and timing of the liability is known with certainty. An example is a liability for which an institution knows that it must pay \$8 million six months from now. Depository institutions know the amount they are committed to pay (principal plus interest) on the maturity date of a fixed-rate deposit, assuming that the depositor does not withdraw funds prior to the maturity date. Type-I liabilities are not

EXHIBIT 1 Classification of Liabilities of Institutional Investors

Liability type	Amount of outlay	Timing of cash outlay	Example
Type I	known	known	Fixed-rate CD issued by a depository institution
Type II	known	uncertain	Standard life insurance policy
Type III	uncertain	known	Floating-rate CD issued by a depository institution
Type IV	uncertain	uncertain	Insurance policy by a property and casualty insurance company

limited to depository institutions. A guaranteed investment contract is an example of this type of liability that is created by a life insurance company.

A Type-II liability is one for which the amount of the cash outlay is known, but the timing of the cash outlay is uncertain. The most obvious example of a Type-II liability is a typical life insurance policy. There are many types of life insurance policies, but the most basic provides that, for an annual premium, a life insurance company agrees to make a specified dollar payment to policy beneficiaries upon the death of the insured. Naturally, the timing of the insured's death is uncertain.

A Type-III liability is one for which the timing of the cash outlay is known, but the amount is uncertain. An example is, a 2-year floating-rate certificate of deposit issued by a depository institution, with an interest rate that is reset quarterly based on a particular market interest rate.

For a Type-IV liability, there is uncertainty as to both the amount and the timing of the cash outlay. Numerous insurance products and pension obligations are in this category. Probably the most obvious examples are automobile and home insurance policies issued by property and casualty insurance companies. When, and if, a payment will be made to the policyholder is uncertain. Whenever an insured asset is damaged, the amount of the payment that must be made is uncertain.

The liabilities of pension plans can also be Type-IV liabilities. For defined benefit plans, retirement benefits depend on the participant's income for a specified number of years before retirement and the total number of years the participant worked. This affects the amount of the cash outlay. The timing of the cash outlay depends on whether the employee remains with the sponsoring plan until retirement and when the employee elects to retire. Moreover, both the amount and the timing depend on how the employee elects to have payments made—over the employee's lifetime or those of the employee and spouse.

2. Bond Market Index as the Investment Objective When there are no liabilities that must be met, the investment objective is often to either match or outperform a designated bond market index. It is important to note that some clients that have a liability structure choose to specify that the manager manage against a bond market index. This is particularly true of pension sponsors. The expectation of the plan sponsor is that the performance of the bond market index selected will generate sufficient cash flow to satisfy the liability structure.

In selecting a benchmark, there are several characteristics a client should consider so that the benchmark can serve as an effective tool for evaluating a portfolio manager. According to Bailey, Richards, and Tierney, the basic characteristics of any useful benchmark are:²

- *Unambiguous*: The names and weights of the securities included in the benchmark are clearly identifiable.
- *Investable*: The client has the option to simply buy-and-hold the benchmark rather than have funds actively managed.
- *Measurable*: The benchmark's return can be calculated on a reasonably frequent basis by either the manager, client, or some third party.
- *Appropriate*: The benchmark is consistent with the manager's investment style.

²Jeffrey V. Bailey, Thomas M. Richards, and David E. Tierney, "Benchmark Portfolios and the Manager/Plan Sponsor Relationship," in Frank J. Fabozzi (ed.), *Current Topics in Investment Management* (New York, NY: Harper & Row Publishers, 1990), p. 70.

- *Reflective of current investment opinions:* The manager has current investment knowledge of the securities included in the benchmark and the way they are categorized within the benchmark.³
- *Specified in advance:* The benchmark is constructed before the beginning of the period for which the manager is evaluated.

The investable characteristic of a benchmark is critical. A benchmark represents the return to a passive investment strategy. The establishment of a benchmark means that active management decisions made by a portfolio manager can be judged. For example, Bailey, Richards, and Tierney note that it is common for consultants and plan sponsors to use as a benchmark the median performance of a group of portfolio managers for the same asset class. Such a benchmark is not investable.

It is essential for a manager to understand the composition and risk profile of a bond market index. Here we review the composition of various bond market indexes. These indexes are classified as broad-based U.S. bond market indexes, specialized U.S. bond market indexes, and global and international bond market indexes. In Chapter 17, we explain the risk profile of a bond market index because the risk profile of a portfolio relative to that of the benchmark bond market index determines relative performance.

a. Broad-Based U.S. Bond Market Indexes The three broad-based U.S. bond market indexes most commonly used by institutional investors are the Lehman Brothers U.S. Aggregate Index, the Salomon Smith Barney (SSB) Broad Investment-Grade Bond Index (BIG), and the Merrill Lynch Domestic Market Index. There are more than 5,500 issues in each index. One study found that the correlation of annual returns between the broad-based bond market indexes is around 98%.⁴

The three broad-based U.S. bond market indexes are computed daily and are “market-value weighted.” This means that, for each issue, the ratio of the market value of an issue relative to the market value of all issues in the index is used as the weight for the issue in all calculations. The securities in the SSB BIG index are all trader priced. For the two other indexes, the securities are either trader priced or model priced. Each index handles intra-month cash flows that must be reinvested in a different way. For the SSB BIG index, these cash flows are assumed to be reinvested at the 1-month Treasury bill rate, while for the Merrill Lynch index, they are assumed to be reinvested in the specific issue. The Lehman index assumes no reinvestment of intra-month cash flows.

Each index is broken into sectors. The Lehman index, for example, is divided into the following six sectors: (1) Treasury sector, (2) agency sector, (3) mortgage passthrough sector, (4) commercial mortgage-backed securities sector, (5) asset-backed securities sector, and (6) credit sector. Exhibit 2 shows the percentage composition of the index as of September 8, 2003.

The agency sector includes agency debentures, but not mortgage-backed or asset-backed securities issued by federal agencies. The mortgage passthrough sector includes agency passthrough securities—Ginnie Mae, Fannie Mae, and Freddie Mac passthrough securities. Recall that the mortgage passthrough securities issued by Ginnie Mae are referred to as

³For example, for an equity benchmark, categorization might be value, growth, capitalization in terms of size. For a bond benchmark it might be the major sectors of the bond index.

⁴Frank K. Reilly and David J. Wright, “Bond Indexes,” Chapter 7 in Frank J. Fabozzi (ed.), *The Handbook of Fixed Income Securities* (New York: McGraw-Hill, 2000).

EXHIBIT 2 Percentage Composition of Lehman Brothers U.S. Aggregate Index as of September 8, 2003

Sector	Percent of market value
Treasury	21.94%
Agency	11.96
Mortgage Passthroughs	34.40
Commercial Mortgage-Backed Securities	2.55
Accept-Backed Securities	1.89
Credit	27.27
Total	100.00%

Source: *Global Relative Value*, Lehman Brothers, *Fixed Income Research*, September 8, 2003.

agency passthroughs and those issued by Fannie Mae and Freddie Mac are called **conventional passthroughs**. However, we shall simply refer to all mortgage passthroughs of these three entities as agency passthroughs. Agency collateralized mortgage obligations and agency stripped mortgage-backed securities are not included in the index. These mortgage derivatives products are not included because inclusion would result in double counting since they are created from agency passthroughs.

In constructing the index for the mortgage sector, for example, the Lehman index groups more than 800,000 individual mortgage pools with fixed-rate coupons into generic aggregates. These **generic aggregates** are defined in terms of agency (i.e., Ginnie Mae, Fannie Mae, and Freddie Mac), program type (i.e., 30-year, 15-year, balloon mortgages, etc.), coupon rate for the passthrough, and the year the passthrough was originated (i.e., vintage). For an issue to be included, it must have a minimum of \$100 million outstanding and a minimum weighted average maturity of one year. Agency passthroughs backed by pools of adjustable-rate mortgages are not included in the mortgage index. (We discuss the composition of this sector in the next chapter.)

The credit sector in the Lehman Brothers index includes corporate issues. In the other two U.S. broad-based bond market indexes, this sector is referred to as the corporate sector.

b. Specialized U.S. Bond Market Indexes The specialized U.S. bond market indexes focus on one sector or sub-sector of the bond market. Indexes on sectors of the market are published by the same three firms that produce the broad-based U.S. bond market indexes. Nonbrokerage firms have created specialized indexes for sectors. For example, Ryan Labs produces a Treasury index. Since none of the broad-based U.S. bond market indexes include noninvestment-grade or high-yield issues, indexes for this sector have been created by the three firms that have created the broad-based indexes and the firms CS First Boston and Donaldson Lufkin and Jenrette. The number of issues included in each high-yield index varies from index to index. The types of issues permitted (e.g., convertible, floating-rate, payment-in-kind) also vary. The index creators treat interim income and default issues differently.

c. Global and International Bond Market Indexes The growth in non-U.S. bond investing has resulted in the proliferation of international bond market indexes. Three types of indexes that include non-U.S. bonds are available. The first is an index that includes both U.S. and non-U.S. bonds. Such indexes are referred to as **global bond indexes** or **world bond indexes**. The second type includes only non-U.S. bonds and is commonly referred to

as **international bond indexes** or **ex-U.S. bond indexes**. Finally, there are specialized bond indexes for particular non-U.S. bond sectors. Indexes can be reported on a hedged currency basis and/or an unhedged currency basis.

There are two types of global bond indexes. The first type restricts each country sector to government bonds. For example, the Merrill Lynch Government Bond Index includes the government sectors of the following countries:

Europe-EMU: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal, and Spain

Europe-Non-EMU: Denmark, Sweden, Switzerland, and U.K.

North America: Canada and U.S.

Japan & Asia/Pacific: Australia, Japan, and New Zealand

Other similar indexes are the Salomon World Government Bond Index and the J.P. Morgan Global Bond Index.

An example of a global bond index that includes government and non-government sectors is the Lehman Brothers Global Index (which Lehman also refers to as its “Core Plus Plus” Portfolio). The index is divided into a U.S. dollar sector and a nondollar sector. The U.S. dollar sector includes all the sectors within the Lehman Brothers U.S. Aggregate Index plus high-yield bonds and dollar-denominated emerging market bonds. The nondollar sector includes the following countries: France, Germany, Italy, Spain, Sweden, and the United Kingdom. An index such as the Lehman Brothers Global Index is an appropriate benchmark for a manager who is permitted to invest in both U.S. and non-U.S. bonds.

An international bond index is an appropriate benchmark for a manager who invests only in non-U.S. bonds. There are two types of non-U.S. indexes. The first is an index that includes government and nongovernment sectors. Other indexes include just the government sector of an international bond index. For example, the Salomon Non-U.S. Government Index is a byproduct of the Salomon World Government Bond Index.

The third type of non-U.S. bond index is a specialized bond index. Examples of such indexes published by two investment banking firms are:

Lehman Brothers

Eurodollar Index

Emerging Markets Index

Pan-European Aggregate Index

Pan-European High-Yield Index

Euro-Aggregate Index

Merrill Lynch

Pan European Broad Market Index

EMU Broad Market Index

European Currencies High Yield Index

Emerging Europe Index

B. Risks

The inability to satisfy an investment objective is called **performance risk**. Let's look at the risks associated with strategies for institutional investors managing funds against a bond market index and those managing funds against a liability structure. A more detailed discussion on quantifying performance risk appears in Chapter 17.

1. Risks Associated with Managing Relative to a Bond Market Index Earlier, the risks associated with investing in individual bonds were discussed. These risks are reviewed in the next chapter, and we also expand our discussion to include risks associated with a portfolio and a benchmark index. We shall refer to the bond market index as the “benchmark index.” It is essential that a manager understand the risk profile of a benchmark index. The portfolio’s relative performance is determined by the differences between the risk profile of the benchmark index and the risk profile of the portfolio.

It is essential that we describe the risk profile of a portfolio and a benchmark index, and that we are able to quantify that risk profile. We will see in the next chapter that there are several measures to quantify a portfolio’s risk. One measure that allows the manager to incorporate all of the major risks associated with a portfolio relative to a benchmark index is **tracking error**. (We postpone discussion of this important measure until the next chapter.) For now, it is important when constructing a portfolio to be able to predict how well the performance of the portfolio will track the future performance of a benchmark index. The larger the tracking error the greater the likelihood that the portfolio’s performance will differ from the performance of the benchmark index.

2. Risks Associated with Managing Against a Liability Structure Some institutional investors want their managers to construct portfolios to meet a liability structure. A liability structure can be a single future liability or multiple liabilities. The risk associated with managing portfolios when the benchmark is liabilities is that the managed portfolio might not generate sufficient cash flow to satisfy the liability structure. Managing relative to a liability structure is popularly referred to as **asset-liability management**. (Our focus here is on managing the assets to satisfy the liability structure, not in managing liabilities.)

As is the case for assets, there are risks associated with liabilities. Here are three examples that should give you a feel for some of these risks. We will see more examples—and more details—in other chapters.

a. Call Risk for Liabilities Consider a liability that can be terminated by the holder at his option. The simplest case is a certificate of deposit specifying that the depositor can withdraw funds prior to the maturity date, but must pay a penalty to do so. If, due to a rise in interest rates, one or more depositors terminate CDs prior to the maturity date, then the depository institution will have to fund itself with new deposits (or borrow in the market) at a higher rate. But, if the depository institution previously invested in a fixed-rate asset paying less than the new borrowing rate, the depository institution realizes a negative spread. This risk is very much like call risk but in this case, the depository institution is concerned with the premature withdrawal of funds when interest rates rise, not when rates fall.

b. Cap Risk Funded investors might invest in a floating-rate bond. Typically, the floater has a cap (i.e., a maximum interest rate). To fund the investment in the floater, the investor might borrow funds on a short-term basis. There is no cap on the borrowing cost. Thus, if rates rise above the cap specified for the floater and there is no cap on the liabilities, then, at some point the funding cost will exceed the rate earned on the floater. This risk is called cap risk.

c. Interest Rate Risk Let’s look at the interest rate risk for an institution as a whole. The **economic surplus** of an entity is the difference between the market value of its assets and the

market value of its liabilities. That is,

$$\text{economic surplus} = \text{market value of assets} - \text{market value of liabilities}$$

While the concept that assets have a market value may not seem unusual, one might ask: What is the market value of liabilities? This value is simply the present value of the liabilities, where the liabilities are discounted at an appropriate interest rate. A rise in interest rates will therefore decrease the present value or market value of the liabilities; a decrease in interest rates will increase the present value or market value of liabilities. Thus, the economic surplus can be expressed as:

$$\text{economic surplus} = \text{market value of assets} - \text{present value of liabilities}$$

For example, consider an institution that has a portfolio comprised of only bonds and liabilities. Let's look at what happens to the economic surplus if interest rates rise. This will cause the bonds to decline in value; but it will also cause the liabilities to decline in value. Since both the assets and liabilities decline, the economic surplus can either increase, decrease, or remain unchanged. The net effect depends on the relative interest-rate sensitivity of the assets and the liabilities.

We can define the duration of liabilities as the responsiveness of the value of the liabilities to a 100 basis point change in interest rates. We define the dollar duration of assets and the liabilities in terms of a 100 basis point change in interest rates per \$100 present value. If the dollar duration of the assets is less than the dollar duration of the liabilities, the economic surplus will decrease if interest rates fall. For example, suppose that the current market value of the asset portfolio is equal to \$100 million and the present value of liabilities is \$90 million. Then the economic surplus is \$10 million. Suppose that the duration of the assets is 3 and the duration of the liabilities is 5. This means that the dollar duration per 100 basis point change in interest rates for the assets per \$100 of market value is \$3 and the dollar duration of the liabilities per \$100 present value is \$5.

Consider the following two scenarios. In the first scenario, interest rates decline by 100 basis points. Because the dollar duration of the assets is \$3, the market value of the assets will increase by approximately \$3 million to \$103 million. The present value of the liabilities will also increase. Since the dollar duration of the liabilities is \$5, the present value of the liabilities will increase by approximately \$4.5 million to \$94.5 million. Thus, the economic surplus decreases from \$10 million to \$8.5 million as a result of a decline in interest rates.

In the second scenario, assume that interest rates rise by 100 basis points. Because the dollar duration of the assets is \$3, the market value of the assets will decrease by approximately \$3 million to \$97 million. The value of the liabilities will also decrease. Since the dollar duration of the liabilities is \$5, the present value of the liabilities will decrease by \$4.5 million to \$85.5 million. The economic surplus therefore increases to \$11.5 million from \$10 million as a result of the rise in interest rates.

Notice that if we looked only at the interest rate risk of the bond portfolio for this institution, our concern is that the assets increase in value if interest rates fall. The interest rate risk for the assets alone derives from a possible increase in interest rates. However, when we analyze the change in interest rates, considering both assets and liabilities, we see that a decline in interest rates is the source of the interest rate risk to this institution because it reduces the economic surplus.

This example is particularly important for corporate pension sponsors since financial accounting rules specify that assets and liabilities must be marked-to-market (FASB 87).

Moreover, the accounting rules require that, if the surplus becomes negative, then the deficit must be reported as a liability on the corporate sponsor's balance sheet.

While our focus has been on the duration of assets, recall that duration is only a first approximation of the sensitivity of an asset or a liability to a change in rates. In analyzing the interest rate sensitivity of assets relative to liabilities, the convexity of both must also be considered. Recall that assets can have negative convexity. This means that, even if the duration of the assets and liabilities is matched, a portfolio of assets with negative convexity will not increase by as much as the liabilities when interest rates decline. This would result in a decline in the economic surplus. Consequently, to determine the true impact on the value of the economic surplus, the convexity of the assets and the liabilities must be considered.

C. Constraints

Clients impose constraints on managers. Examples of constraints a client might impose are a maximum allocation of funds to particular issuer or industry, a minimum acceptable credit rating for issues eligible for purchase, the minimum and maximum duration for the portfolio, whether leverage is permitted, whether shorting is permitted, and limitations on the use of derivative instruments (i.e., futures, options, swaps, caps, and floors).

The constraints imposed should be realistic and consistent with the investment objective. For example, suppose an insurance company issues a 5-year guaranteed investment contract (GIC) with a rate guaranteed 200 basis points over the on-the-run 5-year Treasury issue which has a yield of 6%. The investment objective is then to earn 8% plus a spread for the risk the insurance company incurs. However, if the constraints imposed on the manager require investment only in AAA rated securities and maturities that do not exceed five years, then it will be extremely difficult for the manager to meet the investment objective without excessively trading the portfolio to try to generate short-run returns.

In addition to client-imposed constraints, regulators of state-regulated institutions such as insurance companies (both life and property and casualty companies) may restrict the amount of funds allocated to certain major asset classes. Even the amount allocated within a major asset class may be restricted, depending on the characteristics of the particular asset. Managers of pension funds must comply with ERISA requirements. In the case of investment companies, restrictions on asset allocation are set forth in the prospectus and may be changed only with approval of the fund's board of directors.

Tax implications must also be considered. For example, life insurance companies enjoy certain tax advantages that make investing in tax-exempt municipal securities generally unappealing. Because pension funds too are exempt from taxes, they are not interested in tax-exempt municipal securities.

III. DEVELOPING AND IMPLEMENTING A PORTFOLIO STRATEGY

The second activity in the investment management process is developing and implementing a portfolio strategy. The strategy must be consistent with the investment objective and all constraints. This activity can be divided into the following tasks:

- writing an investment policy
- selecting the type of investment strategy

- formulating the inputs for portfolio construction
- constructing the portfolio

We discuss each of these activities in the following pages.

A. Writing an Investment Policy

The investment policy is a document that links the investor's investment objectives and the types of strategies that the manager (internal and external) may employ in seeking to reach those objectives. The investment policy should specify the permissible risks and the manner in which performance risk is measured.

Typically, the investment policy is developed by the investor in conjunction with a consultant. Given the investment policy, investment guidelines are established for individual managers hired by the investor. For example, if the investment objective is to outperform an aggregate bond index, a different manager might be hired to manage each sector of the index. While at the investor level the investment objective may be to outperform the bond index, the investment objective of each manager hired would be to outperform a specific sector of the index.

The investment guidelines established for each manager are developed in conjunction with the investor, the investor's consultant, and the individual manager. The investment guidelines must be consistent with both the investment policy and the investment philosophy of the manager being retained. The investment guidelines should also define how managers who are engaged will be evaluated.

B. Selecting the Type of Investment Strategy

In the broadest terms, portfolio strategies can be classified as either **active strategies** or **passive strategies**. (A finer breakdown of this classification is provided in Chapter 18.) Essential to all active bond portfolio strategies is the specification of expectations about the factors that influence the performance of an asset class. The risk factors that have historically driven the return on bond portfolios were reviewed, and more detail is provided in the next chapter. In the case of active bond management, this may involve forecasts of interest rates, changes in the term structure of interest rates, interest rate volatility, or yield spreads. Active portfolio strategies involving non-base currency bonds require forecasts of exchange rates as well as local market interest rates.

Passive strategies require minimal expectational input. One popular type of passive strategy is **indexing**, whose objective is to replicate the performance of a designated bond market index. While indexing has been employed extensively in the management of equity portfolios, indexing of bond portfolios is a relatively new practice. Indexing is covered more extensively in Chapter 18.

Several bond portfolio strategies classified as **structured portfolio strategies** have been commonly used. A structured portfolio strategy involves designing a portfolio so as to achieve the same performance as a designated benchmark. Such strategies are frequently followed when trying to satisfy liabilities. Immunization is a strategy designed to generate funds to satisfy a single liability regardless of the course of future interest rates. When the designated benchmark involves satisfying multiple future liabilities regardless of how interest rates change, strategies such as immunization and cash flow matching (or dedication) are being used. These strategies will be covered in Chapter 19.

1. Strategy Selection and Risk Strategy selection begins with a comprehensive analysis of the risk profile of the benchmark index. For that reason, in Chapter 17, we cover not only the various risk characteristics that make up the risk profile of a benchmark index but we also show how these risks can be quantified. Once we understand the risk characteristics of a benchmark index, we can differentiate between the two general types of investing—passive versus active management.

The most common form of passive strategy is bond indexing. In a bond indexing strategy, a portfolio is created to mirror the risk profile of the benchmark index. Typically, it is difficult to replicate the benchmark index precisely, at minimal cost, for the reasons explained in Chapter 18. As a result, the tracking error for an indexed portfolio should be small.

In active management the manager creates a portfolio that departs from the risk characteristics of the benchmark index. The types of departure determine the differences between the risk characteristics of the actively managed portfolio and the benchmark index. The manager makes a decision about the relative risks he wants to accept, compared to the benchmark, based on his expectations. Here tracking error relative to an indexing strategy is expected to be large.

Active management can be differentiated in terms of the “degree” of departure from the risk profile of the benchmark index. Some managers create portfolios with a risk profile that differs in a small way from the benchmark index. Such a management strategy is referred to as “enhanced indexing.” The problem is that “in a small way” is difficult to quantify, so knowing where enhanced indexing ends and active management begins is subjective. (Enhanced indexing is covered in Chapter 18.)

2. Role of Derivatives in Investment Strategies Regardless of whether an active or a passive strategy is selected, a manager must decide whether to employ derivatives in implementing a strategy. Derivatives include futures, forwards, swaps, options, caps, and floors. Of course, the use of derivatives and restrictions on their use are established by the client and/or regulators.

Derivative instruments can be used to control the interest rate risk of a portfolio. The advantages of using derivative instruments rather than cash market instruments are explained in Chapter 22. Bond portfolio strategies employing derivatives to control interest rate risk are also explained in Chapter 22.

C. Formulating the Inputs for Portfolio Construction

Formulating the inputs for portfolio construction in an active portfolio strategy requires two tasks. The first is a forecast by the manager of the inputs that are expected to impact the performance of a security and a portfolio. For many strategies this involves forecasting changes in interest rates, changes in interest rate volatility, changes in credit spreads, and, for international bond portfolios, changes in exchange rates. A discussion of forecasting models is beyond the scope of this book.

The second task is to extrapolate from market data the market’s “expectations.” Recall that a manager’s view is always relative to what is “priced” into the market. We illustrated this when we introduced forward rates. To illustrate the essential point, we can use a simple example of a manager who has a 1-year investment horizon and is deciding between the following two alternatives:

- buy a 1-year Treasury bill

- buy a 6-month Treasury bill, and, when it matures in six months, buy another 6-month Treasury bill

The alternative selected by the manager is not based solely on the manager's forecast of the 6-month Treasury bill rate six months from now. Rather, it is based on that forecast relative to the 6-month rate that is "priced" into the 1-year Treasury bill rate. For example, suppose that the 6-month Treasury bill rate is 3.0% (on a bond-equivalent basis) and the 1-year Treasury bill rate is 3.3% (on a bond-equivalent basis).⁵ Earlier, it was shown that the 6-month interest rate six months from now that would make the manager indifferent to investing in either alternative is 3.6%.⁶ This 3.6% interest rate is sometimes referred to as the "breakeven rate," or more commonly the "forward rate." The manager would be indifferent between the two alternatives if her forecast is that the 6-month rate six months from now will be 3.6%. The manager would prefer the 1-year Treasury bill if her forecast is that the 6-month rate six months from now will be less than 3.6%. The manager would prefer the 6-month Treasury bill if her forecast is that the 6-month rate six months from now will be greater than 3.6%.

As explained in the discussion of forward rates, it is common to refer to forward rates as the market's consensus of future rates; however, from the manager's perspective it is irrelevant whether the rate is truly a "consensus" value. The manager is only concerned with her view of future rates relative to the rates built into today's prices. Consequently, forward rates can be viewed as "hedgeable rates." That is, if a manager purchases the 1-year Treasury bill, she is hedging the 6-month Treasury bill rate six months from now. By doing so, the manager has locked in a 6-month Treasury bill rate six months from now of 3.6%.

Forward rates are just one example of the way in which a portfolio manager can use market information. We will see other examples in some of the chapters that follow.

D. Constructing the Portfolio

Given the manager's forecasts and market-derived information, the manager then assembles the portfolio with specific issues. In active bond portfolio management, asset selection involves identifying opportunities to enhance return relative to the benchmark. In doing so, the manager determines the **relative value** of the securities that are candidates for purchase in the portfolio and candidates for sale from the portfolio.

⁵This is the same example used at earlier. Bond-equivalent basis is computed by doubling the semiannual yield.

⁶To see this, suppose \$100,000 is invested in each alternative. For the 1-year Treasury bill alternative, the manager earns 1.65% (one half the 1-year rate of 3.3%) each 6-month period for two periods (i.e., one year). The total dollars at the end of one year will be:

$$\$100,000 (1.0165)^2 = \$103,327$$

For the 6-month Treasury bill alternative, the manager invests \$100,000 for six months at 1.5% (one half the annual rate) and then reinvests the proceeds for another six months at 1.8% (one half the 3.6% rate). The total dollars at the end of one year will be

$$\$100,000 (1.015)(1.018) = \$103,327.$$

Thus, both alternatives provide the same future value.

According to Jack Malvey, in the bond market the term “relative value” refers to “the ranking of fixed-income investments by sectors, structures, issuers, and issues in terms of their expected performance during some future interval.”⁷ The various methodologies for performing relative value analysis are explained in Chapters 18 and 20.

IV. MONITORING THE PORTFOLIO

Once the portfolio has been constructed, it must be monitored. Monitoring involves two activities. The first is to assess whether there have been changes in the market that might suggest that any of the key inputs used in constructing the portfolio may not be realized. The second task is to monitor the performance of the portfolio.

Monitoring the performance of a portfolio involves two phases. The first is **performance measurement**. This involves the calculation of the return realized by a manager over a specified time interval (the evaluation period). Given a performance measurement over some evaluation period, the second task is **performance evaluation**. This task is concerned with two issues. The first issue is to determine whether the manager added value by outperforming the established benchmark. The second issue is to determine how the manager achieved the observed return. The decomposition of the performance results to explain why those results were achieved is called **return attribution analysis**. Performance evaluation is described in Chapter 18.

V. ADJUSTING THE PORTFOLIO

Investment management is an ongoing process. The activities involved in monitoring the portfolio indicate whether adjustments need be made to the portfolio. By monitoring developments in the capital market, a manager determines whether to revise the inputs used in the portfolio construction process. Based on the new inputs, a manager then constructs a new portfolio. In constructing the new portfolio, the cost of trading issues currently in the portfolio is evaluated. These costs include transaction costs and any adverse tax or regulatory consequences.

Performance measurement indicates how well the manager is performing relative to the investment objectives. Performance is used by the client in deciding whether to retain a manager. However, a client must understand that the time horizon must be adequate to assess the performance of the manager and the strategy selected by the manager. For example, if analysis of the first quarter for a newly hired active manager indicates that he has underperformed the benchmark by 30 basis points, this may not be sufficient time for a fair evaluation. Instead, suppose that a new manager is hired and given cash to invest using a bond indexing strategy. If the same 30 basis point underperformance is observed for this manager, a quarter may be adequate time to assess the manager’s ability to index a portfolio. In fact, even if the manager of an indexed portfolio outperforms the benchmark by 30 basis points, this may be sufficient evidence to question the manager’s ability. An indexing strategy should not have a 30 basis point underperformance or outperformance even for a newly retained manager.

⁷Chapter 20 in this book.

MEASURING A PORTFOLIO'S RISK PROFILE

I. INTRODUCTION

Earlier, we described the risks associated with individual bonds. These risks include:

- Interest rate risk
- Call and prepayment risk
- Yield curve risk
- Reinvestment risk
- Credit risk
- Liquidity risk
- Exchange-rate risk
- Volatility risk
- Inflation or purchasing power risk
- Event risk

It is assumed that the reader is familiar with these risks.

Our focus is on bond portfolio strategies. In this chapter we explain how to measure a portfolio's risk profile. Because we focus on bond portfolio strategies, our interest is on performance relative to a bond market index. We refer to the bond market index as the *benchmark index*. So the process of managing funds relative to a benchmark index requires that a manager thoroughly understand the portfolio's risk profile relative to that of the benchmark index. The difference between the risk profile of the portfolio and the benchmark index results in performance differences. The best way to construct and control the risk profile of a portfolio relative to a benchmark index is to quantify all of the important risks. It is important to understand that a trade can reduce one type of risk while simultaneously increasing another.

II. REVIEW OF STANDARD DEVIATION AND DOWNSIDE RISK MEASURES

The risk associated with an investment can be defined in terms of a well-known statistical measure, the variance, or its conceptual equivalent, the standard deviation (the square root of

the variance). Professor Harry Markowitz was the first to show how to measure the risk of a portfolio when the risks of individual assets are correlated, and how to use that information to build efficient portfolios.¹ The concepts developed by Professor Markowitz, which are now referred to as modern portfolio theory, are covered in more detail elsewhere.² The focus below is on highlighting the application and limitations of the concepts to bond portfolio management.

A. Standard Deviation

In portfolio management, we are interested in the variability of the returns of a portfolio. The standard deviation and the variance are measures of the variability of returns. The larger the standard deviation or the variance, the greater the variability of the returns. The calculation of this variability begins with historical returns. The return for each time period is an observation or data point. The average return is calculated from the historical returns. The **variance** of returns is found by first computing the deviation of each observed return from the average returns and then squaring each deviation. The sum of these squared deviations divided by the number of observations minus one is the variance.

One of the problems with using the variance as a measure of dispersion is that the variance is in terms of squared units of observed returns. Consequently, the square root of the variance, which is called the **standard deviation**, is used as a more understandable measure of the degree of dispersion.

There are some important qualifications to be aware of when using the standard deviation as a risk measure. In many applications of probability theory, it is assumed that the underlying probability distribution is a **normal distribution**. Exhibit 1(a) shows the graph of a normal probability distribution. The average value is also called the **expected value**. The normal distribution has the following properties:

1. The distribution is symmetric around the expected value. That is, the probability of obtaining a value less than the expected value is 50%. The probability of obtaining a value greater than the expected value is also 50%.
2. The probability that an actual outcome will be within a range from one standard deviation below the expected value to one standard deviation above the expected value is 68.3%.
3. The probability that an actual outcome will be within a range from two standard deviations below the expected value to two standard deviations above the expected value is 95.5%.
4. The probability that an actual outcome will be within a range from three standard deviations below the expected value to three standard deviations above the expected value is 99.7%.

For a normal distribution, the expected value and the standard deviation are all the information needed to make statements about the probabilities of realizing certain outcomes. In order to apply the normal distribution to make statements about probabilities, it is necessary

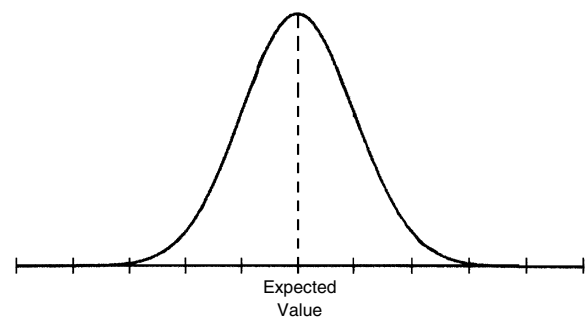
¹Harry M. Markowitz, "Portfolio Selection," *Journal of Finance* (March 1952), pp. 71–91.

²Many of the concepts are covered in Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, and David E. Runkle, *Quantitative Methods for Investment Analysis* (Charlottesville, VA: Association for Investment Management and Research, 2001).

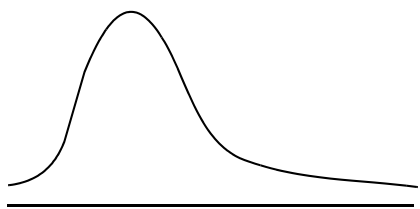
to assess whether a historical distribution (i.e., a distribution created from the observed returns) is normally distributed.

For example, a property of the normal probability distribution is that the distribution is symmetric around the expected value. However, a given probability distribution might be best characterized like those shown in Exhibits 1(b) and 1(c). Such distributions are referred to as **skewed distributions**. In addition to skewness, a historical distribution

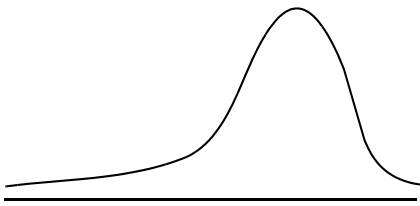
EXHIBIT 1 Probability Distributions



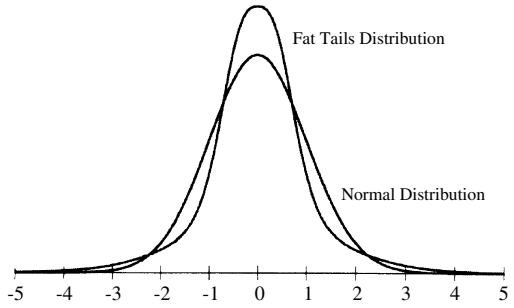
(a) Normal Probability Distribution



(b) Distribution Skewed to the Right (Positively Skewed)



(c) Distribution Skewed to the Left (negatively Skewed)



(d) Fat Tails

may have more outliers (i.e., observations in the “tails”) than the normal distribution predicts. Distributions with this characteristic are said to have **fat tails**. This is depicted in Exhibit 1(d). Notice that, if a distribution does indeed have fat tails but it is assumed to be normally distributed, then the probability of an actual value in a tail is assumed to be less than the true probability.³

In order to determine whether a historical distribution can be characterized as a normal distribution, the following two questions must be addressed:

1. Do the data fit the values predicted by the normal distribution?
2. Are the returns observed today independent of the returns of the prior periods?

Most introductory courses in statistics explain how to test whether historical data can be characterized by a normal distribution and whether historical returns are serially correlated. These tests are beyond the scope of this chapter.⁴

Let's look at the evidence on bond returns. For bonds, there is a lower limit on the loss. For Treasury securities, the limit depends on how high interest rates can rise. Since Treasury rates have never exceeded 15%, this places a lower bound on negative returns from holding a bond. There is a maximum return. Assuming that negative interest rates are not possible, the maximum price for a bond is the undiscounted value of the cash flows (i.e., the sum of the interest payments and maturity value). In turn, this determines the maximum return. On balance, government bond return distributions are negatively skewed. Studies by JP Morgan *RiskMetrics* suggest that this occurs for government bonds. Moreover, government bond returns exhibit fat tails.⁵

Another issue in using a normal distribution is the independence of observations. Casting this in terms of returns, it is important to know whether the return that might be realized today is affected by the return that was realized in a prior period. The term serial correlation is used to describe the correlation between returns in different periods. Studies by JP Morgan's *RiskMetrics* suggest that there is a small positive serial correlation for government bond returns.⁶

B. Downside Risk Measures

Now you understand why the standard deviation is used as a measure of risk and the limitations in using standard deviation as a measure of risk for a distribution that is not normally distributed or symmetric.

Other measures of risk focus on only that portion of an investment's return distribution that is below a specified level. These measures of risk are referred to as **downside risk measures**. For downside risk measures, the portfolio manager defines the **target return**, and returns less than the target return represent adverse consequences. In the case of the standard deviation, the target return is the expected value. However, in practice, this need not be the case. For example, in managing money against a benchmark index, the target return might be X basis

³For a further discussion of skewed distributions, see Chapter 3 in DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*, pp. 138–144.

⁴Tests for serial correlation are explained and illustrated in Chapter 9 of DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*, pp. 450–453.

⁵*RiskMetrics*TM — *Technical Document*, JP Morgan, May 26, 1995, New York p. 48.

⁶*RiskMetrics*TM — *Technical Document*, p. 48.

points over the return of the benchmark index. Outcomes that are less than the return of the benchmark index plus X basis points represent downside risk. In managing money against liabilities, the target return might be the rate guaranteed on liabilities plus a spread of S basis points. Returns with an outcome less than the rate guaranteed on liabilities plus S basis points then represent downside risk.

The three more popular downside risk measures are:

- target semivariance
- shortfall probability
- value at risk.

1. **Target Semivariance** The **target semivariance** is a measure of the dispersion of the outcomes below the target return. A special case of the target semivariance arises when the target return is the expected value. The resulting measure is called the **semivariance**.

While theoretically the semivariance is superior to the variance (standard deviation) as a risk measure, it is not used in bond portfolio management to any significant extent. Ronald Kahn gave the following reasons why semivariance (which he defines as downside risk) is not used:⁷

First, its definition is not as unambiguous as standard deviation or variance, nor are its statistical properties as well known, so it isn't an ideal choice for a universal risk definition. We need a definition which managers, plan sponsors, and beneficiaries can all use.

Second, it is computationally challenging for large portfolio construction problems. In fact, while we can aggregate individual bond standard deviations into a portfolio standard deviation, for other measures of risk we must rely much more on historical extrapolation of portfolio return patterns.

Third, to the extent that investment returns are reasonably symmetric, most definitions of downside risk are simply proportional to standard deviation or variance and so contain no additional information. To the extent that investment returns may not be symmetric, there are problems forecasting downside risk. Return asymmetries are not stable over time, and so are very difficult to forecast. Realized downside risk may not be a good forecast of future downside risk. Moreover, we estimate downside risk with only half the data, losing statistical accuracy.

In Section IIC we review how individual bond standard deviations of return are combined to determine a bond portfolio's standard deviation. In addition, we address the problem of using historical return patterns for individual bonds.

2. **Shortfall Risk** **Shortfall risk** is the probability that an outcome will have a value less than the target return.⁸ From a historical distribution of returns, shortfall risk is the ratio of the number of observations below the target return to the total number of observations. One problem with this risk measure is that the magnitude of the losses below the target return is ignored.

⁷Ronald N. Kahn, "Fixed Income Risk," Chapter 1 in Frank J. Fabozzi (ed.), *Managing Fixed Income Portfolios* (New Hope, PA: Frank J. Fabozzi Associates, 1997), pp. 2–3.

⁸For a further discussion of the measures of shortfall risk discussed here, see DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*, pp. 253–256.

When the target return is zero, shortfall risk is commonly called the **risk of loss**. It is calculated from historical data by dividing the number of observations with a return less than zero by the total number of observations.

Ronald Kahn notes that the use of shortfall risk leads to the same problems identified earlier for target semivariance, namely, “ambiguity, poor statistical understanding, difficulty of forecasting.”⁹

3. Value at Risk To calculate shortfall risk, the portfolio manager specifies a target return and then computes the percentage of observed returns that are less than the target return. A related approach to risk measurement requires that the portfolio manager specify a target probability; the return will not fall below a yet-to-be determined value the percentage of time represented by the target probability.

For example, suppose that the portfolio manager specifies a target probability of 95%. The portfolio manager then determines the return such that the area in the left tail of the probability distribution has a 5% probability. The target return computed is called the **value at risk** (VaR).¹⁰

C. Portfolio Variance

One of the advantages of using the standard deviation or variance as a measure of risk is the ability to compute portfolio risk from the risk of each individual bond in the portfolio. The expected value for a portfolio is the weighted average of the expected values of the individual bonds in the portfolio. The weight assigned to each bond is the market value of the bond as a percentage of the market value of the portfolio. No surprises here. However, the variance of a portfolio is not simply a weighted average of the variances of the bonds comprising the portfolio. A basic principle of modern portfolio theory is that the variance of a portfolio of assets depends not only on the variance of each asset, but also on their covariances or correlations.¹¹

If the standard deviation is used as the measure of risk, the variance of each bond and the covariance for each pair of bonds must be estimated in order to be able to compute the portfolio's standard deviation. Let's look at two major problems with this approach. After we discuss these problems, we will see how to resolve these issues.

The first problem with this approach is that the number of required estimated inputs increases dramatically as the number of bonds in the portfolio or the number of bonds considered for inclusion in the portfolio increases. For example, consider a manager who wants to construct a portfolio for which there are 5,000 bonds that are candidates for inclusion. (If this number sounds large, consider that the broad-based bond market indexes are comprised of far more than 5,000 bonds.) Then the number of variances and covariances that must be estimated is 12,502,500.¹²

⁹Kahn, “Fixed Income Risk,” p. 3.

¹⁰For a further discussion of value at risk, see Chapter 9 in Don M. Chance, *Analysis of Derivatives for the CFA Program* (Charlottesville, VA: Association for Investment Management and Research, 2003), pp. 576–578.

¹¹The calculation of the portfolio variance is described in Chapter 11 of DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*.

¹²The formula for determining the number of variances and covariances is found as follows: $[\text{Number of bonds} \times (\text{Number of bonds} + 1)]/2$.

The second problem is that, whether it is 55 variances and covariances for a 10-bond portfolio or 12,502,500 for a 5,000-bond portfolio, these values must all be estimated. Where does the portfolio manager obtain these values? The answer is that they must be estimated from historical data. While equity portfolio managers have the luxury of working with a long time series of returns on stocks, bond portfolio managers often do not have good sources of historical bond data. In addition, even with time series data on the return for a particular bond, a portfolio manager must question what these returns mean. The reason is that the characteristics of a bond change over time.

For example, consider a 10-year Treasury note issued 8 years ago for which quarterly returns have been calculated. The first quarterly return is the return on a 10-year Treasury note. However, the second quarterly return is the return on a 9.75-year Treasury note. The third quarterly return is the return on a 9.5-year Treasury note, and so on. If the original 10-year Treasury note is in the current portfolio and the manager wants to estimate the standard deviation for that security, the historical standard deviation is not meaningful. Since it was purchased 8 years ago, this security is now a 2-year Treasury note and does not share the return volatility characteristics of the earlier maturities. Not only does the changing time to maturity affect the historical data and render it of limited use, but some securities' characteristics change dramatically over time because of call/prepayment provisions. For example, we discussed mortgage-backed securities. CMO support bonds have average lives that change dramatically due to prepayments, which affects the historical return pattern.

Now, if we couple the problem of a large number of required estimates and the lack of relevant data, we see another major problem. Consider a 100-bond portfolio. There are 5,050 inputs to be estimated. Suppose that just 10% of the inputs are misestimated because of a lack of good historical or meaningful data. This means that there are 505 misestimated numbers which could have a material impact on the estimated portfolio standard deviation.

III. TRACKING ERROR

The measures described in Section II quantify risk exposure in terms of the variability relative to the average value or in the case of downside risk measures, relative to some target return. Consequently, such measures are useful in managing a portfolio compared to liabilities or a target return objective.

When a manager's benchmark is a bond market index, performance relative to that benchmark index is important. A manager who realizes a -1% annual return when the benchmark index returns -3% has performed well. So, it is important that such a manager understand the risk of a portfolio relative to the risk of the benchmark index. The measure used for this purpose is **tracking error**.

A. Measuring Tracking Error

Tracking error measures the dispersion between a portfolio's returns and the returns of a benchmark index. Historical observations are used to compute the potential future tracking error of a portfolio relative to the selected benchmark index. Specifically, the observation used in calculating tracking error is the active return, which is defined as:

$$\text{active return} = \text{portfolio's return} - \text{benchmark index's return}$$

EXHIBIT 2 Calculation of Tracking Error for a Hypothetical Portfolio

Observation period = January 2001-December 2001

Benchmark index = Lehman Aggregate Bond Index

Month in 2001	Portfolio return (%)	Benchmark index return (%)	Active return (%)	Deviation from mean	Squared deviation
Jan	2.10	1.64	0.46	0.26	0.0689
Feb	1.75	0.87	0.88	0.68	0.4658
March	0.88	0.50	0.38	0.18	0.0333
April	-0.73	-0.42	-0.31	-0.51	0.2576
May	0.30	0.60	-0.30	-0.50	0.2475
June	0.43	0.38	0.05	-0.15	0.0218
July	2.91	2.24	0.67	0.47	0.2233
Aug	1.40	1.15	0.25	0.05	0.0028
Sept	1.62	1.17	0.45	0.25	0.0638
Oct	2.16	2.09	0.07	-0.13	0.0163
Nov	-2.10	-1.38	-0.72	-0.92	0.8418
Dec	-0.15	-0.64	0.49	0.29	0.0856
Sum			2.37	0	2.3282
Mean			0.1975		
Variance					0.2117
Standard Deviation = Tracking error					0.4601
Tracking error (in basis points)					46.01

Notes:

Active return = Portfolio return – Benchmark Index return

Variance = (sum of the squares of the deviations from the mean)/11

Division by 11 which is number of observations minus 1

Tracking error = Standard deviation = square root of variance

Tracking error is the standard deviation of the portfolio's active return.

Exhibit 2 shows the calculations of the tracking error for returns to a hypothetical bond portfolio observed for a 12-month period. The benchmark index is the Lehman Aggregate Bond Index. The tracking error is shown at the bottom of the exhibit. Details regarding the interim calculations necessary to compute the standard deviation are not explained here.¹³

A portfolio manager seeking to match the benchmark index will regularly have excess returns close to zero, and therefore a small tracking error will be observed. In theory, if a manager could perfectly match the performance of the portfolio, a tracking error of zero would be realized. An actively managed portfolio that takes positions substantially different from those of the benchmark index (i.e., has a different risk profile) would likely have large observed excess returns, both positive and negative, and thus would have a large observed tracking error.

We will use the portfolio shown in Exhibit 3 to explain tracking error as well as the other risk measures discussed throughout this chapter. There are 45 bonds in the portfolio and, throughout this chapter and the next, we will refer to this portfolio as the “45-bond portfolio.” The 45-bond portfolio was constructed on November 5, 2001 based

¹³See, for example, Example 3–8 in DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*, p. 133. The format in Exhibit 2 uses the format shown in that book.

EXHIBIT 3 45-Bond Portfolio

Cusip	Issuer name	Coupon	Maturity	Credit rating		Sector	Adjusted duration*	Par value (\$000)	Percent of portfolio value
				Moody's	S&P				
057224AF	BAKER HUGHES	8.000	05/15/2004	A2	A	IND	2.26	9,400	1.06
059165BU	BALTIMORE GAS + ELEC	6.500	02/15/2003	A1	AA-	UTL	1.23	28,000	2.94
064057AN	BANK OF NEW YORK	6.500	12/01/2003	A1	A	FIN	1.92	28,000	3.03
097023AL	BOEING CO	6.350	06/15/2003	A1	AA-	IND	1.52	18,600	1.98
191219AY	COCA-COLA ENTERPRISES I	6.950	11/15/2026	A2	A	IND	12.05	93,200	10.20
23383FAK	DAIMLER-BENZ NORTH AMER	6.670	02/15/2002	A3	A+	IND	0.28	37,300	3.84
532457AP	ELI LILLY CO	6.770	01/01/2036	AA3	AA	IND	13.63	9,400	1.03
293561BS	ENRON CORP	6.625	11/15/2005	BAA1	BBB+	UTL	3.36	9,400	0.72
31359CAT	FEDERAL NATL MTG ASSN-GLO	7.400	07/01/2004	AAA+	AAA+	USA	2.41	14,800	1.67
FGG06096	FHLM Gold 7-Years Balloon	6.000	04/01/2026	AAA+	AAA+	FHg	0.38	37,300	3.87
FGD06494	FHLM Gold Guar Single F.	6.500	08/01/2008	AAA+	AAA+	FHd	1.71	43,100	4.55
FGB07098	FHLM Gold Guar Single F.	7.000	05/01/2027	AAA+	AAA+	FHb	0.89	59,800	6.26
FGB06498	FHLN Gold Guar Single F.	6.500	08/01/2027	AAA+	AAA+	FHb	1.81	35,400	3.67
319279BP	FIRST BANK SYSTEM	6.875	09/15/2007	A2	A-	fin	4.92	7,400	0.82
FNA08092	FNMA Conventional Long T.	8.000	01/01/2021	AAA+	AAA+	FNa	-0.24	61,800	6.68
345397GS	FORD MOTOR CREDIT	7.500	01/15/2003	A2	BBB+	FIN	1.14	7,400	0.78
347471AR	FORT JAMES CORP	6.875	09/15/2007	BAA3	BBB-	IND	4.85	7,400	0.73
GNA09490	GNMA I Single Family	9.500	08/01/2019	AAA+	AAA+	GNa	1.65	24,400	2.73
GNA07493	GNMA I Single Family	7.500	04/01/2022	AAA+	AAA+	GNa	0.73	55,900	5.97
GNA06498	GNMA I Single Family	6.500	12/01/2027	AAA+	AAA+	GNa	2.42	9,400	0.98
458182CB	INTER AMERICAN DEV BANK-G	6.375	10/22/2007	AAA	AAA	SUP	5.08	11,300	1.24
524909AS	LEHMAN BROTHERS INC	7.125	07/15/2002	A3	A	FIN	0.68	7,400	0.77
563469CZ	MANITOBA PROV CANADA	8.875	09/15/2021	AA3	AA-	FLA	10.87	7,400	1.00
58013MDE	MCDONALDS CORP	5.950	01/15/2008	AA3	AA	IND	5.19	7,400	0.78
590188 HZ	MERRILL LYNCH & CO. - GLO	6.000	01/12/2003	AA3	AA-	FIN	1.23	9,400	0.98
654106AA	NIKE INC	6.375	12/01/2003	A2	A	IND	1.92	5,500	0.59
655844AJ	NORFOLK SOUTHERN CORP	7.800	05/15/2027	BAA1	BBB	IND	11.48	7,400	0.84
669383CN	NORWEST FINANCIAL INC.	6.125	08/01/2003	AA2	A+	FIN	1.65	7,400	0.78
683234HG	ONTARIO PROV CANADA-GLOBA	7.375	01/27/2003	AA3	AA	FLA	1.18	7,400	0.80

EXHIBIT 3 (Continued)

Cusip	Issuer name	Coupon	Maturity	Credit rating		Sector	Adjusted duration*	Par value (\$000)	Percent of portfolio value
				Moody's	S&P				
744567DN	PUB SVC ELECTRIC + GAS	6.125	08/01/2002	A3	A—	UTL	0.73	5,500	0.57
755111AF	RAYTHEON CO	7.200	08/15/2027	BAA3	BBB—	IND	11.81	14,800	1.53
761157AA	RESOLUTION FUNDING CORP	8.125	10/15/2019	AAA+	AAA+	USA	10.81	31,900	4.24
88731EAF	TIME WARNER ENT	8.375	03/15/2023	BAA1	BBB+	IND	10.75	9,400	1.08
904000AA	ULTRAMAR DIAMOND SHAMROCK	7.200	10/15/2017	BAA2	BBB	IND	9.66	7,400	0.73
912810 DB	US TREASURY BONDS	10.375	11/15/2012	AAA+	AAA+	UST	5.11	18,600	2.58
912810DS	US TREASURY BONDS	10.625	08/15/2015	AAA+	AAA+	UST	8.72	26,100	4.19
912810EQ	US TREASURY BONDS	6.250	08/15/2023	AAA+	AAA+	UST	12.71	55,900	6.50
912827D2	US TREASURY NOTES	7.500	11/15/2001	AAA+	AAA+	UST	0.04	18,600	1.92
9128272P	US TREASURY NOTES	6.625	03/31/2002	AAA+	AAA+	UST	0.41	11,300	1.16
9128273G	US TREASURY NOTES	6.250	08/31/2002	AAA+	AAA+	UST	0.81	18,600	1.94
912827L8	US TREASURY NOTES	5.750	08/15/2003	AAA+	AAA+	UST	1.69	2,000	0.21
912827T8	US TREASURY NOTES	6.500	05/15/2005	AAA+	AAA+	UST	3.13	2,000	0.23
9128273E	US TREASURY NOTES	6.125	08/15/2007	AAA+	AAA+	UST	4.93	2,000	0.23
949740BZ	WELLS FARGO + CO	6.875	04/01/2006	AA3	A	FIN	3.86	9,400	1.03
961214AD	WESTPAC BANKING CORP	7.875	10/15/2002	A1	A+	FIN	0.93	5,500	0.57

* Adjusted duration is the Lehman Brothers term for effective duration.

on prices as of October 31, 2001.¹⁴ The market value (which includes accrued interest) of the portfolio is \$1,001,648,000. The benchmark index is the Lehman Aggregate Bond Index, a broad-based bond market index described in the previous chapter. The market value (which includes accrued interest) of this index is \$6,999,201,792,000.

Lehman Brothers has historical information on excess returns for bonds. Using that information, Lehman Brothers has calculated that the tracking error for the 45-bond portfolio, relative to the Lehman Aggregate Bond Index, is 62 basis points per year. Assuming that the tracking error is normally distributed, the probability is about 68% that the portfolio return over the next year will be within ± 62 basis points of the benchmark index return.

In Section IX we will see how this 62 basis point tracking error is decomposed into the risks that cause this tracking error.

¹⁴I thank Vadim Konstantinovskiy, CFA and Lev Dynkin of the Quantitative Portfolio Strategy Group in Fixed Income Research at Lehman Brothers for providing not only the sample portfolio and its characteristics, but all of the analysis of the portfolio.

B. Actual Versus Predicted Tracking Error

In Exhibit 2, the tracking error calculation is based on the active returns realized. However, the performance shown is the result of the portfolio manager's decisions during those 12 months with respect to portfolio positioning issues such as duration, sector allocations, and credit structure. Hence, we can call the tracking error calculated from these trailing active returns a **backward-looking tracking error**. It is also called the *ex post* tracking error and **actual tracking error**. We will use the term actual tracking error in this chapter and the next.

One problem with the use of actual tracking error is that it does not reflect the effect of the portfolio manager's current decisions on the future active returns and hence the tracking error that may be realized in the future. If, for example, the manager significantly changes the portfolio's effective duration or sector allocations today, then the actual tracking error calculated using data from prior periods would not accurately reflect the current portfolio risks. That is, the actual tracking error will have little predictive value and can be misleading regarding portfolio risks.

The portfolio manager needs a forward looking estimate of tracking error to accurately reflect future portfolio risk. In practice, this is accomplished by using the services of a commercial vendor that has a model, called a multi-factor risk model, that has identified and defined the risks associated with a benchmark index. Such a model is described in Section X. Statistical analysis of historical return data for the bonds in the benchmark index is used to obtain the risk factors and to quantify the risks. (This involves the use of variances and correlations/covariances.) Using the manager's current portfolio holdings, the portfolio's current exposure to the various risk factors can be calculated and compared to the benchmark's exposures to the risk factors. Using the differential factor exposures and the risks of the factors, a **forward looking tracking error** for the portfolio can be computed. This tracking error is also referred to as *ex ante* tracking error or **predicted tracking error**. We will use the term predicted tracking error in this chapter and the next.

There is no guarantee that the predicted tracking error at the start of, say, a year would exactly match the actual tracking error calculated at the end of the year. There are two reasons for this. The first is that, as the year progresses and changes are made to the portfolio, the predicted tracking error changes to reflect the new exposures. The second is that the accuracy of the predicted tracking error depends on stability in the variances and correlations/covariances used in the analysis. These problems notwithstanding, the average of predicted tracking errors obtained at different times during the year will be reasonably close to the actual tracking errors observed at the end of the year.

Both calculations of tracking error have their use. The predicted tracking error is useful in risk control and portfolio construction. The manager can immediately see the likely effect on tracking error of any intended change in the portfolio. Thus, the portfolio manager can do a what-if analysis for various portfolio strategies and eliminate those that would result in tracking errors beyond his tolerance for risk. The actual tracking error can be useful for assessing performance as discussed in the next chapter.

When actual tracking error is reported, the following supplementary information is often provided: average return difference (i.e., average of the active returns), the maximum active return difference (i.e., the highest of the active returns), and the minimum active return difference (i.e., the lowest of the active returns). It should be noted that some market participants refer to the active return for a given observation period as tracking error.¹⁵

¹⁵The author uses this term. See also DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*, p. 236 and the glossary to the book, p. 657.

IV. MEASURING A PORTFOLIO'S INTEREST RATE RISK

A portfolio's duration is used to measure the portfolio's exposure to changes in the level of interest rates under the assumption of a parallel shift in the yield curve. For a portfolio, duration is the approximate percentage change in market value for a 100 basis point change in interest rates, assuming a parallel shift in the yield curve. So, a portfolio duration of 4 means that the portfolio's market value will change by approximately 4% for a 100 basis point change in the interest rate for all maturities. Earlier, we explained how to compute the duration for individual securities.

Three different duration measures are used to quantify a bond's or a portfolio's exposure to a parallel shift in the yield curve—modified duration, Macaulay duration, and effective duration. As explained, **modified duration** assumes that, when interest rates change, the cash flows do not change. This is a limitation when measuring the exposure of bonds with embedded options (e.g., callable bonds, mortgage-backed securities, and some asset-backed securities) to changes in interest rates. **Macaulay duration** is related to modified duration and suffers from the same failure to consider changes in cash flows when interest rates change. In contrast, **effective duration** takes these changes in cash flows into account and thus, is the appropriate measure for bonds with embedded options. Effective duration is also referred to as **option-adjusted duration**.

The calculation of a portfolio's duration begins with the calculation of the duration for each of the individual bonds comprising the portfolio. This calculation requires the use of a valuation model. Duration is found by shocking (i.e., changing) interest rates and computing the new value of the bond will be. *Consequently, the duration measure is only as good as the valuation model.* We explained how the effective duration of a bond with an embedded option is computed using the binomial model and also explained mortgage-backed and asset-backed securities.

For the 45-bond portfolio, the duration of each issue is shown in the column labeled "adjusted duration." Lehman Brothers uses this term instead of effective duration or option-adjusted duration. It is important to understand that the measure used by Lehman Brothers takes into consideration the impact of changes in interest rates on the cash flows of a security. Here we see a clear example of the differences in the terminology used by major bond market participants.

A. Portfolio Duration

A portfolio's duration can be obtained by calculating the weighted average of the durations of the bonds in the portfolio. The weight for a bond is the proportion of the market value of a portfolio represented by a security. Mathematically, a portfolio's duration is calculated as follows:

$$w_1D_1 + w_2D_2 + w_3D_3 + \dots + w_KD_K$$

where

- w_i = market value of bond i /market value of the portfolio
- D_i = duration of bond i
- K = number of bonds in the portfolio

To illustrate this calculation, consider the following 3-bond portfolio in which all three bonds are option free and pay a semiannual coupon:

Bond	Par amount owned	Market value
10% 5-year	\$4 million	\$4,000,000
8% 15-year	5 million	4,231,375
14% 30-year	1 million	1,378,586

In this illustration, it is assumed that the next coupon payment for each bond is six months from now. The market value of the portfolio is \$9,609,961. The market price per \$100 par value, the yield, and the duration for each bond are given below:

Bond	Price (\$)	Yield (%)	Duration
10% 5-year	100.0000	10	3.861
8% 15-year	84.6275	10	8.047
14% 30-year	137.8586	10	9.168

In this illustration, K is equal to 3 and:

$$\begin{aligned}
 w_1 &= \$4,000,000 / \$9,609,961 = 0.416 & D_1 &= 3.861 \\
 w_2 &= \$4,231,375 / \$9,609,961 = 0.440 & D_2 &= 8.047 \\
 w_3 &= \$1,378,586 / \$9,609,961 = 0.144 & D_3 &= 9.168
 \end{aligned}$$

The portfolio's duration is:

$$0.416(3.861) + 0.440(8.047) + 0.144(9.168) = 6.47$$

A portfolio duration of 6.47 means that, for a 100 basis point change in the yield for each of the three bonds, the market value of the portfolio will change by approximately 6.47%. But keep in mind, the yield of each of the bonds must change by 100 basis points for the duration measure to be useful. This is a critical assumption and its importance cannot be overemphasized.

An alternative procedure for calculating the duration of a portfolio is to first calculate the dollar price change for a given number of basis points for each security and then add the price changes. Next, divide the total of the price changes by the initial market value of the portfolio to produce a percentage price change that can be adjusted to obtain the portfolio's duration.

For example, consider the 3-bond portfolio shown above. Suppose we calculate the dollar price change for each bond in the portfolio, based on its duration, for a 50 basis point change in yield. We then have:

Bond	Market value	Duration	Change in value for 50 bp yield change
10% 5-year	\$4,000,000	3.861	\$77,220
8% 15-year	4,231,375	8.047	170,249
14% 30-year	1,378,586	9.168	63,194
Total			\$310,663

Thus, a 50 basis point change in each rate changes the market value of the 3-bond portfolio by \$310,663. Since the market value of the portfolio is \$9,609,961, a 50 basis point change

produces a change in value of 3.23% (\$310,663 divided by \$9,609,961). Since duration is the approximate percentage change for a 100 basis point change in rates, this means that the portfolio duration is 6.46 (found by doubling 3.23). This is the same value found earlier.

If the above formula is applied to the 45-bond portfolio, it can be shown that the portfolio's duration is 4.59. Alternatively, portfolio duration can be computed from the durations of the sectors. For example, below we have the duration and the percentage of the portfolio for each sector:

Sector	% of portfolio	Adjusted duration	% of portfolio $\times$ Adjusted duration
Treasury	18.96	7.21	1.37
Agencies	5.90	8.43	0.50
Financial Inst.	8.20	2.16	0.18
Industrials	20.55	9.61	1.97
Utilities	4.23	1.53	0.06
Non-US Credit	7.45	2.65	0.20
Mortgage	34.70	0.90	0.31
Asset Backed	0.00	0.00	0.00
CMBS	0.00	0.00	0.00
Totals	100.00		4.59

We see that the duration for the 45-bond portfolio is 4.59.

B. Duration of a Bond Market Index

Since a bond market index is simply a portfolio, the effective duration of a bond market index can be computed in the same way that the duration of a portfolio is computed. In Chapter 16 we reviewed the three broad-based-bond market indexes. Here we look at one of these indexes, the Lehman Aggregate Bond Index.

This bond market index consists of various sectors which we described in detail in Chapter 16. A breakdown of the index by detailed sector, as of October 31, 2001, and the effective duration ("adjusted duration" in Lehman Brother's terminology) are given below:

Sector	% of portfolio	Adjusted duration	% of portfolio $\times$ Adjusted duration
Treasury	22.65	6.22	1.41
Agencies	11.55	4.49	0.52
Financial Inst.	7.74	4.61	0.36
Industrials	10.20	6.45	0.66
Utilities	2.29	5.68	0.13
Non-US Credit	6.46	5.41	0.35
Mortgage	35.36	1.46	0.52
Asset Backed	1.71	3.29	0.06
CMBS	2.05	5.10	0.10
Totals	100.00		4.10

Duration is calculated by multiplying each sector weight by the duration for the sector and then summing these products. The third column above shows the product of each sector weight times the duration. The sum of the last column is 4.1. So the duration of the index is 4.1.

Notice that, if the benchmark index for the 45-bond portfolio is the Lehman Aggregate Bond Index, then the portfolio is more sensitive to a parallel shift in interest rates than the benchmark index (4.59 versus 4.10).

C. Contribution to Portfolio Duration and Benchmark Index Duration

Some portfolio managers look at exposure of a portfolio or a benchmark index to an issue or to a sector simply in terms of the market value percentage of that issue or sector in the portfolio. A better measure of exposure to an individual issue or sector is its **contribution to portfolio duration** or **contribution to benchmark index duration**. This is found by multiplying the percentage of the market value of the portfolio represented by the individual issue or sector times the duration of the individual issue or sector. That is:

$$\text{Contribution to portfolio duration} = \text{weight of issue or sector in portfolio} \times \text{duration of issue or sector}$$

$$\begin{aligned} \text{Contribution to benchmark index duration} \\ = \text{weight of issue or sector in benchmark index} \times \text{duration of issue or sector} \end{aligned}$$

To illustrate this, look at Exhibit 4, which shows the sector breakdown of both the 45-bond portfolio and the benchmark index. The contribution to portfolio duration and the contribution to the benchmark index duration are shown.

A portfolio manager who wants to determine the contribution of a sector to portfolio duration relative to the contribution of the same sector in a broad-based market index can compute the difference between the two contributions. The difference in the percentage distribution by sector is not as meaningful as is the difference in the contribution to duration.

D. Convexity

We discussed the limitations to the usefulness of duration. One of the limitations is that duration does not do as good a job of estimating the change in the value of a security, a portfolio, or a benchmark index for a large change in interest rates. A measure that can be used to *improve* the estimate provided by duration is convexity, which is used to estimate the portion of the effect on the change in value when interest rates change that is not explained by duration.

EXHIBIT 4 Contribution to Portfolio and Benchmark Index Duration Based on Sectors

Sector	Portfolio			Benchmark			Difference	
	% of portfolio	Adjusted duration	Contribution to adj. duration	% of portfolio	Adjusted duration	Contribution to adj. duration	% of portfolio	Contribution to adj. duration
Treasury	18.96	7.21	1.37	22.65	6.22	1.41	-3.69	-0.04
Agencies	5.90	8.43	0.50	11.55	4.49	0.52	-5.65	-0.02
Financial Inst.	8.20	2.16	0.18	7.74	4.61	0.36	0.46	-0.18
Industrials	20.55	9.61	1.97	10.20	6.45	0.66	10.35	1.31
Utilities	4.23	1.53	0.06	2.29	5.68	0.13	1.94	-0.07
Non-US Credit	7.45	2.65	0.20	6.46	5.41	0.35	0.99	-0.15
Mortgage	34.70	0.90	0.31	35.36	1.46	0.52	-0.66	-0.21
Asset Backed	0.00	0.00	0.00	1.71	3.29	0.06	-1.71	-0.06
CMBS	0.00	0.00	0.00	2.05	5.10	0.10	-2.05	-0.10
Totals	100.00		4.59	100.00		4.10	0.00	0.49

The convexity of the 45-bond portfolio is $+0.13$, and the convexity of the Lehman Aggregate Bond Index is -0.26 . How does one interpret these two values? An extensive discussion explained why convexity value is difficult to interpret since it can be scaled in different ways. It is important to observe that the portfolio exhibits positive convexity while the benchmark exhibits negative convexity. This means that the manager should expect that, for a large decline in interest rates, the portfolio will outperform the benchmark index. We will confirm this when we describe scenario analysis in the next chapter.

V. MEASURING YIELD CURVE RISK

Duration provides a measure of the exposure of a portfolio or a benchmark index to changes in the level of interest rates. However, duration does not indicate the exposure of a portfolio or a benchmark index to changes in the shape of the yield curve. We previously discussed the various types of yield curve changes (i.e., shifts). Therefore, it is imperative that a manager understand the risk exposure resulting from a shift in the yield curve for both the portfolio and the benchmark index.

One way to get a feel for the risk exposure resulting from yield curve shifts is to analyze the distribution of the present values of the cash flows for the portfolio and the benchmark index.¹⁶ Exhibit 5 shows the distribution of the present values of the cash flows for the 45-bond portfolio and for the Lehman Aggregate Bond Index. The exhibit demonstrates that the portfolio has greater cash flows (in present value terms) with shorter and longer maturities relative to the benchmark index and they are more barbelled.

Another way to see this exposure to changes in the yield curve is to compute the key rate durations of the portfolio and the benchmark. Key rate duration, which was described earlier, is the sensitivity of a portfolio's value to the change in a particular key spot rate. The specific maturities on the spot rate curve for which key rate durations are measured vary from vendor to vendor. Exhibit 6 reports six key rate durations as computed by Lehman Brothers.¹⁷ Consider the 5-year key rate duration for the portfolio, 0.504. This measure is interpreted as follows: For a 100 basis point change in the 5-year rate holding all other rates constant, the portfolio's value will change by approximately 0.5%.

The key rate durations reported in Exhibit 6 support the observation from the analysis of the present values of the cash flows: There is more exposure to changes in the short and long end of the maturity sectors and less exposure to the maturity sectors in between. (Note that the key rate durations shown are based on modified durations, not effective durations. At the time of the analysis, Lehman Brothers' model did not produce key rate durations based on effective duration.)

¹⁶The cash flows are obtained as follows. For the option-free bonds in the portfolio and the benchmark index, the cash flows are to the maturity date. For the callable bonds in the portfolio and the benchmark index, a "cash flow to adjusted duration" was computed. This is a weighted blend of a cash flow to maturity and to call. The weight is selected so that the blended cash flow has a duration equal to the option-adjusted duration. For the MBS in the portfolio and the benchmark, a "cash flow to worst" is a zero volatility cash flow produced using a single path of interest rates from the current yield curve and the Lehman Brothers prepayment model. The yield of each security is used in computing the present value of any of the cash flows for the security.

¹⁷In theory, there is a rate duration for every maturity. In practice, a rate duration is computed for certain "key" maturities. These durations are called key rate durations.

EXHIBIT 5 Present Value of Cash Flow Structure

Year	Portfolio cash flows (%)	Benchmark cash flows (%)	Difference (%)
0.00	3.939	2.277	1.662
0.25	8.873	5.814	3.059
0.50	5.684	4.688	0.996
0.75	5.876	3.705	2.170
1.00	7.404	6.031	1.373
1.50	10.422	8.707	1.715
2.00	6.622	7.105	−0.483
2.50	4.916	6.094	−1.178
3.00	2.970	5.339	−2.368
3.50	2.393	4.805	−2.412
4.00	3.998	7.093	−3.095
5.00	4.088	7.148	−3.060
6.00	6.453	4.993	1.460
7.00	4.477	8.890	−4.412
10.00	6.754	8.635	−1.881
15.00	6.550	4.295	2.255
20.00	4.694	2.519	2.175
25.00	3.632	1.323	2.309
30.00	0.197	0.525	−0.328
40.00	0.056	0.026	0.030

EXHIBIT 6 Key Rate Durations*

Year	Portfolio	Benchmark	Difference
0.50	0.212	0.165	0.047
2.00	0.428	0.590	−0.162
5.00	0.504	1.047	−0.543
10.00	0.624	1.025	−0.401
20.00	2.001	0.861	1.139
30.00	0.862	0.504	0.358
TOTAL	4.631	4.192	0.439

*Based on modified duration.

VI. SPREAD RISK

For non-Treasury securities, the yield is equal to the Treasury yield plus a spread to the Treasury yield curve. Non-Treasury securities are referred to as spread products. The risk that the price of a bond changes due to changes in spreads is referred to as **spread risk**. A measure of how a spread product's price changes if the spread sought by the market changes is called **spread duration**.

A. Types of Spread Duration Measures

The issue in addressing spread duration is in identifying the particular spread that is assumed to change? As explained at earlier, there are three spread measures used for fixed-rate bonds: nominal spread, zero-volatility spread, and option-adjusted spread.

The **nominal spread** is the traditional spread measure. That is, it is the difference between the yield on a spread product and the yield on a comparable maturity Treasury issue. Thus, when spread is defined as the nominal spread, spread duration indicates the approximate percentage change in price for a 100 basis point change in the nominal spread, holding the Treasury yield constant. It is important to note that, for any spread product, spread duration is the same as duration if the nominal spread is used. For example, suppose that the duration of a corporate bond is 5. This means that, for a 100 basis point change in interest rates, the value of the corporate bond changes by approximately 5%. It does not matter whether the change in rates is due to a change in the level of rates (i.e., a change in the Treasury rate) or a change in the nominal spread.

The **zero-volatility spread**, or **static spread**, is the spread that, when added to the Treasury spot rate curve, makes the present value of the cash flows (when discounted at the spot rates plus the spread) equal to the price of the bond plus accrued interest. It is a measure of the spread over the Treasury spot rate curve. When spread is defined in this way, spread duration is the approximate percentage change in price for a 100 basis point change in the zero-volatility spread, holding the Treasury spot rate curve constant.

The **option-adjusted spread** (OAS) is another spread measure¹⁸ that can be interpreted as the approximate percentage change in price of a spread product for a 100 basis point change in the OAS, holding Treasury rates constant. So, for example, if a corporate bond has a spread duration of 3, this means that, if the OAS changes by 20 basis points, then the price of this corporate bond will change by approximately 0.6% ($0.03 \times 0.002 \times 100$).

How do you know whether a spread duration for a fixed-rate bond is a spread based on the nominal spread, zero-volatility spread, or the OAS? You do not know. You must ask the broker/dealer or vendor of the analytical system.

B. Spread Duration for a Portfolio or a Benchmark Index

The spread duration for a portfolio or a bond index is computed as a market weighted average of the spread duration for each sector. The spread duration for the 45-bond portfolio and for the Lehman Aggregate Bond Index are computed in Exhibit 7. Notice that, with the exception of the Treasury sector, the spread duration for all the sectors is the same as the adjusted duration used earlier. The spread duration for the Treasury sector is, of course, zero.

The spread duration for the 45-bond portfolio is 3.22 compared to 2.69 for the benchmark index. That is, there is greater spread risk for the 45-bond portfolio than for the benchmark index. The greater spread duration is the result of the underweight in the Treasury sector combined with the longer adjusted duration in spread sectors, specifically Industrials. Notice that the calculation of spread duration is simply the sum of the contribution to duration for each sector.

VII. CREDIT RISK

The credit risk of a portfolio on a stand alone basis or relative to a benchmark index can be gauged by the allocation to each rating. However, as noted earlier, a better gauge is the contribution to duration by credit rating. Exhibit 8 compares the 45-bond portfolio and the benchmark index by credit quality. Information on the contribution to duration by credit quality is also shown.

¹⁸Calculation of the OAS was covered earlier.

EXHIBIT 7 Spread Duration for 45-Bond Portfolio and Benchmark Index Based on Sectors

Sector	Portfolio			Benchmark		
	% of portfolio	Spread duration	Contribution to spread duration	% of portfolio	Spread duration	Contribution to spread duration
Treasury	18.96	0.00	0.00	22.65	0.00	0.00
Agencies	5.90	8.43	0.50	11.55	4.49	0.52
Financial Inst.	8.20	2.16	0.18	7.74	4.61	0.36
Industrials	20.55	9.61	1.97	10.20	6.45	0.66
Utilities	4.23	1.53	0.06	2.29	5.68	0.13
Non-US Credit	7.45	2.65	0.20	6.46	5.41	0.35
Mortgage	34.70	0.90	0.31	35.36	1.46	0.52
Asset Backed	0.00	0.00	0.00	1.71	3.29	0.06
CMBS	0.00	0.00	0.00	2.05	5.10	0.10
Totals	100.00		3.22	100.01		2.70

EXHIBIT 8 Quality Analysis

Quality	Portfolio			Benchmark			Difference	
	% of portfolio	Adjusted duration	Contribution to adj. duration	% of portfolio	Adjusted duration	Contribution to adj. duration	% of portfolio	Contribution to adj. duration
AAA+	24.86	7.50	1.86	34.09	5.63	1.92	-9.23	-0.06
MBS	34.70	0.90	0.31	35.36	1.46	0.52	-0.66	-0.21
AAA	1.24	5.08	0.06	5.53	4.21	0.23	-4.29	-0.17
AA	6.40	5.68	0.36	4.97	5.01	0.25	1.43	0.11
A	27.16	5.39	1.46	10.91	5.73	0.63	16.25	0.84
BAA	5.64	9.29	0.52	9.13	6.07	0.55	-3.49	-0.03
Totals	100.00		4.57	99.99		4.10	0	0.49

VIII. OPTIONALITY RISK FOR NON-MBS

Some corporate bonds and agency debentures have embedded options—call and put options. These options affect the performance of a portfolio and a benchmark index. The adverse effect on performance resulting from these embedded options is referred to as **optionality risk**. (We address prepayment risk associated with MBS in Section IX.) The optionality risk exposure of a bond occurs because a change in interest rates changes the value of the embedded option, which, in turn changes the value of the bond. The same is true at the portfolio and benchmark index level.

Optionality risk can be quantified using measures commonly used in the option pricing. In option pricing theory, the delta of an option is an estimate of the sensitivity of the value of the option to changes in the price of the underlying instrument.¹⁹ For bonds, delta can be computed for each issue that has an embedded option and then these deltas can be aggregated to obtain an estimate of the delta for a portfolio or a benchmark index.

Exhibit 9 shows the delta of the 45-bond portfolio and the benchmark index, as well as the difference between the two. The percentage of the portfolio represents all holdings

¹⁹The concept of an option delta is described in several chapters in Chance, *Analysis of Derivatives for the CFA Program*.

EXHIBIT 9 Option Delta Analysis

Option Delta	Portfolio			Benchmark			Difference	
	% of portf.	Delta	Cntrb. to delta	% of portf.	delta	Cntrb. to delta	% of portf.	Cntrb. to delta
Bullet	57.08	0.0000	0.0000	48.38	0.0000	0.0000	8.70	0.0000
Callable traded to Maturity	5.64	0.0000	0.0000	8.69	0.0245	0.0021	-3.05	-0.0021
Callable traded to Call	2.58	0.3677	0.0095	3.16	0.6046	0.0191	-0.58	-0.0096
Putable traded to Maturity	0.00	0.0000	0.0000	0.25	0.0792	0.0002	-0.25	-0.0002
Putable traded to Put	0.00	0.0000	0.0000	0.39	0.5104	0.0020	-0.39	-0.0020
Totals	65.30		0.0095	60.88		0.0234	4.42	-0.0140

other than the mortgage sector. (Thus, the value of 65.30% for the portfolio is 100% minus the mortgage holdings of 34.7%.) Notice that the holdings are partitioned in terms of the embedded option and how the securities trade. Bullet bonds do not have embedded options and therefore the delta for these bonds is zero. A security with an embedded call option is classified as trading either to its call date or to its maturity date, depending on its market price. In this context, “trading to” indicates that the price of the security is such that the market is pricing the security either as if it will be called or as if it will not be called. The same is true for securities with a put option—they are classified as either trading to the put date or trading to maturity.

IX. RISKS OF INVESTING IN MORTGAGE-BACKED SECURITIES

Earlier, there was extensive discussion of mortgage-backed securities. In Chapter 16 we described how the MBS sector is presented in the broad-based bond market indexes. The MBS sector is the largest sector of the broad-based bond market index. As of the time of the analysis, the MBS sector of the Lehman Aggregate Bond Index is 35% and includes only agency mortgage passthrough securities.²⁰ Consequently, it is critical for a portfolio manager whose benchmark is a broad-based bond market index to understand the risks associated with this sector.

The three major risks associated with investing in the MBS sector are:

- sector risk
- prepayment risk
- convexity risk

Each of these risks is discussed below.

A. Sector Risk in the MBS Sector

The MBS sector is divided into several subsectors based on the coupon rate. The motivation for this classification is that the coupon rate relative to the prevailing mortgage rate has an impact on prepayments and therefore on the spread at which an MBS trades relative to

²⁰These securities were described earlier.

EXHIBIT 10 MBS Sector Analysis

MBS Seasoning	Portfolio			Benchmark			Difference	
	% of portf.	Adjusted duration	Cntrb. to adj. dur.	% of portf.	Adjusted duration	Cntrb. to adj. dur.	% of portf.	Cntrb. to adj. dur.
COUPON < 6.0%								
Unseasoned	0.00	0.00	0.00	0.78	2.91	0.02	-0.78	-0.02
Mod. Seasoned	0.00	0.00	0.00	0.03	2.18	0.00	-0.03	0.00
Seasoned	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.0% ≤ COUPON < 7.0%								
Unseasoned	8.52	1.23	0.10	16.66	2.17	0.36	-8.15	-0.26
Mod. Seasoned	4.55	1.71	0.08	1.79	1.87	0.03	2.77	0.04
Seasoned	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7.0% ≤ COUPON < 8.0%								
Unseasoned	6.26	0.89	0.06	10.25	0.67	0.07	-4.00	-0.01
Mod. Seasoned	5.97	0.73	0.04	2.79	0.76	0.02	3.17	0.02
Seasoned	0.00	0.00	0.00	0.02	1.34	0.00	-0.02	0.00
8.0% ≤ COUPON < 9.0%								
Unseasoned	0.00	0.00	0.00	1.80	0.13	0.00	-1.80	0.00
Mod. Seasoned	6.68	-0.24	-0.02	0.64	0.16	0.00	6.04	-0.02
Seasoned	0.00	0.00	0.00	0.21	0.78	0.00	-0.21	0.00
9.0% ≤ COUPON < 10.0%								
Unseasoned	0.00	0.00	0.00	0.05	0.51	0.00	-0.05	0.00
Mod. Seasoned	0.00	0.00	0.00	0.03	0.24	0.00	-0.03	0.00
Seasoned	2.73	1.65	0.05	0.26	0.83	0.00	2.48	0.04
COUPON ≥ 10.0%								
Unseasoned	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mod. Seasoned	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Seasoned	0.00	0.00	0.00	0.05	1.42	0.00	-0.05	0.00
Subtotals								
Unseasoned	14.77		0.16	29.55		0.46	-14.78	-0.30
Mod. Seasoned	17.19		0.11	5.28		0.06	11.91	0.05
Seasoned	2.73		0.05	0.53		0.00	2.20	0.04
Totals	34.70		0.31	35.36		0.52	-0.66	-0.21

Unseasoned: Origination date on or after January 1, 1996.

Moderately Seasoned: Origination date between January 1, 1992 and December 31, 1995.

Seasoned: Origination date on or before December 31, 1991.

Treasuries. In turn, prepayments are also affected by how long the underlying mortgage pools have been outstanding. This characteristic is referred to as the seasoning of the underlying mortgage pool. This characteristic can be classified as unseasoned, moderately seasoned, and seasoned.

Exhibit 10 shows the spread risk for the different subsectors of the MBS market for both the 45-bond portfolio and the benchmark index. The risk is measured in terms of contribution to adjusted duration. Notice that the portfolio is substantially underweighted in unseasoned MBS and overweighted in moderately seasoned and seasoned MBS.

B. Prepayment Risk

Prepayment risk is the risk of an adverse price change due to changes in expected prepayments. The benchmark used for prepayments is the Public Securities Association (PSA) prepayment benchmark.²¹ One measure of prepayment risk is **prepayment sensitivity** which is the basis point change in the price of an MBS for a 1% increase in prepayments.

²¹The Public Securities Association is now the Bond Market Association.

For example, suppose that, for some MBS at 500 PSA, the price is 110.08. A 1% increase in the PSA prepayment rate means that PSA increases from 500 PSA to 505 PSA. Suppose that, at 505 PSA, the price is recomputed using a valuation model and found to be 110.00. Therefore, the price decline is -0.08 , in terms of basis points, -8 , so that the prepayment sensitivity is -8 .

Some MBS products increase in value when prepayments increase and some MBS products decrease in value when prepayments increase. Examples of the former are passthrough securities trading at a discount (i.e., passthrough securities whose coupon rate is less than the prevailing mortgage rate) and principal-only mortgage strips. These securities have positive prepayment sensitivity. Examples of MBS products that decrease in value when prepayments increase are passthrough securities trading at a premium (i.e., passthrough securities whose coupon rate is greater than the prevailing mortgage rate) and interest-only mortgage strips. These securities have negative prepayment sensitivity.

Exhibit 11 shows the risk exposure in terms of prepayment sensitivity for both the 45-bond portfolio and the benchmark. The risk exposure is provided for the same

EXHIBIT 11 MBS PSA Sensitivity Analysis

MBS Sector	Portfolio			Benchmark			Difference	
	% of portf.	PSA sens	Cntrb. to PSA sens	% of portf.	PSA sens	Cntrb. to PSA sens	% of portf.	Cntrb. to PSA sens
COUPON < 6.0%								
Conventional	0.00	0.00	0.00	0.10	0.01	0.00	-0.10	-0.00
GNMA 30 yrs	0.00	0.00	0.00	0.02	0.44	0.00	-0.02	-0.00
15 year MBS	0.00	0.00	0.00	0.63	-0.74	-0.00	-0.63	0.00
Balloon	0.00	0.00	0.00	0.05	-1.21	-0.00	-0.05	0.00
6.0% ≤ COUPON < 7.0%								
Conventional	3.67	-2.33	-0.09	11.66	-1.93	-0.22	-7.99	0.14
GNMA 30 yrs	0.98	-1.98	-0.02	2.65	-1.71	-0.05	-1.67	0.03
15 year MBS	4.55	-1.69	-0.08	3.97	-1.82	-0.07	0.58	-0.00
Balloon	3.87	-0.87	-0.03	0.18	-1.92	-0.00	3.70	-0.03
7.0% ≤ COUPON < 8.0%								
Conventional	6.26	-3.70	-0.23	8.32	-4.24	-0.35	-2.06	0.12
GNMA 30 yrs	5.97	-4.47	-0.27	3.48	-3.75	-0.13	2.49	-0.14
15 year MBS	0.00	0.00	0.00	1.22	-3.24	-0.04	-1.22	0.04
Balloon	0.00	0.00	0.00	0.05	-3.31	-0.00	-0.05	0.00
8.0% ≤ COUPON < 9.0%								
Conventional	6.68	-6.45	-0.43	1.48	-6.48	-0.10	5.19	-0.33
GNMA 30 yrs	0.00	0.00	0.00	1.05	-5.56	-0.06	-1.05	0.06
15 year MBS	0.00	0.00	0.00	0.12	-4.11	-0.00	-0.12	0.00
Balloon	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9.0% ≤ COUPON < 10.0%								
Conventional	0.00	0.00	0.00	0.14	-9.05	-0.01	-0.14	0.01
GNMA 30 yrs	2.73	-8.27	-0.23	0.20	-6.54	-0.01	2.54	-0.21
15 year MBS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Balloon	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COUPON ≥ 10.0%								
Conventional	0.00	0.00	0.00	0.01	-11.01	-0.00	-0.01	0.00
GNMA 30 yrs	0.00	0.00	0.00	0.04	-7.88	-0.00	-0.04	0.00
15 year MBS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Balloon	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Subtotals								
Conventional	16.60		-0.75	21.71		-0.69	-5.11	-0.06
GNMA 30 yrs	9.68		-0.51	7.44		-0.25	2.24	-0.26
15 year MBS	4.55		-0.08	5.94		-0.12	-1.39	0.04
Balloon	3.87		-0.03	0.28		-0.01	3.59	-0.03
Totals	34.70		-1.37	35.36		-1.06	-0.66	-0.31

PSA sensitivity: change in price (in units of basis points of price) for a 1% increase in PSA.

coupon subsectors used for the MBS sector risk in Exhibit 10. In addition, the subsectors are further partitioned in terms of conventional issues (i.e., Fannie Mae and Freddie Mac passthroughs), 30-year Ginnie Mae issues, 15-year MBS, and MBS backed by balloon mortgages.²² A summary of the portfolio's exposure relative to the benchmark is provided in the last three lines of the exhibit. The portfolio has a greater percentage invested in the GNMA 30-year and Balloon sectors than the benchmark and less invested in the 15-year MBS sector. However, the portfolio manager is not interested in the percentage difference for the amount invested in each sector, but rather the contribution to prepayment sensitivity. As seen in the exhibit, the prepayment sensitivity difference is primarily due to the greater exposure to the GNMA 30-year sector (−0.26).

C. Convexity Risk

The sector of the MBS area that tends to exhibit negative convexity is mortgage passthroughs. Consequently, a portfolio manager should assess the convexity of a portfolio compared to the benchmark index. A portfolio manager's allocation to the MBS sector might be the same as that of the benchmark index with the same effective duration, and yet performance can be quite different if the portfolio has different exposure to convexity. This concept is referred to as **convexity risk**.

Exposure to convexity risk in the MBS sector is computed from the difference between the convexity of a portfolio and the convexity of the benchmark index. (The appropriate convexity measure is **effective convexity** explained previously.) Convexity is −0.55 for the MBS sector of the 45-bond portfolio and −0.67 for the MBS sector of the benchmark index. So, the 45-bond portfolio has less negative convexity than the benchmark index. Exhibit 12 decomposes the convexity risk by subsectors within the MBS sector and shows the contribution to convexity by subsector.

X. MULTI-FACTOR RISK MODELS

Multi-factor risk models (sometimes referred to simply as “factor models”) can be used by portfolio managers (both indexers and active managers) to quantify the risk exposure of a portfolio or a benchmark index. Below we describe how a multi-factor risk model can be used in bond portfolio management to quantify the risk profile of a portfolio or a benchmark index in terms of the risk factors described in the previous sections, and relate these risk factors to tracking error.²³ In the next chapter, we will see how multi-factor risk models can be used to construct a portfolio and to rebalance a portfolio.

In the fixed income area, several vendors and dealer firms have developed multi-factor risk models. In our illustration, we will use the Lehman Brothers multi-factor risk model.²⁴ While the model covers U.S. dollar-denominated securities in most Lehman Brothers domestic fixed-rate bond indices (Aggregate, High Yield, Eurobond), we will use only the model for the

²²These types of mortgage passthrough securities were described earlier.

²³For a discussion of factor models, see Chapter 11 of DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*, pp. 598–618.

²⁴This model is described in Lev Dynkin, Jay Hyman, and Wei Wu, “Multi-Factor risk Models and Their Applications,” in Frank J. Fabozzi (ed.), *Professional Perspectives on Fixed Income Portfolio Management: Volume 2* (New Hope, PA: Frank J. Fabozzi Associates, 2001).

EXHIBIT 12 MBS Convexity Analysis

MBS Sector	Portfolio			Benchmark			Difference	
	% of portf.	Convexity	Contrb. to conv.	% of portf.	Convexity	Contrb. to conv.	% of portf.	Contrb. to conv.
COUPON < 6.0%								
Conventional	0.00	0.00	0.00	0.10	-1.56	-0.00	-0.10	0.00
GNMA 30 yrs	0.00	0.00	0.00	0.02	-1.22	-0.00	-0.02	0.00
15 year MBS	0.00	0.00	0.00	0.63	-1.36	-0.01	-0.63	0.01
Balloon	0.00	0.00	0.00	0.05	-1.75	-0.00	-0.05	0.00
6.0% ≤ COUPON < 7.0%								
Conventional	3.67	-2.30	-0.08	11.66	-2.41	-0.28	-7.99	0.20
GNMA 30 yrs	0.98	-2.99	-0.03	2.65	-2.95	-0.08	-1.67	0.05
15 year MBS	4.55	-0.81	-0.04	3.97	-1.53	-0.06	0.58	0.02
Balloon	3.87	-0.75	-0.03	0.18	-1.45	-0.00	3.70	-0.03
7.0% ≤ COUPON < 8.0%								
Conventional	6.26	-1.76	-0.11	8.32	-1.47	-0.12	-2.06	0.01
GNMA 30 yrs	5.97	-1.96	-0.12	3.48	-2.29	-0.08	2.49	-0.04
15 year MBS	0.00	0.00	0.00	1.22	-1.02	-0.01	-1.22	0.01
Balloon	0.00	0.00	0.00	0.05	-0.78	-0.00	-0.05	0.00
8.0% ≤ COUPON < 9.0%								
Conventional	6.68	-1.99	-0.13	1.48	-0.80	-0.01	5.19	-0.12
GNMA 30 yrs	0.00	0.00	0.00	1.05	-0.85	-0.01	-1.05	0.01
15 year MBS	0.00	0.00	0.00	0.12	-0.55	-0.00	-0.12	0.00
Balloon	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9.0% ≤ COUPON < 10.0%								
Conventional	0.00	0.00	0.00	0.14	-0.81	-0.00	-0.14	0.00
GNMA 30 yrs	2.73	-0.24	-0.01	0.20	-0.60	-0.00	2.54	-0.01
15 year MBS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Balloon	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COUPON ≥ 10.0%								
Conventional	0.00	0.00	0.00	0.01	-0.54	-0.00	-0.01	0.00
GNMA 30 yrs	0.00	0.00	0.00	0.04	-0.24	-0.00	-0.04	0.00
15 year MBS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Balloon	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Subtotals								
Conventional	16.60		-0.33	21.71		-0.42	-5.11	0.09
GNMA 30 yrs	9.68		-0.15	7.44		-0.17	2.24	0.02
15 year MBS	4.55		-0.04	5.94		-0.08	-1.39	0.05
Balloon	3.87		-0.03	0.28		-0.00	3.59	-0.03
Totals	34.70		-0.55	35.36		-0.67	-0.66	0.13

Lehman Aggregate Bond Index. The historical data needed to estimate the model are updated monthly by Lehman Brothers. The portfolio we will analyze is the 45-bond portfolio that has been used throughout this chapter.

Much like other commercially available or dealer proprietary multi-factor risk models, the Lehman Brothers model focuses on predicted tracking error. Specifically, tracking error is defined as one standard deviation of the difference between the 45-bond portfolio and benchmark annualized returns. For the 45-bond portfolio, the predicted tracking error is 62 basis points per year.

A. Systematic and Non-Systematic Risks

The multi-factor risk model seeks to identify the specific risks that contribute to the *predicted* tracking error. All of the risks are measured in terms of predicted tracking error. The analysis begins with a decomposition of the risks into two general categories—*systematic risk* and *non-systematic risk*. The latter is the risk that remains after removing systematic risk and, is also referred to as *residual risk*. For the portfolio, the following was determined:

predicted tracking error due to systematic risk = 54.8 basis points

predicted tracking error due to non-systematic risk = 28.4 basis points

If you add these two risks, you find a predicted tracking error risk greater than 62 basis points. What is wrong? There is nothing wrong. Remember that predicted tracking errors are standard deviations. We know that it is not correct to add standard deviations when computing the risk of a portfolio. To obtain the predicted tracking error for the portfolio, the sum of the squares of the two predicted tracking errors equals the square of the portfolio's predicted tracking error. The square root of the sum of the squares of the two predicted tracking errors equals the portfolio's predicted tracking error. That is, for the 45-bond portfolio:

$$[54.8^2 + 28.4^2]^{0.5} = 62 \text{ basis points.}$$

The addition of variances assumes a zero correlation between the risk factors (i.e., the risk factors are statistically independent). If this is not the case, the correlation between the risk factors must be taken into account in estimating the predicted tracking error.

B. Components of Systematic Risk

Systematic risk can be decomposed into several risks. The first decomposition treats systematic risk as term structure factor risk and non-term structure factor risks.

1. Term Structure Risk As explained earlier, a portfolio's exposure to changes in the general level of interest rates is measured in terms of exposure to (1) a parallel shift in the yield curve and (2) a nonparallel shift in the yield curve. Taken together, this risk exposure is referred to as *term structure risk*.

From our discussion in Section IV we know that the duration of the 45-bond portfolio is greater than the benchmark duration (4.59 versus 4.10). The difference between the yield curve risk for the portfolio and for the benchmark was shown in Exhibits 5 and 6, which are based on the distribution of the present value of the cash flows and the key rate durations, respectively.

The model indicates that the predicted tracking error due to term structure risk is 49.8 basis points.

2. Non-Term Structure Risk Factors Other systematic risks that are not due to exposure to changes in the term structure are called *non-term structure risk factors*. They include:

- sector risk
- quality risk
- optionality risk
- coupon risk
- MBS risk

MBS risk consists of the risks associated with investing in mortgage-backed securities. As explained in Section IX, MBS risk can be decomposed into sector, prepayment, and convexity risks.

For the 45-bond portfolio, the Lehman model indicates that the predicted tracking error due to the non-term structure risk is 25.5 basis points. We now know that predicted tracking error due to systematic risk is 54.8 basis points, consisting of:

predicted tracking error due to term structure risk = 49.8 basis points

predicted tracking error due to non-term structure risks = 25.5 basis points

Again, the predicted tracking error due to systematic risk is not equal to the sum of these two components of predicted tracking error. If the term structure factor and the non-term structure factors are statistically independent, then the total predicted tracking error from all the systematic factors is 55.9 basis points ($= [(49.8)^2 + (25.5)^2]^{0.5}$). Why is there a difference between the 55.9 basis points just computed and the 54.8 basis points for the systematic risk? The reason is that the correlation (not reported here) between the two systematic risks is not zero. When the correlations are taken into account the predicted tracking error due to systematic risk is 54.8 basis points.

The Lehman model accounts for the correlation between risk factors in determining the predicted tracking errors due to each of the non-term structure risk factors. The results for the 45-bond portfolio are shown below:

Tracking error due to	Predicted tracking error
sector risk	22.7 basis points
quality risk	10.7 basis points
optionality risk	1.3 basis points
coupon risk	1.4 basis points
MBS sector risk	9.3 basis points
MBS volatility risk	8.3 basis points
MBS prepayment risk	8.8 basis points

C. Non-Systematic Risk

Non-systematic risk is divided into those risks that are issuer specific and components that are issue specific. This risk is due to the fact that the portfolio has greater exposure to specific issues and issuers than the benchmark index. To understand these non-systematic risks, look at the last column of Exhibit 3.

The last column of the exhibit reports the percentage of the 45-bond portfolio's market value invested in each issue. Since there are only 45 issues in the portfolio, each issue makes up a non-trivial fraction of the portfolio. Specifically, look at the exposure to two corporate issuers, Coca-Cola Enterprises and Daimler-Benz North America. The former represents about 10% of the portfolio and the latter almost 4%. If either firm is downgraded, this would cause large losses in the 45-bond portfolio, but it would *not* have a significant effect on the benchmark which includes 7,000 issues. Consequently, a large exposure to a specific corporate issuer represents a material mismatch between the exposure of the portfolio and the exposure of a benchmark index that must be taken into account in assessing a portfolio's risk relative to a benchmark index.

As an example of non-systematic risk, suppose that a portfolio included Enron bonds in December 2001. For that month, Enron bonds lost more than three quarters of their

value. The weighting of the bonds in the Merrill Lynch High Grade Index was 0.2%. If these bonds represent 1.6% of the portfolio, this would have resulted in a 100 basis point underperformance relative to the index at a time when sectors such as Consumer Cyclical, Media, Telecom, and Technology outperformed the index by more than 200 basis points.²⁵

²⁵Phil Galdi, Mark Anderson, and Arjun Kondamani, *U.S. High Grade Bond Index*, Merrill Lynch (December 5, 2001).

CHAPTER 18

MANAGING FUNDS AGAINST A BOND MARKET INDEX

I. INTRODUCTION

The benchmark for a manager can be either a bond market index or liabilities. In this chapter, we provide an overview of strategies for managing funds against a bond market index. We restrict our discussion to domestic bond markets, specifically the U.S. bond market, in the illustrations in this chapter. In Chapter 20, we cover global bond investing. In Chapter 19, we discuss strategies for managing funds when the benchmark is one or more liabilities.

II. DEGREES OF ACTIVE MANAGEMENT

In the previous chapter we discussed the risk factors associated with a bond portfolio and a bond market index. One can classify bond portfolio strategies in terms of the degree to which a manager constructs a portfolio with a risk profile that differs from the risk profile of the benchmark index. Kenneth Volpert of the Vanguard Group has classified bond portfolio strategies as follows:¹

1. pure bond index matching
2. enhanced indexing/matching risk factors
3. enhanced indexing/minor risk factor mismatches
4. active management/larger risk factor mismatches
5. unrestricted active management

We discuss each of these strategies below.

¹Kenneth E. Volpert, “Managing Indexed and Enhanced Indexed Bond Portfolios,” Chapter 3 in Frank J. Fabozzi (ed.), *Fixed Income Readings for the Chartered Financial Analyst Program: First Edition* (New Hope, PA: Frank J. Fabozzi Associates, 2000). Volpert uses the term “active management/full-blown active” instead of “unrestricted active management.” I have changed the classification based on the suggestion of David Zahn, CFA, of Citigroup Asset Management.

A. Pure Bond Index Strategy

In terms of risk and return, the strategy with the least risk of underperforming the index is a pure bond index matching strategy.

1. Reasons for Indexing There are several reasons that support a bond indexing strategy. First, empirical evidence suggests that historically the overall performance of active bond managers has been poor. Obviously, this continues to be an ongoing debate in the profession.

The second reason cited in support of indexing is the lower management advisory fees compared to active management advisory fees. Advisory fees charged by active managers typically range from 15 to 50 basis points. The range of fees for indexed portfolios, in contrast, is 1 to 20 basis points (with the upper range representing fees for enhanced indexing, discussed below). Some pension plan sponsors have chosen to eliminate advisory fees by managing some or all of their funds in-house using an indexing strategy. Lower nonadvisory fees, such as custodial fees, is the third reason for the popularity of bond indexing.

However, some active managers (discussed below) index a portion of their portfolio and sometimes index the entire portfolio. A manager might index a particular sector because he believes that either (1) he does not have the skills necessary to outperform a sector of the market or (2) the particular sector is price efficient so that it is futile to attempt to outperform the market. Here are two examples.

In our first example, suppose that a manager's benchmark is the Lehman Aggregate Bond Index and that the manager specializes in credit analysis. Furthermore, the manager believes she can structure both the credit sector and the ABS sector of the portfolio in such a way as to outperform these sectors of the benchmark index. However, the manager has no special skill in the MBS sector. Moreover, the manager believes there are no opportunities to enhance return in the Treasury and Agency sectors. The manager can then index the MBS, Treasury, and Agency sectors. Thus, while the portfolio might be actively managed in general, three sectors are indexed.

In our second example, consider the same manager who not only believes she has the skills to select securities to outperform the credit and ABS sectors of the index, but also that she can be effective in forecasting changes in credit spreads. The manager will continue to pursue an active strategy relative to the credit and ABS sectors while indexing the MBS, Treasury, and Agency sectors, and will alter the allocation to each sector based on her forecasts of changes in credit spreads.

Now let's see why an active manager might periodically pursue an indexing strategy. As we will see when we discuss active strategies, a manager takes a view on some risk factors and positions a portfolio accordingly. Suppose that an active manager has no view on any of the risk factors. In that case, the manager seeks to be neutral with respect to the benchmark index—that is, the manager will temporarily index the portfolio.

Here is a second case where an active manager might temporarily pursue an indexing strategy. Suppose that an active manager is asked to take over a portfolio of an active manager who has been terminated by the client. Until the manager has the opportunity to rebalance the portfolio in a manner consistent with her views, she may temporarily index the portfolio.

2. Logistical Problems with an Indexing Strategy The pure bond indexing strategy requires creating a portfolio that replicates the issues comprising the benchmark index. This means that the indexed portfolio mirrors the benchmark index. However, a manager pursuing this strategy encounters several logistical problems. First, the price of each issue used by the organization that publishes the benchmark index may not be an execution price available to

the manager. In fact, these prices might be materially different from the prices offered by dealers. In addition, the prices used by organizations reporting index values are based on bid prices. However, the prices the manager pays when constructing or rebalancing the portfolio are dealer ask prices. Thus there is a bias between the performance of the benchmark index and the indexed portfolio equal to the bid-ask spread.

Furthermore, there are logistical problems unique to certain sectors of the bond market. Consider first the corporate bond market. There are more than 4,000 issues in the corporate bond sector of a broad-based bond market index. Because many of these issues are illiquid, not only might the prices be unreliable, but many of the issues may not even be available. Next, consider the mortgage sector. There are over 800,000 agency passthrough issues. As explained in Chapter 16, the organizations that publish indexes aggregate these issues into a few hundred generic issues. The manager is then faced with the difficult task of finding passthrough securities with risk/return profiles that are the same as those of the hypothetical generic issues.

Finally, as explained in Section IV, total return depends on the reinvestment rate available on interim cash flows received prior to month end. If the organization publishing the benchmark index regularly overestimates the reinvestment rate, then the indexed portfolio might underperform the benchmark index.

3. Rebalancing an Indexing Portfolio Once an indexed portfolio is constructed, the portfolio must then be rebalanced as the composition and the characteristics of the target index change. For example, over time the duration of an index and the indexed portfolio changes, making rebalancing necessary to bring the portfolio's duration in line with the duration of the index. Transaction costs arise when rebalancing a portfolio. The manager will seek a rebalancing strategy that minimizes transaction costs. The multi-factor risk model described later in this chapter can be used to rebalance an indexed portfolio at minimum cost.

B. Enhanced Indexing/Matching Risk Factors

An enhanced indexing strategy can be used to construct a portfolio that matches the primary risk factors without acquiring each issue in the index. This is a common strategy used by smaller funds because of the difficulties in acquiring all of the issues comprising the index. Generally speaking, the smaller the number of issues used to replicate the benchmark index, the lower the transaction costs but the greater the difficulties in matching the risk factors. In contrast, the more issues purchased to replicate the benchmark index, the greater the transaction costs, but the lower the risk due to the mismatch of risk factors between the indexed portfolio and the benchmark index.

In the spectrum of strategies defined by Volpert, this strategy is called an "enhanced strategy," although some investors refer to this as simply an indexing strategy. Two commonly used techniques to construct a portfolio that replicates an index are:

1. cell matching (stratified sampling)
2. tracking error minimization using a multi-factor risk model

Both techniques assume that the performance of an individual bond depends on a number of systematic factors that affect the performance of all bonds and on an unsystematic factor unique to the individual issue or issuers.

1. **Cell Matching** With the **cell matching technique** (also called the **stratified sampling technique**), the index is divided into cells representing the risk factors. The objective is then to select from all of the issues in the index one or more issues in each cell to represent that entire cell. The total dollar amount purchased of the issues from each cell will be based on a criterion such as the percentage of the index's total market value that the cell represents. For example, if $W\%$ of the market value of the index is comprised of double-A corporate bonds, then $W\%$ of the market value of the indexed portfolio should be composed of double A corporate bond issues. A better approach would be to base selection on the contribution to duration, a measure explained in the previous chapter.

The number of cells the indexer uses depends on the dollar amount of the portfolio to be indexed. In indexing a portfolio of less than \$50 million, for example, a large number of cells would require the purchase of odd lots of issues. This increases the cost of buying the issues to represent a cell, and thus would increase the likelihood of underperforming the benchmark index. Reducing the number of cells to overcome this problem increases predicted tracking error because the risk profile of the indexed portfolio might differ materially from that of the index.

2. **Tracking Error Minimization Using a Multi-Factor Risk Model** In the previous chapter we discussed multi-factor risk models. Specifically, our focus was on using such models to quantify the risk profile of a portfolio or a benchmark index, and describing the risk factors in the model. We used a Lehman model in our illustrations. As mentioned in the previous chapter, a multi-factor risk model can also be used to construct a portfolio with a predicted tracking error acceptable to a manager pursuing an enhanced indexing strategy that matches the risk factors.

C. Enhanced Indexing/Minor Risk Factor Mismatches

Another enhanced strategy is based on constructing a portfolio with minor deviations from the risk factors that affect the performance of the index. For example, such a strategy might be used to create a portfolio with a slight overweighting of issues or sectors that are relative values. This strategy matches the duration of the portfolio with the duration of the benchmark index. That is, there are no duration bets in this strategy, as is the case with the pure index match strategy and the enhanced index with matching risk strategy.

D. Active Management/Larger Risk Factor Mismatches

Active bond strategies attempt to outperform the market by constructing a portfolio that has a greater index mismatch than is the case for enhanced indexing. The decision to pursue an active strategy or to recommend that a client direct a manager to pursue an active strategy must be based on the belief that there is some gain to be derived from such costly efforts; in order for a gain to exist, pricing inefficiencies must exist. The particular strategy chosen depends on the reasons that price inefficiencies exist.

Volpert identifies two types of active strategies. In the more conservative of these strategies, the manager creates larger mismatches, in terms of risk factors, relative to the benchmark index. This includes minor duration mismatches. Typically, there is a limit to the degree of duration mismatch. For example, the manager might be constrained to within ± 1 of the

duration of the index.² So, if the duration of the index is 4, the manager may construct a portfolio with duration between 3 and 5. To take advantage of anticipated reshaping of the yield curve, the manager might create a portfolio such that there are significant differences in cash flow distribution between the benchmark index and the portfolio. As another example, if the manager believes that, within the corporate sector, A rated issues will outperform AA rated issues, the manager might overweight the A rated issues and underweight AA rated issues.

The manager can choose to invest a small percentage of the portfolio in one or more sectors that are not in the bond market index. For example, the portfolio might have a small allocation to nonagency mortgage-backed securities. If the index includes only investment-grade products, the manager can allocate some portion of the portfolio, if permitted by investment guidelines, to non-investment-grade products.

Most active managers fall into this category of active management rather than the unrestricted active management category described below.

E. Unrestricted Active Management

In the case of unrestricted active management, the manager is permitted to make a significant duration bet without any constraint. The portfolio can have a duration of zero (i.e., invested all in cash) or, using leverage, the portfolio can have a duration that is a large multiple of the duration of the benchmark index. The manager can choose not to invest in one or more of the major sectors of the broad-based bond market indexes. The manager can make a significant allocation to sectors not included in the index.

We will discuss various strategies used in active bond portfolio management in the next section.

III. STRATEGIES

Active portfolio strategies seek to generate additional return after adjusting for risk. This additional return is popularly referred to as *alpha*. We shall refer to these strategies as **value added strategies**, which can be classified as strategic strategies and tactical strategies.

Strategic strategies, sometimes referred to as **top down value added strategies**, include the following:

1. interest rate expectations strategies
2. yield curve strategies
3. inter- and intra-sector allocation strategies

Tactical strategies, sometimes referred to as **relative value strategies**, are short-term trading strategies. They include:

1. strategies based on rich/cheap analysis
2. yield curve trading strategies
3. return enhancing strategies employing futures and options

²Actually, some might say that the manager may be constrained to be within “ ± 1 year” of the duration of the index. However, as emphasized earlier, characterizing duration in temporal terms (i.e., years) should be avoided.

We discuss these strategies below. Strategies involving futures and options are not covered in this chapter. These strategies involve identifying mispriced futures and options and taking positions in the cash market and in the futures or options markets to capture the mispricing. The pricing of futures and options was covered earlier.

To explain strategic strategies, we will use the portfolio recommended by Lehman Brothers to its clients in a August 27, 2001 publication (*Global Relative Value*). The recommended portfolio, shown in Exhibit 1, is for a manager whose benchmark index is the Lehman U.S. Aggregate Bond Index. Note that, in this exhibit, the credit sector is basically the corporate sector, as discussed in Chapter 16.

A. Interest Rate Expectations Strategies

A manager who believes that he can accurately forecast the level of interest rates will alter the portfolio's duration based on that forecast. Since duration is a measure of interest rate sensitivity, this strategy requires increasing a portfolio's duration if interest rates are expected to fall and reducing duration if interest rates are expected to rise. For those managers whose benchmark is a bond market index, this means increasing the portfolio duration relative to the benchmark index duration if interest rates are expected to fall, and reducing portfolio duration if interest rates are expected to rise. The degree to which the duration of the managed portfolio is permitted to diverge from that of the benchmark index may be limited by the client. Interest rate expectations strategies are commonly referred to as *duration strategies*.

A portfolio's duration may be altered in the cash market by swapping (or exchanging) bonds in the portfolio for new bonds that will achieve the target portfolio duration. Or, a manager can sell bonds and stay in cash (i.e., invest in money market instruments) if he believes that interest rates are going to increase. Alternatively, a more efficient means for altering the duration of a bond portfolio is to use interest rate futures contracts. As we explain in Chapter 22, buying futures increases a portfolio's duration, while selling futures decreases the duration.

The key to this active strategy is, of course, an ability to forecast the direction of interest rates. The academic literature does not support the view that interest rates can be forecasted with accuracy in order to realize positive active returns on a consistent basis. It is doubtful that betting on future interest rates will provide consistently superior returns.

While a manager might not pursue an active strategy based strictly on future interest rate movements, certain circumstances can lead a manager to make an interest rate bet to cover inferior performance relative to a benchmark index. For example, suppose a manager represents himself to a client as pursuing one of the active strategies discussed later in this

EXHIBIT 1 Recommended Portfolio for U.S. Aggregate Core Portfolio (August 24, 2001)

	Percent of market value by duration range										Contribution to							
	0-2		2-4		4-7		7-9		9+		Total		OAD*			Spread duration		
Sector	Index	Rec	Index	Rec	Index	Rec	Index	Rec	Index	Rec	Index	Rec	Index	Rec	Diff	Index	Rec	Diff
Treasury	5.53	2.01	5.12	3.64	4.80	0.00	0.78	3.22	7.20	4.19	23.44	13.05	1.40	0.99	-0.41	0.00	0.00	0.00
Agency	3.09	0.00	3.53	5.74	2.56	3.44	0.65	1.61	1.42	0.92	11.25	11.71	0.50	0.55	0.05	0.50	0.56	0.06
Mrg.Pass-throughs	6.81	5.54	19.79	22.78	8.22	8.49	0.00	0.00	0.00	0.00	34.82	36.81	1.04	1.22	0.18	1.16	1.32	0.16
CMBS	0.03	0.00	0.37	0.27	1.53	1.87	0.03	0.00	0.00	0.00	1.95	2.14	0.10	0.12	0.02	0.10	0.12	0.02
ABS	0.53	3.19	0.69	2.33	0.48	0.00	0.06	0.00	0.00	0.00	1.75	5.51	0.06	0.12	0.07	0.06	0.12	0.07
Credit	3.01	3.17	7.77	10.35	9.37	9.98	2.28	3.16	4.35	4.11	26.78	30.78	1.46	1.62	0.16	1.46	1.61	0.15
Total	19.00	13.91	37.27	45.11	26.96	23.78	3.81	7.99	12.97	9.22	100.00	100.00	4.56	4.63	0.07	3.28	3.73	0.45

*OAD = option-adjusted duration

Source: *Global Relative Value*, Lehman Brothers, Fixed Income Research, August 27, 2001, p. 2.

chapter. Suppose further that the manager is evaluated over a 1-year investment horizon, and that, three months before the end of the investment horizon, the manager's performance is significantly below the client-specified benchmark index. If the manager believes the account will be lost due to the underperformance, the manager has an incentive to bet on interest rate movements. If the manager is correct, the account will be saved, while an incorrect bet will result in further underperforming the index. In this case, the account will probably be lost regardless of the extent of the underperformance. A client can prevent this type of gaming by imposing constraints on the extent to which the portfolio's duration can diverge from that of the index. Also, in the performance evaluation stage of the investment management process, described later in this chapter, decomposing the portfolio's return into the risk factors that generate the return highlights the extent to which a portfolio's return is attributable to changes in the level of interest rates.

Exhibit 1 shows the option-adjusted duration for the Lehman Brothers Aggregate Index and for the recommended portfolio. In this case, Lehman Brothers uses the term "option-adjusted duration" rather than effective duration, which in other reports, the term "adjusted duration" is used. Exhibit 1 shows that the recommended duration for a U.S. aggregate portfolio is 4.63, compared to 4.56 for the index. That is, the recommended portfolio duration is 102% of the index duration and therefore the portfolio has slightly greater exposure to changes in interest rates.

B. Yield Curve Strategies

As explained previously, the yield curve for U.S. Treasury securities shows the relationship between maturity and yield. The shape of the yield curve changes over time. A shift in the yield curve refers to a change in the yield of each Treasury maturity. A parallel shift in the yield curve refers to a shift in which the change in yield is the same for all maturities. A nonparallel shift in the yield curve means that the yield change is not the same number of basis points for each maturity.

Top down yield curve strategies involve positioning a portfolio to capitalize on expected changes in the shape of the Treasury yield curve. There are three yield curve strategies: (1) a bullet strategy, (2) a barbell strategy, and (3) a ladder strategy. In a bullet strategy, the portfolio is constructed so that the maturities of the bonds in the portfolio are highly concentrated at one point on the yield curve. In a barbell strategy, the maturities of the bonds in the portfolio are concentrated at two extreme maturities. In practice, when managers refer to a barbell strategy, they are referring to a comparison with a bullet strategy. For example, a bullet strategy might be to create a portfolio with maturities concentrated around 10 years, while a corresponding barbell strategy might be a portfolio with 5-year and 20-year maturities. In a ladder strategy, the portfolio has approximately equal amounts of each maturity. So, for example, a portfolio might have equal amounts of bonds with one year to maturity, two years to maturity, etc.

Each of these strategies results in different performance when the yield curve shifts. The actual performance depends on both the type and magnitude of the shift. Thus, no general statements can be made about the optimal yield curve strategy. This conclusion will be demonstrated in Section IV.

When a yield curve strategy is applied by a manager whose benchmark index is a broad-based bond market index, a mismatch of maturities relative to the benchmark index exists in one or more of the bond sectors. Some managers use duration buckets rather than maturity, as a measure of the mismatch. For example, in Exhibit 1, the index and the recommended portfolio are grouped into five duration ranges: 0–2, 2–4, 4–7, 7–9, and 9+. Notice that the

recommended portfolio underweights the 0–2 and 9+ duration buckets. That is, the short and long end of the yield curve are underweighted relative to the benchmark index.

C. Inter- and Intra-Sector Allocation Strategies

A manager might allocate funds among the major bond sectors in a manner that differs from the allocation of the benchmark index. This approach is referred to as an **inter-sector allocation strategy**. For example, from Exhibit 1 we can see the distribution of the Lehman U.S. Aggregate Bond Index along with Lehman’s recommended asset allocation:

Sector	Index	Recommended	Weighting
Treasury	23.44%	13.05%	Underweight
Agency	11.25	11.71	Overweight
Mtg. Passthroughs	34.82	36.81	Overweight
CMBS	1.95	2.14	Overweight
ABS	1.75	5.51	Overweight
Credit	26.78	30.78	Overweight

Basically, this allocation strategy is aimed at benefiting from spread products—products that expose investors to credit risk and prepayment risk for mortgages (and some asset-backed securities). The recommended portfolio has less exposure to the Treasury sector and more exposure to the spread product sectors. Notice that all of the spread product sectors are overweighted.

As explained in the previous chapter, spread duration measures the exposure of a portfolio to changes in spreads. Exhibit 1 provides information about the exposure to spread risk. First is the difference in spread duration between the benchmark index (3.28) and the recommended portfolio (3.73). This differential is to be expected, given the overweighting of the spread sectors in the recommended portfolio relative to the benchmark index. The spread duration quantifies the extent of this overweighting. The second is the difference between the contribution to spread duration for each sector in the index and the corresponding sector in the recommended portfolio.

In an **intra-sector allocation strategy**, the manager’s allocation of funds within a sector differs from that of the index. Exhibit 1 shows the allocation within each sector as a percentage of the total market. Exhibit 2 displays Lehman Brothers’ intra-sector allocation recommendation for the credit sector as of August 24, 2001 in terms of **contribution to spread duration** for each credit quality and by sector. The credit sector is equivalent to the corporate bond sector as previously noted, and is further divided into the following subsectors: industrial, financial, utility, and non-corporate.³

Exhibit 3 shows the recommended allocation for the MBS sector as of the same date. This recommendation is presented in terms of market value and spread duration, by program and price. The MBS sector includes the mortgage passthrough sector and the commercial mortgage-backed securities sector. Let us first discuss the mortgage passthrough sector.

Information is classified by issue—Government National Mortgage Association (Ginnie Mae or GNMA), the Federal National Mortgage Association (Fannie Mae), and the Federal Home Loan Mortgage Corporation (Freddie Mac)—original maturity (30-year and 15-year passthroughs)—and by price. Why is there this detailed information? The differences in allocation, by price, are included to emphasize the differences in prepayment risk exposure

³“Non-corporate issues” is Lehman Brothers’ terminology for non-U.S. corporates.

EXHIBIT 2 Corporate Sector Recommendation in Terms of Contribution to Spread Duration
(August 24, 2001)

	Aaa–Aa			A			Baa			Total		
	Index	Rec.	Diff.	Index	Rec.	Diff.	Index	Rec.	Diff.	Index	Rec.	Diff.
<i>Spread Duration</i>												
0–3	0.04	0.05	0.01	0.05	0.06	0.01	0.03	0.07	0.05	0.12	0.18	0.07
3–5	0.10	0.02	–0.08	0.12	0.16	0.04	0.08	0.11	0.03	0.30	0.29	–0.01
5–7	0.08	0.06	–0.02	0.17	0.27	0.10	0.15	0.13	–0.02	0.40	0.46	0.06
7–10	0.05	0.09	0.04	0.10	0.19	0.09	0.10	0.11	0.01	0.26	0.40	0.14
10+	0.06	0.05	–0.01	0.19	0.23	0.04	0.14	0.00	–0.14	0.40	0.28	–0.12
Total	0.33	0.27	–0.06	0.62	0.91	0.29	0.50	0.43	–0.08	1.46	1.61	0.15
<i>Sector</i>												
Industrial	0.09	0.15	0.05	0.33	0.70	0.37	0.33	0.08	–0.25	0.75	0.92	0.17
Financial	0.12	0.11	–0.02	0.22	0.21	–0.01	0.04	0.00	–0.04	0.38	0.32	–0.06
Utility	0.01	0.00	–0.01	0.04	0.00	–0.04	0.09	0.22	0.13	0.13	0.22	0.08
Non-Corp.	0.09	0.02	–0.07	0.05	0.00	–0.05	0.06	0.13	0.07	0.20	0.15	–0.05
Total	0.31	0.27	–0.04	0.63	0.91	0.28	0.51	0.43	–0.09	1.46	1.61	0.15

Source: *Global Relative Value*, Lehman Brothers, Fixed Income Research, August 27, 2001, p. 2.

EXHIBIT 3 MBS Sector Recommendation in Terms of Contribution to Spread Duration
(August 24, 2001)

Program & Price		Index		Recommended		Difference	
		%Mkt.val.	Spread dur.	%Mkt.val.	Spread dur.	%Mkt.val.	Spread dur.
GNMA							
30-year	< 98	0.02	0.00	0.00	0.00	–0.02	0.00
	98 to <102	2.56	0.11	1.90	0.08	–0.66	–0.04
	102 to <106	4.65	0.15	3.06	0.11	–1.59	–0.05
	106+	0.40	0.01	1.95	0.06	1.55	0.05
15-year	< 98	0.00	0.00	0.00	0.00	0.00	0.00
	98 to <102	0.08	0.00	0.00	0.00	–0.08	0.00
	102 to <106	0.17	0.00	0.00	0.00	–0.17	0.00
	106+	0.00	0.00	0.00	0.00	0.00	0.00
GNMA Summary		7.89	0.29	6.92	0.25	–0.97	–0.04
Fannie Mae and Freddie Mac							
Conventional 30-year							
	< 98	0.79	0.04	2.30	0.12	1.51	0.08
	98 to <102	12.47	0.46	11.70	0.49	–0.77	0.03
	102 to <106	7.79	0.21	5.88	0.16	–1.91	–0.05
	106+	0.26	0.01	1.60	0.05	1.34	0.04
Conventional 15-year							
	< 98	0.02	0.00	0.00	0.00	–0.02	0.00
	98 to <102	2.99	0.10	7.04	0.22	4.05	0.13
	102 to <106	2.34	0.06	1.38	0.04	–0.96	–0.02
	106+	0.00	0.00	0.00	0.00	0.00	0.00
Conventional Summary		26.66	0.87	29.90	1.07	3.24	0.21
Balloons		0.26	0.01	0.00	0.00	–0.26	–0.01
Total Pass Throughs		34.81	1.16	36.81	1.32	2.00	0.16
CMBS		1.95	0.10	2.14	0.12	0.18	0.02
Total		36.77	1.26	38.95	1.44	2.18	0.18

Source: *Global Relative Value*, Lehman Brothers, Fixed Income Research, August 27, 2001, p. 3.

between the benchmark index and the recommended portfolio. For example, in the 30-year programs, we observe an underweighting of premium products (i.e., passthroughs trading above par value). The allocation in the mortgage passthrough sector suggests a concern that prepayments will accelerate, causing premium products (i.e., high-coupon mortgages) to underperform the low-coupon and par-coupon mortgages. This is a result of the negative convexity of the premium products.

1. Spreads Due to Credit Risk Inter- and intra-sector allocations indicate that a manager anticipates certain changes in spreads. Spreads reflect differences in credit risk, call risk (or prepayment risk), and liquidity risk. When the spread for a particular sector or subsector is expected to decline or narrow, a manager might decide to overweight that sector or subsector. The manager will underweight the sector if he expects the spread to increase or widen.

Credit or quality spreads change because of expected changes in economic prospects. Credit spreads between Treasury and non-Treasury issues widen in a declining or contracting economy and narrow during economic expansion. The economic rationale is that in a contracting economy, corporations experience declining revenue and cash flow, making it difficult for corporate issuers to service their contractual debt obligations. To induce investors to hold non-Treasury securities, the yield spread relative to Treasury securities must widen. The converse is that, during economic expansion, revenue and cash flow pick up, increasing the likelihood that corporate issuers will have the capacity to service their contractual debt obligations. Yield spreads between Treasury and government sponsored enterprise securities vary depending on investor expectations about the prospects that an implicit government guarantee will be honored.

Therefore, a manager can use economic forecasts to develop forecasts of credit spreads. Also, some managers base forecasts on historical credit spreads. The underlying principle is that a “normal” credit spread relationship exists. If the current credit spread differs materially from “normal,” then the manager should position the portfolio so as to benefit from a return to the normal credit spread. The assumption is that the normal credit spread is some type of mean value and that “mean reversion” will occur. If, in fact, there has been a structural shift in the marketplace, this reversion may not occur if the normal spread changes.

A manager also analyzes technical factors in order to assess relative value. For example, a manager might analyze the prospective supply and demand for new issues on spreads in individual sectors or issuers to determine whether they should be overweighted or underweighted. This commonly used tactical strategy is referred to as **primary market analysis**. Technical factors are discussed in Chapter 20.

2. Spreads Due to Call or Prepayment Risk Now let’s look at spreads due to call or prepayment risk. Expectations about how these spreads will change affect the inter-sector allocation decision between Treasury securities (which are noncallable securities except for a handful of callable Treasury issues outstanding) and spread products that have call risk. Both corporate and agency bonds include callable and noncallable issues. All mortgages are prepayable. Asset-backed securities include products that are prepayable, although borrowers may be unlikely to exercise this option. Consequently, with sectors having different degrees of call risk, expectations about how spreads will change also affect intra-allocation decisions. They affect (1) the allocation between callable and noncallable bonds within the corporate bond sector and (2) the allocation among premium (i.e., high coupon), par, and discount (i.e., low coupon) bonds within the agency, corporate, mortgage, and ABS sectors.

Spreads due to call risk will change as a result of expected changes in (1) the direction of change in interest rates and (2) interest rate volatility. An expected drop in the level of

interest rates will widen the spread between callable bonds and noncallable bonds as prospects increase that the issuer will exercise the call option. The spread narrows if interest rates are expected to rise. An increase in interest rate volatility increases the value of the embedded call option, and thereby increases the spread between (1) callable bonds and noncallable bonds and (2) premium and discount bonds. Trades that are motivated by the manager's anticipation of better performance resulting from the embedded options of individual issues or sectors are referred to as **structure trades**. Such trades are discussed in Chapter 20.

D. Individual Security Selection Strategies

Once the manager makes the allocation to a sector or subsector, he must then select the specific issues. A manager does not typically invest in all issues within a sector or subsector. Instead, depending on the size of the portfolio, the manager selects a representative number of issues.

At this stage, a manager makes an intra-sector allocation decision to the specific issues. The manager might believe that some securities within a subsector are mispriced and will therefore outperform other issues within the same sector over the time horizon. Managers pursue several different active strategies in order to identify mispriced securities. The most common strategy identifies an issue as undervalued because either (1) its yield is higher than that of comparably rated issues or (2) its yield is expected to decline (and price therefore rise) because the manager's credit analysis leads the manager to believe that an issue will be upgraded by the rating agency before the issue is put on credit watch/rating watch for an upgrade or a positive rating outlook is assigned by a rating agency.

Once a portfolio is constructed, a manager might undertake a swap of one bond for another that is similar in terms of coupon, maturity, and credit quality, but offers a higher yield. This is a **substitution swap**, and it depends on capital market imperfections. Such situations sometimes exist due to temporary market imbalances and the fragmented nature of the non-Treasury bond market. The risk the manager faces in making a substitution swap is that the bond purchased is not identical to the bond for which it is exchanged. Moreover, bonds typically have similar, but not identical, maturity and coupon, which might result in differences in convexity.

An active strategy used in the mortgage-backed securities market is to identify mispriced individual issues of passthroughs, CMO classes, or stripped MBS, given an assumed prepayment rate. In the case of CMOs and stripped MBS, this strategy involves investing in a non-index product—that is, a product not included in the benchmark index.

It is critical when evaluating any potential swaps to assess the impact of the swap on the risk exposure of the portfolio. We will see how a bond swap can be assessed when we discuss multifactor risk models in Section V. Also, we will see how a manager can use a multi-factor risk model to incorporate a security selection strategy in constructing a portfolio.

IV. SCENARIO ANALYSIS FOR ASSESSING POTENTIAL PERFORMANCE

An active manager needs a tool for assessing the potential performance of:

1. a trading strategy,
2. a portfolio, and
3. a portfolio relative to a benchmark index.

In this section, we will explain how scenario analysis can be used to assess potential performance.

A. Assessing Trades

A trade is evaluated in terms of its potential performance. When comparing possible trades, or a trade of a security with a current position, the relative performance of the alternatives must be assessed, where performance means the **expected total return** over the investment horizon. The total return is comprised of three sources:

1. the coupon payments
2. the change in the value of the bond
3. income from reinvestment of coupon payments and principal repayment (in the case of amortizing securities) from the time of receipt to the end of the investment horizon.

For example, suppose that an investor purchases a security for \$90 and expects a return over a 1-year investment horizon from the three sources equal to \$6. Then the expected total rate of return is 6.7%(= \$6/\$90).

When a trade involves borrowing, the interest cost of the borrowed funds is deducted from the dollar return from these three sources. (In Section VII we will discuss how funds can be borrowed in the bond market via a repurchase agreement.) The dollar return, adjusted for the financial cost, is then related to the dollar amount invested. For example, suppose the investor purchased the \$90 security by borrowing \$80 and investing \$10 of his own funds (i.e., the investor's equity). Suppose also that the cost of the borrowed funds is 5%, or \$4. Then the dollar return, after adjusting for the financing cost, is \$2 (\$6 - \$4). The total rate of return is 20% (\$2 return divided by the investor's equity of \$10).

1. Computing the Total Return The total return takes into account all three potential sources of dollar return over the investor's investment horizon. It is the rate of return that will grow the amount invested (i.e., price plus accrued interest) to the projected total dollar return at the investment horizon.⁴ The calculation of the total return requires that the investor specify:

- the investment horizon
- the reinvestment rate
- the price of the bond at the investment horizon.

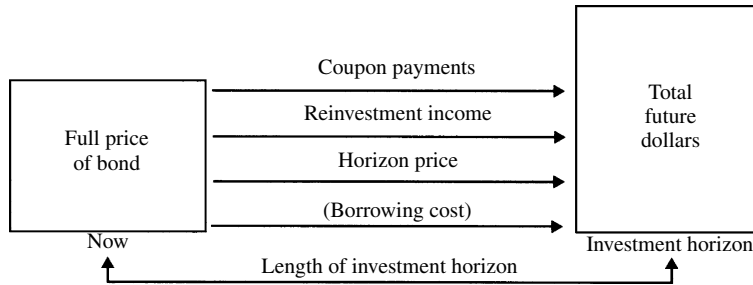
A graphical depiction of the total return calculation is presented in Exhibit 4. More formally, the steps for computing total return are as follows:

Step 1: Compute the total coupon payments plus the reinvestment income based on an expected reinvestment rate. If coupon payments are semiannual, the reinvestment rate is one-half the annual interest rate that the investor assumes can be earned on the reinvestment of coupon interest payments.⁵

⁴The total return is also referred to as the **horizon return**.

⁵An investor can use multiple reinvestment rates for cash flows from the bond over the investment horizon.

EXHIBIT 4 Graphical Depiction of Total Return Calculation



Total return is the interest rate that grows the price of the bond to the total future dollars

Step 2: Determine the projected price of the bond at the investment horizon, which is referred to as the **horizon price**. We explained how the price of a bond is computed given the term structure of default-free interest rates (i.e., the Treasury spot rate curve) and the term structure of credit spreads. Moreover, for bonds with embedded options, the price depends on the option-adjusted spread (OAS). So, to determine the horizon price it is necessary to use a forecasted Treasury spot rate curve, term structure of credit spreads, and OAS at the horizon date. Obviously, the predicted values reflect changes in interest rates and spreads from the current date to the investment horizon. We shall refer to these rates as the **structure of rates at the horizon date**.

However, in the illustrations to follow, we will simplify by assuming a single yield at the horizon date. This yield reflects the Treasury rate plus a spread and is referred to as the **horizon yield**.

Step 3: Add the values computed in Steps 1 and 2. Reduce this value by any borrowing cost to obtain the total future dollars that will be received from the investment, given the predicted reinvestment rate and projected structure of rates at the horizon date (or horizon yield in our illustrations to follow).

Step 4: Compute the *semiannual total return* as follows:

$$\left(\frac{\text{total future dollar}}{\text{full price of bond}} \right)^{1/h} - 1$$

where the full price is the price plus accrued interest and h is the *number of periods in the investment horizon*. For bonds with semiannual coupon payments, h is the number of semiannual periods.

Step 5: For semiannual-pay bonds, double the interest rate found in Step 4. The result is the total return expressed on a bond-equivalent basis. Alternatively, the total return can be computed on an effective rate basis, using the following formula:

$$(1 + \text{semiannual total return})^2 - 1$$

Whether the total return is calculated on a bond-equivalent basis or an effective rate basis depends on the situation. If the total return is compared to a benchmark index whose rate of return is calculated on a bond-equivalent basis, then the total return should be calculated in the same way. However, if the bond is used to satisfy liabilities with rates calculated on an effective basis, then the total return should be calculated on an effective rate basis.

To illustrate the computation of total return, suppose that an investor with a 1-year investment horizon is considering the purchase of a 20-year 6% corporate bond. The issue is selling for \$86.4365 at a yield of 7.3%, and will be purchased for cash (i.e., no funds will be borrowed). Assume that the yield curve is flat and the projected yield for a 20-year Treasury issue is 6.5%. This means that the yield spread over the projected Treasury issue is 80 basis points. The investor expects that:

1. he can reinvest the coupon payments at 6%
2. the Treasury yield curve will shift down by 25 basis points and remain flat at the end of 1 year, so that the yield for the 19-year Treasury issue is 6.25% (6.5% minus 25 basis points)
3. the yield spread to the 19-year Treasury issue is unchanged at 80 basis points, so the horizon yield is 7.05% (6.25% plus 80 basis points)

The calculations are shown below.

Step 1: Compute the total coupon payments plus reinvestment income using an annual reinvestment rate of 6%, or 3% every six months. The semiannual coupon payment is \$3. The future value of an annuity formula can be used for this calculation or, because the investment horizon is only one year, the future value can be computed as follows:

$$\begin{array}{rcl}
 \text{First coupon payment reinvested for six months} & = \$3 (1.03) & = \$3.09 \\
 \text{Second coupon payment} & & = \$3.00 \\
 \text{Total} & & = \underline{\$6.09}
 \end{array}$$

Step 2: The horizon price is determined as follows. The price of this bond when discounted at a flat 7.05% yield is \$89.0992.

Step 3: Adding the amounts in Steps 1 and 2 gives the total future dollars of \$95.1892.

Step 4: Compute the following (h is 2 in our illustration):

$$\left(\frac{\$95.1892}{\$86.4365} \right)^{1/2} - 1 = 4.94\%$$

Step 5: The total return on a bond-equivalent basis and on an effective rate basis are:

$$\begin{array}{ll}
 2 \times 4.94\% = 9.88\% & \text{(BEY)} \\
 (1 + 0.0494)^2 - 1 = 10.13\% & \text{(effective rate basis)}
 \end{array}$$

a. OAS-Total Return A valuation model can be used to incorporate the option-adjusted spread (OAS) into the calculation of the horizon price. The manager must specify the changes he expects in the OAS at the investment horizon. The horizon price can be “backed out” of a valuation model. This technique can be extended to the total return framework by making assumptions about the required variables at the horizon date.

Assumptions about the OAS at the investment horizon reflect the expectations of the portfolio manager. It is common to assume that the OAS at the horizon date will be the same as the OAS at the time of purchase. A total return calculated under this assumption is referred to as a **constant-OAS total return**. Alternatively, managers take positions that reflect their views on how the OAS will change. The total return framework can be used to assess the sensitivity of the performance of a bond with an embedded option to changes in the OAS.

b. Total Return for a Mortgage-Backed and Asset-Backed Security In calculating the total return for mortgage-backed and asset-backed securities, the total future dollars depend on (1) the projected principal repayment (scheduled repayment plus projected prepayments) and (2) the interest earned on reinvestment of projected interest payments and principal payments. To compute the total future dollars, a prepayment rate over the investment horizon must be projected.

The monthly total return for a mortgage-backed security or an asset-backed security with monthly payments is computed as follows:

$$\text{monthly total return} = \left(\frac{\text{total future dollars}}{\text{full price}} \right)^{\frac{1}{\text{number of months in horizon}}} - 1$$

The monthly total return can be annualized on a bond-equivalent yield basis as follows:

$$\text{bond-equivalent annual return} = 2[(1 + \text{monthly total return})^6 - 1]$$

Recall from our discussion that we calculate the bond-equivalent yield for a monthly pay security by first computing the effective 6-month yield and then doubling the effective 6-month yield. This is the calculation indicated by the bond-equivalent annual return formula above.

So, for example, if the monthly total return for a monthly pay mortgage-backed security or asset-backed security is 0.7%, the bond-equivalent annual return is

$$2[(1 + 0.007)^6 - 1] = 0.0855 = 8.55\%$$

Or, the effective annual return can be computed as follows:

$$\text{effective annual return} = (1 + \text{monthly total return})^{12} - 1$$

The effective annual return is computed by compounding the monthly return. For the previous example, the effective annual return is:

$$(1 + 0.007)^{12} - 1 = 0.0873 = 8.73\%$$

c. Scenario Analysis The computation of a total return is based on assumptions regarding interest rates expected at the investment horizon, spreads at the investment horizon, and reinvestment rates during the investment period. A manager does not rely on one set of assumptions when making an investment decision. Instead, a manager computes total return under different sets of assumptions. A set of assumptions is referred to as a **scenario**. Evaluating returns for a strategy under several scenarios is called **scenario analysis**. Regulators also require certain institutions to perform scenario analysis based on assumptions specified by regulations.⁶

Exhibits 5 and 6 provide illustrations of scenario analysis. The bond used in the illustrations is the 6% 20-year corporate bond selling for \$86.4365 at a yield of 7.3%. In both exhibits, the assumptions in the scenario analysis are that the yield curve is flat and that shifts in the yield curve are parallel shifts. In Exhibit 5, we assume that only the Treasury yield curve shifts. In Exhibit 6, we assume that the yield spread changes and that the change varies with shifts in the Treasury yield curve.

EXHIBIT 5 Scenario Analysis Assuming only the Treasury Yield Curve Changes (1-Year Investment Horizon)

	Scenario								
	1	2	3	4	5	6	7	8	9
At trade date									
Treasury rate	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%
Spread (bp)	80	80	80	80	80	80	80	80	80
Initial yield	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%
Coupon rate	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%
Maturity	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
Initial price	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365
At horizon date									
Treasury rate change (bp)	−150	−100	−50	−25	0	25	50	100	150
Spread change (bp)	0	0	0	0	0	0	0	0	0
Horizon yield	5.80%	6.30%	6.80%	7.05%	7.30%	7.55%	7.80%	8.30%	8.80%
Coupon rate	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%
Remaining maturity	19.0	19.0	19.0	19.0	19.0	19.0	19.0	19.0	19.0
Horizon price	102.2846	96.7035	91.5375	89.0992	86.7520	84.4920	82.3155	78.1993	74.3770
Reinvestment rate	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%
Interest + reinvest inc	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09
Total future dollars	108.3746	102.7935	97.6275	95.1892	92.8420	90.5820	88.4055	84.2893	80.4670
Total return (SA)	11.97%	9.05%	6.28%	4.94%	3.64%	2.37%	1.13%	−1.25%	−3.51%
Total return (BEY)	23.95%	18.10%	12.55%	9.88%	7.28%	4.74%	2.27%	−2.50%	−7.03%
Total return (effective)	25.38%	18.92%	12.95%	10.13%	7.41%	4.80%	2.28%	−2.48%	−6.91%

⁶Some broker/dealers, vendors of analytical systems, and regulators refer to scenario analysis as “simulation” even though the two techniques are not equivalent. Simulation is a more powerful tool that takes into consideration the dynamics of interactions of the factors. For a discussion of Monte Carlo simulation applied to common stock portfolio management, see Chapter 5 in Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, and David E. Runkle, *Quantitative Methods for Investment Analysis* (Charlottesville, VA: Association for Investment Management and Research, 2001), pp. 261–266.

EXHIBIT 6 Scenario Analysis Assuming Shift in Treasury Yield Curve and Change in Yield Spread (1-Year Investment Horizon)

	Scenario								
	1	2	3	4	5	6	7	8	9
At trade date									
Treasury rate	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%
Spread (bp)	80	80	80	80	80	80	80	80	80
Required yield	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%	7.30%
Coupon rate	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%
Maturity	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
Horizon price	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365	86.4365
At horizon date									
Treasury rate change (bp)	−150	−100	−50	−25	0	25	50	100	150
Spread change (bp)	40	25	20	10	0	−10	−20	−25	−40
Horizon yield	6.20%	6.55%	7.00%	7.15%	7.30%	7.45%	7.60%	8.05%	8.40%
Coupon rate	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%
Remaining maturity	19.0	19.0	19.0	19.0	19.0	19.0	19.0	19.0	19.0
Horizon price	97.7853	94.0708	89.5795	88.1496	86.7520	85.3858	84.0501	80.2191	77.4121
Reinvestment rate	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%
Interest + reinvest inc	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09
Total future dollars	103.8753	100.1608	95.6695	94.2396	92.8420	91.4758	90.1401	86.3091	83.5021
Total return (SA)	9.62%	7.65%	5.21%	4.42%	3.64%	2.87%	2.12%	−0.07%	−1.71%
Total return (BEY)	19.25%	15.29%	10.41%	8.83%	7.28%	5.75%	4.24%	−0.15%	−3.42%
Total return (effective)	20.18%	15.88%	10.68%	9.03%	7.41%	5.83%	4.28%	−0.15%	−3.39%

2. Controlling for Interest Rate Risk in Trades When assessing strategies, it is critical to compare positions with the same dollar duration, unless the objective of a trade is to alter the duration. To understand why this is so, consider two bonds, X and Y. Suppose that the price of bond X is 80 and the duration is 5, while bond Y has a price of 90 and a duration of 4. A 100 basis point change in the yield of bond X would change its price by about 5%, or a change of about \$4 per \$80 of market value. Thus, the dollar duration for a 100 basis point change in yield is \$4 per \$80 of market value. Similarly, for bond Y, the dollar duration for a 100 basis point change in yield per \$90 of market value is \$3.60. So, if bonds X and Y are being considered as alternative investments, the amount of each bond in the strategy should be such that both have the same *dollar* duration.

To illustrate, suppose that a portfolio manager owns \$10 million par value of bond X which has a market value of \$8 million. The dollar duration of bond X per 100 basis point change in yield is \$400,000. Suppose further that the portfolio manager is considering exchanging bond X, which it owns, for bond Y. If the portfolio manager wants to have the same interest rate exposure (i.e., dollar duration) for bond Y that she currently has for bond X, she will buy a market value amount of bond Y that has the same dollar duration. If the portfolio manager purchased \$10 million par value of bond Y, and therefore \$9 million of market value, the dollar price change per 100 basis point change in yield would be only \$360,000. If, instead, the portfolio manager purchased \$10 million market value of bond Y, the dollar duration per 100 basis point change in yield would be \$400,000. Since bond Y is trading at 90, \$11.11 million par value of bond Y must be purchased to keep the dollar duration of bond Y equal to that of bond X.

Mathematically, the market value of bond Y that is required in order to have the same dollar duration (per 100 basis point change in rates) as bond X is:

$$\text{market value of bond Y} = \frac{\text{dollar duration of bond X}}{\text{duration of bond Y}/100}$$

The par value of bond Y that must be purchased to obtain the same dollar duration as bond X is:

$$\text{par value of bond Y} = \frac{\text{market value of bond Y}}{\text{price of bond Y per \$1 of par value}}$$

Using our previous illustration to demonstrate how to use these formulas, we know that:

$$\text{dollar duration of bond X} = \$400,000$$

$$\text{duration of bond Y} = 4$$

$$\text{market value of bond Y} = \frac{\$400,000}{4/100} = \$10,000,000$$

This means that \$10 million *market* value of bond Y has the same dollar duration as the \$8 million market value position in bond X. The amount of par value of bond Y that must be purchased, given its price of 90, or 0.90 per \$1 of par value, is:

$$\text{par value of bond Y} = \frac{\$10,000,000}{0.90} = \$11.11 \text{ million}$$

Failure to adjust a trade to account for an expected change in yield spread, keeping the dollar duration constant, leads to the result that the outcome of the trade will be affected by the expected change in the yield spread and the change in the yield level. Thus, a manager would be taking a conscious yield spread view and possibly an undesired view on the level of interest rates.

Also note that equating the dollar durations of two positions only means that they will be equal for small changes in rates because of the convexity of a bond.

3. Assessing the Portfolio There is no shortage of trading strategies suggested by bond dealer firms or in the popular press. Investment management firms have developed proprietary trading strategies. All of these strategies are based on a set of expectations about the bond market over the investment horizon. Furthermore, these strategies all involve risk. A trading strategy might involve borrowing in the repo market and/or shorting bonds. Some managers and dealers incorrectly use the term “arbitrage” to refer to these trading strategies they tout to customers. In fact, such strategies incur risk, even though the proponent of the strategy might perceive the risk to be small.

The potential performance of any trading strategy can be quantified using total return analysis. More specifically, scenario analysis is used to determine the total return under different assumptions about what might occur over the investment horizon. Scenario analysis identifies the range of possible outcomes and therefore provides the manager with a feel for the risk associated with a trade.

In this section, we use a basic illustration illustrate how a trade can be evaluated. We begin with three Treasury securities—A, B, and C. Information about each of these three

EXHIBIT 7 Three Hypothetical Treasury Securities

Information on three Treasury securities:

Treasury issue	Coupon rate (%)	Price	Yield to maturity (%)	Maturity (years)
A	6.5	100	6.5	5
B	8.0	100	8.0	20
C	7.5	100	7.5	10

Calculation of duration and convexity (shock rates by 10 basis points):

Treasury issue	Value if rate changes by		Duration	Convexity
	+10 bp	−10 bp		
A	99.5799	100.4222	4.21122	10.67912
B	99.0177	100.9970	9.89681	73.63737
C	99.3083	100.6979	6.94821	31.09724

securities is provided in Exhibit 7. Security A is a short-term Treasury, security B is a long-term Treasury, and security C is an intermediate-term Treasury. Each security is selling at par, and it is assumed that the next coupon payment is six months from now. The duration and convexity for each security are calculated in the exhibit. Since all three securities are trading at par value, the respective durations and convexities are equal to the dollar durations and dollar convexities per \$100 of par value.

Suppose that two Treasury portfolios are constructed. The first portfolio consists entirely of security C, the 10-year issue, and shall be referred to as the **bullet portfolio**, because the principal is returned at one time—the maturity date of the 10-year issue. The second portfolio invested 51.86% in security A and 48.14% in security B. This portfolio shall be referred to as the **barbell portfolio**, because the maturity dates are both shorter than and longer than that of the bullet portfolio.

As shown in Exhibit 7, the duration of the bullet portfolio is 6.94821. The duration of the barbell portfolio is the market value weighted average of the durations of the securities in the portfolio, and is computed below:

$$0.5186 (4.21122) + 0.4814 (9.89681) = 6.94826$$

The duration of the barbell is equal to the duration of the bullet. In fact, the barbell portfolio was designed to produce this result.

Duration is a first approximation of the change in market value resulting from a change in interest rates. As explained, the convexity measure provides an improvement to the duration estimate. The convexity measures of the bullet and barbell portfolios are not equal. We discussed the issues associated with computing the convexity measure previously so we won't repeat the discussion here. The important concept to understand regarding the convexity measure in Exhibit 7 is the relative size of the convexity measures for the two portfolios. The convexity of the bullet portfolio is 31.09724. The convexity of the barbell is a market weighted average of the convexities of the securities in the portfolio. That is:

$$0.5186 (10.67912) + 0.4814 (73.63737) = 40.98722$$

Thus, the convexity of the bullet portfolio is less than that of the barbell portfolio. Below is a summary of the duration and convexity measures of the two portfolios:

Parameter	Treasury portfolio	
	Bullet	Barbell
Dollar duration	6.94821	6.94826
Dollar convexity	31.09724	40.98722

Suppose that a manager is considering the following trade: buy one portfolio and sell the other. In this case, selling a portfolio means that the manager **sells short**, or **shorts**, the security or securities in the portfolio. When an investor shorts a bond, the coupon interest paid to holders of the bond must be paid by the investor to the owner of the bond. Also assume that the manager is basing the trade on a 6-month investment horizon.

Since both portfolios have the same duration and the same dollar amount invested in each portfolio, then the portfolios have the same dollar duration, but different convexity. Suppose the manager believes that there will be significant interest rate volatility over the next six months. It is precisely under such circumstances that a portfolio with higher convexity experiences better performance. Consequently, suppose that the manager decides to buy the barbell portfolio because it has higher convexity, and short the bullet portfolio.

Let's assess this trade for the manager. First, note that the statement that a "large" change is expected is vague. Can the manager be more specific about how much rates must change in order to benefit from the better convexity of the barbell portfolio relative to the bullet portfolio? Also, is there an implicit assumption about what happens to the shape of the yield curve at the investment horizon? This can be quantified by using total return analysis and scenario analysis.

The last column of Exhibit 8 shows the total return over a six-month period for this trading strategy, assuming that the yield curve shifts in a parallel fashion. Since the manager

EXHIBIT 8 Performance of Trading Strategy Over a 6-Month Horizon Assuming a Parallel Yield Curve Shift: Scenario Analysis

Yield change (in bp)	Price plus coupon (\$)			Total return (%)		
	A	B	C	Barbell	Bullet	Trading strategy*
−300	115.6407	141.0955	126.7343	55.79	53.47	2.32
−250	113.4528	133.6753	122.4736	46.38	44.95	1.43
−200	111.3157	126.8082	118.3960	37.55	36.79	0.76
−150	109.2281	120.4477	114.4928	29.26	28.99	0.27
−100	107.1888	114.5512	110.7559	21.47	21.51	−0.05
−50	105.1965	109.0804	107.1775	14.13	14.35	−0.22
−25	104.2176	106.4935	105.4453	10.63	10.89	−0.26
0	103.2500	104.0000	103.7500	7.22	7.50	−0.28
25	102.2935	101.5961	102.0907	3.92	4.18	−0.27
50	101.3481	99.2780	100.4665	0.70	0.93	−0.23
100	99.4896	94.8852	97.3203	−5.45	−5.36	−0.09
150	97.6735	90.7949	94.3050	−11.28	−11.39	0.11
200	95.8987	86.9830	91.4146	−16.79	−17.17	0.38
250	94.1640	83.4271	88.6433	−22.01	−22.71	0.70
300	92.4686	80.1070	85.9857	−26.96	−28.03	1.06

*A negative sign indicates that the bullet portfolio outperformed the barbell portfolio; a positive sign indicates that the barbell portfolio outperformed the bullet portfolio.

owns the barbell portfolio and is short the bullet portfolio, then the difference between the total returns in columns (5) and (6) is the total return for this trading strategy. For example, if the yield curve shifts down by 150 basis points, the barbell portfolio would earn a 6-month total return of 29.26%. If the manager owned the bullet portfolio, then the total return would be 28.99% for the 150 basis point decline in yield. However, the manager is short the bullet portfolio so he must pay 28.99%. Thus, the barbell portfolio earned 29.26% but the manager had to pay 28.99%, resulting in a 27 basis point 6-month total return for this trading strategy.

The total return in the various scenarios shown in the last column allows the manager to quantify the size of the yield curve shift required in order to realize a positive return. The yield curve must change in a parallel fashion by more than 100 basis points (the precise number of basis points is not shown in Exhibit 8) in order to benefit from the better convexity of the barbell portfolio. Thus, the manager now knows more than simply the fact that he is taking a view on a “large” rate change, but, more specifically, a view that rates will change by more than 100 basis points.

Exhibit 9 shows the return for this strategy if the yield curve does not shift in a parallel fashion. In computing the total return for the trading strategy in Exhibit 9, we assume a steepening of the yield curve. There are an infinite number of ways that the yield curve can steepen. The scenario analysis in Exhibit 9 assumes that, for the change in yield for security C shown in the first column, the yield of A will change by the same amount less 30 basis points, whereas the yield of B will change by the same amount plus 30 basis points. The last column shows that the trade results in a loss for all scenarios except a 300 basis point shift.

Thus, we see the power of total return analysis and scenario analysis to sharpen our skills in assessing a trading strategy.

EXHIBIT 9 Performance of Trading Strategy Over a 6-Month Horizon Assuming a Steepening of the Yield Curve: Scenario Analysis

Yield change for C (in bp)	Price plus coupon (\$)			Total return (%)		
	A	B	C	Barbell	Bullet	Trading strategy*
−300	116.9785	136.5743	126.7343	52.82	53.47	−0.65
−250	114.7594	129.4918	122.4736	43.70	44.95	−1.24
−200	112.5919	122.9339	118.3960	35.14	36.79	−1.65
−150	110.4748	116.8567	114.4928	27.09	28.99	−1.89
−100	108.4067	111.2200	110.7559	19.52	21.51	−1.99
−50	106.3863	105.9874	107.1775	12.39	14.35	−1.97
−25	105.3937	103.5122	105.4453	8.98	10.89	−1.91
0	104.4125	101.1257	103.7500	5.66	7.50	−1.84
25	103.4426	98.8243	102.0907	2.44	4.18	−1.74
50	102.4839	96.6046	100.4665	−0.69	0.93	−1.63
100	100.5995	92.3963	97.3203	−6.70	−5.36	−1.34
150	98.7582	88.4758	94.3050	−12.38	−11.39	−0.99
200	96.9587	84.8200	91.4146	−17.77	−17.17	−0.60
250	95.2000	81.4080	88.6433	−22.88	−22.71	−0.17
300	93.4812	78.2204	85.9857	−27.73	−28.03	0.30

Assumptions:

Change in yield of A = Change in yield of C minus 30 bp.

Change in yield of B = Change in yield of C plus 30 bp.

*A negative sign indicates that the bullet portfolio outperforms the barbell portfolio; a positive sign indicates that the barbell portfolio outperforms the bullet portfolio.

B. Assessing Strategies Relative to a Benchmark Index

One way to assess the strategic bets a manager makes is to examine how the portfolio is expected to perform relative to the benchmark index under various scenarios. That is, scenario analysis can be helpful and, again, performance is measured in terms of total return.

To illustrate how this is done, let's consider once again the 45-bond portfolio described in the previous chapter to demonstrate the risk profile of a portfolio and benchmark index and the risk factors. (See Exhibit 3 in Chapter 17 for the portfolio composition.) The analysis was performed as of November 5, 2001, based on prices of October 31, 2001. The benchmark is the Lehman Aggregate Bond Index.

The duration and convexity for the 45-bond portfolio and for the benchmark index are:

	Duration	Convexity
Portfolio	4.59	0.13
Benchmark	4.10	-0.26

The value of the 45-bond portfolio has greater sensitivity to a parallel shift in interest rates than does the benchmark index. The 45-bond portfolio exhibits positive convexity while the benchmark index exhibits negative convexity. Consequently, the manager should expect that for a large decline in interest rates, the 45-bond portfolio will outperform the benchmark index by an amount greater than the duration would indicate.

Now let's do some simple scenario analysis to assess the performance of both the 45-bond portfolio and the benchmark index. The scenario analysis assumes a 1-year investment horizon and a parallel shift in interest rates. The results are shown in Exhibit 10, which reports the 1-year total return of both the portfolio and the benchmark index based on parallel shifts in the yield curve of 0, ± 50 , ± 100 , ± 150 , ± 200 , ± 250 , and ± 300 basis points. Notice that the total return profile is as expected, given the duration and the convexity. Because the duration of the portfolio is greater than the duration of the benchmark index, the portfolio outperforms the benchmark when interest rates decline and underperforms when interest rates rise. Moreover, the positive convexity of the portfolio and the negative convexity of the benchmark imply that the better performance of the portfolio is further enhanced for a large decline in interest rates.

EXHIBIT 10 Scenario Analysis to Compare Performance (Total Return %) of Portfolio to Benchmark Index Assuming a Parallel Shift in Interest Rates

Change in basis points	45-Bond portfolio	Benchmark index	Difference in bps
300	-12.354	-12.081	-27.30
250	-10.374	-10.054	-32.00
200	-8.283	-7.947	-33.60
150	-6.121	-5.822	-29.90
100	-3.898	-3.700	-19.80
50	-1.606	-1.575	-3.10
0	0.744	0.523	22.10
-50	3.157	2.589	56.80
-100	5.672	4.659	101.30
-150	8.335	6.791	154.40
-200	11.181	9.006	217.50
-250	14.237	11.284	295.30
-300	17.592	13.779	381.30

The scenario analysis framework in Exhibit 10 is useful for giving the portfolio manager a feel for the relationship between the performance of the portfolio relative to the benchmark index. However, this analysis considers only one risk: interest rate risk. In addition, the analysis of interest rate risk is limited to a parallel shift in the yield curve so that a nonparallel shift is not considered. Of course, a portfolio manager can “play” with different yield curve shifts to assess the impact on the portfolio and benchmark index. In fact, some portfolio managers seek to identify “realistic yield curve shifts” and analyze a portfolio based on these shifts. The simple scenario analysis of Exhibit 10 also ignores changes in spreads for spread products. Again, one can attempt to assess the performance assuming different changes in spreads. However, a more insightful methodology for quantifying the risk exposure of a portfolio relative to a benchmark index was discussed in the previous chapter—a multi-factor risk model.

V. USING MULTI-FACTOR RISK MODELS IN PORTFOLIO CONSTRUCTION

In the previous chapter we explained how multi-factor risk models can be used to quantify the risk exposure of a portfolio. But a multi-factor risk model is also an invaluable tool for constructing a portfolio, where the term “constructing a portfolio” means either the initial construction or the rebalancing of a portfolio.

In constructing a portfolio using the multi-factor risk model, an optimizer is used. The purpose of an optimizer is to obtain the optimal value for an objective function subject to constraints. Optimizers use mathematical programming, but a discussion of the types of mathematical programming and the algorithms used to solve mathematical programming problems is beyond the scope of this chapter. It is important to understand that, if a portfolio manager specifies an objective, the optimizer or mathematical programming model will compute the optimal solution. We will give two illustrations to see how this is done. In our illustrations, we will use the 45-bond portfolio and continue to use the Lehman U.S. Aggregate Bond Index as the benchmark index.

A. Rebalancing to Construct a More Passive Portfolio

In the previous chapter, we saw that the predicted tracking error for the 45-bond portfolio is 62 basis points per year. Suppose that the portfolio manager wants to reduce the predicted tracking error significantly. That is, the manager wants to rebalance the portfolio in such a manner that its risk exposure is much closer to that of the benchmark index. Depending on the extent to which the manager reduces the predicted tracking error, the manager shifts from actively managing a portfolio to an approach that more closely resembles enhanced indexing. The goal is to rebalance the portfolio in a cost efficient manner. Here is where the optimizer comes in.

An optimizer can be used to identify trades that reduce tracking error in a cost efficient way. Specifically, the objective specified for the optimizer is to minimize tracking error, the constraint is to keep the turnover of the portfolio to a minimum. The optimizer begins with a universe of market prices for acceptable securities. The acceptable securities are those permitted by the investment guidelines. The optimizer is designed to select a trade that identifies a 1-for-1 bond swap with the greatest reduction in tracking error per unit of each bond purchased.

When the optimizer was applied to the 45-bond portfolio, the following bond swap was identified:

Bond swap #1:

Sell:	\$61,567,000 of Coca-Cola Enterprises I	6.950%	11/15/2026
Buy:	\$66,392,000 of Federal Natl Mtg Assn-Global	4.375%	10/15/2006

The trades are stated here in terms of par value, but the cash values for each side of the trade are approximately equal. If this bond swap is undertaken, predicted tracking error for the portfolio would be reduced from 62 basis points to 43 basis points.

The next eight bond swaps identified by the optimizer, along with the resulting predicted tracking error after each trade, are shown below. The trades' impact on the predicted tracking error is cumulative. While the cost of each transaction is not shown, the portfolio manager has the opportunity to compare the reduction in predicted tracking error with the cost of the trade.

Bond swap #2:

Sell:	\$18,600,000 of U.S. Treasury Notes	7.500%	11/15/2001
Buy:	\$43,876,000 of General Motors Acpt Corp	0.000	12/01/2012
New predicted tracking error for portfolio after bond swap:			38 basis points

Bond swap #3:

Sell:	\$26,906,000 of Coca-Cola Enterprises I	6.950	11/15/2026
Buy:	\$29,567,000 of FNMA Conventional Long T.	5.500	2/01/2031
New predicted tracking error for portfolio after bond swap:			34 basis points

Bond swap #4:

Sell:	\$14,407,000 of Daimler-Benz North America	6.670	2/15/2002
Buy:	\$16,324,000 of Pacific Bell	6.625	10/15/2034
New predicted tracking error for portfolio after bond swap:			31 basis points

Bond swap #5:

Sell:	\$19,331,000 of US Treasury Bonds	6.250	8/15/2023
Buy:	\$20,974,000 of Tennessee Valley Authority-Global	5.375	11/13/2008
New predicted tracking error for portfolio after bond swap:			28 basis points

Bond swap #6:

Sell:	\$10,318,000 of Daimler-Benz North America	6.670	2/15/2002
Buy:	\$8,844,000 of Tennessee Valley Authority-Global	7.125	5/1/2030
New predicted tracking error for portfolio after bond swap:			26 basis points

Bond swap #7:

Sell:	\$12,569,000 of US Treasury Bonds	6.250	8/15/2023
Buy:	\$13,659,000 of Canadian Government-Global		11/5/2008
New predicted tracking error for portfolio after bond swap:			24 basis points

Bond swap #8:

Sell:	\$6,554,000 of Daimler-Benz North America	6.670	2/15/2002
Buy:	\$5,618,000 of Tennessee Valley Authority-Global	7.125	5/1/2030
New predicted tracking error for portfolio after bond swap:			23 basis points

Bond swap #9:

Sell:	\$12,788,000 of Raytheon Co.	7.200	8/15/2027
Buy:	\$12,878,000 of Intl Business Machines	5.375	2/1/2009
New predicted tracking error for portfolio after bond swap:			22 basis points

After these nine bond swaps, the portfolio's predicted tracking error would be reduced from 62 basis points to 22 basis points. The total transaction cost of these swaps would have been \$1 million.

The same optimization approach described above can be used to rebalance an indexed portfolio as the index changes.

B. Incorporating Active Strategies

We have shown how a multi-factor risk model and an optimizer can be used to rebalance a portfolio in order to reduce predicted tracking error. An optimizer can also be used to structure a portfolio so as to incorporate a market view (on sectors or on the term structure) or a view on individual issues or issuers. To see how, let's suppose that a manager follows a top down approach to the selection of sectors. Consequently, the manager seeks exposure to sector risk, while at the same time, seeking to minimize exposure to other risk factors such as term structure risk and non-sector non-term structure systematic risk. How can this be done?

An optimizer can be used to rebalance the current portfolio in a manner that maintains the current predicted tracking error for sector risk but reduces the predicted tracking error for the other systematic risks. The optimizer is set up so that there is a substantial penalty for reducing exposure to sector risk.

For example, consider the 45-bond portfolio. As discussed in the previous chapter, the predicted tracking error for the portfolio is 62 basis points and the predicted tracking error due to sector risk is 22.7 basis points. The optimizer was run to keep the predicted tracking error due to sector risk as close as possible to 22.7 basis points while reducing the predicted tracking error due to the other risk factors. The following nine bond swaps were identified by the optimizer:

Bond Swap # 1:

Sell:	\$569,800 of US Treasury Bonds	6.250	8/15/2023
Buy:	\$755,000 of AT&T Corp—Global	6.500	3/15/2029
New predicted tracking error for portfolio after bond swap:		61.4 basis points	

Bond Swap # 2:

Sell:	\$580,400 of Time Warner Ent	8.375	3/15/2023
Buy:	\$634,400 of Tennessee Valley Auth	5.880	4/01/2036
New predicted tracking error for portfolio after bond swap:		59.3 basis points	

Bond Swap # 3:

Sell:	\$382,100 of US Treasury Bonds	6.250	8/15/2023
Buy:	\$440,200 of Burlington Resources Inc	7.200	8/15/2031
New predicted tracking error for portfolio after bond swap:		59.3 basis points	

Bond Swap # 4:

Sell:	\$561,500 of Coca-Cola Enterprises I	6.950	11/15/2026
Buy:	\$565,000 of Italy, Republic of-Global	6.000	2/22/2011
New predicted tracking error for portfolio after bond swap:		57.0 basis points	

Bond Swap # 5:

Sell:	\$490,700 of US Treasury Bonds	6.250	8/15/2023
Buy:	\$559,500 of Motorola Inc	8.000	11/01/2011
New predicted tracking error for portfolio after bond swap:		56.2 basis points	

Bond Swap # 6:

Sell: \$490,000 of AT&T Corp—Global 6.500 3/15/2029
 Buy: \$408,700 of Tennessee Valley Auth 5.880 4/01/2036
 New predicted tracking error for portfolio after bond swap: 55.2 basis points

Bond Swap # 7:

Sell: \$548,400 of US Treasury Bonds 6.250 8/15/2023
 Buy: \$651,600 of AT&T Corp—Global 6.000 3/15/2009
 New predicted tracking error for portfolio after bond swap: 54.2 basis points

Bond Swap # 8:

Sell: \$177,600 of Norfolk Southern Corp 7.800 5/15/2027
 Buy: \$178,900 of US Treasury Notes 6.500 10/15/2006
 New predicted tracking error for portfolio after bond swap: 53.7 basis points

Bond Swap # 9:

Sell: \$265,000 of AT&T Corp - Global 6.500 3/15/2029
 Buy: \$222,600 of Bristol-Myers Squibb-Global 5.750 10/1/2011
 New predicted tracking error for portfolio after bond swap: 53.4 basis points

After these bond swaps, the predicted tracking error for the portfolio declines from 62 basis points to 53 basis points. An analysis of the predicted tracking error risks for the original 45-bond portfolio and for the new portfolio is shown in Exhibit 11. Note first that the predicted tracking error due to sector risk was effectively unchanged (an increase of a mere 0.1 basis points). The major risk eliminated is the term structure risk. The predicted tracking error attributable to this source is reduced by 11 basis points. So, while the tracking error due to sector risk is unchanged, the portfolio's predicted tracking error is reduced from 62 basis points to 53 basis, effectively attributable to a reduction in term structure risk.

VI. PERFORMANCE EVALUATION

Thus far in this chapter we have analyzed various portfolio strategies. In this section we turn our attention to the measurement and evaluation of the performance of a bond portfolio

EXHIBIT 11 Predicted Tracking Error Before and After Nine Bond Swaps Seeking to Hold Tracking Error Due to Sector Risk Constant

Predicted tracking error due to	Predicted tracking error (in bps)		Change in predicted tracking error (in bps)
	Original	After bond swaps	
<i>Systematic risks</i>			
Term structure risk	49.8	38.4	−11.4
Sector risk	22.7	22.8	0.1
Quality risk	10.7	10.4	−0.3
Optionality risk	1.3	2.5	1.2
Coupon risk	1.4	1.3	−0.1
MBS sector risk	9.3	9.3	0
MBS volatility risk	8.3	8.3	0
MBS prepayment risk	8.8	8.8	0
<i>Nonsystematic risks</i>	28.4	26.9	
Total risk	62.0	53.4	

manager. Performance measurement requires the calculation of realized return over a specified time interval, called the **evaluation period**.

Performance evaluation is concerned with two issues. The first is whether the manager added value by outperforming the designated benchmark index. The second is how the manager achieved the realized return. The decomposition of performance results used in order to understand how the results were achieved is called **performance attribution analysis**.

A bond performance and attribution analysis should satisfy three basic requirements.⁷ First, the process should be accurate. For example, as explained below, there are several ways to measure portfolio return. The measure that is used should recognize the timing of each cash flow, resulting in a much more accurate measure of the actual portfolio performance.

Second, the process should be informative. It should measure the managerial skills that go into bond portfolio management. To be informative, the process must effectively address the key management skills, and measure these skills in terms of realized performance. For example, the process should identify the degree to which performance is attributed to changes in the level of interest rates, changes in the shape of the yield curve, changes in spreads, and individual security selection. The effect of transaction costs on performance should also be recognized.

Third, the process should be simple. The output of the process should be understood by manager and client, or others concerned with the performance of the portfolio.

While the process must provide accurate information, this does not mean that analysis needs to be accurate to the n -th decimal place. Rather, it should provide the client with an accurate picture of the major areas, as well as order of magnitude, of outperformance or underperformance. A client engages a manager, at least in part because of claims made by the manager regarding the strategies that will be pursued. Information that indicates whether a manager's performance is consistent with these claims is critical in the decision to retain or discharge the manager.

A. Performance Measurement

The starting point for evaluating a manager's performance is the measurement of return. Essentially the measurement process begins with the calculation of total return for the sub-periods of the evaluation period. For example, if the evaluation period is one year, the sub-period might be a month, so that there would be 12 sub-period returns. Three methodologies that can be used to calculate the average sub-period return in order to compute the return for the evaluation period: (1) the arithmetic average rate of return, (2) the time-weighted rate of return (also called the geometric rate of return), and (3) the dollar-weighted rate of return. These methodologies and their limitations are discussed elsewhere.⁸

Because rate of return is influenced by the client's decisions with respect to the periodic withdrawal and contribution of funds, a return measure should be reflective of the manager's performance without being affected by client decisions to add or take away funds. The proper methodology to resolve this problem is the time-weighted rate of return. The CFA Institute Performance Presentation Standards require the use of time-weighted rates of return that

⁷Frank J. Fabozzi and Gifford Fong, *Advanced Fixed Income Portfolio Management* (Chicago, IL: Probus Publishing, 1994), p. 281.

⁸For an explanation and illustration of dollar-weighted rate of return and time-weighted rate of return, see Chapter 2 of DeFusco, McLeavey, Pinto, and Runkle, *Quantitative Methods for Investment Analysis*, pp. 81–90.

adjust for cash flows (i.e., contributions and withdrawals). Time-weighted rates of return that adjust for daily-weighted cash flows must be used for periods beginning January 1, 2005 and later.

B. Single-Index Performance Evaluation Measures

In the 1960s, several single-index measures were used to evaluate the relative performance of portfolio managers. The three measures are referred to as the Treynor measure, the Sharpe measure, and the Jensen measure. All three measures assume that there is a linear relationship between the portfolio's return and the return on the benchmark index.⁹ These measures did not indicate how or why a portfolio manager outperformed or underperformed a benchmark index.

In early studies of portfolio manager performance, these measures were used primarily to evaluate the performance of equity mutual fund managers. However, these measures are less useful today because of the subsequent development of the performance attribution models discussed next.

C. Performance Attribution Analysis

Single-index performance evaluation measures do not explain how a manager achieved a particular observed return. The reason a client must have the answer to this question is that a manager may tell a client that he plans to pursue an active strategy. The client would then expect that any superior return achieved by the manager results from this active strategy. But how can the client be certain?

For example, suppose a manager who solicits funds from a client claims that he can achieve superior performance by selecting underpriced bonds. Suppose also that this manager generates a superior return compared to the client-designated bond index. The client should not necessarily be satisfied with this performance until the return realized by the manager is decomposed into the components that generated the return. A client may find that the superior performance is due to the manager's timing of the market (i.e., modifying portfolio duration in anticipation of interest rate movements) rather than due to selecting underpriced bonds. In such an instance, the manager may have outperformed the benchmark index, but not by following the strategy that the manager stated that he intended to pursue.

Performance attribution analysis seeks to identify and quantify the active management decisions that contributed to the performance of a portfolio. The performance of a portfolio can be decomposed in terms of the risk factors discussed in Chapter 17.

Several commercial vendors have developed models for performance attribution analysis. The analysis is performed relative to a benchmark index. We will illustrate performance attribution models using the system developed by Global Advanced Technology (G.A.T.).¹⁰ This model decomposes a portfolio's total return into the following factors: (1) static return, (2) interest sensitive return, (3) spread change return, and (4) trading return. The difference between the total return and the sum of the four factors is called the **residual (error)**. Each of the return factors can be further decomposed as described below.

⁹These measures are described in Frank K. Reilly and Keith C. Brown, *Investment Analysis and Portfolio Management* (South-Western College Publishing, 2002), pp. 1109–1117.

¹⁰The illustration is provided by Frank Jones and Leonard Peltzman of Guardian Life. A model developed by another vendor is presented in one of the end of chapter questions.

The **static return** is the portion of a portfolio's total return that is attributable to "rolling down the yield curve." That is, the static return is the portion of the return that would be earned assuming a static (meaning zero volatility) world, in which the yield curve evolves to its implied forward curve. The static return is further decomposed into two components: (1) **risk-free return** and (2) **accrual of OAS return**. The risk-free return is based on the assumption that the portfolio consists of only Treasury strips. The risk-free return is then calculated based on the rolling down of the yield curve. The accrual of OAS return is also calculated from rolling down the yield curve, but, unlike the risk-free return, it is based on investing in spread products.

The **interest sensitive return** is that portion of a portfolio's return attributable to changes in the level, slope, and shape of the yield curve. In turn, this return is decomposed into two components: (1) effective duration return and (2) convexity return. Key rate durations can be used to measure sensitivity to changes in the shape of the yield curve. The **effective duration return** is the sum of the returns attributable to the key rate durations. The **convexity return** is the return due to change in the portfolio's duration over the evaluation period.

The **spread change return** is the portion of a portfolio's return that is due to changes in both (1) bond sector spreads and (2) individual security richness/cheapness. The portion of the spread return attributable to changes in the sector's OAS is called the **delta OAS return** and the spread return due to a widening or tightening of a specific issue's spread is called the **delta rich/cheap return**.

The portion of a portfolio's total return that is attributable to changes in the composition of the portfolio is called the **trading return**. The trading return identifies the manager's value added from changes in the composition of the portfolio, as opposed to a simple buy-and-hold strategy.

In the illustration, a performance attribution analysis is performed on a portfolio of corporate securities. We will refer to this portfolio as Portfolio A. The evaluation period is September 1996, a month when the yield curve shifted downward. The shift was almost a parallel shift. The effective duration of Portfolio A was 7.09. The effective duration of the benchmark index, the Merrill Lynch Corporate Index, was 5.76. Thus, Portfolio A had a higher duration than did the benchmark index.

Exhibit 12 presents the results of the performance attribution analysis for the portfolio and for the individual sectors. The second column provides information about the manager's sector views (i.e., underweighting or overweighting). The third column shows how the allocation paid off for each sector.

Because the yield curve shifted downward in an almost parallel fashion, Portfolio A would be expected to outperform the benchmark index because of the portfolio's higher duration. This is captured in the interest sensitive return, the fifth column of Exhibit 12. This indicates that, holding all other factors constant, the outperformance would have been 37.9 basis points. Portfolio A did not outperform by that much because the spread change return which was -18.5 basis points. The static return and the trading return were minimal.

Exhibit 13 provides a summary of the analysis. The return on Portfolio A was 2.187% and the return on the benchmark index was 1.954%, so that Portfolio A outperformed the index by 23 bps in September 1996.

VII. LEVERAGING STRATEGIES

A manager may be permitted to use leverage as part of a trade or trading strategy. Leverage means that funds are borrowed to purchase some of the securities involved in the strategy.

EXHIBIT 12 Performance Attribution Example

Portfolio A: \$3.0 billion corporate bond portfolio with an effective duration of 7.09
Merrill Corporate Index: Benchmark index with a duration of 5.76

	% of Portfolio	Total return	Static return	Interest sensitive return	Spread change return	Trading return	Residual
Portfolio Totals							
Portfolio A	100.000	2.187	0.453	1.813	-0.087	-0.003	0.011
Merrill Corporate	100.000	1.954	0.452	1.433	0.098	0.000	-0.029
Difference	0.000	0.233	0.001	0.380	-0.185	-0.003	0.040
Sector Analysis*							
Agencies							
Portfolio A	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Merrill Corporate	12.044	2.083	0.476	1.918	-0.118	0.000	-0.193
Difference	-12.044	-2.083	-0.476	-1.918	0.118	0.000	0.193
Industrials							
Portfolio A	31.480	2.325	0.459	1.924	-0.026	-0.059	0.027
Merrill Corporate	26.769	2.121	0.460	1.606	0.108	0.000	-0.053
Difference	4.711	0.204	-0.001	0.318	-0.134	-0.059	0.080
Financials							
Portfolio A	15.580	2.023	0.439	1.560	0.077	-0.057	0.004
Merrill Corporate	37.363	1.707	0.444	1.210	0.060	0.000	-0.007
Difference	-21.783	0.316	-0.005	0.350	0.017	-0.057	0.011
Utilities							
Portfolio A	15.900	0.310	0.528	-0.090	0.540	0.042	-0.710
Merrill Corporate	7.385	2.167	0.469	1.564	0.185	0.000	-0.051
Difference	8.515	-1.857	0.059	-1.654	0.355	0.042	-0.659
Telephones							
Portfolio A	17.080	2.439	0.439	1.843	0.027	0.144	-0.014
Merrill Corporate	4.440	2.331	0.447	1.723	0.201	0.000	-0.040
Difference	12.640	0.108	-0.008	0.120	-0.174	0.144	0.026
Oil							
Portfolio A	4.940	0.562	0.467	2.035	-1.939	0.000	0.001
Merrill Corporate	1.670	2.123	0.462	1.513	0.194	0.000	-0.046
Difference	3.270	-1.561	0.005	0.522	-2.133	0.000	0.045
Internationals							
Portfolio A	14.180	2.264	0.443	1.891	-0.079	0.021	-0.012
Merrill Corporate	10.022	2.118	0.446	1.597	0.095	0.000	-0.020
Difference	4.158	0.146	-0.003	0.294	-0.174	0.021	0.008
Miscellaneous							
Portfolio A	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Merrill Corporate	0.308	0.796	0.416	0.384	-0.025	0.000	0.021
Difference	-0.308	-0.796	-0.416	-0.384	0.025	0.000	-0.021

*In the sector analyses, we compare the constituents of Portfolio A that fall into a particular sector with the constituents of the benchmark that fall into the same sector. For example, the industrials from Portfolio A are evaluated against the industrials from the Merrill Corporate Index.

Source: G.A.T. Integrative Bond System

So, we begin this section with a discussion of the advantages and disadvantages of leverage, and then we discuss the use of repurchase agreements as a source of borrowed funds. Scenario analysis, discussed in Section IV, can then be used to assess a trade or a portfolio strategy that uses leverage.

A. The Principle of Leverage

Leveraging is the investment approach of borrowing funds with the expectation of earning a return in excess of the cost of funds. The attractive feature of leveraging is that it magnifies the

EXHIBIT 13 Summary of Performance Attribution Analysis

Risk factor	Portfolio A returns (bps)	Merrill Corporate index returns (bps)	Difference	% of total return difference
Static Return	45.3	45.2	0.1	0.4%
Interest Sensitive Return	181.3	143.3	38.0	163.1%
Spread Change Return	−8.7	9.8	−18.5	−79.4%
Trading Return	−0.3	0.0	−0.3	−1.3%
Residual	1.1	−2.9	4.0	17.2%
Total	218.7	195.4	23.3	100.0%

return realized from investment in a security for a given change in the price of that security. That's the good news. The bad news is that leveraging also magnifies losses.

To illustrate, consider an investor who plans to purchase a 30-year U.S. Treasury bond in anticipation of a decline in interest rates six months from now. The investor has \$1 million to invest, which is referred to as the **investor's equity**. Assuming that the coupon rate for the Treasury bond is 8%, the next coupon payment is six months from now, and the bond can be purchased at par value, then the investor can purchase \$1 million par value of the Treasury bond with the equity available.

Exhibit 14 shows the return that will be realized for various assumed yields six months from now for the Treasury bond. The dollar return consists of the coupon payment six months from now plus the change in the value of the Treasury bond. There is no reinvestment income. At the end of six months, the 30-year Treasury bond is a 29.5-year Treasury bond. The percent return is found by first dividing the dollar return by the \$1 million of investor's equity and then multiplying by 2 to annualize so the return is computed on a bond-equivalent basis. Notice that the range for the annualized rate of return is from −29.8% to +63.0%.

In our illustration, the investor does not borrow any funds, so that the strategy is referred to as an **unleveraged strategy**. Now suppose that the investor borrows \$1 million in order to purchase an additional \$1 million of par value of the Treasury bond. Assume further that the loan agreement specifies that:

1. the maturity of the loan is six months
2. the annual interest rate for the loan is 9%, and

EXHIBIT 14 Annual Return from a \$1 Million Investment in a 30-Year 8% Coupon Treasury Bond Held for Six Months

Assumed yield six months from now (%)	Price per \$100 par value (\$)*	Market value per \$1 million par value (\$)*	Semiannual coupon payment (\$)	Dollar return (\$)	Annualized percent return (%)**
10.00	81.12	811,200	40,000	−148,800	−29.8
9.50	85.23	852,300	40,000	−107,700	−21.5
9.00	89.72	897,200	40,000	−62,800	−12.6
8.50	94.62	946,200	40,000	−13,800	−2.8
8.00	100.00	1,000,000	40,000	40,000	8.0
7.50	105.91	1,059,100	40,000	99,100	19.8
7.00	112.41	1,124,100	40,000	164,100	32.8
6.50	119.58	1,195,800	40,000	235,800	47.2
6.00	127.51	1,275,100	40,000	315,100	63.0

*This is the price and market value six months later, rounded to the nearest \$100.

** Annualized by doubling the semiannual return.

EXHIBIT 15 Annual Return from a \$2 Million Investment in a 30-Year 8% Coupon Treasury Bond Held for Six Months Using \$1 Million of Borrowed Funds

Assumed yield six months from now (%)	Market value per \$100 par value (\$)*	Market value per \$2 million par value (\$)*	Semiannual coupon payment (\$)	Dollar return to equity (\$)***	Annualized percent return (%)***
10.00	81.12	1,622,400	80,000	−342,600	−68.5
9.50	85.23	1,704,600	80,000	−260,400	−52.1
9.00	89.72	1,794,400	80,000	−170,600	−34.1
8.50	94.62	1,892,400	80,000	−72,600	−14.5
8.00	100.00	2,000,000	80,000	35,000	7.0
7.50	105.91	2,118,200	80,000	153,200	30.6
7.00	112.41	2,248,200	80,000	283,200	56.6
6.50	119.58	2,391,600	80,000	426,600	85.3
6.00	127.51	2,550,200	80,000	585,200	117.0

*This is the price and market value six months later, rounded to the nearest \$100.

**After deducting interest expense of \$45,000 ($\$1 \text{ million} \times 9\%/2$).

***Annualized by doubling the semiannual return.

3. \$1 million par value of the 30-year 8% coupon Treasury bond is used as collateral for the loan

Therefore, the loan is a collateralized loan.

The amount invested is then \$2 million, which comes from the investor's equity of \$1 million and \$1 million of borrowed funds. In this strategy, the investor uses leverage. Since the investor has the use of \$2 million in proceeds and has equity of \$1 million, this is called "2-to-1 leverage." (This means \$2 invested for \$1 in investor's equity.)

Exhibit 15 shows the annual rate of return for this leveraged strategy for the same yields shown in Exhibit 14. The return is measured relative to the investor's equity of \$1 million, not the \$2 million. The dollar return shown in the exhibit adjusts for the cost of borrowing.

Because of the use of borrowed funds, the range for the annualized percent return is wider (−68.5% to +117.0%) than in the unleveraged case (−29.8% to 63.0%). This example clearly shows that leverage is a two-edged sword—it magnifies returns both up and down. Notice that, if the market yield does not change at the end of six months, then the unleveraged strategy generates an 8% annual return. That is, for the \$1 million invested, the coupon interest is \$40,000 for six months. Since there is no change in the market value of the security, this produces a 4% semiannual return, or 8% on a simple annual basis (i.e., a bond-equivalent basis). In contrast, consider what happens if \$2 million is invested in the 2-for-1 leveraging strategy. Since \$2 million is invested, the coupon interest is \$80,000 for six months. But the interest cost of the \$1 million loan for six months is \$45,000 ($\$1 \text{ million} \times 9\%/2$). Thus, the dollar return after the financing cost is \$35,000 ($\$80,000 - \$45,000$). Hence, the return on the investor's \$1 million equity is 3.5% for six months ($\$35,000/\$1 \text{ million}$) and 7% annualized. Without leverage, the investor earns 8% if interest rates do not change but only 7% in the same scenario in the 2-for-1 leveraging strategy.

Suppose that, instead of borrowing \$1 million, the investor is able to borrow \$11 million for six months at an annual interest rate of 9%. The investor can now purchase \$12 million of Treasury bonds, using \$1 million of investor's equity and \$11 million of borrowed funds. The lender requires that the \$11 million of Treasury bonds be used as collateral for this loan.

EXHIBIT 16 Annual Return from a \$12 Million Investment in a 30-Year 8% Coupon Treasury Bond Held for Six Months Using \$11 Million of Borrowed Funds

Assumed yield six months from now (%)	Price per \$100 par value (\$)*	Market value per \$2 million par value (\$)*	Semiannual coupon payment (\$)	Dollar return to equity (\$)***	Annualized percent return (%)***
10.00	81.12	9,734,900	480,000	−2,280,100	−456.0
9.50	85.23	10,227,900	480,000	−1,787,100	−357.4
9.00	89.72	10,766,000	480,000	−1,249,000	−249.8
8.50	94.62	11,354,700	480,000	−660,300	−132.1
8.00	100.00	12,000,000	480,000	−15,000	−3.0
7.50	105.91	12,708,800	480,000	693,800	138.8
7.00	112.41	13,489,100	480,000	1,474,100	294.8
6.50	119.58	14,349,600	480,000	2,334,600	466.9
6.00	127.51	15,300,700	480,000	3,285,700	657.1

*This is the price and market value six months later, rounded to the nearest 100.

**After deducting interest expense of \$495,000 (\$11 million $\times$ 9%/2).

***Annualized by doubling the semiannual return.

Since there is \$12 million invested and \$1 million of investor's equity, this strategy is said to have "12-to-1 leverage."

Exhibit 16 shows the annual return assuming the same yields used in Exhibits 14 and 15. Notice the considerably wider range of annual returns for the 12-to-1 leverage strategy compared to the 2-to-1 leverage strategy or the unleveraged strategy. When the yield remains at 8%, the 12-to-1 strategy results in an annual return of −3%. This result occurs because the coupon interest earned on the \$12 million invested for six months is \$480,000 (\$12 million $\times$ 8%/2) but the interest expense is \$495,000 (\$11 million borrowed $\times$ 9%/2). The dollar return to the investor for the 6-month period is then −\$15,000 or −1.5% (−\$15,000/\$1 million). Doubling the −1.5% semiannual return gives the −3% annual return.

Exhibit 17 shows the range of outcomes for different degrees of leverage. The greater the leverage, the wider the range of potential outcomes.

B. Borrowing Funds via Repurchase Agreements

A **repurchase agreement** is the sale of a security with a commitment by the seller to buy back the same security from the purchaser at a specified price at a designated future date. The price

EXHIBIT 17 Annual Return for Various Degrees of Leverage

Assumed yield six months from now (%)	Annual return for \$1 million of equity and debt of X\$ million (%)					
	\$0	\$1	\$2	\$3	\$5	\$11
10.00	−29.8	−68.5	−107.3	−146.0	−223.6	−456.0
9.50	−21.5	−52.1	−82.6	−113.2	−174.2	−357.4
9.00	−12.6	−34.1	−55.7	−77.2	−120.4	−249.8
8.50	−2.8	−14.5	−26.3	−38.0	−61.6	−132.1
8.00	8.0	7.0	6.0	5.0	3.0	−3.0
7.50	19.8	30.6	41.5	52.3	73.9	138.8
7.00	32.8	56.6	80.5	104.3	151.9	294.8
6.50	47.2	85.3	123.5	161.6	238.0	466.9
6.00	63.0	117.0	171.1	225.1	333.1	657.1

at which the seller must subsequently repurchase the security is called the **repurchase price** and the date by which the security must be repurchased is called the **repurchase date**. Basically, a repurchase agreement is a **collateralized loan**, where the collateral is the security that is sold and subsequently repurchased.¹¹ The agreement is best described with an illustration.

Suppose a government securities dealer has purchased \$10 million of a particular Treasury security. The dealer can finance the position with its own funds or by borrowing from a bank. Typically, however, the dealer uses the repurchase agreement or “repo” market to obtain financing. In the repo market, the dealer uses the \$10 million Treasury security as collateral for the loan. The term of the loan and the interest rate the dealer agrees to pay are specified. The interest rate is called the **repo rate**. When the term of the loan is one day, it is called an **overnight repo** (or RP); a loan for more than one day is called a **term repo** (or term RP). The difference between the purchase (repurchase) price and the sale price is the dollar interest cost of the loan.

Suppose now that a customer of the dealer firm has \$10 million. The dealer firm agrees to deliver (“sell”) \$10 million of the Treasury security to the customer in exchange for \$10 million and simultaneously agree to buy back (i.e., “repurchase”) the same Treasury security the next day for \$10 million plus interest.

The dollar amount of the interest is determined by the repo rate, the number of days that the funds are borrowed (i.e., the term of the loan), and the amount borrowed. The formula for the dollar interest is:

$$\text{dollar interest} = \text{amount borrowed} \times \text{repo rate} \times \text{repo term}/360$$

Notice that the dollar interest is computed on an actual/360-day basis.

In our example, if the repo rate is 5%, then we know:

$$\begin{aligned} \text{amount borrowed} &= \$10,000,000 \\ \text{repo rate} &= 0.05 \\ \text{repo term} &= 1 \text{ day} \end{aligned}$$

Therefore the dollar interest is:

$$\text{dollar interest} = \$10,000,000 \times 0.05 \times 1/360 = \$1,388.89$$

So, the dealer sells the Treasury security to the customer for \$10 million and agree to repurchase it the next day for \$10,001,388.89 (\$10,000,000 + \$1,388.89).

The advantage for the dealer in using the repo market to borrow on a short-term basis is that the repo rate is lower than the cost of bank financing. (The reason for this is explained below.) From the customer’s perspective, the repo market offers an attractive yield on a short-term secured transaction that is highly liquid.

While the example illustrates the use of the repo market to finance a dealer’s long position, dealers can also use the repo market to cover a short position. For example, suppose a government dealer sold short \$10 million of Treasury securities two weeks ago and must now cover the position—that is, deliver the securities. The dealer can do a **reverse repo** (agree to

¹¹A special type of repurchase agreement used in the mortgage-backed securities market is called a “dollar roll.” For a description of a dollar roll, see Chapter 9 in Frank J. Fabozzi and David Yuen, *Managing MBS Portfolios* (Hoboken, NJ: John Wiley & Sons, 1998).

buy the securities and then sell them back). Of course, the dealer eventually would have to buy the Treasury security in the market in order to cover the short position. In this case, the dealer actually makes a collateralized loan to the customer. The customer (or other dealer) then uses the funds obtained from the collateralized loan to create leverage.

1. Industry Jargon A good deal of industry jargon is used to describe repo transactions. In order to understand the terminology, remember that one party lends money and accepts a security as collateral for the loan; the other party borrows money and provides collateral in the form of a security.

We discuss the following terminology below:

1. “repo” versus “reverse repo”
2. “reversing out a security” and “reversing in a security”
3. “selling collateral” and “buying collateral”

First, if a party loans a security and receives cash, then, from this party’s perspective, the transaction is called a “repo.” If a party loans cash and receives a security as collateral, this is a “reverse repo” transaction from this party’s perspective.

When someone lends a security (i.e., uses a security as collateral) in order to receive cash (i.e., borrow money), that party is said to be “reversing out” a security. A party that lends money with a security as collateral is said to be “reversing in” a security.

Finally, the expressions “selling collateral” and “buying collateral” are used to describe a party financing a security with a repo on the one hand, and lending on the basis of collateral, on the other.

To summarize, the following expressions are used for the transaction:

Borrower of funds	Lender of funds
repo	reverse repo
reversing out a security	reversing in a security
selling collateral	buying collateral

Rather than using industry jargon, investment guidelines should be clear as to what a manager is permitted to do. For example, a client may have no objections to its portfolio manager using a repo as a short-term investment—that is, the portfolio manager may lend funds on a short-term basis. The investment guidelines will set forth how the loan arrangement should be structured to protect against credit risk. We’ll discuss this below. However, if a client does not want a money manager to use the repo agreement as a vehicle for borrowing funds (thereby, creating leverage), it should state so.

2. Margin and Marking to Market Despite the fact that high-quality collateral underlies a repo transaction, both parties to the transaction are exposed to credit risk. Why does credit risk exist in a repo transaction? Consider our initial example of the dealer who uses \$10 million of government securities as collateral for a loan. If the dealer cannot repurchase the government securities, the customer keeps the collateral; if interest rates on government securities increase subsequent to the repo transaction, however, the market value of the government securities declines, and the customer then owns securities with a market value less than the amount of the loan. If the market value of the security rises instead, the dealer will be concerned with the return of the collateral, which then has a market value greater than the amount of the loan.

Repos should be carefully structured to reduce credit risk exposure. The amount lent should be less than the market value of the security used as collateral, thereby providing the lender with cushion should the market value of the security decline. The amount by which the market value of the security used as collateral exceeds the value of the loan is called **repo margin** or simply **margin**. Margin is also referred to as the “haircut.” Repo margin is generally between 1% and 3%, but can be 10% or more for borrowers of lower credit worthiness and/or when less liquid or more price sensitive securities are used as collateral.

For example, consider the dealer who needs to borrow \$10 million to finance the purchase of a Treasury security. Suppose that the repo margin is 2%. Then for a Treasury security with a market value of \$10 million, only 98% of that amount, \$9.8 million, will be lent. That is, the dealer will agree to deliver (sell) \$10 million of the Treasury security to the customer for \$9.8 million and agree to repurchase the \$10 million of the Treasury security the next day for \$9.8 million plus the dollar interest. The dollar interest for this overnight repo, assuming a repo rate of 5%, is:

$$\text{dollar interest} = \$9,800,000 \times 0.05 \times 1/360 = \$1,361.11$$

Note that the dollar interest is based on \$9.8 million (the amount actually lent by the customer), not \$10 million as in our earlier example.

Another practice used to limit credit risk is to mark the collateral to market on a regular basis. Marking a position to market means recording the value of a position at its market value. When the market value changes by a certain percentage, the repo position is adjusted accordingly. The decline in market value below a specified amount results in a **margin deficit**. In such cases, the borrower of funds typically has the option to resolve the margin deficit either by providing additional cash or by transferring additional acceptable securities to the lender. When the market value rises above the amount required, **excess margin** will result. When this occurs, the lender of funds has the option to give cash to the borrower equal to the amount of the excess margin or to transfer purchased securities to the borrower.

3. Delivery and Credit Risk One concern of the parties to a repo is delivery of the collateral to the lender. The most obvious procedure is for the borrower to deliver the collateral to the lender or to the lender’s clearing agent. In such instances, the collateral is said to be “delivered out.” At the end of the repo term, the lender returns the collateral to the borrower in exchange for the principal and interest payment. This procedure may be expensive though, particularly for short-term repos, because of costs associated with delivering the collateral. The cost of delivery would be factored into the repo rate. The lender’s risk in terms of taking possession of the collateral derives from the fact that the borrower might sell the security, or go under so that the lender has nothing to liquidate, or use the same security as collateral for a repo with another party.

As an alternative to delivering out the collateral, the lender might agree to allow the borrower to hold the security in a segregated customer account. Of course, in this arrangement, the lender still faces the risk that the borrower might use the collateral fraudulently by offering it as collateral for another repo transaction. If the borrower holds the collateral, then the transaction is called a **hold-in-custody repo** (HIC repo). Despite the credit risk associated with a HIC repo, this arrangement is sometimes used in transactions when the collateral is difficult to deliver or when the transaction amount is small and the lender is comfortable with the reputation of the borrower.

Another arrangement is for the borrower to deliver the collateral to the lender’s custodial account at the borrower’s clearing bank. The custodian then has possession of the collateral

on behalf of the lender. This practice reduces the cost of delivery because it requires only a transfer within the borrower's clearing bank. If, for example, a dealer enters into an overnight repo with Customer A, the collateral is transferred back to the dealer the next day. The dealer can then enter into a repo with Customer B for, say, five days without having to redeliver the collateral. The clearing bank simply establishes a custodian account for Customer B and holds the collateral in that account. This specialized type of arrangement is called a **tri-party repo**. Tri-party repos account for about half of all repo arrangements.

The third party bank is responsible for marking the collateral to market and reporting these values each day to the two parties. Also, if the borrower wishes to substitute collateral (i.e., change the specific Treasury securities collateralizing the loan), the third-party agent verifies that the collateral satisfies the requirements set forth in the repo agreement.

4. Determinants of the Repo Rate The repo rate varies from transaction to transaction depending on a variety of factors, including:

- quality of collateral
- term of the repo
- delivery requirement
- availability of collateral
- prevailing federal funds rate

The higher the credit quality and liquidity of the collateral, the lower the repo rate. The repo rate varies with the term of the repo agreement. This is basically the very short-term end of the yield curve. The maturity of the security used as collateral for the repo does not affect the repo rate. If delivery of the collateral to the lender is required, the repo rate is lower.

The more difficult it is to obtain the collateral, the lower the repo rate. To understand why this is so, remember that the borrower (or equivalently the seller of the collateral) has a security that lenders of cash want. Such collateral is referred to as **hot collateral** or **special collateral** (or just as “on special”). Collateral that does not have this characteristic is referred to as **general collateral**. The party that needs the hot collateral will be willing to lend at a lower rate in order to obtain the collateral.

While these factors determine the repo rate on a particular transaction, the federal funds rate determines the general level of repo rates in the United States. Banks borrow funds from each other via the federal funds market. The interest rate charged on such borrowing is called the **federal funds rate** (or “fed funds”). The repo rate is generally lower than the federal funds rate because a repo is a form of collateralized borrowing, while a federal funds transaction is unsecured borrowing.

C. Computing Duration for a Leveraged Portfolio

When a manager borrows to leverage a portfolio, the following procedure should be used to compute the portfolio's duration. The portfolio is then composed of the assets and the liabilities. The change in the portfolio value that includes borrowed funds is equal to:

$$= \frac{\text{dollar price change of all the bonds when rates change} - \text{dollar price change of the liabilities when rates change}}{\text{total change in the portfolio value when rates change}}$$

Dividing the total change in the portfolio value by the initial value of the portfolio and adjusting for the basis point change in rates equals the duration of the portfolio. If the liabilities are short term, then the duration of the liability is low.

To illustrate these calculations, consider the following three-bond portfolio, with duration and change in value for a 50-basis point change as shown:

Bond	Market value	Duration	Change in value for 50 bp yield change
10% 5-year	\$4,000,000	3.861	\$77,220
8% 15-year	4,231,375	8.047	170,249
14% 30-year	1,378,586	9.168	63,194
Portfolio	\$9,609,961	6.470	\$310,663

Note that this is the portfolio used in Chapter 17 to illustrate the duration calculation for an unlevered portfolio. Suppose that \$2 million was borrowed to buy the securities in the portfolio, so that the amount of the client's funds invested (i.e., the equity) is \$7,609,961. The client is interested in how the rate changes affect the equity investment. Suppose further that the funds are borrowed using a 3-month repurchase agreement so that the duration of the liabilities is close to zero and the dollar change in the value of the liabilities for a 50 basis point change in rates is close to zero. Then the change in the value of the portfolio for a 50 basis point change in rates is computed as follows:

$$\begin{array}{rcl}
 \text{dollar price change of the bonds} & = & \$310,663 \\
 - \text{dollar price change of the liabilities} & = & 0 \\
 \hline
 \text{total change in portfolio value} & = & \$310,663
 \end{array}$$

Thus, the percentage change in the portfolio's value for a 50 basis point change in rates is 4.08% (\$310,663 divided by \$7,609,961), so the portfolio's duration is 8.16. The higher duration that results from the \$2 million short-term reverse repo borrowing (8.16 versus 6.47) is due to the leveraging of the portfolio.

CHAPTER 19

PORTFOLIO IMMUNIZATION AND CASH FLOW MATCHING

I. INTRODUCTION

In this chapter we will explain strategies for managing bond portfolios to satisfy predetermined liabilities. The two strategies we will discuss are immunization and cash flow matching. **Immunization** is a hybrid strategy having elements of both active and passive strategies. It is used to minimize reinvestment risk over a specified investment horizon. Immunization can be employed to structure a portfolio designed to fund a single liability or multiple liabilities. **Cash flow matching** is used to construct a portfolio that will fund a schedule of liabilities from a portfolio's cash flows, with the portfolio's value diminishing to zero after payment of the last liability.

II. IMMUNIZATION STRATEGY FOR A SINGLE LIABILITY

Classical immunization can be defined as the process by which a bond portfolio is created to have an assured return for a specific time horizon irrespective of interest rate changes.¹ The fundamental principle underlying immunization is to structure a portfolio that balances the change in the value of the portfolio at the end of the investment horizon with the return from the reinvestment of portfolio cash flows (both coupon and principal payments). That is, immunization offsets interest rate risk and reinvestment risk. The general principle of immunization is summarized in Exhibit 1.

To accomplish this balancing requires controlling portfolio duration. By setting the portfolio duration equal to the desired portfolio time horizon, positive and negative incremental return sources offset one another. This is a necessary condition for effectively immunized portfolios.

¹The classical theory of immunization is set forth in F.M. Reddington, "Review of the Principles of Life Insurance Valuations," *Journal of the Institute of Actuaries*, 1952; and Lawrence Fisher and Roman Weil, "Coping with Risk of Interest Rate Fluctuations: Returns to Bondholders from Naive and Optimal Strategies," *Journal of Business* (October 1971), pp. 408–431.

EXHIBIT 1 General Principle of Classical Immunization

Objective: Lock in a minimum target rate of return and target an accumulated value regardless of how interest rates change over an investment horizon.

Risk when interest rates change:

Reinvestment risk

Interest rate or price risk

Assumption: Parallel shift in the yield curve (i.e., all yields rise and fall uniformly)

Principle:

Scenario 1: Interest rates increase

Implications:

1. Reinvestment income increases
2. Value of portfolio of bonds with maturities greater than the investment horizon declines in value

Result: Gain in reinvestment income $\geq$ loss in portfolio value

Scenario 2: Interest rates decline

Implications:

1. Reinvestment income decreases
2. Value of portfolio of bonds with maturities greater than the investment horizon increases in value

Result: Loss in reinvestment income $\leq$ gain in portfolio value

A. Illustration

To demonstrate the principle of immunization, consider the situation faced by a life insurance company that sells a guaranteed investment contract (GIC). This policy specifies that a life insurance company guarantees that a specified dollar amount will be paid to the policyholder at a specified future date. Or, equivalently, the life insurance company guarantees a specified rate of return on the payment date (referred to as the “crediting rate”).

For example, suppose that a life insurance company sells a 5-year GIC that guarantees an interest rate of 7.5% per year on a bond-equivalent yield basis (or, equivalently, 3.75% every six months for the next ten 6-month periods). Suppose that the payment made by the policyholder to purchase the GIC is \$9,642,899. Then the value that the life insurance company has guaranteed to the policyholder five years from now is \$13,934,413. When investing the \$9,642,899, the target accumulated value for the life insurance company is \$13,934,413 after five years, which is the same as a target yield of 7.5% on a bond-equivalent basis.

Suppose the life insurance company buys \$9,642,899 par value of a bond selling at par with a 7.5% yield to maturity that matures in five years. The life insurance company will not be assured of realizing a total return at least equal to the target return of 7.5% because to realize 7.5% the coupon interest payments must be reinvested at a minimum rate of 3.75% every six months. That is, the accumulated value will depend on the reinvestment rate.

To demonstrate this, assume that immediately after investing the \$9,642,899 in the 7.5% 5-year bond, yields in the market change and stay at the new level for the remainder of the five years. Exhibit 2 illustrates what happens at the end of five years. The first column shows

EXHIBIT 2 Total Accumulated Value and Total Return After Five Years for a 5-Year 7.5% Bond Selling to Yield 7.5%

Investment horizon:	5 years	Price:	100.00
Coupon rate:	7.50%	Par value purchased:	\$9,642,899
Maturity:	5 years	Purchase price:	\$9,642,899
Yield to maturity:	7.50%	Target accumulated value:	\$13,934,413

New yield (%)	Coupon (\$)	Reinvestment income (\$)	Price of bond (\$)	Accumulated value (\$)	Total return (%)
11.00	3,616,087	1,039,753	9,642,899	14,298,739	8.04
10.50	3,616,087	985,615	9,642,899	14,244,601	7.96
10.00	3,616,087	932,188	9,642,899	14,191,175	7.88
9.50	3,616,087	879,465	9,642,899	14,138,451	7.80
9.00	3,616,087	827,436	9,642,899	14,086,423	7.73
8.50	3,616,087	776,093	9,642,899	14,035,079	7.65
8.00	3,616,087	725,426	9,642,899	13,984,412	7.57
7.50	3,616,087	675,427	9,642,899	13,934,413	7.50
7.00	3,616,087	626,087	9,642,899	13,885,073	7.43
6.50	3,616,087	577,398	9,642,899	13,836,384	7.35
6.00	3,616,087	529,352	9,642,899	13,788,338	7.28
5.50	3,616,087	481,939	9,642,899	13,740,925	7.21
5.00	3,616,087	435,153	9,642,899	13,694,139	7.14
4.50	3,616,087	388,985	9,642,899	13,647,971	7.07
4.00	3,616,087	343,427	9,642,899	13,602,414	7.00

the new yield level. The second column shows the total coupon payments. The third column gives the reinvestment earned over the five years if the coupon payments are reinvested at the new yield level shown in the first column. The price of the bond at the end of five years shown in the fourth column is the par value. The fifth column is the accumulated value from all three sources: coupon interest, reinvestment income, and bond price. The total return is shown in the last column.²

If yields do not change so that the coupon payments can be reinvested at 7.5% (3.75% every six months), the life insurance company will achieve the target accumulated value. If market yields rise, an accumulated value (total return) higher than the target accumulated value (target yield) will be achieved. This is because the coupon payments can be reinvested at a higher rate than the initial yield to maturity. Contrast this with what happens when the yield declines. The accumulated value (total return) will be less than the target accumulated value (target yield). Therefore investing in a coupon bond with a yield to maturity equal to the target yield and a maturity equal to the investment horizon does not assure that the target accumulated value will be achieved.

Suppose that instead of investing in a bond maturing in five years the life insurance company invests in a 12-year bond with a coupon rate of 7.5% selling at par to yield 7.5%. Exhibit 3 presents the accumulated value and total return in five years if the market yield changes immediately after the bond is purchased and remains at the new yield level. The fourth column of the exhibit is the market price of a 7.5% 7-year bond (since five years have passed at the horizon date). If the market yield increases, the portfolio will fail to achieve the target accumulated value; if the market yield decreases, the accumulated value (total return) will exceed the target accumulated value (target yield).

²The value in this column is found as follows: $2 \left[\left(\frac{\text{Accumulated value}}{\$9,642,899} \right)^{1/10} - 1 \right]$

EXHIBIT 3 Total Accumulated Value and Total Return After Five Years for a 12-Year 7.5% Bond Selling to Yield 7.5%

Investment horizon:	5 years	Price:	100.00
Coupon rate:	7.50%	Par value purchased:	\$9,642,899
Maturity:	12 years	Purchase price:	\$9,642,899
Yield to maturity:	7.50%	Target accumulated value:	\$13,934,413

New yield (%)	Coupon (\$)	Reinvestment income (\$)	Price of bond (\$)	Accumulated value (\$)	Total return (%)
11.00	3,616,087	1,039,753	8,024,639	12,680,479	5.55
10.50	3,616,087	985,615	8,233,739	12,835,440	5.80
10.00	3,616,087	932,188	8,449,754	12,998,030	6.06
9.50	3,616,087	879,465	8,672,941	13,168,494	6.33
9.00	3,616,087	827,436	8,903,566	13,347,090	6.61
8.50	3,616,087	776,093	9,141,907	13,534,087	6.90
8.00	3,616,087	725,426	9,388,251	13,729,764	7.19
7.50	3,616,087	675,427	9,642,899	13,934,413	7.50
7.00	3,616,087	626,087	9,906,163	14,148,337	7.82
6.50	3,616,087	577,398	10,178,367	14,371,852	8.14
6.00	3,616,087	529,352	10,459,851	14,605,289	8.48
5.50	3,616,087	481,939	10,750,965	14,848,992	8.82
5.00	3,616,087	435,153	11,052,078	15,103,318	9.18
4.50	3,616,087	388,985	11,363,569	15,368,642	9.54
4.00	3,616,087	343,427	11,685,837	15,645,352	9.92

EXHIBIT 4 Change in Reinvestment Income and Price Due to Interest Rate Change After Five Years for a 12-Year 7.5% Bond Selling to Yield 7.5%

New yield (%)	Change in reinvestment income (\$)	Change in price (\$)	Total change in accumulated value (\$)
11.0	364,326	(1,618,260)	(1,253,934)
10.5	310,188	(1,409,160)	(1,098,972)
10.0	256,762	(1,193,145)	(936,383)
9.5	204,039	(969,958)	(765,919)
9.0	152,010	(739,333)	(587,323)
8.5	100,666	(500,992)	(400,326)
8.0	49,999	(254,648)	(204,649)
7.5	—	—	—
7.0	(49,340)	263,264	213,924
6.5	(98,029)	535,468	437,439
6.0	(146,075)	816,952	670,877
5.5	(193,487)	1,108,066	914,579
5.0	(240,273)	1,409,179	1,168,905
4.5	(286,441)	1,720,670	1,434,229
4.0	(331,999)	2,042,938	1,710,939

The reason for this result can be seen in Exhibit 4 that summarizes the change in reinvestment income and the change in price resulting from a change in the market yield. For example, if the market yield rises instantaneously by 200 basis points, from 7.5% to 9.5%, reinvestment income will be \$204,039 greater; however, the market price of the bond will decrease by \$969,958. The net effect is that the accumulated value will be \$765,919 less than the target accumulated value. The reverse will be true if the market yield decreases. The change

EXHIBIT 5 Total Accumulated Value and Total Return After Five Years for a 6-Month 7.5% Bond Selling to Yield 7.5%

Investment horizon:	5 years	Price:	100.00
Coupon rate:	7.50%	Par value purchased:	\$9,642,899
Maturity:	0.50 years	Purchase price:	\$9,642,899
Yield to maturity:	7.50%	Target accumulated value:	\$13,934,413

New yield (%)	After one period (\$)	Accumulated value (\$)	Total return (%)
11.00	10,004,508	16,198,241	10.65
10.50	10,004,508	15,856,037	10.20
10.00	10,004,508	15,520,275	9.75
9.50	10,004,508	15,190,848	9.30
9.00	10,004,508	14,867,650	8.85
8.50	10,004,508	14,550,580	8.40
8.00	10,004,508	14,239,534	7.95
7.50	10,004,508	13,934,415	7.50
7.00	10,004,508	13,635,117	7.05
6.50	10,004,508	13,341,549	6.60
6.00	10,004,508	13,053,613	6.15
5.50	10,004,508	12,771,214	5.70
5.00	10,004,508	12,494,259	5.25
4.50	10,004,508	12,222,656	4.80
4.00	10,004,508	11,956,313	4.35

in the price of the bond will more than offset the decline in reinvestment income, resulting in an accumulated value that exceeds the target accumulated value. Now we can see what is happening to the accumulated value. There is a trade-off between interest rate (or price) risk and reinvestment risk. For this 12-year bond, the target accumulated value will be realized only if the market yield does not increase.

Because neither a coupon bond with the same maturity nor a bond with a longer maturity provides the target accumulated value, maybe a bond with a maturity shorter than five years will. Consider a 7.5% bond with six months remaining to maturity selling at par. Exhibit 5 shows the accumulated value and total return over the 5-year investment horizon. The second column shows the accumulated value after six months. The third column shows the accumulated value after five years by reinvesting the value accumulated after six months at the yield shown in the first column.³

By investing in this 6-month bond, the manager incurs no price risk, although there is reinvestment risk. The target accumulated value will be achieved only if the market yield remains at 7.5% or rises. Once again, the life insurance company is not assured of achieving the target accumulated value.

If we assume there is a one-time instantaneous change in the market yield, is there a coupon bond that the life insurance company can purchase to equal the target accumulated value whether the market yield rises or falls? The life insurance company should look for a coupon bond so that regardless of how the market yield changes, the change in reinvestment income will offset the change in price.

Consider a 6-year 6.75% bond selling at \$96.42899 to yield 7.5%. Suppose \$10 million of par value of this bond is purchased for \$9,642,899. Exhibit 6 provides the same information

³This value is found as follows: $\$10,004,508 (1 + \text{new yield}/2)^9$.

for this bond as Exhibits 2, 3, and 5 do for the previous bonds considered for purchase. Looking at the last two columns, we see that the accumulated value and the total return are never less than the target accumulated value and target yield.⁴ Thus, the target accumulated value is assured regardless of what happens to the market yield.

Exhibit 7 shows why. When the market yield rises, the change in the reinvestment income more than offsets the decline in price. When the market yield declines, the increase in price exceeds the decline in reinvestment income. What characteristic of this bond assures that the target accumulated value will be realized regardless of how the market yield changes? The duration for each of the four bonds is shown in Exhibit 8.

The duration of the liability is 4.82.⁵ Notice that the 6-year 6.75% bond which equals the target accumulated value regardless of what happens to the market yield, has a duration equal to the duration of the liability, 4.82. This is the key. To immunize a portfolio's target accumulated value (target yield) against a change in the market yield, the life insurance company must invest in a bond (or a bond portfolio) such that (1) the portfolio's duration is equal to the liability's duration, and (2) the initial present value of the cash flows from the bond (or bond portfolio) equals the present value of the future liability.

The two bonds with a duration shorter than the duration of the liability expose the portfolio to reinvestment risk. The one bond with a duration greater than the investment horizon exposes the portfolio to price risk.

When bonds with embedded options are included in an immunized portfolio, it is the effective duration that is the appropriate duration measure. The effective duration of the liability is the same as the modified duration of the liability. Thus, the requirements for

EXHIBIT 6 Total Accumulated Value and Total Return After Five Years for a 6-Year 6.75% Bond Selling to Yield 7.5%

Investment horizon:	5 years	Price:	100.00
Coupon rate:	6.75%	Par value purchased:	\$10,000,000
Maturity:	6 years	Purchase price:	\$9,642,899
Yield to maturity:	7.5%	Target accumulated value:	\$13,934,413

New yield (%)	Coupon (\$)	Reinvestment income (\$)	Price of bond (\$)	Accumulated value (\$)	Total return (%)
11.00	3,375,000	970,432	9,607,657	13,953,089	7.53
10.50	3,375,000	919,903	9,652,592	13,947,495	7.52
10.00	3,375,000	870,039	9,697,846	13,942,885	7.51
9.50	3,375,000	820,831	9,743,423	13,939,253	7.51
9.00	3,375,000	772,271	9,789,325	13,936,596	7.50
8.50	3,375,000	724,350	9,835,556	13,934,906	7.50
8.00	3,375,000	677,061	9,882,119	13,934,180	7.50
7.50	3,375,000	630,395	9,929,017	13,934,413	7.50
7.00	3,375,000	584,345	9,976,254	13,935,599	7.50
6.50	3,375,000	538,902	10,023,832	13,937,734	7.50
6.00	3,375,000	494,059	10,071,755	13,940,814	7.51
5.50	3,375,000	449,808	10,120,027	13,944,835	7.52
5.00	3,375,000	406,141	10,168,650	13,949,791	7.52
4.50	3,375,000	363,051	10,217,628	13,955,679	7.53
4.00	3,375,000	320,531	10,266,965	13,962,495	7.54

⁴There is a mathematical proof that shows this. We will not present the proof here. In the illustration shown in Exhibits 6 and 7, at 8% the accumulated value is slightly less than the target (\$13,934,180 versus \$13,934,413). This difference is due to rounding.

⁵The duration of a zero-coupon liability is equal to the number of years to maturity of the liability divided by 1 plus one-half the yield. In our illustration, it is 5 divided by $(1 + 0.075/2)$.

EXHIBIT 7 Change in Reinvestment Income and Price Due to Interest Rate Change after Five Years for a 6-Year 6.75% Bond Selling to Yield 7.5%

New yield (%)	Change in reinvestment income (\$)	Change in price (\$)	Total change in accumulated value (\$)
11.0	340,036	(321,360)	18,676
10.5	289,507	(276,426)	13,081
10.0	239,643	(231,171)	8,472
9.5	190,435	(185,595)	4,840
9.0	141,875	(139,692)	2,183
8.5	93,955	(93,461)	494
8.0	46,666	(46,898)	(232)
7.5	—	—	—
7.0	(46,050)	47,237	1,187
6.5	(91,493)	94,815	3,322
6.0	(136,336)	142,738	6,402
5.5	(180,587)	191,009	10,422
5.0	(224,254)	239,632	15,378
4.5	(267,344)	288,611	21,267
4.0	(309,865)	337,947	28,082

EXHIBIT 8 Duration for the Four Bonds Analyzed as Potential Candidates for Immunization

Bond	Duration	Risk
5-year, 7.5% coupon at par	4.11	Reinvestment
12-year, 7.5% coupon at par	7.83	Price
6-month, 7.5% coupon at par	0.48	Reinvestment
6-year, 6.75% coupon at 96.42899	4.82	

immunization can be restated in more general terms as: (1) the portfolio's effective duration is equal to the liability's effective duration and (2) the initial present value of the projected cash flows from the bond (or bond portfolio) equals the present value of the future liability.

B. Rebalancing an Immunized Portfolio

Our illustration of the principles underlying immunization assumes a one-time instantaneous change in the market yield. In practice, the market yield will fluctuate over the investment horizon. As a result, the duration of the portfolio will change as the market yield changes. In addition, the duration will change simply because of the passage of time. In any interest rate environment different from a flat term structure, the duration of a portfolio will change at a different rate than time.

Even in the face of changing market yields, a portfolio can be immunized if it is rebalanced periodically so that its duration is readjusted to the duration of the liability. For example, if the investment horizon is initially five years and the yield is 7.5%, the initial portfolio should have a duration of 4.82. After six months the investment horizon will be 4.5 years and the liability's duration will be 4.34 ($= 4.5/1.0375$). However, the duration of the portfolio will probably be different from 4.34. Thus, the portfolio must be rebalanced so that its duration is equal to 4.34. Six months later the portfolio must be rebalanced again so that its duration will equal the duration of a liability due in four years.

How often should the portfolio be rebalanced to adjust its duration? On one hand, more frequent rebalancing increases transaction costs, thereby reducing the likelihood of achieving the target return. On the other hand, less frequent rebalancing will result in the portfolio's duration wandering from the target duration (i.e., the duration of the liability), which will also reduce the likelihood of achieving the target return. Thus, a manager faces a trade-off: some transaction costs must be accepted to prevent the portfolio duration from wandering too far from its target; but some misalignment in the portfolio duration must be tolerated, or transaction costs will become prohibitively high.

C. Application Considerations

In the actual process of constructing an immunized portfolio, the selection of the universe is extremely important. The lower the credit quality of the securities considered, the higher the potential risk and return. Immunization theory assumes there will be no defaults and that securities will be responsive only to overall changes in interest rates. The lower the credit quality, the greater the possibility these assumptions will not be met. Further, securities with embedded options such as call features or mortgage-backed prepayments complicate and may even prevent the accurate measure of cash flows and hence duration, frustrating the basic requirements of immunization. Finally, liquidity is a consideration for an immunized portfolio because, as noted above, the portfolio must be rebalanced over time and principal liquidated on the target investment horizon to pay the liability.

Optimization procedures can be used to construct an immunized portfolio. Typically, immunization takes the form of minimizing the initial portfolio cost subject to the constraint of having sufficient cash to satisfy the liability at the horizon date. Further considerations such as average credit quality, minimum and maximum concentration constraints, and, perhaps, issuer constraints may also be included. Throughout this process the need to establish realistic guidelines and objectives is critical. In addition, because the optimization is very sensitive to the pricing of the universe being considered, accurate pricing and the interface of an experienced trader are valuable. Because of the many inputs and variations that are typically available, the optimization process should be approached in an iterative manner where the final solution is the result of a number of trials.

Transaction costs are important in meeting the target rate for an immunized portfolio. Transaction costs must be considered not only in the initial immunization (i.e., when the immunized portfolio is first created), but also in the periodic rebalancing necessary to avoid duration drift. The manager does not want to get into a situation where the portfolio will incur a substantial number of trades and enjoy only marginal benefits from risk minimization. Fortunately, transaction costs can be included in the optimization framework such that a trade-off between transaction costs and risk minimization can be evaluated.

D. Extensions of Classical Immunization Theory

The sufficient condition for classical immunization is that the duration of the portfolio match the duration of the liability. Classical theory is based on the following assumptions:

Assumption 1. Any changes in the yield curve are parallel changes, (i.e., interest rates move either up or down by the same amount for all maturities).

Assumption 2. The portfolio is valued at a fixed horizon date and there are no cash inflows or outflows during the time horizon except for coupon income and reinvestment income.

Assumption 3. The target value of the investment is defined as the portfolio value at the horizon date if the interest rate structure does not change (i.e., no change in forward rates).

Perhaps the most critical assumption of classical immunization techniques concerns Assumption 1—the type of interest rate change anticipated. A property of a classically immunized portfolio is that the target value of the investment is the lower limit of the value of the portfolio at the horizon date if there are parallel interest rate changes.⁶ This would appear to be an unrealistic assumption since such interest rate behavior is rarely, if ever, experienced in reality. According to the theory, if there is a change in interest rates that does not correspond to this shape preserving shift, matching the duration of the portfolio to the duration of the liability no longer assures immunization.⁷

A natural extension of classical immunization theory is a technique for modifying the assumption of parallel shifts in interest rates. One approach is to develop a strategy to handle an arbitrary interest rate change so that it is not necessary to specify an alternative duration measure. The approach, developed by Gifford Fong and Oldrich Vasicek, establishes an **immunization risk** measure against an arbitrary interest rate change.⁸ The immunization risk measure can then be minimized subject to the constraint that the portfolio duration be equal to the investment horizon resulting in a portfolio with minimum exposure to any interest rate movements.

One way of minimizing immunization risk is shown in Exhibit 9. The spikes in the two panels of Exhibit 9 represent actual portfolio cash flows. The taller spikes depict the actual cash flows generated by principal payments while the smaller spikes represent coupon payments. Both portfolio A and portfolio B are composed of two bonds whose weighted durations equal the investment horizon. Portfolio A is, in effect, a “barbell” portfolio—a portfolio comprised of short and long maturities and interim coupon payments. For portfolio B, the two bonds mature very close to the investment horizon and the coupon payments are nominal over the investment horizon. When a portfolio has the characteristics of portfolio B it is called a “bullet” portfolio.

It is not difficult to see why the barbell portfolio should have greater immunization risk than the bullet portfolio. Assume that both portfolios have durations equal to the liability’s duration, so that both portfolios are immune to parallel rate changes. This immunity is attained as a consequence of balancing the effect of changes in reinvestment rates on payments received during the investment horizon against the effect of changes in capital value of the portion of the portfolio still outstanding at the end of the investment horizon. When interest rates change in an arbitrary nonparallel way, however, the effect on the two portfolios is very different. Suppose, for instance, that short rates decline while long rates go up. Both portfolios would realize a decline of the portfolio’s value at the end of the investment horizon below the target accumulated value, since they experience a capital loss in addition to lower reinvestment rates. The decline, however, would be substantially higher for the barbell portfolio for two reasons. First, the lower reinvestment rates are experienced for the bonds in the barbell

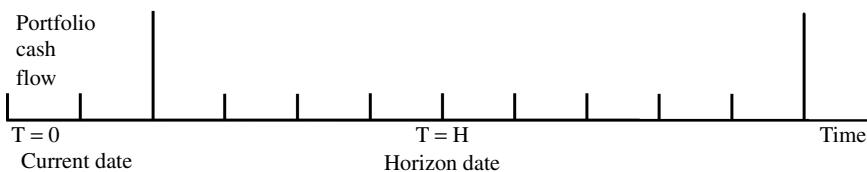
⁶Fisher and Weil, “Coping with Risk of Interest Rate Fluctuations: Returns to Bondholders from Naive and Optimal Strategies.”

⁷For a more complete discussion of these issues, see John C. Cox, Jonathan E. Ingersoll Jr., and Stephen A. Ross, “Duration and the Measurement of Basis Risk,” *Journal of Business* (January 1979), pp. 51–61.

⁸H. Gifford Fong and Oldrich A Vasicek, “A Risk Minimizing Strategy for Portfolio Immunization,” *Journal of Finance* (December 1984), pp. 1541–1546.

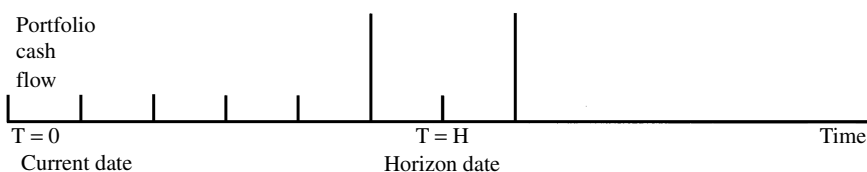
EXHIBIT 9 Illustration of Immunization Risk Measure

Portfolio A: High-risk immunized portfolio:



Note: Portfolio duration matches horizon length. Portfolio's cash flow dispersed.

Portfolio B: Low-risk immunized portfolio:



Portfolio duration matches horizon length. Portfolio's cash flow concentrated around horizon dates.

portfolio for the shorter maturities than on the bullet portfolio, so that the reinvestment risk is much greater. Second, the longer maturity bond in the barbell portfolio at the end of the investment horizon is much longer than the longer maturity bond in the bullet portfolio, which means that the same rate increase generates a greater capital loss. Thus, the bullet portfolio has less exposure to the change in the interest rate term structure than the barbell portfolio.

It should be clear from the foregoing discussion that immunization risk is reinvestment risk. The portfolio that has the least reinvestment risk will have the least immunization risk. When there is a high dispersion of cash flows around the horizon date, as in the barbell portfolio, the portfolio is exposed to higher reinvestment risk. In contrast, when the cash flows are concentrated around the horizon date, as in the bullet portfolio, the portfolio is subject to minimum reinvestment risk.

An example of a zero immunization risk portfolio is a portfolio consisting of zero-coupon bonds maturing at the investment horizon. This is because there is no reinvestment risk. Moving from these bonds to coupon-paying bonds, the portfolio manager encounters the problem of how to select coupon-paying bonds that provide the best protection against immunization risk. The foregoing discussion indicates that the manager should select bonds which have most of their cash flow payments around the horizon date. Therefore, if the manager can construct a portfolio that replicates zero-coupon bonds that mature at the investment horizon, that portfolio will be the lowest immunization risk portfolio.

Now let us formalize the measure of immunization risk. As explained earlier, the target accumulated value of an immunized portfolio is a lower bound on the terminal value of the portfolio at the investment horizon if yields on all maturities change by the same amount. If yields of different maturities change by different amounts, then the target accumulated value is not necessarily the lower bound on the investment value. Fong and Vasicek demonstrate that if

the yield curve changes in any arbitrary way, the relative change in the portfolio value depends on the product of two terms.⁹ The first term depends solely on the structure of the investment portfolio while the second term is a function of interest rate movement only. The second term characterizes the nature of the yield curve shift. Since this shift can be arbitrary, this term is an uncertain quantity and therefore outside of the control of the manager. However, the first term is under the control of the manager since it depends solely on the composition of the portfolio. This first term is thus a measure of risk for immunized portfolios and defined as follows:

$$\text{Immunization risk measure} = \frac{\text{PVCF}_1(1 - H)^2 + \text{PVCF}_2(2 - H)^2 + \dots + \text{PVCF}_n(n - H)^2}{\text{Initial investment value}}$$

where

PVCF_t = present value of the cash flow in period t discounted at the prevailing yield

H = length of the investment horizon

n = time to receipt of the last portfolio cash flow

The immunization risk measure agrees with the intuitive interpretation of risk discussed earlier. For portfolio A in Exhibit 9, the barbell portfolio, the portfolio payments are widely dispersed in time and the immunization risk measure would be high. The portfolio payments occur close to the investment horizon for portfolio B in Exhibit 9, the bullet portfolio, so that the immunization risk measure is low.

Given the measure of immunization risk that is to be minimized and the constraint that the duration of the portfolio equals the duration of the liability as well as any other applicable investment constraints, the optimal immunized portfolio can be found using optimization models.¹⁰

III. CONTINGENT IMMUNIZATION

Contingent immunization consists of identifying both the available immunization target rate and a lower **safety net level return** with which a client would be minimally satisfied or is a minimum required rate of return.¹¹ The manager can continue to pursue an active strategy until an adverse investment experience drives the then available potential return—combined active return (from actual past experience) and immunized return (from expected future experience)—down to the safety net level; at such time the manager would be obligated to completely immunize the portfolio and lock in the safety net level return. As long as this safety net return is not violated, the manager can continue to actively manage the portfolio. Once the immunization mode is activated because the safety net return is violated, the manager can no longer return to the active mode unless the contingent immunization plan is abandoned.

⁹Fong and Vasicek, "A Risk Minimizing Strategy for Portfolio Immunization."

¹⁰More specifically, linear programming can be employed because the risk measure is linear in the portfolio payments.

¹¹Martin L. Leibowitz and Alfred Weinberger, "Contingent Immunization—Part I: Risk Control Procedures," *Financial Analysts Journal* (November-December 1982), pp. 17–31.

A. Key Considerations

The key considerations in implementing a contingent immunization strategy are:

1. establishing well defined immunized initial and ongoing available target returns
2. identifying a suitable and immunizable safety net
3. implementing an effective monitoring procedure to ensure that the safety net return is not violated

B. An Illustration

To illustrate the basic principles of the contingent immunization strategy suppose a plan sponsor is willing to accept a 6% return over a 5-year investment horizon at a time when the possible immunized rate of return is 7.5%. The 6% rate of return is called the safety net (or minimum target or floor) return. The difference between the possible immunized rate of 7.5% and the safety net return is called the **cushion spread** or **excess achievable return**. It is this cushion spread of 150 basis points in our example that offers the manager latitude in pursuing an active strategy. The greater the cushion spread the more scope the manager has for an active management policy.

Assuming an initial portfolio of \$100 million, the required terminal asset value when the safety net return is 6% is \$134.39 million. The general formula for the required terminal value assuming semiannual compounding is:

$$\text{Required terminal value} = I(1 + s/2)^{2H}$$

where

I = initial portfolio value

s = safety net rate

H = number of years in the investment horizon

In our example, the initial portfolio value is \$100 million, s is 6%, and H is five years. Therefore, the required terminal value is:

$$\$100 \text{ million}(1.03)^{10} = \$134.39 \text{ million}$$

Since the current available return is assumed to be 7.5%, the assets required at the inception of the plan in order to generate the required terminal value of \$134.39 million are \$93 million. This is found as follows assuming semiannual compounding. The required assets at any given point in time, t , necessary to achieve the required terminal value are:

$$\text{Required assets at time } t = \frac{\text{Required terminal value}}{(1 + i_t)^{2(H-t)}}$$

where

i_t = the immunized semiannual yield available at time t

The other variables were defined earlier.

Since in our example the required terminal value is \$134.39 million and the market yield that can be realized if the immunization mode is activated is 7.5%, the required assets are:

$$\frac{\$134.39 \text{ million}}{(1.0375)^{10}} = \$93 \text{ million}$$

Consequently, the safety cushion of 150 basis points translates into an initial dollar safety margin of \$7 million (\$100 million – \$93 million).

Now suppose that the portfolio manager invested the initial \$100 million in a portfolio of 30-year par bonds with a coupon of 7.5%. Let's look at what happens if there is a change in the yield level immediately following the purchase of these bonds.

First assume the yield level decreases to 5.6% from 7.5%. The value of the portfolio of 30-year bonds would increase to \$127.46 million. However, the asset value required to achieve the required terminal value if the portfolio is immunized at a 5.6% rate is \$102 million. This is found by using the previous formula. The required terminal value is \$134.39 million and the market yield for immunizing (following the yield change) is 5.6%, therefore:

$$\frac{\$134.39 \text{ million}}{(1.028)^{10}} = \$101.96 \text{ million}$$

The amount by which the \$127.46 million exceeds the required asset value (i.e., the dollar safety margin) in this case is \$25.5 million (\$127.46 million – \$101.96 million). This amount is \$18.46 million greater than the initial dollar safety margin of \$7 million and therefore allows the manager more freedom to pursue active management.

Suppose instead of a decline in the yield level the immediate change is an increase in the yield level to 8.6%. At that yield level the portfolio of 30-year bonds would decline in value to \$88.23 million. The required asset value to achieve the terminal value of \$134.39 million is \$88.21 million. Consequently, an immediate rise in the yield by 110 basis points to 8.6% will decrease the dollar safety margin to zero. At this yield level the immunization mode would be triggered with an immunization target rate of 8.6% to ensure that the required terminal value will be realized. If this were not followed, further adverse movements of interest rates would jeopardize the required terminal value for the portfolio of \$134.39 million. The yield level at which the immunization mode becomes necessary is called the **trigger point**.

C. Controlling and Monitoring the Strategy

For purposes of monitoring a contingent immunization plan, it is useful to recast the dollar safety margin in terms of the potential return. This return, also called the **return achievable with an immunization strategy**, measures the yield that would be realized if, at any given point in time, the current value of the portfolio is immunized at the prevailing market yield.

Since duration approximates the price sensitivity of a portfolio to changes in market yields, trigger yields can be computed for portfolios of different durations so that the manager would know how much leeway there is for a given risk position with respect to an adverse movement in the market yield; that is, how much of an adverse movement in the market yield can be tolerated before the immunization mode must be activated. However, when using duration to compute trigger points it is important to remember the limitation of duration—it assumes a small parallel shift in the yield curve. In practice, the specific rate movements need

to be monitored because the skewness of any particular movement should be expected to change the portfolio's trigger point.

The key to a contingent immunization plan is the ability to control and monitor the performance of the portfolio over time so that the manager knows how much leeway he has to actively manage the portfolio, and when the portfolio should be immunized in order to achieve the minimum target return.

An accurate immunization target is critical in determining not only the basis for the initial problem set-up (e.g., the safety net return will usually be a certain basis point difference from the target over a specified time period), but also in determining what immunization levels are available over the investment horizon. A safety net return too close to the initial target return makes triggering the immunization process highly likely, while too low a safety net defeats the purpose of the process since the very low satisfactory minimum return would probably never trigger immunization. Finally, without an adequate monitoring procedure, the benefits of the strategy may be lost because of the inability to know when action is appropriate.

In spite of good control and monitoring procedures, attainment of the minimum target return may not be realized due to factors beyond the control of the manager. There are two reasons for this.¹² The first is that there is the possibility of a rapid adverse large movement in market yields that the manager does not have enough time to shift from an active to an immunization strategy at the rate needed to achieve the minimum target return. Frequent jumps of market yields of several hundred basis points would hinder the effective implementation of a contingent immunization strategy. The second reason why the minimum target return may not be attained is that, if the immunization strategy is implemented, there is no guarantee that the immunized rate will be achieved even if the portfolio is reconstructed at the required rate.

IV. IMMUNIZATION FOR MULTIPLE LIABILITIES

Immunization with respect to a single investment horizon is applicable to situations where the objective of the investment is to preserve the value of the investment at the horizon date. This may be the case when a single given liability is payable at the horizon date or a target investment value is to be attained at that date. More often, however, investment funds pay a number of liabilities and no single horizon corresponds to the schedule of liabilities (i.e., multiple liabilities). Examples of such liabilities are pension fund payments, insurance policy annuity payments, and contractual payments under a structured settlement in a legal case.

Two strategies can be employed in seeking to satisfy these liabilities. The first is an extension of the single-period immunization strategy discussed earlier. The second is a cash flow matching strategy. We discuss the immunization strategy first.

A portfolio is said to be immunized with respect to a given liability stream if there are enough funds to pay all of the liabilities when due even if interest rates change by a parallel shift. However having the portfolio duration match the liabilities duration is not sufficient for immunizing multiple liabilities.¹³

¹²Martin L. Leibowitz and Alfred Weinberger, "Contingent Immunization—Part II: Problem Areas," *Financial Analysts Journal* (January-February 1983).

¹³G.O. Bierwag, George G. Kaufman, and Alden Toevs, "Immunization for Multiple Planning Periods," unpublished paper, Center for Capital Market Research, University of Oregon, October 1979.

A. Conditions for Immunizing Multiple Liabilities

The necessary and sufficient conditions to assure multiple liability immunization in the case of parallel rate shifts are as follows:¹⁴

- The *first condition* is that the present value of the assets must equal the present value of the liabilities.
- The *second condition* is that the (composite) portfolio duration must equal the (composite) liabilities duration.

The composite liabilities duration is a weighted average, found as follows:

$$\frac{(1)\text{PVL}_1 + (2)\text{PVL}_2 + \dots + (m)\text{PVL}_m}{\text{Total present value of liabilities}}$$

where

$$\begin{aligned}\text{PVL}_t &= \text{present value of the liability occurring at time } t \\ m &= \text{time of the last liability payment}\end{aligned}$$

- The *third condition* is that the distribution of durations of individual assets in the portfolio must have a wider range than the distribution of the liabilities.

An implication of the second condition is that if there are liabilities beyond 30 years, it is not necessary to have a duration for the portfolio that is 30 years in order to immunize the entire liability stream. The condition requires the manager construct a portfolio so the portfolio duration matches the weighted average of the liability durations. This is important because in any reasonable interest rate environment it is unlikely that a portfolio of investment-grade coupon bonds can be constructed with a duration in excess of 15 years. However, in a corporate pension fund situation, the liability stream is typically a diminishing amount liability stream. That is, liabilities in the earlier years are the greatest and liabilities further out toward the 30-year end are generally lower. By taking a weighted average duration of the liabilities, the manager can usually construct the portfolio duration at something that is more manageable, say, 8 or 9.

The third condition requires that when selecting securities to be included in a portfolio, the manager not only has to keep track of matching assets and liabilities duration, but also must track the specified distribution of portfolio assets.

The three conditions for multiple liability immunization assure immunity against parallel rate shifts only. In a series of articles, Reitano has explored the limitations of the parallel shift assumption.¹⁵ He has also developed models that generalize the immunization of multiple liabilities to arbitrary yield curve shifts. His research makes it clear that classical multiple-period

¹⁴H. Gifford Fong and Oldrich A Vasicek, "A Risk Minimizing Strategy for Portfolio Immunization," *Journal of Finance* (December 1984), pp. 1541–1546.

¹⁵Robert R. Reitano, "A Multivariate Approach to Immunization Theory," *Actuarial Research Clearing House*, Vol. 2 (1990), and "Multivariate Immunization Theory," *Transactions of the Society of Actuaries*, Vol. XLIII, 1991. For a detailed illustration of the relationship between the underlying yield curve shift and immunization, see Robert R. Reitano, "Non-Parallel Yield Curve Shifts and Immunization," *Journal of Portfolio Management* (Spring 1992), pp. 36–43.

immunization can disguise the risks associated with nonparallel yield curve shifts and that a model that protects against one type of yield curve shift may introduce reinvestment and price risk to other interest rate shifts.

B. Acceptable Universe of Securities and the Cost of Immunizing

When discussing the actual process to construct a single-period immunization strategy, several factors affecting portfolio return were reviewed. Effectively, the greater the credit risk the manager is willing to accept, the higher the potential yield that can be locked in when immunizing a single liability. Equivalently, the higher the potential yield, the lower the cost of immunizing a single-period liability (i.e., the lower the present value of the liability).

The situation is the same when seeking to immunize multiple liabilities. However, a portfolio manager does not think in terms of a single “yield” that can be locked in via immunization since there are multiple liabilities. Instead, a portfolio manager thinks in terms of the cost of creating the initial immunized portfolio. Again, the greater the credit risk that the portfolio manager/client is willing to accept, the lower the cost of the immunized portfolio. In fact, it is not simply credit risk where the portfolio manager can pick up additional potential yield. If a portfolio manager/client is willing to accept call risk or prepayment risk, for example, there is potential to create a lower cost immunized portfolio.

C. Immunizing Defined Benefit Obligations

Sponsors of defined benefit plans represent the major potential users of the multiple liability immunization strategy.¹⁶ As just explained, a plan sponsor will work with a portfolio manager in deciding the universe of securities that are permitted in the immunized portfolio. In turn, this depends on the degree of risk the plan sponsor is willing to accept.

A special case of immunization for multiple liabilities is when the plan sponsor wants to immunize the surplus of the defined benefit plan against an adverse movement in interest rates. Recall that surplus is the difference between the present value of the plan assets and the present value of the liabilities. When interest rates change, this will affect the value of the plan assets and the plan liabilities. To determine the value of the liabilities of a defined benefit plan a suitable interest rate or interest rates must be employed. Let’s look at this issue of valuing liabilities.

First, it should be clear that the liabilities should not be discounted at a single interest rate for the same reason that it is inappropriate to discount cash inflows for an asset using a single interest rate. Each liability is unique in terms of when it is paid in the future and therefore requires its own interest rate—the spot rate. There is no controversy in the finance literature about the use of spot rates for discounting liabilities. Where there is a controversy is the spot rates that should be used.

Historically, in determining the value of liabilities actuaries would somehow figure out what single interest rate they believe that a plan sponsor could earn on plan assets. That single interest rate would then be used to determine the present value of the plan liabilities. Not only is the valuation of liabilities using a single interest rate wrong, but the interest rate was subjectively determined (i.e., it was not market determined). This approach to determining the value of plan liabilities is unacceptable.

¹⁶In a defined benefit plan, the projected benefit payments estimated by an actuary represent the legal obligation of the plan sponsor.

There are two yield curves that have been proposed for valuing liabilities based on market yields and multiple interest rates: (1) the Treasury spot rate curve and (2) the Treasury yield curve plus a spread. In the United States, this issue is unresolved as of year-end 2003. Advocates of the Treasury spot rate curve approach to valuing liabilities argue that liabilities should be reported for financial reporting purposes based on how much it would cost the plan sponsor to pay them off today by buying a portfolio of zero-coupon Treasuries.¹⁷ That is, if a corporate sponsor of a defined benefit pension plan became bankrupt and the liabilities had to be paid off, this would be the cost to pay off those liabilities without any risk that they would not be satisfied.

Those who advocate a spread over the Treasury yield curve—an approach that has been advocated by the U.S. Department of the Treasury—argue that it is reasonable to expect a return could be earned on plan assets greater than that offered on Treasury securities in the long run. More specifically, an investment-grade rated corporate bond index yield curve is mentioned as the appropriate interest rates at which to discount liabilities. There are at least three problems with this approach. First, the yields are for coupon-bearing instruments and therefore are not spot rates. Second, using a corporate bond index yield curve means that as corporate credit spreads in the market increase (assuming Treasury yields do not decline to offset this), the value of plan liabilities will decrease. This does not make economic sense. Third, there is credit risk and therefore no assurance that the liabilities will be satisfied.¹⁸

V. CASH FLOW MATCHING FOR MULTIPLE LIABILITIES

Cash flow matching is an alternative to immunizing portfolio to match liabilities. It is an intuitively appealing strategy because the manager need only select securities to match liabilities.

Cash flow matching can be described intuitively as follows. A bond is selected with a maturity that matches the last liability. The amount invested in this bond is such that the principal plus final coupon payment is equal to the last liability. The remaining elements of the liability stream are then reduced by the coupon payments on this bond, and another bond is chosen for the next to last liability, adjusted for any coupon payments of the first bond selected. Going backward in time, this sequence is continued until all liabilities have been matched by payments on the securities selected for the portfolio. Exhibit 10 provides a simple illustration of this process for a 5-year liability stream. Linear programming techniques can be employed to construct a least-cost cash flow matching portfolio from an acceptable universe of bonds.

A. Cash Flow Matching Versus Multiple Liability Immunization

In the special case where all of the liability flows were perfectly matched by the asset flows of the portfolio, the resulting portfolio would have no reinvestment risk and, therefore, no

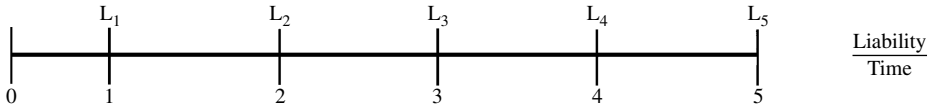
¹⁷Ronald Ryan and Frank J. Fabozzi, “Rethinking Pension Liabilities and Asset Allocation,” *Journal of Portfolio Management* (Summer 2002), pp. 7–15.

¹⁸Moreover, as of the end of 2003, the corporate bond index yield curves that had been advocated are not good proxies for corporate bond market yields. For a more detailed discussion of the problems with this approach, see Ronald Ryan and Frank J. Fabozzi, “Pension Fund Crisis Revealed,” *Journal of Investing* (Fall 2003), pp. 43–48.

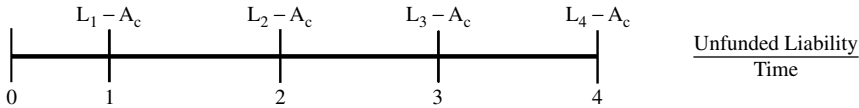
EXHIBIT 10 Illustration of Cash Flow Matching Process

Assume: 5-year liability stream

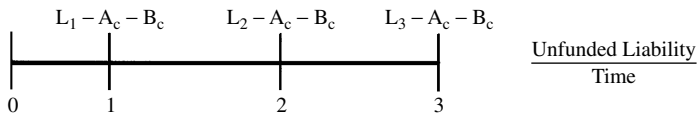
Cash flow from bonds are annual.

**Step 1:**Cash flow from Bond A selected to satisfy L_5 Coupons = A_c ; Principal = A_p and $A_c + A_p = L_5$

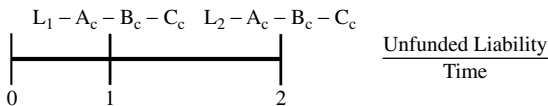
Unfunded liabilities remaining:

**Step 1:**Cash flow from Bond B selected to satisfy L_4 Unfunded liability = $L_4 - A_c$ Coupons = B_c ; Principal = B_p and $B_c + B_p = L_4 - A_c$

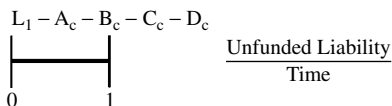
Unfunded liabilities remaining:

**Step 3:**Cash flow from Bond C selected to satisfy L_3 Unfunded liability = $L_3 - A_c - B_c$ Coupons = C_c ; Principal = C_p and $C_c + C_p = L_3 - A_c - B_c$

Unfunded liabilities remaining:

**Step 4:**Cash flow from Bond D selected to satisfy L_2 Unfunded liability = $L_2 - A_c - B_c - C_c$ Coupons = D_c ; Principal = D_p and $D_c + D_p = L_2 - A_c - B_c - C_c$

Unfunded liabilities remaining:

**Step 5:**Select Bond E with a cash flow of $L_1 - A_c - B_c - C_c - D_c$

immunization or cash flow matching risk. However, given typical liability schedules and bonds available for cash flow matching, perfect matching is unlikely. Under such conditions, a minimum immunization risk approach would, at worst, be equal to cash flow matching and would probably be better, because an immunization strategy is less costly to fund liabilities. This is due to two factors.

First, a relatively conservative rate of return assumption for short-term cash, which may occasionally be substantial, must be made throughout the life of the plan in cash flow matching, whereas an immunized portfolio is essentially fully invested at the remaining horizon duration. Second, funds from a cash flow matched portfolio must be available when each liability is due and, because of the difficulty in perfect matching, usually before. An immunized portfolio need only have sufficient value on the date of each liability because funding is achieved by a rebalancing of the portfolio. Because the reinvestment assumption for excess cash for cash flow matching is for many years into the future, a conservative assumption is appropriate.

Thus, even with the sophisticated linear programming techniques used in cash flow matching, in most cases it will be more costly than immunization. However, cash flow matching is easier to understand than multiple liability immunization, and this has occasionally led to its selection as the strategy for meeting liabilities (i.e., called **dedicated portfolio strategies**).

B. Extensions of Basic Cash Flow Matching

In the basic cash flow matching technique described above, only asset cash flows occurring prior to a liability date can be used to satisfy the liability. The technique can be extended to handle situations in which cash flows occurring both before and after the liability date can be used to meet a liability.

A popular variation of multiple liability immunization and cash flow matching to fund liabilities is one that combines the two strategies. This strategy, referred to as **combination matching** or **horizon matching**, creates a portfolio that is duration-matched with the added constraint that it be cash matched in the first few years, usually five years. The advantage of combination matching over multiple liability immunization is that liquidity needs are provided for in the initial cash flow matched period. Also, most of the positive slope or inversion of a yield curve tends to take place in the first few years. Cash flow matching the initial portion of the liability stream reduces the risk associated with nonparallel shifts of the yield curve. The disadvantage of combination matching over multiple liability immunization is that the cost is greater.

RELATIVE-VALUE METHODOLOGIES FOR GLOBAL CREDIT BOND PORTFOLIO MANAGEMENT*

I. INTRODUCTION

Corporate bonds are the second oldest and, for most asset managers, the most demanding and fascinating subset of the global debt capital markets. The label, “corporate,” understates the scope of this burgeoning asset class. As commonly traded and administered within the context of an overall debt portfolio, the “corporate asset class” actually encompasses much more than pure corporate entities. Instead of the title, “corporate asset class,” this segment of the global bond market really should be classified as the “credit asset class,” including any nonagency mortgage-backed securities (MBS), commercial mortgage-backed securities (CMBS), asset-backed securities (ABS). Sovereigns and government-controlled entities with foreign currency debt issues thought to have more credit risk than the national government should also be included. In keeping with conventional practice in the fixed-income market however, the application of the term “credit asset class” in this chapter will pertain only to corporate bonds, sovereigns, and government-controlled entities.

From six continents, thousands of organizations (corporations, government agencies, projects and structured pools of debt securities) with different credit “stories” have sold debt to sustain their operations and to finance their expansion. These borrowers use dozens of different types of debt instruments (first mortgage bonds, debentures, equipment trust certificates, subordinated debentures, medium-term notes, floating rate notes, private placements, preferred stock) and in multiple currencies (dollars, yen, euros, Swiss francs, pounds) from maturities ranging from one year to even a thousand years. Sometimes these debt structures carry embedded options, which may allow for full or partial redemption prior to maturity at the option of either the borrower or the investor. Sometimes, the coupon payment floats with short-term interest rates or resets to a higher rate after a fixed interval or a credit rating change.

*This chapter is authored by Jack Malvey, CFA.

Investors buy credit assets because of the presumption of higher long-term returns despite the assumption of credit risk. Except near and during recessions, credit products usually outperform U.S. Treasury securities and other higher-quality “spread sectors” like U.S. agency securities, mortgage-backed securities, and asset-backed securities. In the 30-year period since the beginning of the Lehman indices (1973 through 2002), investment-grade credit outperformed U.S. Treasuries by 30 basis points (bp) per year on average (9.42% versus 9.12%).¹ As usual, an average masks the true daily, weekly, monthly, and annual volatility of credit assets relative performance. Looking at the rolling 5-year excess returns of U.S. investment-grade credit from 1926 through early 2003 in Exhibit 1, one can observe extended periods of generous and disappointing returns for credit assets. Perhaps more meaningful, an examination of volatility-adjusted (Sharpe ratio) excess returns over Treasuries over a rolling 5-year period shown in Exhibit 2 further underscores the oscillations in relative credit performance.

Global credit portfolio management presents a complex challenge. Each day, hundreds of credit portfolio managers face thousands of choices in the primary (new issue) and secondary markets. In addition to tracking primary and secondary flows, investors have to keep tabs on ever-varying issuer fundamentals, creditworthiness, acquisitions, earnings, ratings, etc. The task of global credit portfolio management is to process all of this rapidly changing information about the credit markets (issuers, issues, dealers, and competing managers) and to construct the portfolio with the best return for a given risk tolerance. This discipline combines the qualitative tools of equity analysis with the quantitative precision of fixed-income analysis. This chapter provides a brief guide to methodologies that may help portfolio managers meet this formidable challenge.

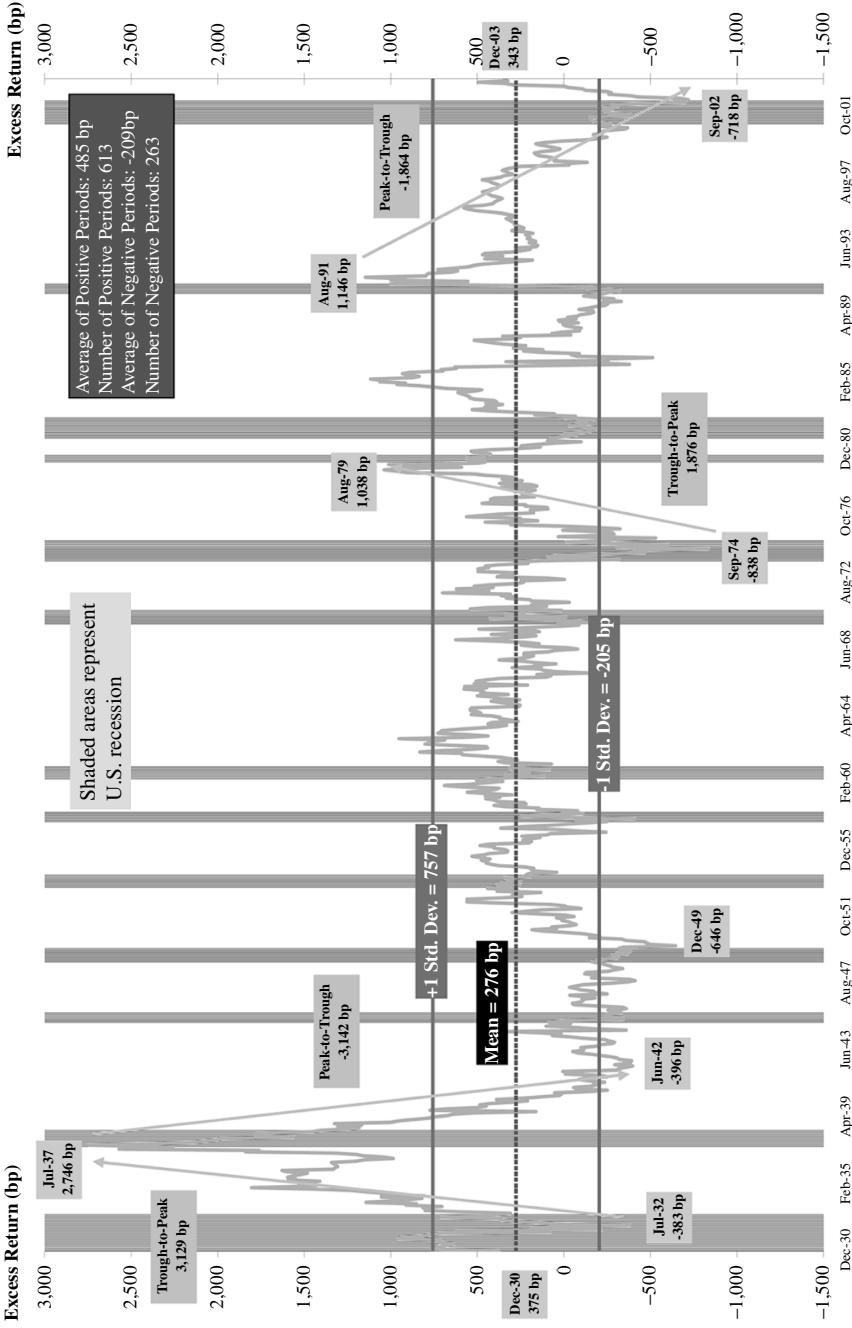
II. CREDIT RELATIVE-VALUE ANALYSIS

Credit portfolio management represents a major subset of the multi-asset global portfolio management process illustrated in Exhibit 3. After setting the currency allocation (in this case, dollars were selected for illustration convenience) and distribution among fixed-income asset classes, bond managers are still left with a lengthy list of questions to construct an optimal credit portfolio. Some examples are:

- Should U.S. investors add U.S. dollar-denominated bonds of non-U.S. issuers?
- Should central banks add high-quality euro-denominated corporate bonds to their reserve holdings?
- Should LIBOR-funded London-based portfolio managers buy fixed-rate U.S. industrial paper and swap into floating-rate notes?
- Should Japanese mutual funds own euro-denominated telecommunications debt, swapped back into dollars or yen using currency swaps?
- Should U.S. insurers buy perpetual floaters (i.e., floaters without a maturity date) issued by British banks and swap back into fixed-rate coupons in dollars using a currency/interest rate swap?
- When should investors reduce their allocation to the credit sector and increase allocation to governments, pursue a “strategic upgrade trade” (sell Baa/BBBs and buy higher-rated Aa/AA credit debt), rotate from industrials into utilities, switch from consumer cyclicals to

¹Based on absolute returns of key Lehman indices from 1973.

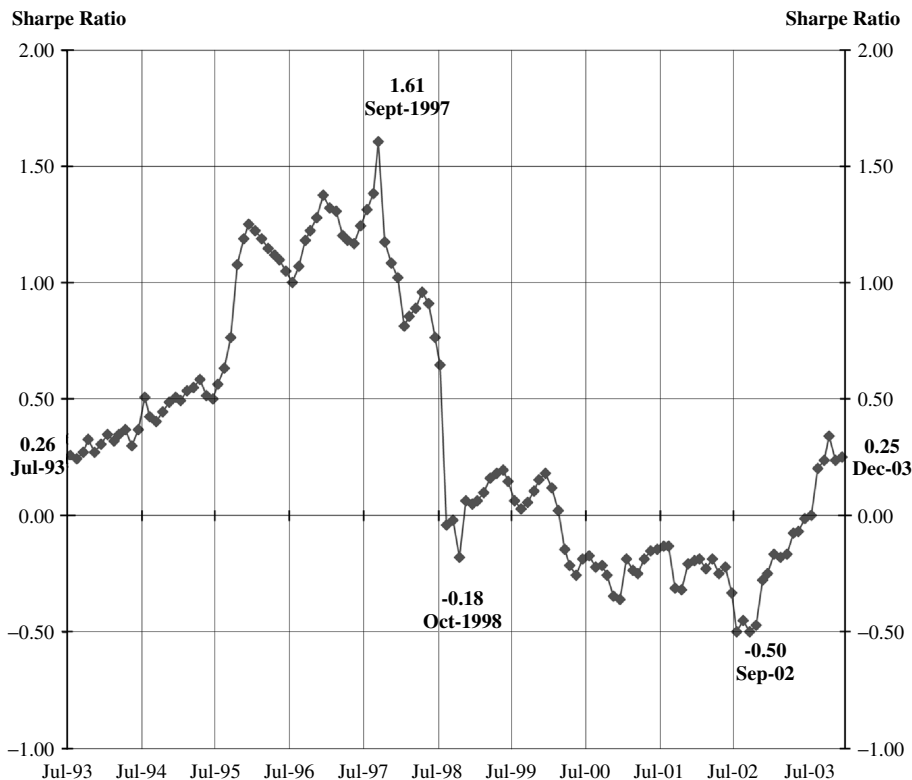
EXHIBIT 1 Rolling 5-Year U.S. Investment-Grade Credit Index Excess Returns* (bp) January 1926 through December 31, 2003



*Excess returns represent the difference, positive or negative, between the total return of all credit securities and Treasury securities along a set of key rate duration points across the term structure. This single statistic, excess return, therefore normalizes for the duration differential among debt asset classes, in this case between longer-duration credit and shorter duration Treasuries.

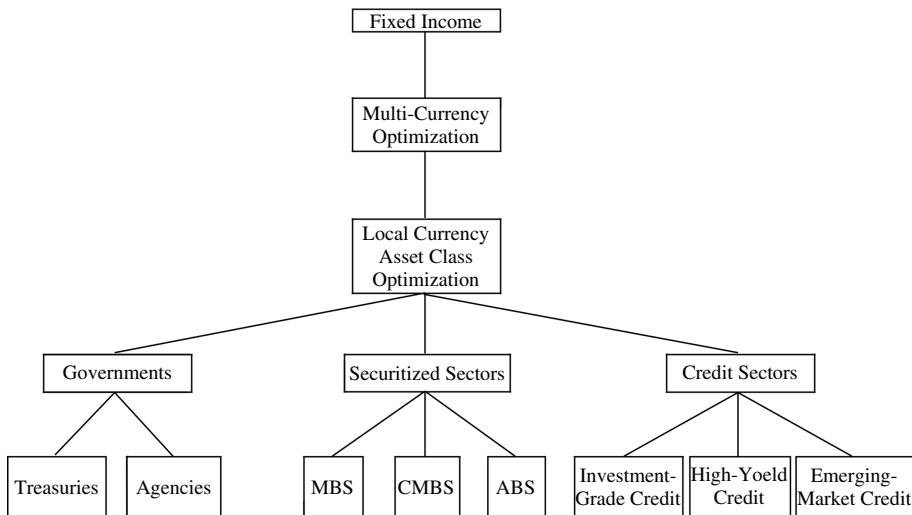
Source: Data series from Ibbotson Associates prior to August 1988, Lehman Brothers data thereafter

EXHIBIT 2 U.S. Credit 5-Year Rolling Sharpe Ratio: July 1993 through December 31, 2003



Source: Lehman Brothers U.S. Investment-Grade Credit Index

EXHIBIT 3 Fixed Income Portfolio Management Process



non-cyclicals, overweight airlines and underweight telephones, or deploy a credit derivative² (e.g., short the high-yield index or reduce a large exposure to a single issuer by selling an issuer-specific credit default swap) to hedge their portfolios?

To respond to such questions, managers need to begin with an analytical framework (relative-value analysis) and to develop a strategic outlook for the global credit markets.

A. Relative Value

Economists have long debated the concept and measurement of “value.” But fixed-income practitioners, perhaps because of the daily pragmatism enforced by the markets, have developed a consensus about the definition of value. In the bond market, **relative value** refers to *the ranking of fixed-income investments by sectors, structures, issuers, and issues in terms of their expected performance during some future period of time.*

For a day trader, relative value may carry a maximum horizon of a few minutes. For a dealer, relative value may extend from a few days to a few months. For a total return investor, the relative value horizon typically runs from 1–3 months. For a large insurer relative value usually spans a multi-year horizon. Accordingly, **relative-value analysis** refers to the methodologies used to generate such rankings of expected returns.

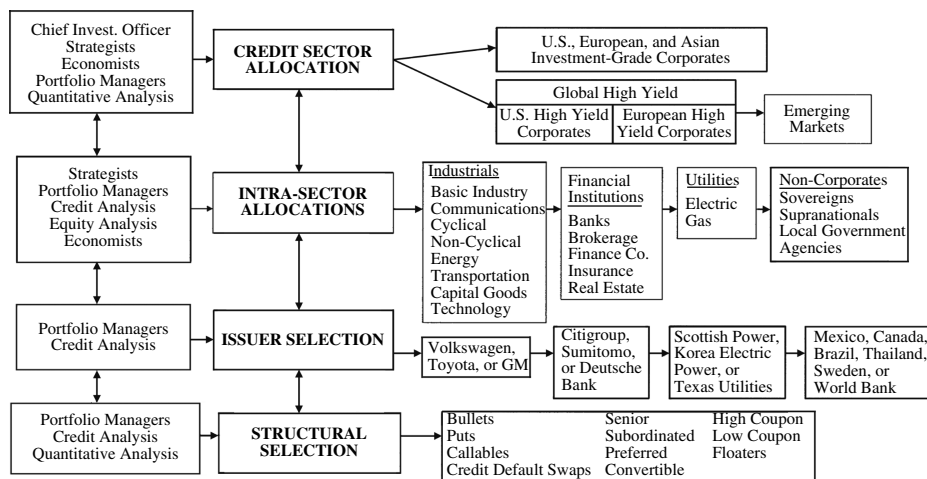
B. Classic Relative-Value Analysis

There are two basic approaches to global credit bond portfolio management—**top-down approach** and **bottom-up approach**. The top-down approach focuses on high-level allocations among broadly defined credit asset classes. The goal of top-down research is to form views on large-scale economic and industry developments. These views then drive asset allocation decisions (overweight certain sectors, underweight others). The bottom-up approach focuses on individual issuers and issues that will outperform their peer groups. Managers follow this approach hoping to outperform their benchmark due to superior security selection, while maintaining neutral weightings to the various sectors in the benchmark.

Classic relative-value analysis is a dialectical process combining the best of top-down and bottom-up approaches as shown in Exhibit 4. This process blends the macro input of chief investment officers, strategists, economists, and portfolio managers with the micro input of credit analysts, quantitative analysts, and portfolio managers. The goal of this methodology is to pick the sectors with the most potential upside, populate these favored sectors with the best representative issuers, and select the structures of the designated issuers at the yield curve points that match the investor’s for the benchmark yield curve.

For many credit investors, using classic relative-value analysis provides a measure of portfolio success. Although sector, issuer, and structural analyses remain the core of superior relative-value analysis, the increased availability of information and technology has transformed the analytical process into a complex discipline. Credit portfolio managers have far more data than ever on the total returns of sectors, issuers, and structures, quantity and composition of new-issue flows, investor product demand, aggregate credit-quality movements, multiple sources of fundamental and quantitative credit analyses on individual issuers, and yield spread data to assist them in their relative-value analysis.

²Credit derivatives are discussed in Chapter 24.

EXHIBIT 4 Credit Sector Portfolio Management Process: Classic, Dialectical Relative Value Analysis

C. Relative-Value Methodologies

The main methodologies for credit relative-value maximization are:

- total return analysis
- primary market analysis
- liquidity and trading analysis
- secondary trading rationales and constraints analysis
- spread analysis
- structure analysis
- credit curve analysis
- credit analysis
- asset allocation/sector analysis

In the sections that follow, we discuss each of these methodologies.

III. TOTAL RETURN ANALYSIS

The goal of global credit portfolio management for most investors is to optimize the risk-adjusted total return of their credit portfolio. The best place to start is naturally total return analysis. Accordingly, credit relative-value analysis begins with a detailed dissection of past returns and a projection of expected returns. For the entire asset class and major contributing sub-sectors (such as banks, utilities, pipelines, Baa/BBB's, etc.), how have returns been formed? How much is attributed to credit spread movements, sharp changes in the fundamental fortunes of key issuers, and yield curve dynamics? If there are macro determinants of credit returns (the total return of the credit asset class), then credit markets may display regular patterns. For instance, the macroeconomic cycle is the major determinant of overall credit spreads. During recessions, the escalation of default risk widens spreads (which are risk premiums over underlying, presumably default-free, government securities (or swaps))

and reduces credit returns relative to Treasuries. Conversely, economic prosperity reduces bankruptcies and enhances overall credit fundamentals of most issuers. Economic prosperity usually leads to tighter credit spreads and boosts credit returns relative to Treasuries. For brief intervals, noncyclical technical factors can offset fundamentals. For example, the inversion of the U.S. Treasury yield curve in 2000 actually led to wider credit spreads and credit underperformance despite solid global economic growth and corporate profitability.

Thanks to the development of total return indices for credit debt (databases of prices, spreads, issuer, and structure composition), analyses of monthly, annual, and multi-year total returns have uncovered numerous patterns (i.e., large issue versus small issue performance variation, seasonality, election-cycle effects, and government benchmark auction effects) in the global credit market. Admittedly, these patterns do not always re-occur. But an awareness and understanding of these total-return patterns are essential to optimizing portfolio performance.

IV. PRIMARY MARKET ANALYSIS

The analysis of primary markets centers on new issue supply and demand. Supply is often a misunderstood factor in tactical relative-value analysis. Prospective new supply induces many traders, analysts, and investors to advocate a defensive stance toward the overall corporate market as well as toward individual sectors and issuers. Yet the premise, “supply will hurt spreads,” which may apply to an individual issuer, does not generally hold up for the entire credit market. Credit spreads are determined by many factors; supply, although important, represents one of many determinants. During most years, increases in issuance (most notably during the first quarter of each year) are associated with market-spread contraction and strong relative returns for credit debt. In contrast, sharp supply declines are accompanied frequently by spread expansion and a major fall in both relative and absolute returns for credit securities. For example, this counter-intuitive effect was most noticeable during the August-October 1998 interval when new issuance nearly disappeared in the face of the substantial increase in credit spreads. (This period is referred to as the “Great Spread-Sector Crash.”)

In the investment-grade credit market, heavy supply often compresses spreads and boost relative returns for credit assets as new primary valuations validate and enhance secondary valuations. When primary origination declines sharply, secondary traders lose reinforcement from the primary market and tend to reduce their bid spreads. Contrary to the normal supply-price relationship, relative credit returns often perform best during periods of heavy supply. For example, 2001 will be recalled for both the then all-time record for new credit origination as well as the best relative performance for U.S. credit securities in nearly two decades.

A. The Effect of Market-Structure Dynamics

Given their immediate focus on the deals of the day and week, portfolio managers often overlook short-term and long-term market-structure dynamics in making portfolio decisions. Because the pace of change in market structure is often gradual, market dynamics have less effect on short-term tactical investment decision-making than on long-term strategy.

The composition of the global credit bond market has shifted markedly since the early 1980s. Medium-term notes (MTN) dominate issuance in the front end of the credit yield curve. Structured notes and swap products have heralded the introduction of derivative instruments into the mainstream of the credit market. The high-yield corporate sector has become an accepted asset class. Global origination has become more popular for U.S. government agencies, supranationals (e.g., The World Bank), sovereigns, and large corporate borrowers.

Although the ascent of derivatives and high-yield instruments stands out during the 1990s, the true globalization of the credit market was the most important development. The rapid development of the Eurobond market since 1975, the introduction of many non-U.S. issuers into the dollar markets during the 1990s, and the birth of the euro on January 1, 1999, have led to the proliferation of truly transnational credit portfolios.

These long-term structural changes in the composition of the global credit asset class arise due to the desire of issuers to minimize funding costs under different yield curve and yield spread, as well as the needs of both active and asset/liability bond managers to satisfy their risk and return objectives. Portfolio managers will adapt their portfolios either in anticipation of or in reaction to these structural changes across the global credit markets.

B. The Effect of Product Structure

Partially offsetting this proliferation of issuers since the mid-1990s, the global credit market has become structurally more homogeneous. Specifically, bullet and intermediate-maturity structures have come to dominate the credit market. A **bullet maturity** means that the issue is not callable, puttable, or sinkable prior to its scheduled final maturity. The trend toward bullet securities does not pertain to the high-yield market, where callables remain the structure of choice. With the hope of credit-quality improvement, many high-yield issuers expect to refinance prior to maturity at lower rates.

There are three strategic portfolio implications for this structural evolution. First, the dominance of bullet structures translates into scarcity value for structures with embedded call and put features. That is, credit securities with embedded options have become rare and therefore demand a premium price. Typically, this premium (price) is not captured by option-valuation models. Yet, this “scarcity value” should be considered by managers in relative-value analysis of credit bonds.

Second, bonds with maturities beyond 20 years are a small share of outstanding credit debt. This shift reduced the effective duration of the credit asset class and cut aggregate sensitivity to interest-rate risk. For asset/liability managers with long time horizons, this shift of the maturity distribution suggests a rise in the value of long credit debt and helps to explain the warm reception afforded, initially at least, to most new offerings of issues with 100-year maturities in the early and mid-1990s.

Third, the use of credit derivatives has skyrocketed since the early 1990s. The rapid maturation of the credit derivative market will lead investors and issuers to develop new strategies to match desired exposures to credit sectors, issuers, and structures.

V. LIQUIDITY AND TRADING ANALYSIS

Short-term and long-term liquidity needs influence portfolio management decisions. Citing lower expected liquidity, some investors are reluctant to purchase certain types of issues such as small-sized issues (less than \$1.0 billion), private placements, MTNs, and non-local corporate issuers. Other investors gladly exchange a potential liquidity disadvantage for incremental yield. For investment-grade issuers, these liquidity concerns often are exaggerated.

As explained earlier, the liquidity of credit debt changes over time. Specifically, liquidity varies with the economic cycle, credit cycle, shape of the yield curve, supply, and the season. As in all markets, unknown shocks, like a surprise wave of defaults, can reduce credit debt liquidity as investors become unwilling to purchase new issues at any spread and dealers become reluctant to position secondary issues except at very wide spreads. In reality, these

transitory bouts of illiquidity mask an underlying trend toward heightened liquidity across the global credit asset class. With a gentle push from regulators, the global credit asset class is well along in converting from its historic “over-the-counter” domain to a fully transparent, equity/U.S. Treasury style marketplace. In the late 1990s, new technology led to creating ECNs (electronic communication networks), essentially electronic trading exchanges. In turn, credit bid/ask spreads generally have trended lower for very large, well-known corporate issues. This powerful twin combination of technological innovation and competition promises the rapid development of an even more liquid and efficient global credit market during the early 21st Century.

VI. SECONDARY TRADE RATIONALES

Capital market expectations constantly change. Recessions may arrive sooner rather than later. The yield curve may steepen rather than flatten. The auto and paper cycles may be moving down from their peaks. Higher oil and natural gas prices may benefit the credit quality of the energy sector. An industrial may have announced a large debt-financed acquisition, earning an immediate ratings rebuke from the rating agencies. A major bank may plan to repurchase 15% of its outstanding common stock (great for shareholders but leading to higher financial leverage for debtholders). In response to such daily information flows, portfolio managers amend their holdings. To understand trading flows and the real dynamics of the credit market, investors should consider the most common rationales of whether to trade and not to trade.

A. Popular Reasons for Trading

There are dozens of rationales to execute secondary trades when pursuing portfolio optimization. Several of the most popular are discussed below. The framework for assessing secondary trades is the total return framework explained in Chapter 18.

1. Yield/Spread Pickup Trades **Yield/spread pickup trades** represent the most common secondary transactions across all sectors of the global credit market. Historically, at least half of all secondary swaps reflect investor intentions to add additional yield within the duration and credit-quality constraints of a portfolio. If 5-year, Baa1/BBB General Motors paper trades at 150 bp, 10 bp more than 5-year, Baa1/BBB–Ford Motor, some investors will determine the rating differential irrelevant and purchase General Motors bond and sell the Ford Motor (an issue swap) for a spread gain of 10 bp per annum.

This “yield-first psychology” reflects the institutional yield need of long-term asset/liability managers. Despite the passage of more than three decades, this investor bias toward yield maximization also may be a methodological relic left over from the era prior to the introduction and market acceptance of total-return indices in the early-1970s. The limitations of yield measures as an indicator of potential performance were explained and the total return framework was demonstrated to be a superior framework for assessing potential performance for a trade.

2. Credit-Upside Trades **Credit-upside trades** take place when the debt asset manager expects an upgrade in an issuer’s credit quality that is not already reflected in the current market yield spread. In the illustration of the General Motors and Ford Motor trade described above, some investors may swap based on their view of potential credit-quality improvement for General Motors. Obviously, such trades rely on the credit analysis skills of the investment management team. Moreover, the manager must be able to identify a potential upgrade before

the market, otherwise the spread for the upgrade candidate will already exhibit the benefits of a credit upgrade.

Credit-upside trades are particularly popular in the crossover sector—securities with ratings between Ba2/BB and Baa3/BBB- by two major rating agencies. In this case, the portfolio manager is expressing an expectation that an issue of the highest speculative grade rating (Ba1/BB+) has sufficiently positive credit fundamentals to be upgraded to investment grade (i.e., Baa3/BBB-). If this upgrade occurs, not only would the issue's spread narrow based on the credit improvement (with an accompanying increase in total return, all else equal), but the issue also would benefit from improved liquidity, as managers prohibited from buying high-yield bonds could then purchase that issue. Further, the manager would expect an improvement in the portfolio's overall risk profile.

3. Credit-Defense Trades **Credit-defense trades** become more popular as geopolitical and economic uncertainty increase. Secular sector changes often generate uncertainties and induce defensive positioning by investors. In anticipating greater competition, in the mid-1990s some investors reduced their portfolio exposures to sectors like electric utilities and telecommunications. As some Asian currencies and equities swooned in mid-1997, many portfolio managers cut their allocation to the Asian debt market. Unfortunately because of yield-maximization needs and a general reluctance to realize losses by some institutions (i.e., insurers), many investors reacted more slowly to credit-defensive positioning. But after a record number of “fallen angels” in 2002, which included such major credit bellwether issuers as WorldCom, investors became more quick to jettison potential problem credits from their portfolios. Ironically once a credit is downgraded by the rating agencies, internal portfolio guidelines often dictate security liquidation immediately after the loss of single-A or investment-grade status. This is usually the worst possible time to sell a security and maximizes losses incurred by the portfolio.

4. New Issue Swaps **New-issue swaps** contribute to secondary turnover. Because of perceived superior liquidity, many portfolio managers prefer to rotate their portfolios gradually into more current and usually larger sized on-the-run issues. This disposition, reinforced by the usually superior market behavior of newer issues in the U.S. Treasury market (i.e., the on-the-run issues), has become a self-fulfilling prophecy for many credit issues. In addition, some managers use new issue swaps to add exposure to a new issuer or a new structure.

5. Sector-Rotation Trades **Sector-rotation trades**, within credit and among fixed-income asset classes, have become more popular since the early 1990s. In this strategy, the manager shifts the portfolio from a sector or industry that is expected to underperform to a sector or industry which is believed will outperform on a total return basis. With the likely development of enhanced liquidity and lower trading transaction costs across the global bond market in the early 21st Century, sector-rotation trades should become more prevalent in the credit asset class.

Such intra-asset class trading already has played a major role in differentiating performance among credit portfolio managers. For example, as soon as the Fed launched its preemptive strike against inflation in February 1994, some investors correctly exchanged fixed-rate corporates for floating-rate corporates. In 1995, the specter of U.S. economic weakness prompted some investors in high-yield corporates to rotate from consumer-cyclical sectors like autos and retailing into consumer non-cyclical sectors like food, beverage, and healthcare. Anticipating slower U.S. economic growth in 1998 induced a defensive tilt by some portfolio managers away from other cyclical groups like paper and energy. The resurrection of Asian and European

economic growth in 1999 stimulated increased portfolio interest in cyclicals, financial institutions, and energy debt. Credit portfolio managers could have avoided a great deal of portfolio performance disappointment in 2002 by underweighting utilities and many industrial sectors.

6. Curve Adjustment Trades **Yield curve-adjustment trades**, or simply, **curve-adjustment trades** are taken to reposition a portfolio's duration. For most credit investors, their portfolio duration is typically within a range from 20% below to 20% above the duration of the benchmark index. If credit investors could have predicted U.S., euro, and yen yield curve movements perfectly in 2002, then they would have increased their credit portfolio duration at the beginning of 2002 in anticipation of a decrease in interest rates. Although most fixed-income investors prefer to alter the duration of their aggregate portfolios in the more-liquid Treasury market, strategic portfolio duration tilts also can be implemented in the credit market.

This is also done with respect to anticipated changes in the credit term structure or credit curve. For example, if a portfolio manager believes credit spreads will tighten (either overall or in a particular sector), with rates in general remaining relatively stable, they might shift the portfolio's exposure to longer spread duration issues in that sector.

7. Structure Trades **Structure trades** involve swaps into structures (e.g., callable structures, bullet structures, and puttable structures) that are expected to have better performance given expected movements in volatility and the shape of the yield curve. Here are some examples of how different structures performed in certain periods in the 1990s.

- During the second quarter of 1995, the rapid descent of the U.S. yield curve contributed to underperformance of high-coupon callable structures because of their negative convexity property.
- When the yield curve stabilized during the third quarter of 1995, investors were more willing to purchase high-quality callable bonds versus high-quality bullet structures to earn an extra 35 bp of spread.
- The sharp downward rotation of the U.S. yield curve during the second half of 1997 contributed to poor relative performance by puttable structures. The yield investors had sacrificed for protection against higher interest rates instead constrained total return as rates fell.
- The plunge in U.S. interest rates and escalation of yield-curve volatility during the second half of 1998 again restrained the performance of callable structures compared to bullet structures.
- The upward rebound in U.S. interest rates and the fall in interest-rate volatility during 1999 contributed to the relative outperformance of callable structures versus bullet structures.

These results follow from the price/yield properties of the different structures that were explained previously. Structural analysis is also discussed in Section VIII of this chapter.

8. Cash Flow Reinvestment **Cash flow reinvestment** forces investors into the secondary market on a regular basis. During 2003, the sum of all coupon, maturity, and partial redemptions (via tenders, sinking funds, and other issuer prepayments) equaled approximately 100% of all new gross issuance across the dollar bond market. Before the allocation of any net new investment in the bond market, investors had sufficient cash flow reinvestment to absorb nearly all new bond supply. Some portfolio cash inflows occur during interludes in the primary market or the composition of recent primary supply may not be compatible

with portfolio objectives. In these periods, credit portfolio managers must shop the secondary market for investment opportunities to remain fully invested or temporarily replicate the corporate index by using financial futures. Portfolio managers who incorporate analysis of cash flow reinvestment into their valuation of the credit market can position their portfolios to take advantage of this cash flow reinvestment effect on spreads.

B. Trading Constraints

Portfolio managers also should review their main rationales for not trading. Some of the best investment decisions are not to trade. Conversely, some of the worst investment decisions emanate from stale views based on dated and anachronistic constraints (e.g., avoid investing in bonds rated below Aa/AA). The best portfolio managers retain very open minds, constantly self-critiquing both their successful and unsuccessful methodologies.

1. Portfolio Constraints Collectively, portfolio constraints are the single biggest contributor to the persistence of market inefficiency across the global credit market. Here are some examples:

- Because many asset managers are limited to holding securities with investment-grade ratings, they are forced to sell immediately the debt of issuers who are downgraded to speculative-gradings (Ba1/BB+ and below). In turn, this selling at the time of downgrade provides an opportunity for investors with more flexible constraints to buy such newly downgraded securities at a temporary discount (provided, of course, that the issuer's creditworthiness stabilizes after downgrade).
- Some U.S. state employee pension funds cannot purchase credit securities with ratings below A3/A—due to administrative and legislative guidelines.
- Some U.S. pension funds also have limitations on their ownership of MTNs and non-U.S. corporate issues.
- Regulators have limited U.S. insurance companies investment in high-yield corporates.
- Many European investors are restricted to issues rated at least single-A and sometimes Aa3/AA—and above, created originally in annual-pay Eurobond form.
- Many investors are confined to their local currency market—yen, sterling, euro, U.S. dollar. Often, the same issuer, like Ford, will trade at different spreads across different geographic markets.
- Globally, many commercial banks must operate exclusively in the floating-rate realm: all fixed-rate securities, unless converted into floating-rate cash flows via an interest rate swap, are prohibited.

2. “Story” Disagreement “Story” disagreement can work to the advantage or disadvantage of a portfolio manager. Traders, salespersons, sell-side analysts and strategists, and buy-side credit research have dozens of potential trade rationales that supposedly will benefit portfolio performance. The proponents of a secondary trade may make a persuasive argument, but the portfolio manager may be unwilling to accept the “shortfall risk”³ if the investment recommendation does not provide its expected return. For example in early 1998, analysts and investors alike were divided equally on short-term prospects for better valuations of

³Shortfall risk is explained in Chapter 17. Shortfall risk is the probability that the outcome will have a value less than the target return.

Asian sovereign debt. After a very disappointing 1997 for Asian debt performance, Asia enthusiasts had little chance to persuade pessimists to buy Asian debt at the beginning of 1998. Technically, such lack of consensus in the credit market signals an investment with great outperformance potential. Indeed, most Asian debt issues recorded exceptional outperformance over the full course of 1998 and 1999. After a difficult 2002, the same “rebound effect” was observed in electric utilities during 2003. Of course, “story” disagreement can also work in the other direction. For example, Enron was long viewed as a very solid credit before its sudden bankruptcy in late 2001. An asset manager wedded to this long-term view might have been reluctant to act on the emergence of less favorable information about Enron in the summer of 2001.

3. **Buy-and-Hold** Although many long-term asset/liability managers claim to have become more total return focused in the 1990s, accounting constraints (cannot sell positions at a loss compared with book cost or take too extravagant a gain compared with book cost) often limit the ability of these investors to trade. Effectively, these investors (mainly insurers) remain traditional “buy-and-hold” investors. Some active bond managers have converged to quasi-“buy-and-hold” investment programs at the behest of consultants to curb portfolio turnover. In the aftermath of the “Asian Contagion” in 1997–1998, this disposition toward lower trading turnover was reinforced by the temporary reduction in market liquidity provided by more wary bond dealers. As shown in 2000–2002, however, a buy-and-hold strategy can gravely damage the performance of a credit portfolio. At the first signs of credit trouble for an issuer, many credit portfolios would have improved returns by reducing their exposure to a deteriorating credit.

4. **Seasonality** Secondary trading slows at month ends, more so at quarter ends, and the most at the conclusion of calendar years. Dealers often prefer to reduce their balance sheets at fiscal year-end (November 30, December 31, or March 31 (Japan)). Also, portfolio managers take time to mark their portfolios, prepare reports for their clients, and chart strategy for the next investment period. During these intervals, even the most compelling secondary offerings can languish.

VII. SPREAD ANALYSIS

By custom, some segments of the high-yield and emerging (EMG) debt markets still prefer to measure value by bond price or bond yield rather than spread. But for the rest of the global credit market, nominal spread (the yield difference between corporate and government bonds of similar maturities) has been the basic unit of both price and relative-value analysis for more than two centuries.

A. Alternative Spread Measures

Many U.S. practitioners prefer to value investment-grade credit securities in terms of option-adjusted spreads (OAS) so they can be more easily compared to the volatility (“vol”) sectors (mortgage-backed securities and U.S. agencies).⁴ But given the rapid reduction of credit

⁴These sectors are referred to as “vol” sectors because the value of the securities depends on expected interest rate volatility. These “vol” securities have embedded call options and the value of the options, and hence the value of the securities, depends on expected interest rate volatility.

structures with embedded options since 1990 (see structural discussion above), the use of OAS in primary and secondary pricing has diminished within the investment-grade credit asset class. Moreover, the standard one-factor binomial models⁵ do not account for credit spread volatility. Given the exclusion of default risk in OAS option-valuation models, OAS valuation has seen only limited extension into the higher-risk markets of the quasi-equity, high-yield corporate, and EMG-debt asset classes.

Starting in Europe during the early 1990s and gaining momentum during the late 1990s, interest rate swap spreads have emerged as the common denominator to measure relative value across fixed- and floating-rate note credit structures. The U.S. investment-grade and high-yield markets eventually may switch to swap spreads to be consistent with Europe and Asia.

Other U.S. credit spread calculations have been proposed, most notably using the U.S. agency benchmark curve. These proposals emanate from the assumption of a persistent U.S. budgetary surplus and significant liquidation of outstanding U.S. Treasury securities during the first decade of the 21st Century. As again demonstrated by 2002, history teaches that these budget assumptions may unfortunately prove to be faulty. Although some practitioners may choose to derive credit-agency spreads for analytical purposes, this practice will be unlikely to become standard market convention.

Credit-default swap spreads have emerged as the latest valuation tool during the great stresses in the credit markets of 2000–2002. Most likely, credit-default swap spreads will be used as a companion valuation reference to nominal spreads, OAS, and swap spreads. The market, therefore, has an ability to price any credit instrument using multiple spread references. These include the spread measures discussed earlier—nominal spread, static or zero-volatility spread, OAS, credit-swap spreads (or simply swap spreads), and credit default spreads. The spread measures used the Treasury yield curve or Treasury spot rate curve as the benchmark. Given the potential that swap spreads will become the new benchmark, these same measures can be performed relative to swaps rather than relative to U.S. Treasuries. However, using swap rates as a benchmark has been delayed by the decoupling of traditional credit spreads (credit yield minus government yield) from swap spreads over 2000–2002. Effectively, credit risk during a global recession and its aftermath superseded the countervailing influence of strong technical factors like lower and steeper yield curves which affected the interest rate swap market differently.

B. Closer Look at Swap Spreads

Swap spreads became a popular valuation yardstick for credit debt in Europe during the 1990s. This practice was enhanced by the unique nature of the European credit asset class. Unlike its American counterpart, the European credit market has been consistently homogeneous. Most issuance was of high quality (rated Aa3/AA—and above) and intermediate maturity (10 years and less). Consequently, swap spreads are a good proxy for credit spreads in such a uniform market. Most issuers were financial institutions, natural swappers between fixed-rate and floating-rate obligations. And European credit investors, often residing in financial institutions like commercial banks, have been much more willing to use the swap methodology to capture value discrepancies between the fixed- and floating-rate markets.

Structurally, the Asian credit market more closely resembles the European than the U.S. credit market. As a result, the use of swap spreads as a valuation benchmark also became common in Asia.

⁵The model is referred to as a one-factor model because only the short-term rate is the factor used to construct the tree.

The investment-grade segment of the U.S. credit market may well be headed toward an embrace of swap spreads. The U.S. MBS, CMBS, agency, and ABS sectors (accounting for about 55% of the U.S. fixed-income market) made the transition to swap spreads as a valuation benchmark during the second half of the 1990s. Classical nominal credit spreads derived directly from the U.S. Treasury yield curve were distorted by the special effects of U.S. fiscal surpluses and buybacks of U.S. Treasury securities in 2000 and 2001. Accordingly, many market practitioners envision a convergence to a single global spread standard derived from swap spreads.

Here is an illustration of how a bond manager can use the interest rate swap spread framework. Suppose that a hypothetical Ford Motor Credit 7 1/2x's of 2008 traded at a bid price (i.e., the price at which a dealer is willing to buy the issue) of 113 bp over the 5-year U.S. Treasury yield of 6.43%. This equates to a yield-to-maturity of 7.56% (6.43% + 113 bp). On that date, 5-year swap spreads were 83 bp (to the 5-year U.S. Treasury). Recall that swaps are quoted where the fixed-rate payer pays the yield on a Treasury with a maturity equal to the initial term of the swap plus the swap spread. The fixed-rate payer receives LIBOR flat—that is, no increment over LIBOR. So, if the bond manager invests in the Ford Motor Credit issue and simultaneously enters into this 5-year swap, the following would result:

Receive from Ford Motor Credit (6.43% + 113 bp)	7.56%
– Pay on swap (6.43% + 83 bp)	7.26%
+ Receive from swap	LIBOR
Net	LIBOR + 30 bp

Thus, a bond manager could exchange this Ford Motor Credit bond's fixed coupon flow for LIBOR + 30 bp. On the trade date, LIBOR was 6.24%, so that the asset swapper would earn 6.54% (= 6.24% + 30 bp) until the first reset date of the swap. A total return manager would want to take advantage of this swap by paying fixed and receiving floating if he expects interest rates to increase in the future.

The swaps framework allows managers (as well as issuers) to more easily compare securities across fixed-rate and floating-rate markets. The extension of the swap spread framework may be less relevant for speculative-grade securities, where default risk becomes more important. In contrast to professional money managers, individual investors are not comfortable using bond valuation couched in terms of swap spreads. The traditional nominal spread framework is well understood by individual investors, has the advantages of long-term market convention, and works well across the entire credit-quality spectrum from Aaa's to B's. However, this nominal spread framework does not work very well for investors and issuers when comparing the relative attractiveness between the fixed-rate and floating-rate markets.

C. Spread Tools

Investors should also understand how best to evaluate spread levels in their decision-making. Spread valuation includes mean-reversion analysis, quality-spread analysis, and percent yield spread analysis.

1. Mean-Reversion Analysis The most common technique for analyzing spreads among individual securities and across industry sectors is **mean-reversion analysis**. The “mean” is the average value of some variable over a defined interval (usually one economic cycle for the credit market). The term “mean reversion” refers to the tendency for some variable's value to revert (i.e., move towards) its average value. Mean-reversion analysis is a form of relative-value

analysis based on the assumption that the spread between two sectors or two issuers will revert back to its historical average. This would lead investors to buy a sector or issuer identified as “cheap” because historically the spread has been tighter and will eventually revert back to that tighter spread. Also, this would lead investors to sell a sector or issuer identified as “rich” because the spread has been wider and is expected to widen in the future.

Mean-reversion analysis involves the use of statistical analysis to assess whether the current deviation from the mean spread is significant. For example, suppose the mean spread for an issuer is 80 basis points over the past six months and the standard deviation is 12 basis points. Suppose that the current spread of the issuer is 98 basis points. The spread is 18 basis points over the mean spread or equivalently 1.5 standard deviations above the mean spread. The manager can use that information to determine whether or not the spread deviation is sufficient to purchase the issue. The same type of analysis can be used to rank a group of issuers in a sector.

Mean-reversion analysis can be instructive as well as misleading. The mean is highly dependent on the interval selected. There is no market consensus on the appropriate interval and “persistence” frequents the credit market meaning cheap securities, mainly a function of credit uncertainty, often tend to become cheaper. Rich securities, usually high-quality issues, tend to remain rich.

2. Quality-Spread Analysis **Quality-spread analysis** examines the spread differentials between low and high-quality credits. For example, portfolio managers would be well advised to consider the “credit upgrade trade” discussed in Section VI when quality-spreads collapse to cyclical troughs. The incremental yield advantage of lower-quality products may not compensate investors for lower-quality spread expansion under deteriorating economic conditions. Alternatively, credit portfolio managers have long profited from overweighting lower-quality debt at the outset of an upward turn in the economic cycle.

3. Percent Yield Spread Analysis Dating from the early 20th Century, **percent yield spread analysis** (the ratio of credit yields to government yields for similar duration securities) is another popular technical tool used by some investors. This methodology has serious drawbacks that undermine its usefulness. Percent yield spread is more a derivative than an explanatory or predictive variable. The usual expansion of credit percent yield spreads during low-rate periods like 1997, 1998, and 2002 overstates the risk as well as the comparative attractiveness of credit debt. And the typical contraction of credit percent yield spreads during upward shifts of the benchmark yield curve does not necessarily signal an imminent bout of underperformance for the credit asset class. Effectively, the absolute level of the underlying benchmark yield is merely a single factor among many factors (demand, supply, profitability, defaults, etc.) that determine the relative value of the credit asset class. These other factors can offset or reinforce any insights derived from percent yield spread analysis.

VIII. STRUCTURAL ANALYSIS

As explained earlier in this chapter, there are bullet, callable, puttable, and sinking fund structures. **Structural analysis** is simply analyzing the performance of the different structures discussed throughout this chapter. While evaluating bond structures was extremely important in the 1980s, it became less influential in credit bond market since the mid-1990s for several reasons. First, the European credit bond market almost exclusively features intermediate

EXHIBIT 5 Changing Composition of the U.S. Investment-Grade Credit Markets*

	1990 (%)	2003 (%)
Bullets	24	94
Callables	72	3
Sinking Funds	32	1
Putables	5	2
Zeros	4	N/A

*Note: Figures in table do not add to 100% given that some structures may have contained multiple options (e.g., a callable corporate bond may also have a sinking fund and put provision).

Source: Lehman Brothers U.S. Investment-Grade Credit Index

bullets. Second, as can be seen in Exhibit 5, the U.S. credit and the global bond markets have moved to embrace this structurally homogeneous European bullet standard. Plenty of structural diversity still resides within the U.S. high yield and EMG debt markets, but portfolio decisions in these speculative-grade sectors understandably hinge more on pure credit differentiation than the structural diversity of the issue-choice set.

Still, structural analysis can enhance risk-adjusted returns of credit portfolios. As we discussed, leaving credit aside, issue structure analysis and structural allocation decisions usually hinge on yield curve and volatility forecasts as well as interpretation of option-valuation model outputs (see the discussion below). This is also a key input in making relative value decisions among structured credit issues, mortgage-backed securities, and asset-backed securities. In the short run and assuming no change in the perceived creditworthiness of the issuer, yield curve and volatility movements will largely influence structural performance. Investors should also take into account long-run market dynamics that affect the composition of the market and, in turn, credit index benchmarks.

Specifically, callable structures have become rarer in the U.S. investment-grade credit bond market with the exception of the 2000 inversion. This is due to an almost continuously positively-sloped U.S. term structure since 1990 and the yield curve's intermittent declines to approximately multi-decade lows in 1993, 1997, 1998, and 2002. As a result, the composition of the public U.S. corporate bond market converged toward the intermediate-bullet Eurobond and euro-denominated bond market. To see this, we need only look at the structure composition of Lehman's U.S. Investment-Grade Credit Bond Index. Bullets increased from 24% of this index at the start of 1990 to 94% (principal value basis) by 2003. Over this interval, callables declined at a remarkable rate from 72% to just a 3% index share. Sinking-fund structures, once the structural mainstay of natural-gas pipelines and many industrial sectors, are on the "structural endangered species list" with a drop from 32% of the public bond market in 1990 to only 1% in 2003. Despite several brief flurries of origination in the mid-1990s and the late-1990s introduction of callable/putable structures, putable structure market share fell from 5% in 1990 to 2% by 2003. Pure corporate zeros are in danger of extinction with a fall from 4% market share in 1990 to negligible by 2003.

A. Bullets

Here is a review of how different types of investors are using bullet structures with different maturities.

Front-end bullets (i.e., bullet structures with 1- to 5-year maturities) have great appeal for investors who pursue a “barbell strategy.” A barbell strategy was described in which both the short and long end of the barbell are U.S. Treasury securities. There are “barbellers” who use credit securities at the front or short-end of the curve and Treasuries at the long-end of the yield curve. There are non-U.S. institutions who convert short bullets into floating-rate products by using interest rate swaps. The transactions are referred to as “asset swaps,” and the investors who employ this transaction are referred to as “asset swappers.”

Intermediate credit bullets (5- to 12-year maturities), especially the 10-year maturity sector, have become the most popular segment of the U.S. and European investment-grade and high-yield credit markets. Fifteen-year maturities, benchmarked off the 10-year bellwether Treasury, are comparatively rare and have been favored by banks that occasionally use them for certain types of swaps. Because new 15-year structures take five years to descend along a positively sloped yield curve to their underlying 10-year bellwether, 15-year maturities hold less appeal for many investors in search of return through price appreciation emanating from benchmark rolldown. In contrast, rare 20-year structures have been favored by many investors. Spreads for these structures are benched off the 30-year Treasury. With a positively sloped yield curve, the 20-year structure provides higher yield than a 10-year or 15-year security and less vulnerability (lower duration) than a 30-year security.

The 30-year maturity is the most popular form of long-dated security in the global credit market. In 1992, 1993, late 1995, and 1997, there was a minor rush to issue 50-year (half-Centuries) and 100-year (Centuries) securities in the U.S. credit bond market. These longer-dated securities provide investors with extra positive convexity for only a modest increase in effective (or modified-adjusted) duration.⁶ In the wake of the “Asian Contagion” and especially the “Great Spread-Sector Crash” of August 1998, the cyclical increases in risk aversion and liquidity premiums greatly reduced both issuer and investor interest in these ultra-long maturities.

B. Callables

Typically after a 5-year or 10-year wait (longer for some rare issues), credit structures are callable at the option of the issuer at any time. Call prices usually are set at a premium above par (par + the initial coupon) and decline linearly on an annual basis to par by 5–10 years prior to final scheduled maturity. The ability to refinance debt in a potentially lower-interest rate environment is extremely valuable to issuers. Conversely, the risk of earlier-than-expected retirement of an above-current market coupon is bothersome to investors.

In issuing callables, issuers pay investors an annual spread premium (about 20 bp to 40 bp for high-quality issuers) for being long (from an issuer’s perspective) the call option. Like all security valuations, this call premium varies through time with capital market conditions. Given the higher chance of exercise, this call option becomes much more expensive during low rate and high volatility periods. Since 1990, this call premium has ranged from approximately 15 bp to 50 bp for investment-grade issuers. Callables significantly underperform bullets when interest rates decline because of their negative convexity. When the bond market rallies, callable structures do not fully participate given the upper boundary imposed by call prices. Conversely, callable structures outperform bullets in bear bond markets as the probability of early call diminishes.

⁶Recall that the longer the maturity, the greater the convexity.

C. Sinking Funds

A sinking fund structure allows an issuer to execute a series of partial calls (annually or semiannually) prior to maturity. Issuers also usually have an option to retire an additional portion of the issue on the sinking fund date, typically ranging from 1 to 2 times the mandatory sinking fund obligation. Historically, especially during the early 1980s, total return investors favored the collection of sinking fund structures at sub-par prices. These discounted sinking funds retained price upside during interest rate rallies (provided the indicated bond price remained below par), and, given the issuers' requirement to retire at least annually some portion of the issue at par, the price of these sinking fund structures did not fall as much compared to callables and bullets when interest rates rose. It should be noted that astute issuers with strong liability management skills can sometimes satisfy such annual sinking fund obligations in whole or in part through prior open market purchases at prices below par. Nonetheless, this annual sinking fund purchase obligation by issuers does limit bond price depreciation during periods of rising rates.

D. Puttables

Conventional put structures are simpler than callables. Yet in trading circles, put bond valuations often are the subject of debate. American-option callables grant issuers the right to call an issue at any time at the designated call price after expiration of the non-callable or non-redemption period. Put bonds typically provide investors with a one-time, one-date put option (European option) to demand full repayment at par. Less frequently, put bonds include a second or third put option date. A very limited number of put issues afford investors the privilege to put such structures back to the issuers at par in the case of rating downgrades (typically to below investment-grade status).

Thanks to falling interest rates, issuers shied away from new put structures as the 1990s progressed. Rather than incur the risk of refunding the put bond in 5 or 10 years at a higher cost, many issuers would prefer to pay an extra 10 bp to 20 bp in order to issue a longer-term liability.

Put structures provide investors with a partial defense against sharp increases in interest rates. Assuming that the issuer still has the capability to meet its sudden obligation, put structures triggered by a credit event enable investors to escape from a deteriorating credit. Perhaps because of its comparative scarcity, the performance and valuation of put structures have been a challenge for many portfolio managers. Unlike callable structures, put prices have not conformed to expectations formed in a general volatility-valuation framework. Specifically, as explained, the implied yield volatility of an option can be computed from the option's price and a valuation model. In the case of a puttable bond, the implied volatility can be obtained using a valuation model such as the binomial model. The implied volatility should be the same for both puts and calls, all factors constant. Yet, for puttable structures, implied volatility has ranged between 4%–9% since 1990, well below the 10%–20% volatility range associated with callable structures for the same time period. This divergence in implied volatility between callables (high) and puttables (low) suggests that asset managers, often driven by a desire to boost portfolio yield, underpay issuers for the right to put a debt security back to the issuer under specified circumstances. In other words, the typical put bond should trade at a lower yield in the market than is commonly the case.

Unless put origination increases sharply, allowing for greater liquidity and the creation of more standardized trading conventions for this rarer structural issue, this asymmetry in implied volatility between puttable and corporate structures will persist. Meanwhile, this

structure should be favored as an outperformance vehicle only by those investors with a decidedly bearish outlook for interest rates.

IX. CREDIT CURVE ANALYSIS

The rapid growth of credit derivatives⁷ since the mid-1990s has inspired a groundswell of academic and practitioner interest in the development of more rigorous techniques to analyze the term structure (1–100 years) and credit structure (Aaa/AAA through B2/B's) of credit spread curves (higher risk higher-yield securities trade on a price rather than a spread basis).

Credit curves, both term structure and credit structure, are almost always positively sloped. In an effort to moderate portfolio risk, many portfolio managers take credit risk in short and intermediate maturities and to substitute less-risky government securities in long-duration portfolio buckets. This strategy is called a **credit barbell strategy**. Accordingly, the application of this strategy diminishes demand for longer-dated credit risk debt instruments by many total return, mutual fund, and bank portfolio bond managers. Fortunately for credit issuers who desire to issue long maturities, insurers and pension plan sponsors often meet long-term liability needs through the purchase of credit debt with maturities that range beyond 20 years.

Default risk increases non-linearly as creditworthiness declines. The absolute risk of issuer default in any one year remains quite low through the investment-grade rating categories (Aaa/AAA to Baa3/BBB–). But investors constrained to high-quality investments often treat downgrades like quasi-defaults. In some cases like a downgrade from single-A to the Baa/BBB category, investors may be forced to sell securities under rigid portfolio guidelines. In turn, investors justifiably demand a spread premium for the increased likelihood of potential credit difficulty as rating quality descends through the investment-grade categories.

Credit spreads increase sharply in the high-yield rating categories (Ba1/BB+ through D). Default, especially for weak single-Bs and CCCs, becomes a major possibility. The credit market naturally assigns higher and higher risk premia (spreads) as credit and rating risk escalate. Exhibit 6 shows the credit curve for two credit sectors (Baa and single-A industrials) and also illustrates a higher spread is required as maturity lengthens.

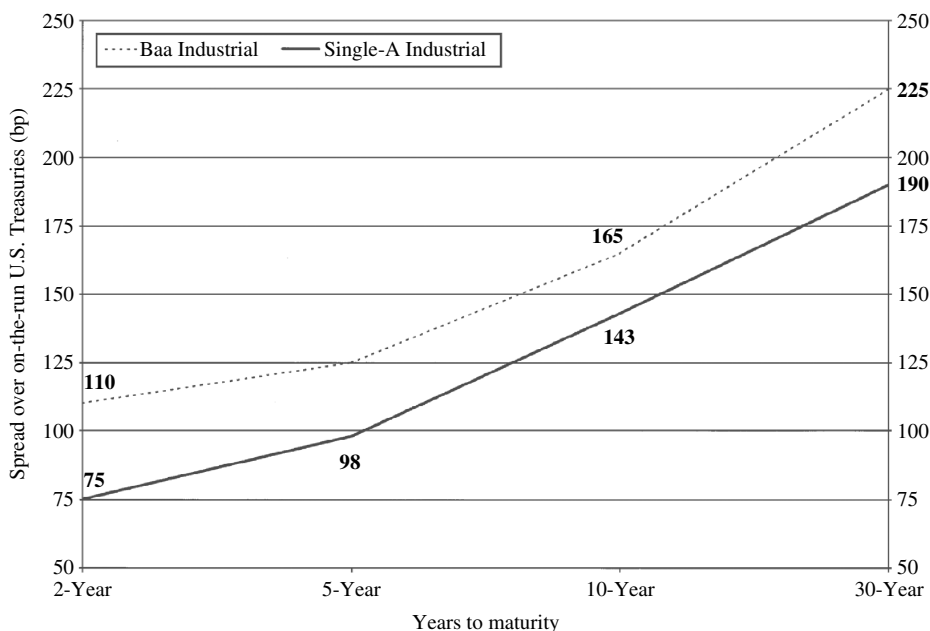
In particular, the investment-grade credit market has a fascination with the slope of issuer credit curves between 10-year and 30-year maturities. Like the underlying Treasury benchmark curve, credit spread curves change shape over the course of economic cycles. Typically, spread curves steepen when the bond market becomes more wary of interest rate and general credit risk. Spread curves also have displayed a minor propensity to steepen when the underlying benchmark curve flattens or inverts. This loose spread curve/yield curve linkage reflects the diminished appetite for investors to assume both curve and credit risk at the long end of the yield curve when higher total yields may be available in short and intermediate credit products.

X. CREDIT ANALYSIS

In the continuous quest to seek credit upgrades and contraction in issuer/issue spread resulting from possible upgrades and, more importantly, to avoid credit downgrades resulting in an increase in issuer/issue spread, superior credit analysis has been and will remain the

⁷Credit derivatives are covered in Chapter 24.

EXHIBIT 6 Illustration of Two Typical U.S. Investment-Grade Curves



Source: Lehman Brothers U.S. Investment-Grade Credit Index, based on average corporate curves 1990–2003

most important determinant of credit bond portfolio relative performance. Credit screening tools tied to equity valuations, relative spread movements, and the Internet (information available tracking all related news on portfolio holdings) can provide helpful supplements to classic credit research and rating agency opinions. But self-characterized credit models, relying exclusively on variables like interest-rate volatility and binomial processes imported from option-valuation techniques, are not especially helpful in ranking the expected credit performance of individual credits like IBM, British Gas, Texas Utilities, Pohang Iron & Steel, Sumitomo, and Brazil.

Credit analysis is both non-glamorous and arduous for many top-down portfolio managers and strategists, who focus primarily on macro variables. Genuine credit analysis encompasses actually studying issuers' financial statements and accounting techniques, interviewing issuers' managements, evaluating industry issues, reading indentures and charters, and developing an awareness of (not necessarily concurrence with) the views of the rating agencies about various industries and issuers.

Unfortunately, the advantages of such analytical rigor may clash with the rapid expansion of the universe of issuers of credit bonds. There are approximately 5,000 different credit issuers scattered across the global bond market. With continued privatization of state enterprises, new entrants to the high-yield market, and expected long-term growth of the emerging-debt markets, the global roster of issuers could swell to 7,500 by 2010. The sorting of this expanding roster of global credit issues into outperformers, market performers, and underperformers demands establishing and maintaining a formidable credit-valuation function by asset managers.

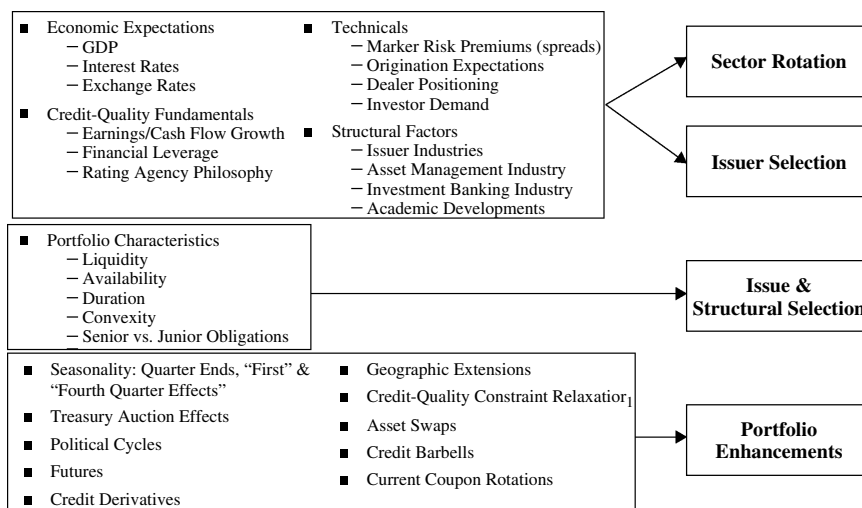
XI. ASSET ALLOCATION/SECTOR ROTATION

Sector rotation strategies have long played a key role in equity portfolio management. In the credit bond market, “macro” sector rotations among industrials, utilities, financial institutions, sovereigns, and supranationals also have a long history. During the last quarter of the 20th Century, there were major variations in investor sentiment toward these major credit sectors. Utilities endured market wariness about heavy supply and nuclear exposure in the early-to-mid 1980s. U.S. and European financial institutions coped with investor concern about asset quality in the late 1980s and early 1990s. Similar investor skittishness affected demand for Asian financial institution debt in the late 1990s. Industrials embodied severe “event risk” in the mid-to-late 1980s, recession vulnerability during 1990–1992, a return of event risk in the late 1990s amid a general boom in corporate mergers and acquisitions, and a devastating series of accounting and corporate governance blows during 2001–2002. Sovereigns were exposed to periodic market reservations about the implications of independence for Quebec, political risk for various countries (i.e., Russia), the effects of the “Asian Contagion” during 1997–1998, and outright defaults like Argentina (2001).

In contrast, “micro” sector rotation strategies have a briefer history in the credit market. A detailed risk/return breakdown (i.e., average return and standard deviation) of the main credit sub-sectors (i.e., banks, brokerage, energy, electrics, media, railroads, sovereigns, supranationals, technology) was not available from credit index providers until 1993 in the United States and until 1999 in Europe. Beginning in the mid-1990s, these “micro” sector rotation strategies in the credit asset class have become much more influential as portfolio managers gain a greater understanding of the relationships among intra-credit sectors from these statistics.

Exhibit 7 illustrates the main factors bearing on sector rotation and issuer selection strategies. For example, an actual or perceived change in rating agency philosophy toward a sector and a revision in profitability expectations for a particular industry represent just two of many factors that can influence relative sectoral performance.

EXHIBIT 7 Some Outperformance Methodologies



Common tactics to hopefully enhance credit portfolio performance are also highlighted in Exhibit 7. In particular, seasonality deserves comment. The annual rotation toward risk aversion in the bond market during the second half of most years contributes to a “fourth-quarter effect”—that is, there is underperformance of lower-rated credits, B’s in high-yield and Baa’s in investment-grade, compared to higher-rated credits. A fresh spurt of market optimism greets nearly every New Year. Lower-rated credit outperforms higher-quality credit—this is referred to as the “first-quarter effect.” This pattern suggests a very simple and popular portfolio strategy: underweight low-quality credits and possibly even credit products altogether until the mid-third quarter of each year and then move to overweight lower-quality credits and all credit product in the fourth quarter of each year.

CHAPTER 21

INTERNATIONAL BOND PORTFOLIO MANAGEMENT*

I. INTRODUCTION

Management of an international bond portfolio poses more varied challenges than management of a domestic bond portfolio. Differing time zones, local market structures, settlement and custodial issues, and currency management all complicate the fundamental decisions facing every fixed income manager in determining how the portfolio should be positioned with respect to duration, sector, and yield curve.

In Chapter 16, the fundamental steps in the investment management process were explained. These steps include:

1. setting investment objectives
2. developing and implementing a portfolio strategy
3. monitoring the portfolio
4. adjusting the portfolio

The added complexities of cross-border investing magnify the importance of a well defined, disciplined, investment process. This chapter is organized to address these challenges for steps 1, 2, and 4.

To provide a broad overview of the many aspects of international fixed income investing in one chapter implies that many topics do not receive the depth of discussion they deserve. For example, the topic of currency management is extensive and we provide only the fundamental principles here. However, the same principles involved with currency management apply equally to international equity portfolio management.

While many of the examples and illustrations in this chapter apply to international investing from the perspective of a U.S. manager investing in bond markets outside of the United States, it is important to keep in mind that the principles apply to any cross-border manager investing outside of his or her domestic bond market. The same issues faced by U.S. managers regarding currency management apply to managers throughout the world when they invest in bonds in which the cash flows are not denominated in their local currency.

*This chapter is authored by Christopher B. Steward, CFA, J. Hank Lynch, CFA, and Frank J. Fabozzi, PhD, CFA, CPA.

II. INVESTMENT OBJECTIVES AND POLICY STATEMENTS

Most investors are attracted to global bonds as an asset class because of their historically higher returns than U.S. bonds. Others are drawn to global bonds because of their diversification value in reducing overall portfolio risk.¹ *An investor's rationale for investing in international bonds is central to developing appropriate return objectives and risk tolerances for a portfolio.*

Broadly speaking, investor specifications include:

1. return objectives
2. risk tolerances

Each of these investment objectives has implications for the management of an international bond portfolio and should be reflected in the investment policy statement.

Global bonds are usually a small part of an overall portfolio added for both return and diversification. The strategic asset allocation for the portfolio is made up of benchmarks that both define the asset class and provide a performance target that each investment manager strives to outperform. Return objectives are often expressed in terms of the benchmark return, e.g., benchmark return plus 100 basis points over a market cycle. The return objectives and risk tolerances will indicate not only the most appropriate benchmark, but also the most suitable investment management style. Investors who are primarily concerned with diversification may wish to place tight limits on the size of positions taken away from the benchmark to ensure diversification is not weakened. A total-return oriented investor might be far less concerned with diversification and focused on absolute return rather than on benchmark relative return.

Investment policy statements should be flexible enough to allow the portfolio manager sufficient latitude for active management while keeping the portfolio close enough to the benchmark to ensure that the portfolio remains diversified. The policy statements should address allowable investments including:

1. the countries in the investment universe, including emerging markets
2. allowable instruments, including mortgages, corporate bonds, asset-backed securities, and inflation-adjusted bonds
3. minimum credit ratings
4. the currency benchmark position and risk limits
5. the use of derivatives such as forwards, futures, options, swaps, and structured notes

The time horizon for investment performance is also important. A short-term time horizon, such as a calendar quarter, may encourage more short-term trading which could

¹Some investors were concerned that the diversification benefits of global bond investing would be substantially diminished by the commencement of European Monetary Union (EMU) in 1999. But, in fact, the economies of continental Europe were already very closely tied together before EMU with most European central banks following the interest rate policies of the German Bundesbank for several years before the move to a single currency. Thus, the impact on diversification of a global bond portfolio caused by EMU has been a small one. EMU, however, has created a much more robust credit market in Europe as issuers and investors, no longer confined to their home markets, have access to a larger, more liquid pan-European bond market. Corporate bond issuance has increased sharply in Europe, and seems likely to continue, building toward a broader range of credits and instruments similar to those available in the U.S. bond market. This was discussed in Chapter 20.

diminish the natural diversification benefit from international bonds as an asset class. Investors who emphasize the risk reduction, or diversification aspect of international bond investing, should have a longer time horizon of perhaps three to five years. As differences between economic cycles can be prolonged, this provides enough time for a full economic cycle to add any diversification benefit.

A. Benchmark Selection

Benchmark selection for an international bond portfolio has many ramifications and should be done carefully. As is the case when choosing an international equity benchmark index, the choice of a pure capitalization (market value) weighted index may create a benchmark that exposes the investor to a disproportionate share in the Japanese² market relative to the investor's liabilities or diversification preferences.³ While international equity indices chosen for benchmarks are most often quoted in the investor's local currency (i.e., unhedged), international bond benchmarks may be hedged, unhedged, or partially hedged depending on the investor's objectives. The choice of a hedged, unhedged, or partially hedged benchmark will likely alter the risk and return profile of the investment portfolio and should reflect the rationale for investing in international bonds.

B. Available Benchmarks

Benchmarks can be selected from one, or a combination of the many existing bond indices:

- global (all countries, including home country)
- international (*ex*-home country)
- government-only
- multi-sector or broad (including corporates and mortgages)
- currency-hedged
- G7 only
- maturity constrained, e.g., 1–3 year, 3–5 year, 7–10 year
- emerging markets

Alternatively, a customized index or “normal” portfolio can be created.

The most frequently used fixed-income benchmarks are the **Citigroup World Government Bond Index (WGBI)** and the **Lehman Global Aggregate**. As discussed above, the benchmark often provides both the return objective and the measure of portfolio risk.

C. Benchmark Currency Position

Currency management is a matter of much debate in the academic literature. Investing internationally naturally generates foreign currency exposures. These currency exposures can be managed either passively or actively, although most global bond managers utilize active management to some degree.

Many managers are attracted to active currency management because of the large gains that can be attained through correctly anticipating currency movements. As currency returns

²The Japanese bond market has historically offered lower yields than most other bond markets.

³While in the equity market where growth in a company's market capitalization generally indicates financial strength, a company or country that issues a large amount of debt (especially relative to its equity in the case of a company or gross national product in the case of a country) may find itself in a more precarious financial position.

are much more volatile than bond market returns, even modest positions in currencies can result in significant tracking error (see Chapter 17). Traditionally, the bond manager has handled currency exposures assuming the same fundamental economic factors (identified later in this chapter) influence currency levels. However, many managers are hiring foreign exchange specialists because bonds and currencies can behave quite differently in reaction to the same stimulus. Both the risks and opportunities posed by currency movements suggest that some specialization in currency is warranted and that a joint optimization of the bond and currency decision provides better risk-adjusted returns.⁴ Research has also shown that active management by currency specialists can consistently add to returns.

The first task is to determine the neutral or strategic foreign currency exposure appropriate for the investment objectives. Most of the academic research on currency hedging for U.S. dollar-based investors suggests that a partially hedged benchmark offers superior risk-adjusted returns as compared with either a fully hedged or unhedged benchmark.⁵ This research has led some to recommend a 50% hedged benchmark for either a passively managed currency strategy, or as a good initial hedged position for an active currency manager. Once the benchmark has been selected, a suitable currency hedge position needs to be determined. For example, a U.S. dollar-based fixed income manager whose primary goal is risk reduction might adopt a hedged or mostly hedged benchmark which has historically shown greater diversification benefit from international bonds. Despite a higher correlation with the U.S. bond market than unhedged international bonds, hedged international bonds offer better risk reduction due to a lower standard deviation of bond returns than an only U.S. bond market portfolio.⁶ In addition, this lesser volatility of hedged international bonds results in more predictable returns. Conversely, an investor who has a total return objective, and a greater risk tolerance, would be more likely to adopt an unhedged, or mostly unhedged benchmark, and allow more latitude for active currency management.

From the perspective of a U.S. investor, Exhibit 1 shows that for the 18-year period 1985–2002 the currency component of investing in unhedged international bonds accounted for much of the total return volatility. The international bond index used is the Citigroup WGBI excluding the United States (denoted by “non-U.S. WGBI”). Investing in international bonds on a hedged basis reduced the return in most periods, but also substantially reduced the return volatility. As can be seen in Exhibit 1, over the 18-year history of the WGBI, hedged international bonds returned less than unhedged international bonds and even lagged the U.S. component of the WGBI slightly. However, the volatility of the hedged non-U.S. WGBI was one third that of the unhedged index, and three quarters that of the U.S. component.

To compare returns on a risk-adjusted basis we can use the Sharpe ratio.⁷ Despite the higher return of the unhedged non-U.S. WGBI, its risk-adjusted return was lower than the hedged index and the U.S. bond component alone for the 1985 through 2002 period.

⁴See Philippe Jorion, “Mean/Variance Analysis of Currency Overlays,” *Financial Analysts Journal* (May/June 1994), pp. 48–56. Jorion argues that currency overlays, although they can add value, are inferior to an integrated approach to currency management.

⁵See Gary L. Gastineau, “The Currency Hedging Decision: A Search for Synthesis in Asset Allocation,” *Financial Analysts Journal* (May–June 1995), pp. 8–17 for a broad overview of the currency hedging debate. For a full discussion of the benefits of utilizing a partially hedged benchmark, see the currency discussion in Steve Gorman, *The International Equity Commitment* (Charlottesville, VA: AIMR 1998).

⁶Recall from modern portfolio theory the important role of correlation in determining the benefits from diversification.

⁷The Sharpe ratio measures returns in excess of the risk-free rate, per unit of standard deviation.

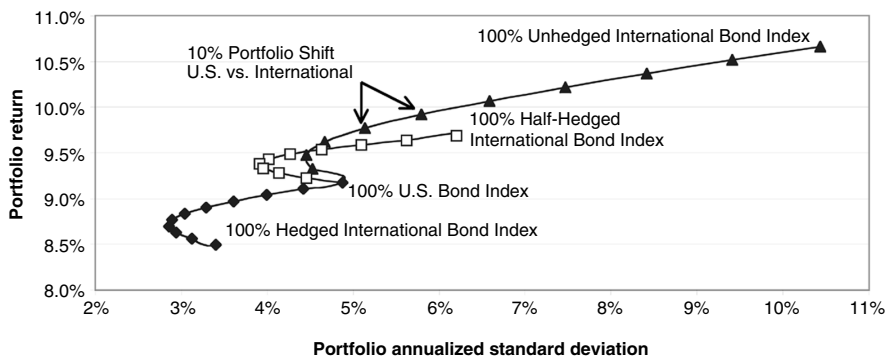
EXHIBIT 1 Hedged and Unhedged Returns: 1985–2002

	Non-U.S. WGBI	U.S.	non-U.S. WGBI hedged	50% Hedged
1985–2002				
Return	10.66%	9.17%	8.49%	9.69%
Volatility	10.4%	4.9%	3.4%	6.2%
Sharpe	0.47	0.71	0.82	0.64
1985–1990				
Return	18.23%	11.27%	8.01%	13.18%
Volatility	13.6%	6.0%	4.3%	8.4%
Sharpe	0.77	0.60	0.07	0.65
1991–1996				
Return	10.78%	8.23%	9.49%	10.23%
Volatility	8.6%	4.3%	3.2%	5.0%
Sharpe	0.69	0.78	1.44	1.09
1997–2002				
Return	3.46%	8.05%	7.97%	5.78%
Volatility	8.6%	4.3%	2.4%	4.8%
Sharpe	−0.14	0.80	1.38	0.24

As noted above, using a 50% hedged portfolio offers a compromise in that its return is virtually midway between the return of the unhedged non-US WGBI and the U.S. bond component with substantially lower volatility than the unhedged index, giving it a higher Sharpe ratio than the unhedged index. Of course the relative performance of the hedged versus the unhedged index depends upon the performance of the home currency (here the U.S. dollar) which can experience long cycles of strength or weakness.

The advantage of using a partially hedged benchmark versus a fully hedged or fully unhedged benchmark is illustrated in a mean-variance framework in Exhibit 2. The 50% hedged portfolio offers better diversification with some small reduction in return when a modest allocation to international bonds is added to U.S. bond portfolios.

EXHIBIT 2 Risk-Return for Unhedged and Hedged International Bond Portfolios (U.S. Investor Perspective) Using 1985–2002 Historical Returns



D. Risk Limits

Many investment guidelines will include explicit risk limits on bond and currency positions as well as duration and credit risk. Exposure limits can be either expressed as absolute percentages, or weights relative to a benchmark. Increasingly, tracking error limits have also been used to set risk limits in investment guidelines.

Bond markets can be divided into four trading blocs:

1. dollar bloc (the U.S., Canada, Australia, and New Zealand)
2. European bloc
3. Japan
4. emerging markets

The European bloc is subdivided into two groups:

1. euro zone market bloc which has a common currency (Germany, France, Holland, Belgium, Luxembourg, Austria, Italy, Spain, Finland, Portugal, and Greece)
2. non-euro zone market bloc (Norway, Denmark, and Sweden)

The United Kingdom often trades more on its own, influenced by both the euro zone and the U.S., as well as its own economic fundamentals.

The trading bloc construct is useful because each bloc has a benchmark market that greatly influences price movements in the other markets. Investors are often focused more on the spread level of, say, Denmark to Germany, than the absolute level of yields in Denmark. (Since the beginning of the European Monetary Union (EMU) in 1999, the euro zone bond markets have traded in a much tighter range.)

Limits on investment in countries outside the benchmark should also be specified at the outset. Despite the pitfalls of using duration to measure interest rate risk across countries, risk limits on duration are nonetheless useful and should be established. Typically, the range of allowable exposures is wider for bond exposures than currency exposures.

Credit risk limits, usually a minimum weighted average credit rating from the major credit rating agencies, and limits on the absolute amount of low or non-investment grade credits, should also be included. Apart from default risk, the illiquidity of lower rated securities may hamper a manager's ability to alter exposures as desired. In the past, due to the lack of a liquid corporate bond market in many countries, and the relative illiquidity of Eurobonds compared to domestic government bond markets, most credit risk in international bond portfolios was concentrated in U.S. and emerging market bonds. However, this difference in liquidity between U.S. corporate bonds and those in other countries has diminished significantly in recent years due to strong growth in the European corporate bond market since European Monetary Union.

III. DEVELOPING A PORTFOLIO STRATEGY

Once the investment policy statement is established, the portfolio manager needs to develop a portfolio strategy appropriate to the investor's objectives and risk tolerances. Just as in many other areas of investment management, portfolio managers often subscribe to different management styles, or investment disciplines.

Since the performance of most portfolio managers is judged against a benchmark return, managers are constantly seeking opportunities to outperform a benchmark. There are a number of means by which portfolio managers can add to returns; however, the bulk of excess

returns relative to the benchmark comes from broad bond market and currency allocation decisions. A disciplined investment approach, based upon fundamental economic factors or market indicators of value, facilitates the market and currency selection process. Because of the historical high volatility of currency returns, the approach to currency management should be a primary concern.

A. Styles of International Bond Portfolio Management

The challenges faced by international fixed income managers are different from those facing domestic fixed income managers. First, the international fixed income portfolio manager must operate in the U.S. bond market plus 10 to 20 other markets, each with their own market dynamics. Second, changes in interest rates generally affect different sectors of the U.S. bond market in much the same way (with the exception of mortgage-backed securities), although the magnitude of the changes may vary. Like the equity market, where it is not unusual to have some industries or market sectors move in opposite directions, international bond markets may also move in different directions depending upon economic conditions and investor risk tolerances.

International bond managers also utilize one or more different management styles. These can be divided into four general categories:

1. the experienced trader
2. the fundamentalist
3. the black box
4. the chartist

We discuss each management style below.

1. The Experienced Trader The **experienced trader** uses his or her experience and intuition to identify market opportunities. The experienced trader tends to be an active trader, trying to anticipate the next market shift by international fixed income and hedge fund managers. The basis for these trades is derived from estimates of competitors' positions and risk tolerances bolstered by observation of market price movements and flow information. The experienced trader is often a contrarian, looking to profit from situations where many investors may be forced to stop themselves out of losing positions.

2. The Fundamentalist The **fundamental style** rests upon a belief that bonds and currencies trade according to the economic cycle. Sector rotation within corporate bonds also will be affected by the economic cycle as different sectors perform relatively better at different points in the cycle. Some of these managers believe that the economic cycle is forecastable, and rely mostly upon economic analysis and forecasts in selecting bond markets and currencies. These managers tend to have less portfolio turnover as the economic fundamentals have little impact on short-term price movements. "Bottom-up" security selection in corporate bonds could also be characterized as a fundamentalist approach even though it rests upon issuer-specific fundamental analysis rather than broad economic fundamentals.

3. The Black Box The **black box approach** is used by quantitative managers who believe that computer models can identify market relationships that people cannot. These models can rely exclusively on economic data, price data, or some combination of the two. Quantitative managers believe using computer models can create a more disciplined investment approach

which, because of other managers' emotional attachment to positions, their lack of trading disciplines, or their inability to process more than a few variables simultaneously, will provide superior investment results.

4. **The Chartist** Some investors called **chartists** or technicians may rely primarily on technical analysis to determine which assets to buy or sell.⁸ Chartists will look at daily, weekly, and monthly charts to try to ascertain the strength of market trends, or to identify potential turning points in markets. Trend-following approaches, such as moving averages, aim to allow the portfolio manager to exploit market momentum. Counter-trend approaches, such as relative strength indices and oscillators try to identify when recent price trends are likely to reverse.

5. **Combining Styles** Very few international bond portfolio managers rely on only one of these management styles, but instead use some combination of each. Investment managers that rely on forecasts of the economic cycle to drive their investment process will from time to time take positions contrary to their medium-term strategy to take advantage of temporary under or overvaluation of markets identified by technical analysis, or estimates of market positions. Even "quant shops" that rely heavily on computer models for driving investment decisions will sometimes look to other management styles to add incremental returns. Regardless of the manager's investment style, investment decisions must be consistent with the investor's return objectives and risk tolerances, and within the investment guidelines.

International bond portfolio managers would do well to maintain a disciplined approach to buy and sell decisions. This would require each allocation away from the benchmark to have a specified price target (or more often yield spread or exchange rate level), and stated underlying rationale. Depending upon the management style, the size of the position should reflect the strength of the investor's conviction or model's signal. As long as the investment rationale that supported the initial decision remained unchanged, the position would be held, or potentially increased, if the market moves in the opposite direction. Each trade should be designed with consideration for the relevant bond yield or exchange rate's behavior through time. For example, an exchange rate that exhibits a tendency to trend will require a different buy and sell discipline than one that tends to consistently revert back to an average exchange rate.

B. Sources of Excess Return

The baseline for any international bond portfolio is the benchmark. However, in order to earn returns in excess of the benchmark, after management fees, the portfolio manager must find ways to augment returns. These excess returns can be generated through a combination of five broad strategies:

1. currency selection
2. duration management/yield curve plays
3. bond market selection
4. sector/credit/security selection
5. investing in markets outside the benchmark (if permitted)

Each of these strategies can add to returns; however, currency and bond market selections generally provide the lion's share of returns. We discuss each of these sources of excess return on the following page.

⁸Technical analysis is covered in equity management textbooks.

1. Currency Selection Most investment guidelines will allow for some active management of currency exposures. The attraction of active currency management is strong because potential gains are so large. The spread between the top and bottom performing bond markets in local currency terms is 13% on average. When currency movements are added to the local currency bond market returns, the average spread between the best and worst performing markets more than doubles to 28%. Thus, international bond portfolio managers may significantly enhance returns by overweighting the better performing bond markets and currencies in the index.

However, as the volatility of currency returns is generally higher than that of bond market returns, the incremental returns gained from currency exposures must be evaluated relative to the additional risk incurred. For an active currency management strategy to consistently provide superior risk-adjusted performance, a currency forecasting method is required that can predict future spot rates (i.e., future exchange rates) better than forward foreign exchange rates (i.e., rates that can be locked in today using the market for forward contracts). As shown later, forward foreign exchange rates are not forecasts of future spot foreign exchange rates, but are determined by short-term interest rate differentials between currencies.

Academic studies have shown that several strategies have been successful in generating consistent profits through active currency management. The fact that forward foreign exchange rates are poor predictors of future spot exchange rates is well established. Historically, discount currencies (i.e., those with higher interest rates than the investor's local currency) have depreciated less than the amount implied by the forward rates, providing superior returns from holding unhedged positions in currencies with higher interest rates. Overweighting currencies with high real interest rates versus those with lower real interest rates has also been shown to provide incremental returns.⁹

In addition, some currency movements are not a random walk, but exhibit serial correlation (i.e., currency movements have a tendency to trend).¹⁰ In a market that tends to trend, simple technical trading rules may provide opportunities for incremental currency returns.¹¹ These findings in several academic studies demonstrate that excess currency returns can be generated consistently, providing a powerful incentive for active currency management.

2. Duration Management Although closely aligned with the bond market selection decision, duration management can also enhance returns. Bullet versus barbell strategies in a curve steepening or flattening environment within a particular country's bond market can enhance yield and total return.¹² In addition to these strategies that are also available to managers investing in their domestic bond market, the international fixed income portfolio manager has the option of shifting duration between markets while leaving the portfolio's overall duration unchanged.

⁹See Gastineau, "The Currency Hedging Decision," pp. 13–14.

¹⁰One suggestion as to why currency markets trend is that central banks attempt to smooth foreign exchange rate movements through intervention. Thus, because central bank participation in the foreign exchange market is not motivated by profit, their actions keep the market from being truly efficient. See Robert D. Arnott and Tan K. Pham, "Tactical Currency Allocation," *Financial Analysts Journal*, (May/June 1993) pp. 47–52.

¹¹See Richard M. Levich and Lee R. Thomas, "The Merits of Active Currency Risk Management: Evidence from International Bond Portfolios," *Financial Analysts Journal*, (September/October 1993) pp. 63–70.

¹²See Chapter 18 for a discussion of this strategy and how it is analyzed.

Duration management has become easier in international markets in recent years. Many countries have concentrated their debt in fewer, more liquid, bond issues. Official strip markets (which separate government bond cash flows into individual interest and principal payments) now exist in at least nine countries. Interest rate futures, available in most markets, offer a liquid and low-cost vehicle for changing duration or market exposure quickly. The interest rate swaps market, used extensively by large institutional investors, is generally very liquid across international bond markets. Following European Monetary Union, the swap curve, rather than individual country yield curves, has increasingly been used as a reference for some markets. Increasingly, countries have set up professional debt management offices that are independent from both central banks and finance ministries. These debt management offices have become significant users of derivatives themselves to minimize borrowing costs, alter the maturity structure or currency composition of outstanding debt, and to promote liquidity in their domestic market.¹³

3. Bond Market Selection Excess returns over the benchmark index from overweighting the best performing bond markets can be extremely large. As we saw above, the annual local currency return differential between the best and worst performing developed bond markets has been 13% on average, providing significant opportunity for generating excess returns. The process for making the bond market selection decision is discussed further in Section III C.

4. Sector/Credit/Security Selection The corporate bond market experienced significant growth in many countries, especially in Europe following European Monetary Union. According to data collected by Merrill Lynch on the size and structure of the world bond market, government bonds account for 55% of the \$30 trillion market for developed country bonds. Corporate bonds account for 25% of the bond market, and about 20% excluding the United States. Some global bond indices include only government bonds, but others, like the Lehman Global Aggregate and the Citigroup Global Broad Investment Grade Indices, include other instruments including corporate bonds and mortgages.

5. Investing in Markets Outside the Index If allowed by investment guidelines, allocating assets to markets outside the index can significantly enhance returns without dramatically altering the risk profile of the portfolio. Here are two examples. First, Finland was one of the best performing bond markets during 1995, but, because of its small size, was not included in the Citigroup World Government Bond Index (WGBI) until June 1996. Second, investing in emerging markets debt as represented by the JP Morgan EMBI + Index would have boosted returns substantially in 1999 and 2000 when it outperformed all developed bond markets on a local currency or U.S. dollar basis.¹⁴

The process for selecting an out-of-index market is similar to that followed by an active manager for a domestic bond portfolio manager when deciding whether or not to construct a portfolio with allocations different from the benchmark index and whether or not to invest outside the index. The manager will assess the potential performance on a total return basis of the markets outside of the index relative to that of the markets to be underweighted in order to allocate funds to out-of-index markets. An international bond portfolio manager, however,

¹³See "OECD Public Debt Markets: Trends and Recent Structural Changes" OECD 2002.

¹⁴The JP Morgan EMBI + Index is comprised of mostly U.S. dollar-denominated sovereign debt issued by emerging market countries. Therefore, credit risk, and to a lesser extent interest rate risk, are the predominate risks associated with the index. For a U.S. investor, currency risk is virtually zero.

must also take into account the affect of currency movements and hedging decision of an investment outside or within the index.

As we saw above, exposure to emerging markets can significantly add to returns. For example, a portfolio composed of 80% exposure to the Citigroup Non-U.S. Government Bond Index and 20% exposure to the J.P. Morgan EMBI+ Index from 1994 through 2002 would have added 120 basis points to the return of the international index and reduced the standard deviation of returns by 12%. A 20% allocation to emerging markets in an international bond portfolio that was half-hedged against foreign exchange rate changes would have increased returns by 223 basis points while decreasing the standard deviation of returns by 37%.

C. A Fundamental-Based Approach to Investing

The portfolio strategy is often composed of

1. a medium-term strategic allocation and
2. a shorter-term tactical allocation

The **strategic allocation** is composed of positions held for one to three months, or longer designed to take advantage of longer-term economic trends. A fundamental-based approach is used to develop the portfolio's strategic allocation. The investment style used in the fundamental-based approach is, of course, the fundamental style, but can also be combined with a quantitative or black box style to forecast relevant strategic factors. The **tactical allocation** generally relies on technical analysis or flow information to identify shifts in market prices that are likely to occur within a few days to several weeks. Tactical allocations are often contrarian in nature, driven by expectations of a reversal in a recent price trend.¹⁵ Of course, the experienced trader, black box, and chartist investment styles most often use technical analysis combinations in their tactical allocation decisions.

The strategic decision of which bond markets and currencies to overweight usually begins with an economic outlook and bond and currency forecasts in each of the markets considered for investment. The long-run economic cycle is closely correlated with changes in bond yields, and trends in both the economic cycle and bond yields tend to persist for a year or longer. The millions of dollars spent each year by money management firms, banks, and brokerage houses in forecasting economic trends is testimony to the potential returns that can be achieved by correctly forecasting economic growth or turning points in the economic cycle.

Forecasting interest rates, however, is extremely difficult. Academic literature generally holds that interest rate forecasts are unable to generate consistent risk-adjusted excess returns. This is partly because market prices can deviate substantially over the short term from the level consistent with the economic fundamentals. Economic fundamentals impact bond and currency prices over the medium to long term. Also, the volatile nature of certain economic data series may result in exaggerated market reactions to individual data releases that may be different from the actual trend in the economy. These deviations may persist for several months until either the initial figure is revised, or several subsequent data releases reveal the error in the initial interpretation.

¹⁵However, tactical allocations can also be momentum following, especially if a breakout of a technical range appears likely. Again, such technical strategies are discussed in investment management textbooks.

The creation of an independent economic outlook can be useful in several ways. First, it can help identify when market interpretations of the economic data are too extreme, or add value through correctly anticipating economic shifts not reflected in the market consensus. Second, as it is often not absolute changes in interest rates, but changes in interest rates relative to other markets that determine the margin of performance in international fixed income investing, an independent economic outlook does not require accurate growth forecasts for each individual market, but only economic growth differentials to be able to add value. Whether the portfolio will invest in U.S. bonds or not, the large influence of the U.S. dollar and the Treasury market on foreign markets underlines the importance of an independent outlook on the U.S. economy.

Thus, the economic outlook forms the foundation for bonds and currencies *strategic* allocation. An economic outlook for each country should be constructed to assist in ranking the relative attractiveness of markets. However, even though economic fundamentals in a particular country may be extremely bond supportive, bond prices may be too high to make it an attractive investment. Likewise, bonds are sometimes excessively cheap in countries with poor economic fundamentals, yet may still provide an attractive investment opportunity. Thus, the economic outlook must be compared with either consensus economic forecasts, or some market value measure to identify attractive investment opportunities.

The strategic allocation decision regarding which markets to overweight or underweight relative to the benchmark is thus a complex interaction of expected returns derived from assessing economic trends, and technical and value factors. Each set of variables is defined and explored below, beginning with the fundamental factors used to create the economic outlook.

1. Fundamental Economic Factors The seven main fundamental economic factors are:

1. cyclical economic indicators
2. inflation
3. monetary policy
4. fiscal policy
5. debt
6. balance of payments
7. politics

Each factor needs to be evaluated against market expectations to determine its likely impact on bond prices and currency rates. Each of these factors is covered in considerable detail in books on macroeconomics and international economics. Some of these factors were also discussed in the context of the factors rating agencies consider when assigning credit ratings to sovereign issuers.

2. Value and Technical Indicators Identifying trends in economic fundamentals can help identify attractive investment opportunities in markets, but some yardstick to measure relative value is needed. Determining relative value is highly subjective. Three relatively objective value measures—real yields, technical analysis, and market sentiment surveys—are discussed below.

a. Real Yields A **real yield** is the inflation-adjusted rate of return demanded by the market for holding long-term fixed income securities. Real yields can quickly erode from sustained inflation. Real yields are impacted by a variety of factors including supply and demand for

capital as well as inflation expectations. Real yields are nominal bond yields minus expected inflation; however, expected inflation is often difficult to quantify. Some countries, including the United States, have inflation-indexed bonds that pay a real rate of interest above the inflation rate.¹⁶ These bonds not only provide investors with protection against a surge in inflation but also offer a means of gauging investor inflation expectations.¹⁷

Nominal bond yields deflated by current inflation, although not a precise measure of the market's real interest rate premium, are easily measurable and can still provide some useful insight into bond valuation. Real yields can be compared across markets or against their long-run averages, such as 5 or 10 years, in each market. The usefulness of real yields as a measure of relative value has diminished as global inflation rates have converged to very low levels.

b. Technicals Technical analysis can be as simple as drawing a trend line on a chart or as complicated as calculating the target of the third impulse wave of an Elliott wave analysis. In addition to valuing bonds and currencies, technical analysis can be used to value everything from stocks, to gold, to pork bellies. What all technical analysis has in common is that it tries to predict future prices solely from examining past price movements. Most technical analysis models fall into one of two camps: trend following or counter trend. The former try to identify trends that should persist for some period of time, and the latter attempt to predict when a recent trend is likely to change. We will not discuss these models here because they are typically described in investment management textbooks.

c. Market Sentiment Market sentiment can be used as a contra-indicator of value in the following way. A heavy overweight of a particular country's bond market implies that fewer managers are likely to add to that market, and more managers, at least eventually, are likely to sell. Market sentiment can be estimated by investor sentiment surveys, or by estimates of investment flows.

Historic trends, as well as the overall levels, should be taken into account when assessing market sentiment. For example, an indication that managers are underweighting Japanese bonds might lead some to conclude that Japanese bonds are due for a rally even though international fixed income managers have consistently underweighted the Japanese market, in part due to its low nominal yields. Sentiment surveys, however, may not capture all market participants such as retail investors, who can also move markets.

IV. PORTFOLIO CONSTRUCTION

Translating the strategic outlook into a portfolio allocation requires a framework for assessing expected returns against incremental portfolio risk. The following discussion on sources of return illustrates how returns can be separated into three components: **excess returns on bonds**, **excess returns on currencies**, and the **short-term risk-free interest rate**. This

¹⁶Inflation-indexed bonds were explained earlier.

¹⁷Nominal yield to maturity is composed of a real yield and an inflation expectations component (Yield to Maturity = Real Yield to Maturity + Expected Inflation to Maturity). In these markets the nominal government bond yield and the real yield offered by inflation-indexed debt of the same maturity can be used to calculate the expected inflation rate to the maturity, sometimes called the breakeven inflation rate.

component methodology can assist identifying where market prices are most out of line with the economic outlook and whether bond market currency exposures should be hedged or left unhedged.

A. Components of Return

To explain the total return components of an international bond portfolio,¹⁸ we will use the following notation. We will let “home currency” mean the currency of the manager. So, for a U.S. manager it is U.S. dollars. For a Japanese portfolio manager it is yen. In the notation, the subscript “*H*” will denote home currency.

We will let “local currency” be the currency of the country where the manager has invested and use the subscript “*L*” to denote the local currency. So, to a U.S. portfolio manager, yen would be the local currency for bonds purchased in the Japanese bond market and denominated in yen, while for a Japanese portfolio manager, U.S. dollars would be the local currency for bonds purchased in the U.S. and denominated in U.S. dollars.

The expected total return of an unhedged international bond portfolio in terms of the home currency depends on three factors:

1. the weight of each country’s bonds in the overall portfolio
2. the expected bond market return for each country in local currency
3. the expected exchange rate percentage change between the home currency and the local currency

Mathematically, the expected total return of an *unhedged* bond portfolio in terms of the home currency can be expressed as follows:¹⁹

$$\begin{aligned} &\text{total expected portfolio return in manager's home currency} \\ &= W_1 \times (r_1 + e_{H,1}) + W_2 \times (r_2 + e_{H,2}) + \cdots + W_N \times (r_N + e_{H,N}) \end{aligned} \quad (1)$$

where

N = number of countries whose bonds are in the portfolio

W_i = weight of country i 's bonds in the portfolio ($i = 1, 2, \dots, N$)

r_i = expected bond return for country i in local currency ($i = 1, 2, \dots, N$)

$e_{H,i}$ = expected *percentage* change of the home currency with country i 's local currency

We will refer to $e_{H,i}$ as the **currency return**.

The expected portfolio return as given by equation (1) is changed to the extent the manager alters exposure to each country’s exchange rate. A common instrument used to alter exposure to exchange rates is a **currency forward contract**. So, let’s look at these contracts and how they are priced. This will lead us to an important relationship that we will use in the balance of this chapter, called **interest rate parity**.

¹⁸The structure of this discussion is taken from Brian D. Singer and Denis S. Karnosky, *The General Framework for Global Investment Management and Performance Attribution* (Charlottesville, VA: The Research Foundation of the Institute of Chartered Financial Analysts, 1994). The notation used is consistent with that of the authors.

¹⁹The relationship in equation (1) is approximate because bond market and currency returns of a foreign investment is more accurately expressed as the compounded gain of the two components: $(1 + r_i) \times (1 + e_{\$,i}) - 1$.

1. Currency Forward Contracts and Their Pricing A forward contract is an agreement where one party agrees to buy “something,” and another party agrees to sell that same “something” at a designated date in the future. Forward contracts are used extensively for currency hedging.

Most currency forward contracts have a maturity of less than one year. For longer-dated forward contracts, the bid-ask spread increases; that is, the size of the bid-ask spread for a given contract increases with the length of the time to contract settlement. Consequently, currency forward contracts become less attractive for hedging long-dated foreign exchange exposure. Other instruments, such as currency swaps,²⁰ can be used for hedging.

A manager can use currency forward contracts to lock in an exchange rate at the future delivery date. In exchange for locking in a foreign exchange rate, the manager forgoes the opportunity to benefit from any advantageous foreign exchange rate movement but eliminates downside risk.

Earlier, the relationship between spot prices and forward prices was demonstrated. Arbitrage arguments can also be used to derive the relationship for currency forward contracts. Consider a U.S. manager with a 1-year investment horizon who has two choices:

Alternative 1: Deposit \$100,000 in a U.S. bank that pays 6% compounded annually for one year.

Alternative 2: Deposit the U.S. dollar equivalent of \$100,000 in some country outside the U.S. where the bank pays 5% compounded annually for one year. We will refer to this country as country i .

Which is the best alternative? It will be the alternative that produces the largest number of U.S. dollars one year from now. Ignoring U.S. and country i 's taxes on interest income or any other taxes, we need to know two things in order to determine the best alternative:

- the spot exchange rate between U.S. dollars and country i 's local currency

and

- the spot exchange rate one year from now between U.S. dollars and country i 's local currency

The first is known; the second is not. However, we can determine the spot exchange rate one year from now between U.S. dollars and country i 's local currency that will make the U.S. manager indifferent between the two investment alternatives.

For alternative 1: The amount of U.S. dollars available one year from now would be \$106,000 (\$100,000 times 1.06).

For alternative 2: Assume that the spot rate is \$0.6757 per one local currency unit. Denoting the local currency units by “LC,” and ignoring the bid/ask spread, \$100,000 can be exchanged for LC 147,995 (\$100,000 divided by 0.6757). The amount of local currency units available at the end of one year would be LC 155,395 (LC 147,995 times 1.05).

²⁰For an explanation of currency swaps, see Chapter 5 in Don M. Chance, *Analysis of Derivatives for the CFA Program* (Charlottesville, VA: Association for Investment Management and Research, 2003).

The number of U.S. dollars that the LC 155,395 can be exchanged for depends on the exchange rate one year from now. Let F denote the exchange rate between these two currencies one year from now. Specifically, F will denote the number of U.S. dollars that can be exchanged for one unit of the local currency one year from now and is called the **forward exchange rate**. Thus, the number of U.S. dollars at the end of one year from the second alternative is:

$$\text{amount of U.S. dollars one year from now} = \text{LC } 155,395 \times F$$

The investor will be indifferent between the two alternatives if the number of U.S. dollars is \$106,000, equal to the dollars resulting from alternative 1. That is,

$$\$106,000 = \text{LC } 155,395 \times F, \text{ or } F = \$106,000 / \text{LC } 155,395$$

Solving, we find that F is equal to \$0.6821. Since the spot rate is \$0.6757 and the forward exchange rate (F) is \$0.6821, then the implied appreciation for the local currency versus the U.S. dollar is 0.95% [$(\$0.6821 / \$0.6757) - 1$]. When there is an implied appreciation, it is called a **forward exchange rate premium** (or simply **forward premium**). If, instead, there had been an implied depreciation, it would be referred to as a **forward exchange rate discount** (or simply **forward discount**).

Thus, if one year from now the spot exchange rate is \$0.6821 per one local currency unit, then the two alternatives will produce the same number of U.S. dollars.²¹ If the local currency has appreciated by more than 0.95%, i.e., one local currency unit can be exchanged for more than \$0.6821, then there will be more than \$106,000 at the end of one year. An exchange rate of \$0.6910 per one local currency unit, for example, would produce \$107,378 (LC 155,395 times \$0.6910). The opposite is also true if one local currency unit can be exchanged for less than \$0.6821. For example, if the future exchange rate is \$0.6790, there will be \$105,513 (LC 155,395 times \$0.6790).

Now suppose that a dealer quotes a 1-year forward exchange rate between the two currencies. The 1-year forward exchange rate fixes today the exchange rate one year from now. Thus, if the 1-year forward exchange rate quoted is \$0.6821 per one local currency unit, investing in the bank in country i will provide no arbitrage opportunity for the U.S. investor. If the 1-year forward rate quoted is more than \$0.6821 per one local currency unit, the U.S. manager can earn an arbitrage profit by selling the local currency forward (and buying U.S. dollars forward for the local currency). In this example, assume the borrowing and lending rates within each country are equal.

To understand this arbitrage opportunity, consider how a portfolio manager could take advantage of a mispricing in the market.²² Under the conditions in the above example, assume that the bid for a 1-year forward contract in local currency is quoted \$0.6910 per local currency unit. The portfolio manager could generate an arbitrage profit by using the following strategy:

Strategy: Borrow \$100,000 for one year at the U.S. rate of 6% compounded annually and enter into a forward contract agreeing to deliver LC 155,395 one year from now at \$0.6910 per local currency.

²¹The forward rate can also be derived by looking at the alternatives from the perspective of a portfolio manager in country i .

²²A portfolio manager in any country, not just one located in one of the countries whose currency is mispriced, could take advantage of this arbitrage opportunity.

That is, one year from now the manager is agreeing to deliver LC 155,395 in exchange for \$107,378 (LC 155,395 multiplied by \$0.6910). To generate the LC 155,395, the \$100,000 that was borrowed can be exchanged for LC 147,995 at today's spot rate of \$0.6757 to one local currency unit, which can be invested in country i at 5% to yield LC 155,395 in one year.

Let's look at the outcome of this strategy at the end of one year:

From investment in country i :

LC from country i investment	LC 155,395

From forward contract:

U.S. \$ from delivery (\$0.6910 per local currency) of LC 155,395 at forward rate	\$107,378

Profit after loan repayment:

U.S. \$ available to repay loan	\$107,378

Loan repayment (principal plus interest)	\$106,000

<i>Profit</i>	\$1,378

Assuming the counterparty to the forward contract does not default, this is a riskless arbitrage situation because a \$1,378 profit is generated without taking any market risk. This will result in the U.S. dollar rising relative to the local currency in the forward exchange rate market, or possibly some other adjustment.²³

Now consider the case where the 1-year forward exchange rate quoted is less than \$0.6821 and see how a portfolio manager can exploit the situation by buying the local currency forward (or, equivalently, selling U.S. dollars forward). Suppose that the 1-year forward exchange rate is \$0.6790 and assume the borrowing and lending rates within each country are equal. The strategy implemented by the portfolio manager is:

Strategy: Borrow LC 100,000 for one year at the local rate of 5% compounded annually and enter into a forward contract agreeing to deliver US \$71,624 one year from now at \$0.6790 per local currency.

When the portfolio manager receives the LC 100,000 borrowed, she can exchange it for US \$67,570. Recall that the spot foreign exchange rate per one local currency unit equals US \$0.6757. So, LC 100,000 multiplied by the spot rate of 0.6757 gives US \$67,570. This amount of U.S. dollars is then invested in the United States at an interest rate of 6% compounded annually and will generate US \$71,624 at the end of one year ($\text{US } \$67,570 \times 1.06$).

Let's look at the outcome of this strategy at the end of one year:

²³As investors move to exploit this arbitrage opportunity, their very actions will serve to eliminate it. This can occur through the combination of a number of factors: (1) the U.S. dollar will depreciate relative to the local currency (i.e., the spot exchange rate expressed in U.S. dollars per local currency unit will rise) as investors sell dollars and buy the local currency; (2) interest rates will rise in the U.S. as investors borrow in the U.S. and invest in country i ; (3) interest rates in country i will fall as more is invested in country i ; and (4) the 1-year forward rate for U.S. dollars will show an appreciation relative to the local currency (i.e., the forward exchange rate expressed in U.S. dollars per local currency unit will fall) to eliminate the arbitrage opportunity as investors buy U.S. dollars forward. In practice, the last factor will dominate.

From investment in the United States:

US \$ from investment in U.S.	US \$71,624

From forward contract:

LC from delivery (\$0.6790 per local currency) of US \$71,624 at forward rate	LC 105,485

Profit after loan repayment:

LC available to repay loan	LC 105,485

Loan repayment (principal plus interest)	<u>LC 105,000</u>

<i>Profit</i>	LC 485

Once again, assuming the counterparty to the forward contract does not default, this is a riskless arbitrage situation because a LC 485 profit is generated with no initial investment. This will result in the U.S. dollar falling relative to the local currency in the forward exchange rate market, or possibly some other adjustment.

The conclusion is the 1-year forward exchange rate must be \$0.6821 because any other forward exchange rate would result in an arbitrage opportunity.

2. Interest Rate Parity and Covered Interest Arbitrage Our illustration indicates that the spot exchange rate and the short-term interest rates in two countries will determine the forward exchange rate. The relationship among the spot exchange rate, the interest rates in two countries, and the forward rate is called **interest rate parity**. It says that a manager, after hedging in the forward exchange rate market, will realize the same domestic return whether investing domestically or in a foreign country. The arbitrage process that forces interest rate parity is called **covered interest arbitrage**.

It can be demonstrated that the forward exchange rate between an investor's home currency, denoted " H " and the currency of country i , is equal to

$$F_{H,i} = S_{H,i} \left(\frac{1 + c_H}{1 + c_i} \right) \quad (2)$$

where

$F_{H,i}$ = forward exchange rate between investor's home currency and the currency of country i

$S_{H,i}$ = spot (or cash) exchange rate between investor's home currency and the currency of country i

c_H = short-term interest rate in the home country which matches the maturity of the forward contract

c_i = short-term interest rate in country i which matches the maturity of the forward contract

c_H and c_i are called the **cash rate**. The cash rate is generally the eurodeposit rate (i.e., offshore deposit rate) for funds deposited in that currency which matches the maturity of the forward contract. The London Interbank Offered Rate, LIBOR, is the most quoted offshore (eurodeposit) rate. LIBOR deposit rates are available for U.S. dollars and most other major currencies, including EURIBOR for euro-denominated deposits.

In our earlier illustration involving the U.S. dollar and the exchange rate of country i , we know that

$$S_{H,i} = 0.6757 \quad c_H = 6\% = 0.06 \quad c_i = 5\% = 0.05$$

$$F_{H,i} = 0.6757 \left(\frac{1.06}{1.05} \right) = 0.6821$$

This value for the 1-year forward exchange rate agrees with the value derived earlier.

By rearranging the above terms, the forward exchange rate discount or premium (or the percentage change of the forward rate from the spot exchange rate), denoted by $f_{H,i}$, *approximately* equals the differential between the short-term interest rates of the two countries. That is,²⁴

$$f_{H,i} = \frac{F_{H,i} - S_{H,i}}{S_{H,i}} \approx c_H - c_i \quad (3)$$

That is, for the return on cash deposits to be equal in both currencies, the lower interest rate currency must appreciate to the forward foreign exchange rate.

The forward rate can also be expressed in “points” or the difference between the forward and spot rate, $F_{H,i} - S_{H,i}$. When interest rates are lower in the foreign country (i.e., the forward points are positive), the forward foreign exchange rate trades at a premium.

B. The Currency Hedge Decision

If a global bond portfolio is fully hedged, the portfolio return of equation (1) changes. Specifically, if the manager hedged the currency exposure in all countries using currency forward contracts, the total return for a fully hedged portfolio into the home currency can be expressed as follows:

$$\begin{aligned} &\text{total expected portfolio return fully hedged into investor's home currency} \\ &= W_1 \times (r_1 + f_{H,1}) + W_2 \times (r_2 + f_{H,2}) + \cdots + W_N \times (r_N + f_{H,N}) \end{aligned} \quad (4)$$

where

$f_{H,i}$ = the forward exchange rate discount or premium between the home currency
and country i 's local currency

That is, instead of being exposed to some expected percentage change of the home currency to country i 's currency, the manager will have locked in the percentage change of the forward exchange rate from the spot exchange rate (the forward discount/premium) at the time of the hedge.

²⁴Equation (2) assumes that exchange rates are quoted in “direct terms,” i.e., the value of the home currency for one unit of the local currency, though quote conventions vary by market. Over-the-counter forward contracts use market convention, most of which for the U.S. dollar are in indirect terms (local currency units per one dollar). Using indirect terms, the forward discount or premium in equation (3) becomes $f_{H,i} = c_i - c_H$. To avoid the complexities of compounding, the time period is assumed to be one year.

Now, what will determine whether or not the manager will hedge the exposure to a given country's exchange rate using a currency forward contract? The decision is based on the expected return from holding the foreign currency relative to the forward premium or discount. That is, if the manager expects that the percentage return from exposure to a currency is greater than the forward discount or premium, then the manager will not use a forward contract to hedge the exposure to that currency. Conversely, if the manager expects the currency return to be less than the forward discount or premium, the manager will use a forward contract to hedge the exposure to a currency.

In the case where the manager expects that the percentage return from exposure to a currency is greater than the forward discount or premium, the unhedged return for country i can be expressed as:

$$\text{unhedged expected return for country } i, R_{H,i} = r_i + e_{H,i} \quad (5)$$

In the case where the manager expects the currency return to be less than the forward discount or premium, we can express the hedged return for a country in terms of the forward exchange rate between the home and local currencies using the interest rate parity relationship. As equation (3) showed, the forward premium or discount is effectively equal to the short-term interest rate differential; thus,

$$f_{H,i} \approx c_H - c_i$$

By substituting the above relationship into equation (4) for the forward hedge, the equation for an individual country's hedged return (HR) is:

$$\text{hedged expected return for country } i, HR_{H,i} = r_i + f_{H,i} \approx r_i + c_H - c_i \quad (6)$$

There remain, however, two further hedging choices for the manager: **cross hedging** and **proxy hedging**. We explain each of these below.

1. Cross Hedging Cross hedging is a bit of a misnomer as it does not reduce foreign currency exposure but only replaces the currency exposure to country i 's currency with currency exposure to country j 's currency. (We explain what cross hedging is in Chapter 22.) For example, suppose a U.S. manager has an unwanted currency exposure in country i that arose from an attractive bond investment in country i . Rather than hedging with a forward contract between U.S. dollars and the currency of country i and eliminating the foreign currency exposure, the manager elects to swap exposure in country i 's currency for exposure to country j 's currency. This is accomplished by entering into a forward contract that delivers the currency of country j in exchange for the currency of country i where the manager has an unwanted currency exposure.

Why would a manager want to undertake a cross hedge? A manager would do so if she expects her home currency to weaken, so she does not want to hedge the currency exposure to country i , but at the same time she expects that country j 's currency will perform better than country i 's currency.

When there is a cross hedge, the hedged return for country i , $HR_{H,i}$, in equation (6) can be rewritten as follows:

$$\text{cross hedged expected return for country } i, CR_{H,i} = r_i + f_{j,i} + e_{H,j}$$

where $f_{j,i}$ is the forward discount or premium between country j and country i . The above expression says that the cross hedged return for country i depends on (1) the expected bond return for country i , (2) the currency return locked in by the cross hedge between country j and country i , and (3) the currency return between the home currency and country j .

We can rewrite the above equation in terms of short-term interest rates as given by interest rate parity. That is, for $f_{j,i}$ we substitute $c_j - c_i$. Doing so and rearranging terms gives:

$$\text{cross hedged expected return for country } i, CR_{H,i} \approx (r_i - c_i) + (c_j + e_{H,j}) \quad (7)$$

Equation (7) says that the cross hedged expected return for country i depends on (1) the differential between country i 's bond return and country i 's short-term interest rate plus (2) the short-term interest rate in country j , and (3) the currency return between the home currency and country j .

2. Proxy Hedging Proxy hedging keeps the currency exposure in country i , but creates a hedge by establishing a short position in country j 's currency. Why would a manager want to undertake a proxy hedge? This strategy would normally be considered only where the currencies of country i and country j are highly correlated, and the hedge costs in country j are lower than in country i . A proxy hedge can also represent a bullish view on the home currency, with a more negative view on country j 's currency than country i 's currency.

When there is a proxy hedge, the hedged return for country i , $HR_{H,i}$, in equation (6) can be rewritten as follows:

$$\text{proxy hedged expected return for country } i, PR_{H,i} = r_i + e_{H,i} + f_{H,j} - e_{H,j}$$

where $f_{H,j}$ is the forward discount or premium between the home country and country j .

Notice that in the above equation, there is still the exposure to the exchange rate between the home currency and country i . The proxy hedge comes into play by the shorting of the currency return between the home currency and country j .

Based on interest rate parity we can replace $f_{H,j}$ with the difference in short-term interest rates, $c_H - c_j$, to get

$$\text{proxy hedged expected return for country } i, PR_{H,i} \approx r_i + e_{H,i} + c_H - c_j - e_{H,j}$$

This is equivalent to

$$\begin{aligned} &\text{proxy hedged expected return for country } i, \\ &PR_{H,i} \approx (r_i - c_i) + (c_i + e_{H,i}) + [(c_H - c_j) - e_{H,j}] \end{aligned} \quad (8)$$

Equation (8) states the expected return for country i using proxy hedging depends on

1. the differential between the bond return for country i and the short-term interest rate for country i
2. the short-term interest rate for country i adjusted for the currency return for country i relative to the home currency
3. the differential in the short-term interest rates between the home currency and country j adjusted for the short currency position in country j .

3. Recasting Relationships in Terms of Short-Term Interest Rates When we substituted short-term interest rate differentials for the forward premia or discounts above, it becomes apparent from equations (6), (7), and (8) that the difference in return between hedging, cross hedging, and proxy hedging is entirely due to differences in short-term interest rates and currency exposure.²⁵ This is also true for the unhedged return for a country as given by equation (5). This can be seen by simply rewriting equation (5) as follows:

$$\text{unhedged expected return for country } i, R_{H,i} = (r_i - c_i) + (c_i + e_{H,i})$$

The unhedged expected return is thus equal to (1) the differential between the bond return in country i and the short-term interest rate in country i and (2) the short-term interest rate in country i adjusted for the currency return.

These equations show how integral the short-term interest rate differential is to the currency hedge decision. This means that (1) the short-term interest rate differential should relate to the currency decision and (2) bond market returns should be an excess return, calculated less the local short-term interest rate. This can be made explicit by adding and subtracting the home currency short-term interest rate to the four return relationships—unhedged, hedged, cross hedged, and proxy hedged. (The derivations are provided in the appendix to this chapter.) By doing so, this allows the forward premium ($f_{H,i} = c_H - c_i$) to be inserted into the currency term giving:

$$\text{unhedged expected return for country } i, R_{H,i} = c_H + (r_i - c_i) + (e_{H,i} - f_{H,i}) \quad (9)$$

$$\text{hedged expected return for country } i, HR_{H,i} = c_H + (r_i - c_i) \quad (10)$$

cross hedged expected return for country i ,

$$CR_{H,i} = c_H + (r_i - c_i) + (e_{H,j} - f_{H,j}) \quad (11)$$

proxy hedged expected return for country i ,

$$PR_{H,i} = c_H + (r_i - c_i) + [(e_{H,i} - e_{H,j}) - f_{j,i}] \quad (12)$$

From equations (9) through (12), we see the return for each strategy can be divided into three distinct return components:

Component 1: the short-term interest rate for the home currency: (c_H)

Component 2: the excess bond return of country i over the short-term interest rate of country i : ($r_i - c_i$)

Component 3: the excess currency return, either unhedged, cross-hedged, or proxy hedged

The first two components, c_H and $(r_i - c_i)$, are the same for each strategy. The excess currency return (the third component) becomes the currency return in excess of the forward premium (or discount) and becomes the basis for the decision of currency hedging. (We will illustrate this below.) The bond decision is purely a matter of selecting the markets which offer the best expected excess return ($r_i - c_i$) and the bond and currency allocation decisions are entirely independent. In a sense, the hedged expected return can be considered the base expected return as it is a component of the unhedged, cross hedged, and proxy hedged expected returns. Thus,

²⁵The derivation of the relationships presented in this section are provided in the appendix to this chapter.

the excess currency returns in the third component are assessed to see if they can add any value over the baseline hedged expected return. This method of analyzing sources of return in effect treats bond and currency returns as if they were synthetic futures or forward positions.

It is important to note that only the hedged position eliminates all currency risk. The cross hedge substitutes one currency exposure for another, but maintains foreign currency exposure. The proxy hedge leaves the portfolio exposed to “basis” risk if the proxy hedge currency appreciates relative to the investment currency.

4. Illustration Let’s illustrate the above relationships using a U.S. portfolio manager given a specific market outlook. Since this illustration uses a U.S. portfolio manager, the home currency is U.S. dollars and therefore “ H ” in the notation is the U.S. dollar, denoted by “US\$.” The outlook is for country i ’s bond market to outperform country j ’s bond market, but for country i ’s currency to provide a higher return than country j ’s currency.

Exhibit 3 illustrates an example comparing the explicit return forecasts for government bonds with a duration of 5 in both countries. Total returns are the sum of the excess bond market return plus the excess return due to currency, consistent with the approach explained in equations (9) through (12). These equations are restated below using US\$ for the home currency, H , using currency j for the cross hedge and proxy hedge, and remembering f_{US,i} \approx C_{US\$} - C_i$:

unhedged expected return for country i , R_{US,i} = c_{US\$} + (r_i - c_i) + (e_{US$,i} - f_{US$,i})$

hedged expected return for country i , HR_{US,i} = c_{US\$} + (r_i - c_i)$

cross hedged expected return for country i , CR_{US,i} = c_{US\$} + (r_i - c_i) + (e_{US$,j} - f_{US$,j})$

proxy hedged expected return for country i ,

$$PR_{US$,i} = c_{US\$} + (r_i - c_i) + [(e_{US$,i} - e_{US$,j}) - f_{j,i}]$$

The interest rates and expected returns are as follows:

$$r_i = 3.5\%$$

$$c_i = 3.0\%$$

EXHIBIT 3 Illustration 1

	Hedged	Unhedged	Cross hedged	Proxy hedged
Expected Returns				
Cash	$c_{US\$}$ = 5.5%	$c_{US\$}$ = 5.5%	$c_{US\$}$ = 5.5%	$c_{US\$}$ = 5.5%
Excess Bond	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%
Excess Currency		e_{US,i} - (c_{US\$} - c_i)$ = 2.3% - (5.5% - 3.0%) = 2.3% - 2.5% = -0.2%	e_{US,j} - (c_{US\$} - c_j)$ = 2.0% - (5.5% - 2.9%) = 2.0% - 2.6% = -0.6%	$(e_{US$,i} - e_{US$,j}) - (c_j - c_i)$ = (2.3% - 2.0%) - (2.9% - 3.0%) = 0.3% - (-0.1%) = 0.4%
Total Return	= 6.0%	= 5.8%	= 5.4%	= 6.4%

$$e_{US\$,i} = 2.3\%$$

$$c_j = 2.9\%$$

$$e_{US\$,j} = 2.0\%$$

$$c_{US\$} = 5.5\%$$

As mentioned earlier, the first two components of the above equations, the U.S. cash rate and the expected excess bond return in country i , are identical in all four equations, and equal to the expected hedged bond return. Thus, we can begin with the hedged bond return and compare the excess currency returns (the third component of the equations) of the unhedged, cross hedged, and proxy hedged strategies. The hedged bond return is

$$c_{US\$} + (r_i - c_i) \text{ or } 5.5\% + (3.5\% - 3.0\%) = 6.0\%.$$

Let's look at this component for the unhedged strategy. From the first equation:

$$\text{excess currency return for unhedged strategy for country } i = (e_{US\$,i} - f_{US\$,i})$$

or equivalently, since from interest rate parity $f_{US\$,i} = c_{US\$} - c_i$, we can rewrite the expression as

$$\text{excess currency return for unhedged strategy for country } i = e_{US\$,i} - (c_{US\$} - c_i)$$

Thus, the performance relative to the hedged currency strategy depends on whether the expected currency appreciation is greater than the short-term interest rate differential [i.e., $e_{US\$,i} > (c_{US\$} - c_i)$] or less than the interest rate differential [i.e., $e_{US\$,i} < (c_{US\$} - c_i)$]. In the former case, the unhedged strategy is expected to outperform the hedged strategy.

Turning to our illustration, the expected return on currency i of 2.3% is less than the short-term interest rate differential of 2.5% over the 1-year horizon (5.5% in the U.S. versus 3.0% in country i). Stated another way, the expected excess currency return component to a U.S. dollar-based investor from an unhedged bond holding in currency i is -0.2% . Consequently, the position would offer a higher return when hedged back into U.S. dollars.

Now consider a cross hedging strategy. Cross hedging allows the portfolio manager to create a currency exposure that can vary substantially from the underlying bond market exposure. A cross hedge replaces one foreign currency exposure with another that usually has a higher expected return. The excess currency component from the cross hedge strategy is:

$$\text{excess currency return for cross hedged strategy for country } i = (e_{US\$,j} - f_{US\$,j})$$

or equivalently, since from interest rate parity $f_{US\$,j} \approx c_{US\$} - c_j$, we can rewrite the expression as

$$\text{excess currency return for cross hedged strategy for country } i = e_{US\$,j} - (c_{US\$} - c_j)$$

Compared to a hedged strategy, a cross hedge is attractive if the short-term interest rate of the country used for the cross hedge plus the expected return in the cross currency is greater than the U.S. dollar short-term interest rate. If the U.S. dollar short-term interest rate is greater than the sum of these two terms, a cross hedged strategy is less attractive than a hedged strategy.

In the illustration, the short-term interest rate differential of 2.6% between the U.S. and country j ($c_{US\$} - c_j$) is greater than the 2.0% expected appreciation of the currency of country j versus the U.S. ($e_{US\$,j}$). In this case, the expected excess currency return for the cross hedge

strategy for country i using country j (i.e., $e_{US\$,j} - (c_{US\$} - c_j)$) is -0.6% . The expected return from cross hedging is 5.4% , so a cross hedge with country j will not be used because the expected return is less than the unhedged position and a straight hedge of currency i .

Finally, let's look at the proxy hedging strategy. From the return of the proxy hedged strategy we know that

$$\text{excess currency return for proxy hedged strategy for country } i = [(e_{US\$,i} - e_{US\$,j}) - f_{j,i}]$$

or equivalently, since from interest rate parity $f_{j,i} \approx c_j - c_i$, we can rewrite the expression as

$$\text{excess currency return for proxy hedged strategy for country } i = [(e_{US\$,i} - e_{US\$,j}) - (c_j - c_i)]$$

To interpret the above equation, let's understand the currency position of the U.S. investor. The investor is long currency i . Consequently, the investor benefits if currency i appreciates but is hurt if currency i depreciates. In a proxy hedge, the investor is still long currency i but the investor is also short currency j . Since the investor is short currency j , the investor is adversely affected if currency j appreciates, but benefits if currency j depreciates relative to currency i .

In our illustration, both currency i and currency j are expected to appreciate relative to the U.S. dollar. The relative currency appreciation between currency i and currency j is what is important according to the equation. If the appreciation for currency i —which the investor is long—is greater than the appreciation for currency j —which the investor is short—an investor will benefit from a proxy hedge. In our illustration, country i 's expected appreciation is 2.3% while country j 's is only 2.0% . Thus, there will be an expected currency return from this proxy hedging strategy of 30 basis points. This is what the first bracketed term in the excess currency return equation above says.

Just looking at the expected currency return from a proxy hedging strategy, however, is not sufficient. The above equation shows the expected currency return for the proxy hedging strategy must be adjusted to determine the excess currency return for the proxy hedging strategy. The adjustment is obtained by subtracting the short-term interest rate differential in countries j and i from the expected currency return from proxy hedging. If that differential is less than the expected currency return from proxy hedging, then proxy hedging is attractive. If it is greater than the expected currency return from proxy hedging, then proxy hedging is unattractive.

In our illustration, the proxy hedging strategy is attractive because the short-term interest rate differential between country j and country i is -10 basis points, which is less than the 30 basis point currency return for the proxy hedging strategy. The excess return from the proxy hedging strategy is then 30 basis points minus the -10 basis point short-term interest rate differential. So, the excess return from the proxy hedging strategy in our illustration is 40 basis points. The proxy hedged expected return is 6.4% , which is greater than the three other alternatives—unhedged, hedge with currency i , and cross hedge with currency j .

In Exhibit 4 we have changed the example in Exhibit 3 by altering one number. In this illustration, the expected appreciation for currency j is now 3.2% rather than 2.0% . Thus, the expected appreciation for currency j is greater than for currency i . The expected currency return for the proxy hedging strategy is then -90 basis points ($2.3\% - 3.2\%$). After adjusting for the short-term interest differential of -10 basis points, the excess currency return using country j in a proxy hedge is -80 basis points. Consequently, the proxy hedge return of 5.2% is unattractive, as it is less than the three other alternatives analyzed. In this illustration, the cross hedge presents the best choice based on the expected returns.

EXHIBIT 4 Illustration 2

	Hedged	Unhedged	Cross hedged	Proxy hedged
Expected Returns				
Cash	$c_{US\$}$ = 5.5%	$c_{US\$}$ = 5.5%	$c_{US\$}$ = 5.5%	$c_{US\$}$ = 5.5%
Excess Bond	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%	$(r_i - c_i)$ = (3.5% - 3.0%) = 0.5%
Excess Currency		$e_{US\$,i} - (c_{US\$} - c_i)$ = 2.3% - (5.5% - 3.0%) = 2.3% - 2.5%	$e_{US\$,j} - (c_{US\$} - c_j)$ = 3.2% - (5.5% - 2.9%) = 3.2% - 2.6%	$(e_{US\$,i} - e_{US\$,j}) - (c_j - c_i)$ = (2.3% - 3.2%) = -0.9% - (-0.1%)
	= 0.0%	= -0.2%	= 0.6%	= -0.8%
Total Return	= 6.0%	= 5.8%	= 6.6%	= 5.2%

C. Adjusting Bond Yields for Coupon Payment Frequency

In the United States and most other dollar bloc countries, coupon payments are made semiannually. There are other markets that follow this practice. Computing the yield for a semiannual-pay bond was explained using two steps. First, the semiannual interest rate that will make the present value of the semiannual cash flows equal to the price plus accrued interest is determined. Second, since the interest rate is semiannual, it is annualized by multiplying by 2. The resulting annualized yield is referred to as a **bond-equivalent yield**.

In European markets (except for the United Kingdom) and Japan, coupon payments are made annually rather than semiannually. Thus, the yield is simply the interest rate that makes the present value of the cash flows equal to the price plus accrued interest. No annualizing is necessary.

The yield quoted in terms of the home market's convention for payments is called the **conventional yield**. For example, Exhibit 5 displays data from the J.P. Morgan Europe (MEUR) page from Reuters' market information service. The column "CNV. YLD" is the conventional yield. So, the U.S. and U.K. yields of 4.20% and 4.68%, respectively, shown in Exhibit 5 are based on the bond-equivalent yield convention of doubling a semiannual yield since coupon payments are made semiannually. In countries where coupon payments are made annually, in Germany and Japan, for example, the conventional yield is simply the annual yield.

Despite the limitations of yield measures discussed earlier, managers compare yields within markets of a country and between countries. (We will give one example in the next section.) Holding aside the problem of potential changes in exchange rates, yield comparisons begin by adjusting conventional yield (i.e., the yield as quoted in the home market) to be consistent with the way the yield is computed for another country. For example, a French government bond pays interest annually while a U.S. government bond pays interest semiannually. If the U.S. government bond yield is being compared to a French government bond yield either (1) the U.S. government bond yield must be adjusted to the yield on an annual-pay basis or (2) the French government bond yield must be adjusted to a yield on a bond-equivalent yield basis.

EXHIBIT 5 10-Year Benchmark Bond Spreads: December 3, 2002

03DEC02 JP MORGAN BENCHMARKS JP MORGAN UK13148							
Country	Coupon	ISSUE		PRICE	CNV YLD	O/US T	O/GER
US	4.00	15-11-12	T	98.39-41	4.20		-23
JAPAN	1.00	20-12-12	JGB	99.73-82	1.02	-322	-345
GERMANY	5.00	04-07-12	BUND	103.98-02	4.47	+23	
FRANCE	4.75	25-10-12	OAT	101.56-62	4.55	+30	+7
UK	5.00	07-03-12	GILT	102.36-42	4.68	+49	+26
ITALY	4.75	01-02-13	BTP	100.89-91	4.69	+45	+22
SPAIN	5.00	30-07-12	BONO	103.42-46	4.55	+31	+7
BELGIUM	5.00	28-09-12	OLO	103.13-17	4.59	+35	+12
HOLLAND	5.00	15-07-12	DSL	103.57-61	4.52	+28	+5
SWEDEN	5.50	08-10-12	SGB	103.44-59	5.04	+79	+56
DENMARK	5.00	15-11-13	DGB	102.00-08	4.76	+51	+28
FINLAND	5.38	04-07-13	FINL	106.34-42	4.60	+35	+12
AUSTRIA	5.00	15-07-12	AGB	103.36-42	4.55	+31	+8
GREECE	5.25	18-05-12	GGB	103.97-03	4.71	+47	+23

Source: MEUR page of Reuters' market information service

Authors' notes to exhibit: "CNV YLD" means conventional yield, or how the yield is quoted in the home market. For example, both the U.S. and U.K. bond markets are semiannual pay, whereas most of Europe is annual pay. However, in Italy, even though bonds are semiannual pay, they are quoted on an annual basis. The spreads ("O/UST" = spread over U.S. Treasuries and "O/GER" = spread over German government bonds) first convert the semiannual-pay markets (the U.S., and the U.K.) to an annual-pay basis before calculating the spread between markets.

The adjustment is done as follows. Given the yield on an annual-pay basis, its bond-equivalent yield (i.e., a yield computed for a semiannual-pay bond) is computed as follows:

$$\text{bond-equivalent yield of an annual-pay bond} = 2[(1 + \text{yield on annual-pay bond})^{0.5} - 1]$$

For example, the conventional yield on a French government bond shown in Exhibit 5 is 4.55%. The bond-equivalent yield is 4.50% as shown below:

$$2[(1 + 0.0455)^{0.5} - 1] = 0.0450 = 4.50\%$$

Notice that the bond-equivalent yield of an annual-pay bond is less than that of the conventional yield.

To adjust the bond-equivalent yield of a semiannual-pay bond to that of an annual-pay basis so that it can be compared to the yield on an annual-pay bond, the following formula can be used:

$$\begin{aligned} &\text{yield on an annual-pay basis of a bond-equivalent yield} \\ &= (1 + \text{yield on a bond-equivalent yield}/2)^2 - 1 \end{aligned}$$

For example, the conventional yield of a U.S. government bond as shown in Exhibit 5 is 4.20%. The yield on an annual-pay basis is:

$$(1 + 0.0420/2)^2 - 1 = 0.0424 = 4.24\%$$

Notice that the yield on an annual-pay basis will be greater than the conventional yield.

Yield spreads are typically computed between a country's yield and that of a benchmark. As explained, the U.S. government bond market and the German government bond market are the two most common benchmarks used. The next-to-the-last column in Exhibit 5, labeled "O/US T," shows the spread between a country's yield and the U.S. Treasury yield. Notice that for the French government bond, the spread is shown as +30 basis points. This spread is obtained by subtracting the French government bond of 4.54% (the conventional yield reported in Exhibit 5) from the adjusted U.S. government bond yield of 4.24% (as computed above).

The last column in Exhibit 5, labeled "O/GER," shows the spread between a country's yield and the German government bond yield. For example, when the spread of the French government bond yield over the German government bond yield is computed, since both markets pay coupon interest annually, the spread is simply the difference in their conventional yields. Since the yield on the French government bond is 4.55% and the yield on a German government bond is 4.47%, the spread is 8 basis points. (Exhibit 5 shows a 7 basis point spread. The difference is due to rounding.)

D. Forward Rates and Breakeven Analysis

As explained earlier, there are various methods of evaluating relative value in international bond markets. Before these can be translated into a market allocation, a manager must compare their strategic outlook to that which is already priced into the market. This can be accomplished by either converting the economic outlook into point forecasts for bond and currency levels, or looking at the forward rates implied by current market conditions and comparing them with the economic outlook.

Bond and currency breakeven rates, the rate which make two investments produce identical total returns, are usually calculated versus a benchmark market return over a specific time horizon. A large yield spread between two markets implies a larger "cushion" (the required spread widening to equate total returns in both markets, or the breakeven rate) over the investment time horizon.

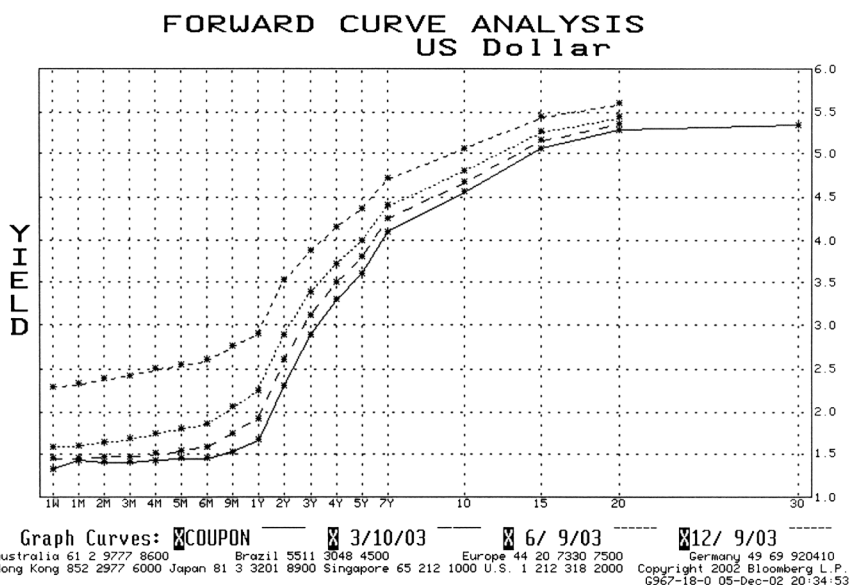
Comparing of forward interest rates can be instrumental in identifying where differences between the strategic outlook and market prices may present investment opportunities. As explained previously, forward interest rates use the shape of the yield curve to calculate implied forward bond rates and allow a quick comparison of what is required, in terms of yield shifts in each market, to provide a return equal to the short-term risk-free rate (a zero excess return). This would correspond to a bond excess return of zero in equations (9) through (12), or $(r_i - c_i) = 0$. Forward interest rates represent a breakeven rate, not across markets, but within markets. The strategic bond allocation can then be derived by increasing exposure to markets where the expected bond return over the short-term interest rate is most positive—that is, where the expected bond yield is furthest below the forward yield. Forward rate calculators are also available on systems such as Bloomberg as illustrated in Exhibit 6.

The forward foreign exchange rate represents a breakeven rate between hedged and unhedged currency returns as previously shown in the components of return analysis. In terms of equations (9), (11), and (12), currency excess return is zero when the percentage change in the currency equals the forward premium or discount. As forward foreign exchange rates are determined by short-term interest rate differentials, they can be estimated from the interest rates on deposits, specifically, Eurodeposit rates as in equations (2) and (3), which can be easily obtained from market data services such as Bloomberg and Reuters.

EXHIBIT 6 Forward Yield Curve Analysis: Germany

<HELP> for explanation.

P067 Govt FWCV

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Breakeven analysis provides another tool for estimating relative value between markets. Because the prices of benchmark bonds are influenced by coupon effects and changes in the benchmark, many international fixed income traders and portfolio managers find it easier to keep pace with changes in yield relationships than price changes in each market. A constant spread between markets when yield levels are shifting, however, may result in a variation in returns as differing benchmark bond maturities and coupons result in a wide spread of interest rate sensitivity across markets. For example, of the benchmark 10-year bonds listed in Exhibit 5, durations range from a low of 7.2 for Greece to 9.4 for Japan where yields equal one third of the next lowest yielding market. (The duration for the U.S. bond was 8.1.) Thus, market duration must be taken into account when determining breakeven spread movements.

Since European Monetary Union, yield differentials within Europe have remained extremely tight. Holding Italian versus German bonds provides a yield advantage of only 22 basis points per year. Obviously, this small amount of additional yield can be easily offset by an adverse price movement between the two markets. In the mid 1990s before European Monetary Union, Italian bonds would have yielded several hundred basis points more than German bonds as the additional currency risk involved in holding Italian bonds had a substantial impact on nominal yield spreads. Even a fairly wide yield cushion, however, can also quickly evaporate.

To illustrate this and show how breakeven analysis is used, look at the yield spread between the 10-year U.S. and Japanese government bonds on December 3, 2002 as shown in Exhibit 5. The spread is 322 basis points, providing Japanese investors who purchased the U.S. benchmark Treasury with additional yield income of 80 basis points per quarter. This

additional yield advantage can be eliminated by the spread widening substantially less than the 80 basis points. The widening can occur in one of the following two ways:

- yields in Japan can decrease, resulting in price appreciation of the Japanese government bond
- yields in the United States can increase, resulting in a price decline of the U.S. Treasury bond

Of course, a combination of the two can also occur. To quantify the amount of spread widening that would erase the yield advantage from investing in a higher yielding market, we need to conduct a breakeven analysis.

It is important to note this breakeven analysis is not a total return analysis; it applies only to bond returns in local currency and ignores currency movements. This breakeven analysis is effective in comparing bond markets that share a common currency, as within the euro zone; however, currency must be taken into account when applying breakeven analysis to markets with different currencies. The additional yield advantage in the example above is erased if the U.S. dollar depreciates by more than 0.80% during the quarter. Below, we illustrate how a hedged breakeven analysis can be calculated using hedged returns, or simply the forward foreign exchange premium or discount between the two currencies.

We know that the duration of the Japanese bond is 9.4.²⁶ This means that for a 100 basis point change in yield the approximate percentage price change for the Japanese bond will be 9.4%. For a 50 basis point change in yield, the percentage price change for the Japanese bond will change by approximately 4.7%. We can generalize this as follows:

$$\text{change in price} = 9.4 \times \text{change in yield}$$

If we let W denote the spread widening, we can rewrite the above equation as:

$$\text{change in price} = 9.4 \times W$$

We want to determine the amount of yield movement in Japan that would exactly offset the yield advantage of 0.80% from investing in U.S. bonds. Thus, we need to calculate what decline in Japanese bond yields would generate exactly 0.80% in price appreciation to make the Japanese investor indifferent between the two investments (ignoring any potential currency movements). Thus, the equation becomes

$$0.80\% = 9.4 \times W$$

Solving for W ,

$$W = 0.80\%/9.4 = 0.085\% = 8.5 \text{ basis points}$$

Therefore, a spread widening of 8.5 basis points due to a decline in the yield in Japan would negate the additional yield from buying the U.S. Treasury issue. In other words, a change in

²⁶This is the modified duration for the issue. Since the Japanese bond and the U.S. bonds are option-free, the modified duration is close to the effective duration. Duration is only a first approximation of the approximate change in value when interest rates change. By only considering duration, the analysis above ignores the impact of convexity on returns.

yields of only 8.5 basis points is needed in this case to delete the 3-month yield advantage of 80 basis points.

We refer to this yield spread change as the **breakeven spread movement**. Note that the breakeven spread movement must (1) be related to an investment horizon and (2) utilize the higher of the two countries' modified durations. Using the highest modified duration provides the minimum spread movement that would offset the additional yield from investing in a higher yielding market. So, in our example, the 3-month breakeven spread movement due to Japanese yields is 8.5 basis points, meaning that it is the spread movement due to a drop in Japanese rates by 8.5 basis points that would eliminate the 3-month additional yield from investing in U.S. Treasury bonds. The breakeven spread movement using the 8.1 duration in the U.S. would be 9.9 basis points ($0.8/8.1 = 9.9$); a difference of only 1.4 basis points.

The breakeven spread movement described above completely ignores the affect of currency movements on returns. It also ignores the implied appreciation or depreciation of the currency reflected in the forward premium or discount. If we subscribe to the methodology discussed earlier in the chapter of attributing cash returns to the currency decision, and measuring bond market returns as the local return minus the cash rate, the results of the breakeven spread movement analysis on a hedged basis may be quite different. We can easily calculate the hedged breakeven spread movement by adding in the forward foreign exchange discount or premium. At the time of this breakeven analysis, 3-month interest rates were 0.0675% in Japan and 1.425% in the United States. With this information we can obtain the embedded forward rate using equation (3); that is,

$$f_{\$,¥} \approx c_{¥} - c_{\$} = (0.0675\% - 1.425\%)/4 = -0.34\%$$

The expected hedged return over the 3-month period, assuming no change in rates, is the sum of the nominal spread differential (0.80%) and the forward premium (-0.34%), or 0.46%. Thus, the breakeven spread movement on a hedged basis is a mere 5 basis points ($0.46\% = 9.4 \times W$) instead of the 8.5 basis points of potential widening calculated on a local currency basis. Consequently, a Japanese investor would have to expect either that spreads would not widen by more than 5 basis points or believe that the dollar would depreciate versus the yen by less than the embedded forward rate to make the trade attractive. Because currency hedge costs (i.e., the forward premium or discount) are determined by short-term interest rates, the breakeven spread movement on a hedged basis will always be smaller when hedging a currency with higher short-term interest rates.

Alternatively, we could use equation (10) to calculate the expected hedged return to a yen-based investor over a 3-month period and compare it to the return on a Japanese 10-year bond over the same period. In order to do so, it is first necessary to adjust the U.S. government bond yield (which is quoted on a bond-equivalent yield basis) to an annual-pay yield basis because the Japanese yield is based on an annual basis. Earlier we showed that the conventional yield of 4.20% for the U.S. government bond as reported in Exhibit 5 is 4.24% on an annual-pay basis. Assuming no change in rates, the expected hedged return is:

$$[(r_{\$} - c_{\$}) + c_{¥}]/4 = [(4.24\% - 1.43\%) + 0.07\%]/4 = [2.88\%]/4 = 0.72\%$$

and the expected Japanese bond return is $(1.02\%/4)$, or 0.26%. Thus, the expected return on a hedged basis is 0.44%, which is close to the 0.46% in the first answer that we calculated.

E. Security Selection

Once the bond market allocation decisions have been made and the optimal duration and yield curve profile selected for each market, the overall portfolio structure needs to be constructed through the purchase or sale of individual securities. Many international bond investors prefer to trade only benchmark issues since they are more liquid than other similar maturity bonds. This can sometimes lead to a “dip” in the yield curve as investors prefer a certain issue or maturity sector. The same phenomenon can result from a squeeze of certain issues in the repo market, or short-term demand imbalances for bonds deliverable into short bond futures positions.

Taxation issues also need to be taken into account when selecting individual bonds for purchase. For example, several markets have tax systems that encourage investors to hold lower coupon bonds, hence certain bonds will tend to trade rich or cheap to the curve depending on their coupon. In markets that impose withholding taxes on coupon payments, international fixed income portfolio managers often minimize their tax liability by replacing a bond that is near its coupon date with another bond of similar maturity. Market anomalies can also arise from differing tax treatment within markets. For example, Italian Eurobonds issued before 1988 are exempt of withholding tax for Italian investors, hence they tend to trade at a lower yield than similar maturity bonds issued after 1988.

APPENDIX

The purpose of this appendix is to show how equations (9), (10), (11), and (12) in the chapter were derived. All equation numbers refer to equations in this chapter.

Unhedged expected return:

To derive the unhedged expected return given by equation (9), we begin with equation (5):

$$\text{unhedged expected return for country } i, R_{H,i} = r_i + e_{H,i}$$

adding and subtracting c_i on the right-hand side of the equation we get:

$$R_{H,i} = r_i - c_i + c_i + e_{H,i}$$

We know that $f_{H,i} = c_H - c_i$, so $c_i = c_H - f_{H,i}$. Substituting for the second c_i in the above equation and rearranging terms we have

$$R_{H,i} = c_H + (r_i - c_i) + (e_{H,i} - f_{H,i})$$

The above equation is equation (9), which states that the unhedged expected return for country i is equal to the short-term interest rate for the home country, the excess bond return of country i , and the unhedged excess currency return of country i 's currency. The excess currency return is the currency return in country i relative to the home country less the short-term interest rate differential between the home country and country i .

Hedged expected return:

The hedged expected bond return for country i , equation (6), is derived by adding a currency hedge $(-e_{H,i} + c_H - c_i)$ to the unhedged return, equation (5). By doing so we get:

$$\text{hedged expected return for country } i, HR_{H,i} = r_i + e_{H,i} - e_{H,i} + c_H - c_i$$

the two currency terms drop out yielding equation (6)

$$HR_{H,i} = r_i + c_H - c_i$$

To derive the expected return given by equation (10), we simply rearranged the terms in equation (6) above obtaining:

$$HR_{H,i} = c_H + (r_i - c_i)$$

The above equation is equation (10), which states that the hedged expected return for country i is equal to the short-term interest rate for the home country and the excess bond return of country i . There is no currency return component since it has been hedged out.

Cross hedged expected return:

To get the cross hedged expected return given by equation (11) we begin with equation (5) and enter into a currency forward that creates a short position in currency i and long position in currency j ($f_{j,i} - e_{i,j}$). The currency position $e_{i,j}$ combined with the original long exposure to currency i , $e_{H,i}$, leaves a net long position in currency j versus the home currency, $e_{H,j}$. Thus,

$$\text{cross hedged expected return for country } i, CR_{H,i} = r_i + e_{H,i} + f_{j,i} - e_{i,j} \text{ or } r_i + f_{j,i} + e_{H,j}$$

We know that $f_{j,i} \approx c_j - c_i$, so we can rewrite the above equation as:

$$CR_{H,i} = r_i + c_j - c_i + e_{H,j}$$

Rearranging terms we get:

$$CR_{H,i} = (r_i - c_i) + (c_j + e_{H,j})$$

The above is equation (7). We know that $f_{H,j} \approx c_H - c_j$ and therefore $c_j = c_H - f_{H,j}$. Substituting for c_j in the above equation and rearranging terms we get

$$CR_{H,i} = c_H + (r_i - c_i) + (e_{H,j} - f_{H,j})$$

The above equation is equation (11), which states that the cross hedged expected return for country i is equal to the short-term interest rate for the home country, the excess bond return of country i , and the currency return of country j over the home country less the short-term interest rate differential between the home country and country j .

Proxy hedged expected return:

To derive the proxy hedged expected return given by equation (12), we begin with the unhedged return, equation (5), and add a short currency position in currency j ($f_{H,j} - e_{H,j}$) to obtain the rewritten proxy hedged expected return equation given in the chapter:

$$\text{proxy hedged expected return for country } i, PR_{H,i} = r_i + e_{H,i} + f_{H,j} - e_{H,j}$$

We know that $f_{H,j} \approx c_H - c_j$ and therefore substituting for $f_{H,j}$ in the above equation we get:

$$PR_{H,i} = r_i + e_{H,i} + c_H - c_j - e_{H,j}$$

adding and subtracting c_i from the right hand side of the equation we get:

$$PR_{H,i} = r_i + e_{H,i} + c_H - c_j - e_{H,i} + c_i - c_i$$

Rearranging terms we get

$$PR_{H,i} = (r_i - c_i) + (c_i + e_{H,i}) + [(c_H - c_j) - e_{H,j}]$$

which is equation (8) in the chapter. Rearranging terms, this equation can be expressed as

$$PR_{H,i} = c_H + (r_i - c_i) + (e_{H,i} - e_{H,j}) - (c_j - c_i)$$

Since $f_{j,i} \approx c_j - c_i$ we can substitute $f_{j,i}$ for $(c_j - c_i)$ and rearrange terms to obtain equation (12):

$$PR_{H,i} = c_H + (r_i - c_i) + [(e_{H,i} - e_{H,j}) - f_{j,i}]$$

Equation (12) states that the proxy hedged expected return for country i is equal to the short-term interest rate for the home country, the excess bond return of country i , and the difference between the currency return of country i and the home country relative to country j and the home country, less the short-term interest rate differential between countries j and i .

The fact that equations (9), (10), (11), and (12) differ only by their last term emphasizes that the bond market decision is unrelated to the currency hedging decision.

CHAPTER 22

CONTROLLING INTEREST RATE RISK WITH DERIVATIVES*

I. INTRODUCTION

Previously, the features and characteristics of interest rate futures and swaps were explained. In addition, at Level II, the valuation of these derivative instruments was explained. In this chapter, our focus is on using derivatives to control the interest rate risk of a portfolio. Other uses for derivatives include speculating on interest rate movements and speculating on changes in interest rate volatility. These strategies are similar to those employed in the equity market and are not covered in this book.

II. CONTROLLING INTEREST RATE RISK WITH FUTURES

The price of an interest rate futures contract moves in the opposite direction from the change in interest rates: when rates rise, the futures price falls; when rates fall, the futures price rises. Buying a futures contract increases a portfolio's exposure to a rate change. That is, the portfolio's duration increases. Selling a futures contract decreases a portfolio's exposure to a rate change. Equivalently, selling a futures contract that reduces the portfolio's duration. Consequently, buying and selling futures can be used to alter the duration of a portfolio.

While managers can alter the duration of their portfolios with cash market instruments (buying or selling Treasury securities), using interest rate futures has the following four advantages:

Advantage 1: Transaction costs for trading futures are lower than trading in the cash market.

Advantage 2: Margin requirements are lower for futures than for Treasury securities; using futures thus permits greater leverage.

Advantage 3: It is easier to sell short in the futures market than in the cash market.

*This chapter is authored by Frank J. Fabozzi, PhD, CFA, CPA, Shrikant Ramamurthy, and Mark Pitts, PhD.

Advantage 4: Futures can be used to construct a portfolio with a longer duration than is available using cash market securities.

To appreciate the last advantage, suppose that in a certain interest rate environment a pension fund manager must structure a portfolio to have a duration of 15 to accomplish a particular investment objective. Bonds with such a long duration may not be available. By buying the appropriate number and kind of interest rate futures contracts, a pension fund manager can increase the portfolio's duration to the target level of 15.

A. General Principles of Interest Rate Risk Control

The general principle in controlling interest rate risk with futures is to combine the dollar exposure of the current portfolio and the dollar exposure of a futures position so that the total dollar exposure is equal to the target dollar exposure. This means that the manager must be able to accurately measure the dollar exposure of both the current portfolio and the futures contract employed to alter the risk profile.

There are two commonly used measures for approximating the change in the dollar value of a bond or bond portfolio resulting from changes in interest rates: price value of a basis point (PVBP) and duration. PVBP is the dollar price change resulting from a one-basis-point change in yield. Duration is the approximate percentage change in price for a 100-basis-point change in rates. (Given the percentage price change, the dollar price change for a given change in interest rates can be computed.) There are two measures of duration: *modified* and *effective*. Effective duration is the appropriate measure for bonds with embedded options. In this chapter when we refer to duration, we mean effective duration. Moreover, since the manager is interested in dollar price exposure, the effective *dollar* duration should be used. For a one basis point change in rates, PVBP is equal to the effective dollar duration for a one-basis-point change in rates.

As emphasized in earlier chapters, a good valuation model is needed to estimate the effective dollar duration. The valuation model is used to determine the new values for the bonds in the portfolio if rates change. Consequently, the starting point in controlling interest rate risk is the development of a reliable valuation model. A reliable valuation model is also needed to value the derivative contracts that the manager wants to use to control interest rate exposure.

Suppose that a manager seeks a **target duration** for the portfolio based on either expectations of interest rates or client-specified exposure. Given the target duration, a target dollar duration for a small basis point change in interest rates can be computed. For a 50 basis point change in interest rates the target dollar duration can be found by multiplying the target duration by the dollar value of the portfolio and then dividing by 200. (We divide by 2 because we are dealing with one half of a 100 basis point change.) For example, suppose that the manager of a \$500 million portfolio wants a target duration of 6. That is, the manager seeks a 3% change in the value of the portfolio for a 50 basis point change in rates (assuming a parallel shift in rates of all maturities). Multiplying the target duration of 6 by \$500 million and dividing by 200 gives a target dollar duration of \$15 million.

The manager must then determine the dollar duration of the current portfolio. The current dollar duration for a 50 basis point change in interest rates is calculated by multiplying the current duration by the dollar value of the portfolio and dividing by 200. For our \$500 million portfolio, suppose that the current duration is 4. The current dollar duration is then \$10 million (4 times \$500 million divided by 200).

The target dollar duration is then compared to the current dollar duration; the difference is the dollar exposure that must be provided by a position in the futures contract. If the

target dollar duration exceeds the current dollar duration, a futures position must increase the dollar exposure by the difference. To increase the dollar exposure, an appropriate number of futures contracts must be purchased. If the target dollar duration is less than the current dollar duration, an appropriate number of futures contracts must be sold. That is,

If target dollar duration – current dollar duration > 0, buy futures

If target dollar duration – current dollar duration < 0, sell futures

Once a futures position is taken, the **portfolio's dollar duration** is equal to the sum of the **current dollar duration without futures** plus the **dollar duration of the futures position**. That is,

$$\begin{aligned} \text{portfolio's dollar duration} &= \text{current dollar duration without futures} \\ &+ \text{dollar duration of futures position} \end{aligned}$$

The objective is to control the portfolio's interest rate risk by establishing a futures position such that the portfolio's dollar duration is equal to the target dollar duration. Thus,

$$\text{portfolio's dollar duration} = \text{target dollar duration}$$

Or, equivalently,

$$\begin{aligned} \text{target dollar duration} &= \text{current dollar duration without futures} \\ &+ \text{dollar duration of futures position} \end{aligned} \tag{1}$$

Over time, as interest rates change, the portfolio's dollar duration will move away from the target dollar duration. The manager can alter the futures position to adjust the portfolio's dollar duration back to the target dollar duration.

Our focus above is on duration. However, duration measures price sensitivity to changes in interest rates assuming a parallel shift in the yield curve. For bond portfolios, nonparallel shifts in the yield curve should be taken into account. The same is true for individual mortgage-backed securities because, as explained in the next chapter, these securities are sensitive to changes in the yield curve. In the next chapter, a framework will be presented that shows how to hedge both a change in the level of interest rates and a change in the shape of the yield curve.

1. Determining the Number of Contracts Each futures contract calls for delivery of a specified par value of the underlying instrument. When interest rates change, the market value of the underlying instrument changes, and therefore the value of the futures contract changes. The amount of change in the futures dollar value must be estimated. This amount is called the **dollar duration per futures contract**. For example, assume the price of an interest rate futures contract is 70 and that the underlying interest rate instrument has a par value of \$100,000. Thus, the futures delivery price is \$70,000 (0.70 times \$100,000). Suppose that a change in interest rates of 100 basis points results in a futures price change of about 3% per contract. Then the dollar duration per futures contract is \$2,100 (0.03 times \$70,000).

The dollar duration of a futures position is then the number of futures contracts multiplied by the dollar duration per futures contract. That is,

$$\begin{aligned} \text{dollar duration of futures position} &= \text{number of futures contracts} \\ &\times \text{dollar duration per futures contract} \end{aligned} \tag{2}$$

How many futures contracts are needed to obtain the target dollar duration? Substituting equation (2) into equation (1), we get

$$\begin{aligned} &\text{number of futures contracts} \times \text{dollar duration per futures contract} \\ &= \text{target dollar duration} - \text{current dollar duration without futures} \end{aligned} \quad (3)$$

Solving for the number of futures contracts we have:

$$\begin{aligned} &\text{number of futures contracts} \\ &= \frac{\text{target dollar duration} - \text{current dollar duration without futures}}{\text{dollar duration per futures contract}} \end{aligned} \quad (4)$$

Equation (4) gives the approximate number of futures contracts needed to adjust the portfolio's dollar duration to the target dollar duration. A positive number means that futures contracts must be purchased; a negative number means that futures contracts must be sold. Notice that if the target dollar duration is greater than the current dollar duration without futures, the numerator is positive and futures contracts are purchased. If the target dollar duration is less than the current dollar duration without futures, the numerator is negative and futures contracts are sold.

2. Dollar Duration for a Futures Position Now we discuss how to measure the dollar duration of a bond futures position. Keep in mind that the goal is to measure the sensitivity of the bond futures value to a change in rates.

Given a valuation model, the general methodology for computing the dollar duration of a futures position for a given change in interest rates is straightforward. The procedure is used for computing the dollar duration of any cash market instrument—shock (change) interest rates up and down by the same number of basis points and compute the average dollar price change.

An adjustment is needed for the Treasury bond and note futures contracts. As explained, the pricing of the futures contract depends on the cheapest-to-deliver (CTD) issue.¹ The calculation of the dollar duration of a Treasury bond or note futures contract requires computing the effect of a change in interest rates on the price of the CTD issue, which in turn determines the change in the futures price. The dollar duration of a Treasury bond and note futures contract is determined as follows:

$$\begin{aligned} &\text{dollar duration of futures contract} = \text{dollar duration of the CTD issue} \\ &\times \frac{\text{dollar duration of futures contract}}{\text{dollar duration of the CTD issue}} \end{aligned}$$

Recall that a conversion factor is specified for each issue that is acceptable for delivery under the futures contract. For each deliverable issue, the product of the futures price and the conversion factor is the adjusted futures price for the issue. This adjusted price is called the **converted price**. Relating this to the equation above, the second term is approximately equal to the reciprocal of the conversion factor of the cheapest-to-deliver issue. Thus, we can write:

$$\text{dollar duration of futures contract} = \frac{\text{dollar duration of the CTD issue}}{\text{conversion factor for the CTD issue}}$$

¹The cheapest-to-deliver issue is the one issue from among all those that are eligible for delivery on a contract that has the highest return in a cash and carry trade. This return is called the implied repo rate.

B. Hedging with Interest Rate Futures

Hedging with futures calls for taking a futures position as a temporary substitute for transactions to be made in the cash market at a later date. If cash and futures prices are perfectly positively correlated, any loss realized by the hedger from one position (whether cash or futures) will be exactly offset by a profit on the other position. *Hedging is a special case of controlling interest rate risk. In a hedge, the manager seeks a target duration or target dollar duration of zero.*

A **short hedge** (or **sell hedge**) is used to protect against a decline in the cash price of a bond. To execute a short hedge, futures contracts are sold. By establishing a short hedge, the manager has fixed the future cash price and transferred the price risk of ownership to the buyer of the futures contract. To understand why a short hedge might be executed, suppose that a pension fund manager knows that bonds must be liquidated in 40 days to make a \$5 million payment to beneficiaries. If interest rates rise during the 40-day period, more bonds will have to be liquidated at a lower price than today to realize \$5 million. To guard against this possibility, the manager can lock in a selling price by selling bonds in the futures market.²

A **long hedge** (or **buy hedge**) is undertaken to protect against an increase in the cash price of a bond. In a long hedge, the manager buys a futures contract to lock in a purchase price. A pension fund manager might use a long hedge when substantial cash contributions are expected and the manager is concerned that interest rates will fall. Also, a money manager who knows that bonds are maturing in the near future and expects that interest rates will fall before receipt can employ a long hedge to lock in a rate for the proceeds to be reinvested.

In bond portfolio management, typically the bond or portfolio to be hedged is not identical to the bond underlying the futures contract. This type of hedging is referred to as **cross hedging**.

The hedging process can be broken down into four steps:

Step 1: Determining the appropriate hedging instrument.

Step 2: Determining the target for the hedge.

Step 3: Determining the position to be taken in the hedging instrument.

Step 4: Monitoring and evaluating the hedge.

We discuss each step below.

1. Determining the Appropriate Hedging Instrument A primary factor in determining which futures contract will provide the best hedge is the correlation between the price on the futures contract and the interest rate that creates the underlying risk that the manager seeks to eliminate. For example, a long-term corporate bond portfolio can be better hedged with Treasury bond futures than with Treasury bill futures because long-term corporate bond rates are more highly correlated with Treasury bond futures than Treasury bill futures. Using the correct delivery month is also important. A manager trying to lock in a rate or price for September will use September futures contracts because they will give the highest degree of correlation.

²When hedging more than just the level of interest rates (e.g., hedging changes in the slope also), more than one hedging instrument is used. One of the hedging instruments could require a long position even though the instrument to be hedged is a long position.

Correlation is not, however, the only consideration if the hedging program is of significant size. If, for example, a manager wants to hedge \$600 million of a cash position in a distant delivery month, liquidity becomes an important consideration. In such a case, it might be necessary for the manager to spread the hedge across two or more different contracts.

While our focus in this chapter is on hedging changes in the level of interest rates, when more than one type of change in the yield curve is to be hedged, then more than one hedging instrument must be used. For example, hedging a mortgage security is covered in the next chapter. As explained in the next chapter, because of the characteristic of mortgage securities, hedging both a change in the level and slope of the yield curve would be more effective. In such cases, two hedging instruments are used. One can also hedge the level, slope, and curvature changes of the yield curve. In that case, three hedging instruments are used.

2. Determining the Target for the Hedge Having determined the correct contract and the correct delivery months, the manager should then determine what is expected from the hedge—that is, what rate will, on average, be locked in by the hedge. This is the **target rate** or **target price**. If this target rate is too high (if hedging a future sale) or too low (if hedging a future purchase), hedging may not be the right strategy for dealing with the unwanted risk. Determining what is expected (calculating the target rate or price for a hedge) is not always simple. We'll see how a manager should approach this problem for both simple and complex hedges.

a. Risk and Expected Return in a Hedge When a manager enters into a hedge, the objective is to “lock in” a rate for the sale or purchase of a security. However, there is much disagreement about which rate or price a manager should expect to lock in when futures are used to hedge. Here are the two views:

View 1: The manager can, on average, lock in the rate at which the futures contracts are bought or sold.

View 2: The manager can, on average, lock in the current spot rate for the security (i.e., current rate in the cash market).

Reality usually lies somewhere between these two views. However, as the following cases illustrate, each view is appropriate for certain situations.

b. The Target for Hedges Held to Delivery Hedges that are held until the futures delivery date provide an example of a hedge that locks in the futures rate (i.e., the first view). The **futures rate** is the interest rate corresponding to the futures delivery price of the deliverable instrument. The complication in the case of using Treasury bond futures and Treasury note futures to hedge the value of intermediate- and long-term bonds, is that because of the delivery options the manager does not know for sure when delivery will take place or which bond will be delivered. This is because of the delivery options granted to the short.³

To illustrate how a Treasury bond futures contract held to the delivery date locks in the futures rate, assume for the sake of simplicity that the manager knows which Treasury bond will be delivered and that delivery will take place on the last day of the delivery month. Suppose that for delivery on the September 1999 futures contract, the conversion factor for a deliverable Treasury issue—the 11 $\frac{1}{4}$ % of 2/15/15—is 1.283, implying that the investor who

³These delivery options were explained earlier.

delivers this issue would receive from the buyer 1.283 times the futures settlement price plus accrued interest. An important principle to remember is that at delivery, the spot price and the futures price times the conversion factor must converge. **Convergence** refers to the fact that at delivery there can be no discrepancy between the spot price and futures price for a given security. If convergence does not take place, arbitrageurs would buy at the lower price and sell at the higher price and earn risk-free profits. Accordingly, a manager could lock in a September 1999 sale price for this issue by selling Treasury bond futures contracts equal to 1.283 times the par value of the bonds. For example, \$100 million face value of this issue would be hedged by selling \$128.3 million face value of bond futures (1,283 contracts).

The sale price that the manager locks in would be 1.283 times the futures price. This is the **converted price**. Thus, if the futures price is 113 when the hedge is set, the manager locks in a sale price of 144.979 (113 times 1.283) for September 1999 delivery, regardless of where rates are in September 1999. Exhibit 1 shows the cash flows for various final prices for this issue and illustrates how cash flows on the futures contracts offset gains or losses relative to the target price of 144.979.

Let's look at each of the columns in Exhibit 1 and explain the computations for one of the scenarios—that is, for one actual sale price for the $11\frac{1}{4}\%$ of 2/15/15 Treasury issue. Consider the first actual sale price of 140. By convergence, at the delivery date the final futures price shown in Column (2) must equal the Treasury bond's actual sale price adjusted by the conversion factor. Thus, the final futures price in Column (2) of Exhibit 1 is computed as follows:

$$\text{final futures price} = \frac{\text{Treasury bonds actual sale price}}{\text{conversion factor}}$$

Since the conversion factor is 1.283 for the $11\frac{1}{4}\%$ of 2/15/15 Treasury issue, for the first actual sale price of 140, the final futures price is

$$\text{final futures price} = \frac{140}{1.283} = 109.1193$$

Column (3) shows the market value of the Treasury bonds. This is found by dividing the actual sale price in Column (1) by 100 to obtain the actual sale price per \$1 of par value and then multiplying by the \$100 million par value. That is,

$$\text{market value of Treasury bonds} = (\text{actual sale price}/100) \times \$100,000,000$$

For the actual sale price of 140, the value in Column (3) is

$$\text{market value of Treasury bonds} = (140/100) \times \$100,000,000 = \$140,000,000$$

Column (4) shows the value of the futures position at the delivery date. This value is computed by first dividing the futures price shown in Column (2) by 100 to obtain the futures price per \$1 of par value. Then this value is multiplied by the par value per contract of \$100,000 and further multiplied by the number of futures contracts. That is,

$$\begin{aligned} \text{value of futures position} &= (\text{final futures price}/100) \times \$100,000 \\ &\times \text{number of futures contracts} \end{aligned}$$

EXHIBIT 1 Treasury Issue Hedge Held to Delivery

Instrument to be hedged: \$100 million 11 $\frac{1}{4}$ % Treasury Bonds of 2/15/15

Conversion factor for September 1999 = 1.283

Price of futures contract when sold = 113

Target price = $(1.283 \times 113) = 144.979$

Par value hedged = \$100,000,000

Number of futures contracts = 1,283

Futures position = Target = \$144,979,000

(1)	(2)	(3)	(4)	(5)	(6)
Actual price for 11.25% T-bonds	Final futures price ¹	Market value of Treasury bonds	Value of futures position ²	Gain or loss from futures position ²	Effective sale price ³
140	109.1192518	140,000,000	140,000,000	4,979,000	144,979,000
141	109.8986750	141,000,000	141,000,000	3,979,000	144,979,000
142	110.6780982	142,000,000	142,000,000	2,979,000	144,979,000
143	111.4575214	143,000,000	143,000,000	1,979,000	144,979,000
144	112.2369447	144,000,000	144,000,000	979,000	144,979,000
145	113.0163679	145,000,000	145,000,000	-21,000	144,979,000
146	113.7957911	146,000,000	146,000,000	-1,021,000	144,979,000
147	114.5752143	147,000,000	147,000,000	-2,021,000	144,979,000
148	115.3546376	148,000,000	148,000,000	-3,021,000	144,979,000
149	116.1340608	149,000,000	149,000,000	-4,021,000	144,979,000
150	116.9134840	150,000,000	150,000,000	-5,021,000	144,979,000
151	117.6929072	151,000,000	151,000,000	-6,021,000	144,979,000
152	118.4723305	152,000,000	152,000,000	-7,021,000	144,979,000
153	119.2517537	153,000,000	153,000,000	-8,021,000	144,979,000
154	120.0311769	154,000,000	154,000,000	-9,021,000	144,979,000
155	120.8106002	155,000,000	155,000,000	-10,021,000	144,979,000

¹ By convergence, must equal bond price divided by the conversion factor.

² Bond futures trade in increments of 1/32. Accordingly, the futures prices and margin flows are only approximate.

³ Transaction costs and the financing of margin flows are ignored.

In our illustration, the number of futures contracts is 1,283. For the actual sale price of the bond at 140, the final futures price calculated earlier is 109.1193. So, the value shown in Column (4) is

$$\begin{aligned}\text{value of futures position} &= (109.1193/100) \times \$100,000 \times 1,283 \\ &= \$140,000,062\end{aligned}$$

The value shown in Column (4) is \$140,000,000 because the final futures price of 109.1193 was rounded. Using more decimal places the value would be \$140,000,000.

Now let's look at the gain or loss from the futures position. This value is shown in Column (5). Recall that the futures contract was shorted. The futures price at which the contracts were sold was 113. So, if the final futures price exceeds 113, this means that there is a loss on the futures position—that is, the futures contract is purchased at a price greater than that at which it was sold. In contrast, if the futures price is less than 113, this means that there is a gain on the futures position—that is, the futures contract is purchased at a price less than that at which it was sold. The gain or loss is determined by the following formula:

$$([113 - \text{final futures price}]/100) \times \$100,000 \times \text{number of futures contracts}$$

In our illustration, for a final futures price of 109.1193 and 1,283 futures contracts, we have

$$([113 - 109.1193]/100) \times \$100,000 \times 1,283 = \$4,978,938.1$$

The value shown in Column (5) is \$4,979,000 because that is the more precise value using more decimal places for the final futures price than shown in Exhibit 1. The value is positive indicating a gain in the futures position. Note that for all the final futures prices above 113 in Exhibit 1, there is a negative value which means that there is a loss on the futures position.

Finally, Column (6) shows the effective sale price for the Treasury bond. This value is computed as follows:

$$\begin{aligned} \text{effective sale price for Treasury bond} &= \text{actual sale price of Treasury bond} \\ &+ \text{gain or loss on futures position} \end{aligned}$$

which is the sum of the numbers in each of the rows of Columns (3) and (5). For the actual sale price of \$140 million, the gain is \$4,979,000. Therefore the effective sale price for the Treasury bond is

$$\$140,000,000 + \$4,979,000 = \$144,979,000$$

Note that this is the target price for the Treasury bond. In fact, it can be seen from Column (6) of Exhibit 1 that the effective sale price for all the actual sale prices for the Treasury bond is the target price. However, the target price is determined by the futures price, so the target price may be higher or lower than the cash (spot) market price when the hedge is set.

When we admit the possibility that bonds other than the issue used in our illustration can be delivered, and that it might be advantageous to do so, the situation becomes somewhat more involved. In this more realistic case, the manager may decide not to deliver this issue, but if she does decide to deliver it, the manager is still assured of receiving an effective sale price of approximately 144.979. If the manager does not deliver this issue, it would be because another issue can be delivered more cheaply, and thus the manager does better than the targeted price.

In summary, if a manager establishes a futures hedge that is held until delivery, the manager can be assured of receiving an effective price dictated by the futures rate (not the spot rate) on the day the hedge is set.

c. The Target for Hedges with Short Holding Periods When a manager must lift (remove) a hedge prior to the delivery date, the effective rate that is obtained is more likely to approximate the current spot rate than the futures rate and this likelihood increases the shorter the term of the hedge. The critical difference between this hedge and the hedge held to the delivery date is that convergence will generally not take place by the termination date of the hedge.

To illustrate why a manager should expect the hedge to lock in the spot rate rather than the futures rate for very short-lived hedges, let's return to the simplified example used earlier to illustrate a hedge maintained to the delivery date. It is assumed that this issue is the only deliverable instrument for the Treasury bond futures contract. Suppose that the hedge is set three months before the delivery date and the manager plans to lift the hedge one day later. It is much more likely that the spot price of the bond will move parallel to the converted futures price (that is, the futures price times the conversion factor), than that the spot price and the converted futures price will converge by the time the hedge is lifted.

A 1-day hedge is, admittedly, an extreme example. Other than underwriters, dealers, and traders who reallocate assets very frequently, few money managers are interested in such a short horizon. The very short-term hedge does, however, illustrate a very important point: *when hedging, a manager should not expect to lock in the futures rate (or price) just by hedging with futures contracts.* The futures rate is locked in only if the hedge is held until delivery, at which point convergence must take place. If the hedge is held for only one day, the manager should expect to lock in the 1-day forward rate,⁴ which will very nearly equal the spot rate. Generally hedges are held for more than one day, but not necessarily to delivery.

d. How the Basis Affects the Target Rate for a Hedge The proper target for a hedge that is to be lifted prior to the delivery date depends on the basis. The **basis** is simply the difference between the spot (cash) price of a security and its futures price; that is:

$$\text{basis} = \text{spot price} - \text{futures price}$$

In the bond market, a problem arises when trying to make practical use of the concept of the basis. The quoted futures price does not equal the price that one receives at delivery. For the Treasury bond and note futures contracts, the actual futures price equals the quoted futures price times the appropriate conversion factor. Consequently, to be useful, the basis in the bond market should be defined using actual futures delivery prices rather than quoted futures prices. Thus, the price basis for bonds should be redefined as:

$$\text{price basis} = \text{spot price} - \text{futures delivery price}$$

For hedging purposes it is also frequently useful to define the basis in terms of interest rates rather than prices. The **rate basis** is defined as:

$$\text{rate basis} = \text{spot rate} - \text{futures rate}$$

where spot rate refers to the current rate on the instrument to be hedged and the futures rate is the interest rate corresponding to the futures delivery price of the deliverable instrument.

The rate basis is helpful in explaining why the two views of hedging described earlier are expected to lock in such different rates. To see this, we first define the **target rate basis**. This is defined as the expected rate basis on the day the hedge is lifted. That is,

$$\text{target rate basis} = \text{spot rate on date hedge is lifted} - \text{futures rate on date hedge is lifted}$$

The target rate for the hedge is equal to

$$\text{target rate for hedge} = \text{futures rate} + \text{target rate basis}$$

Substituting for target rate basis in the above equation

$$\begin{aligned} \text{target rate for hedge} &= \text{futures rate} + \text{spot rate on date hedge is lifted} \\ &\quad - \text{futures rate on date hedge is lifted} \end{aligned}$$

⁴Forward rates were covered earlier.

Consider first a hedge lifted on the delivery date. On the delivery date, the spot rate and the futures rate will be the same by convergence. Thus, the target rate basis if the hedge is expected to be removed on the delivery date is zero. Substituting zero for the target rate basis in the equation above we have the following:

$$\text{target rate for hedge} = \text{futures rate}$$

That is, if the hedge is held to the delivery date, the target rate for the hedge is equal to the futures rate.

Now consider if a hedge is lifted prior to the delivery date. Let's consider the case where the hedge is removed the next day. One would not expect the basis to change very much in one day. Assume that the basis does not change. Then,

$$\text{spot rate on date hedge is lifted} = \text{spot rate when hedge was placed}$$

$$\text{futures rate on date hedge is lifted} = \text{futures rate when hedge was placed}$$

The spot rate when the hedge was placed is simply the spot rate and the futures rate when the hedge was placed is simply the futures rate. So, we can write

$$\text{target rate basis} = \text{spot rate} - \text{futures rate}$$

and substituting the right-hand side of the equation into the target rate for hedge

$$\text{target rate for hedge} = \text{futures rate} + (\text{spot rate} - \text{futures rate})$$

or

$$\text{target rate for hedge} = \text{spot rate}$$

Thus, we see that when hedging for one day (and assuming the basis does not change in that one day), the manager is locking in the spot rate (i.e., the current rate).

If projecting the basis in terms of price rather than rate is easier (as is often the case for intermediate- and long-term futures), it is easier to work with the target price basis instead of the target rate basis. The **target price basis** is the projected price basis for the day the hedge is to be lifted. For a deliverable security, the target for the hedge then becomes

$$\text{target price for hedge} = \text{futures delivery price} + \text{target price basis}$$

The idea of a target price or rate basis explains why a hedge held until the delivery date locks in a price, and other hedges do not. The examples have shown that this is true. For the hedge held to delivery, there is no uncertainty surrounding the target basis; by convergence, the basis on the day the hedge is lifted will be zero. For the short-lived hedge, the basis will probably approximate the current basis when the hedge is lifted, but its actual value is not known in advance. For hedges longer than one day but ending prior to the futures delivery date, there can be considerable basis risk because the basis on the day the hedge is lifted can end up being anywhere within a wide range. Thus, the uncertainty surrounding the outcome of a hedge is directly related to the uncertainty surrounding the basis on the day the hedge is lifted (i.e., the uncertainty surrounding the target basis).

The uncertainty about the value of the basis at the time the hedge is removed is called **basis risk**. *For a given investment horizon, hedging substitutes basis risk for price risk.* Thus, one

trades the uncertainty of the price of the hedged security for the uncertainty of the basis. A manager would be willing to substitute basis risk for price risk if the manager expects that basis risk is less than price risk. Consequently, when hedges do not produce the desired results, it is common to place the blame on basis risk. However, basis risk is the real culprit only if the target for the hedge is properly defined. Basis risk should refer only to the unexpected or unpredictable part of the relationship between cash and futures prices. The fact that this relationship changes over time does not in itself imply that there is basis risk.

Basis risk, properly defined, refers only to the uncertainty associated with the target rate basis or target price basis. Accordingly, it is imperative that the target basis be properly defined if one is to correctly assess the risk and expected return in a hedge.

3. Determining the Position to Be Taken The final step that must be determined before the hedge is set is the number of futures contracts needed for the hedge. This is called the **hedge ratio**. Usually the hedge ratio is expressed in terms of relative par amounts. Accordingly, a hedge ratio of 1.20 means that for every \$1 million par value of securities to be hedged, one needs \$1.2 million par value of futures contracts to offset the risk. *In our discussion, the values are defined so that the hedge ratio is the number of futures contracts.*

Earlier, we defined a cross hedge in the futures market as a hedge in which the security to be hedged is not deliverable on the futures contract used in the hedge. (A bond that does not meet the specific criteria for delivery to satisfy a particular futures contract is referred to as a **nondeliverable bond**.) For example, a manager who wants to hedge the sale price of long-term corporate bonds might hedge with the Treasury bond futures contract, but since non-Treasury bonds cannot be delivered in satisfaction of the contract, the hedge would be considered a cross hedge. A manager might also want to hedge a rate that is of the same quality as the rate specified in one of the contracts, but that has a different maturity. For example, it might be necessary to cross hedge a Treasury bond, note, or bill with a maturity that does not qualify for delivery on any futures contract. Thus, when the security to be hedged differs from the futures contract specification in terms of either quality or maturity, one is led to the cross hedge.

Conceptually, cross hedging is somewhat more complicated than hedging deliverable securities, because it involves two relationships. First, there is the relationship between the cheapest-to-deliver (CTD) issue and the futures contract. Second, there is the relationship between the security to be hedged and the CTD issue. Practical considerations may at times lead a manager to shortcut this two-step relationship and focus directly on the relationship between the security to be hedged and the futures contract, thus ignoring the CTD issue altogether. However, in so doing, a manager runs the risk of miscalculating the target rate and the risk in the hedge. Furthermore, if the hedge does not perform as expected, the shortcut makes it difficult to tell why the hedge did not work out as expected.

The key to minimizing risk in a cross hedge is to choose the right number of futures contracts. This depends on the relative dollar duration of the bond to be hedged and the dollar duration of the futures position. Equation (4) indicated the number of futures contracts required to achieve a particular target dollar duration. The objective in hedging is to make the target dollar duration equal to zero. Substituting zero for target dollar duration in equation (4) we obtain:

$$\text{number of futures contracts} = \frac{\text{current dollar duration without futures}}{\text{dollar duration per futures contract}} \quad (5)$$

To calculate the dollar duration of a bond, the manager must know the precise point in time that the dollar duration is to be calculated (because price volatility generally declines as a bond matures) as well as the price or yield at which to calculate dollar duration (because higher

yields generally reduce dollar duration for a given yield change). *The relevant point in the life of the bond for calculating price volatility is the point at which the hedge will be lifted. Dollar duration at any other point in time is essentially irrelevant because the goal is to lock in a price or rate only on that particular day. Similarly, the relevant yield at which to calculate dollar duration initially is the target yield. Consequently, the numerator of equation (5) is the dollar duration on the date the hedge is expected to be lifted.* The yield that can be used on this date in order to determine the dollar duration is the forward rate.

Let's look at how we apply equation (5) when using the Treasury bond futures contract to hedge. The number of futures contracts will be affected by the dollar duration of the CTD issue. We can modify equation (5) as follows:

$$\begin{aligned} & \text{number of futures contracts} \\ &= - \frac{\text{current dollar duration without futures}}{\text{dollar duration of the CTD issue}} \times \frac{\text{dollar duration of the CTD issue}}{\text{dollar duration per futures contract}} \quad (6) \end{aligned}$$

As noted earlier, the conversion ratio for the CTD issue is a good approximation of the second ratio. Thus, equation (6) can be rewritten as

$$\begin{aligned} & \text{number of futures contracts} \\ &= - \frac{\text{current dollar duration without futures}}{\text{dollar duration of the CTD issue}} \times \text{conversion factor for the CTD issue} \quad (7) \end{aligned}$$

a. An Illustration An example for a single bond shows why dollar duration weighting leads to the correct number of contracts for a hedge. The hedge illustrated is a cross hedge. Suppose that on 6/24/99, a manager owned \$10 million par value of a 6.25% Fannie Mae (FNMA) option-free bond maturing on 5/15/29 selling at 88.39 to yield 7.20%. The manager wants to sell September 1999 Treasury bond futures to hedge a future sale of the FNMA bond. At the time, the price of the September Treasury bond futures contract was 113. The CTD issue was the 11.25% of 2/15/15 issue that was trading at 146.19 to yield 6.50%. The conversion factor for the CTD issue was 1.283. To simplify, assume that the yield spread between the FNMA bond and the CTD issue remains at 0.70% (i.e., 70 basis points) and that the anticipated sale date is the last business day in September 1999.

The target price for hedging the CTD issue would be 144.979 (113×1.283), and the target yield would be 6.56% (the yield at a price of 144.979). Since the yield on the FNMA bond is assumed to stay at 0.70% above the yield on the CTD issue, the target yield for the FNMA bond would be 7.26%. The corresponding price for the FNMA bond for this target yield is 87.76. At these target levels, the dollar durations for a 50 basis point change in rates for the CTD issue and the FNMA bond per \$100 of par value are \$6.255 and \$5.453, respectively. As indicated earlier, all these calculations are made using a settlement date equal to the anticipated sale date, in this case the end of September 1999. The dollar duration for a 50 basis point change in rates for \$10 million par value of the FNMA bond is then \$545,300 ($[\$10 \text{ million}/100] \times \$5.453$). Per \$100,000 par value for the CTD issue, the dollar duration per futures contract is \$6,255 ($[\$100,000/100] \times \$6.255$).

Thus, we know

$$\begin{aligned} \text{current dollar duration without futures} &= \text{dollar duration of the FNMA bond} = \$545,300 \\ \text{dollar duration of the CTD issue} &= \$6,255 \\ \text{conversion factor for CTD issue} &= 1.283 \end{aligned}$$

Substituting these values into equation (7) we obtain

$$\text{number of futures contracts} = \frac{\$545,300}{\$6,255} \times 1.283 = -112 \text{ contracts}$$

Consequently, to hedge the FNMA bond position, 112 Treasury bond futures contracts must be shorted.

Exhibit 2 uses scenario analysis to show the outcome of the hedge based on different prices for the FNMA bond at the delivery date of the futures contract. Let's go through each of the columns. Column (1) shows the assumed sale price for the FNMA bond and Column (2) shows the corresponding yield based on the actual sale price in Column (1). This yield is found from the price/yield relationship. Column (3) shows the yield for the CTD issue, computed on the assumed 70 basis point yield spread between the FNMA bond and CTD issue. So, by subtracting 70 basis points from the yield for the FNMA bond in Column (2), the yield on the CTD issue (the 11.25% of 2/15/15) is obtained. Given the yield for the CTD issue in Column (3), the price per \$100 of par value of the CTD issue can be computed. This CTD price is shown in Column (4).

Now we move from the price of the CTD issue to the futures price. As explained in the description of the columns in Exhibit 1, the futures price is computed by dividing the price for the CTD issue shown in Column (4) by the conversion factor of the CTD issue (1.283). This price is shown in Column (5).

EXHIBIT 2 Hedging a Nondeliverable Bond to a Delivery Date with Futures

Instrument to be hedged: \$10 million FNMA 6.25% of 05/15/29

Price of FNMA as of hedge date (6/24/99) = 88.39

Conversion factor for September 1999 = 1.283

Price of futures contract when sold = 113

Target price for FNMA bonds = 87.76

Par value hedged = \$10,000,000

Number of futures contracts = 112

Futures position = \$12,656,000

Target market value for FNMA bonds = \$8,776,000

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Actual sale price of FNMA bonds	Yield at sale	Yield of 11.25% Treasury bond ¹	Price of 11.25% Treasury bond	Futures price ²	Value of futures position	Gain or loss on futures position ³	Effective sale price ³
8,000,000	8.027%	7.327%	135.812	105.85510	11,855,771	800,229	8,800,229
8,100,000	7.922%	7.222%	137.024	106.79953	11,961,547	694,453	8,794,453
8,200,000	7.818%	7.118%	138.226	107.73658	12,066,497	589,503	8,789,503
8,300,000	7.717%	7.017%	139.419	108.66640	12,170,637	485,363	8,785,363
8,400,000	7.617%	6.917%	140.603	109.58913	12,273,983	382,017	8,782,017
8,500,000	7.520%	6.820%	141.778	110.50490	12,376,549	279,451	8,779,451
8,600,000	7.424%	6.724%	142.944	111.41384	12,478,350	177,650	8,777,650
8,700,000	7.330%	6.630%	144.102	112.31608	12,579,401	76,599	8,776,599
8,800,000	7.238%	6.538%	145.251	113.21175	12,679,716	(23,716)	8,776,284
8,900,000	7.148%	6.448%	146.392	114.10096	12,779,307	(123,307)	8,776,693
9,000,000	7.059%	6.359%	147.524	114.98383	12,878,188	(222,188)	8,777,812
9,100,000	6.971%	6.271%	148.649	115.86047	12,976,373	(320,373)	8,779,627
9,200,000	6.886%	6.186%	149.766	116.73101	13,073,873	(417,873)	8,782,127
9,300,000	6.801%	6.101%	150.875	117.59554	13,170,700	(514,700)	8,785,300
9,400,000	6.719%	6.019%	151.977	118.45417	13,266,868	(610,868)	8,789,132
9,500,000	6.637%	5.937%	153.071	119.30702	13,362,386	(706,386)	8,793,614

¹ By assumption, the yield on the cheapest-to-deliver issue is 70 basis points lower than the yield on the FNMA bond.

² By convergence, the futures price equals the price of the cheapest-to-deliver issue divided by 1.283 (the conversion factor).

³ Transaction costs and the financing of margin flows are ignored.

The value of the futures position is found in the same way as in Exhibit 1. First the futures price per \$1 of par value is computed by dividing the futures price by 100. Then this value is multiplied by \$100,000 (the par value for the contract) and the number of futures contracts. That is,

$$\text{value of futures position} = (\text{futures price}/100) \times \$100,000 \times \text{number of futures contracts}$$

The values in Column (6) are derived using this formula. Using the first sale price for the FNMA of \$8 million as an example, the corresponding futures price in Column (5) is 105.8551. Since the number of futures contracts sold is 112, the value of the futures position is

$$\text{value of futures position} = (105.8551/100) \times \$100,000 \times 112 = \$11,855,711$$

Now let's calculate the gain or loss on the futures position shown in Column (7). Note that the negative values in Column (7) for all futures prices above 113 mean there is a loss on the futures position. Since the futures price at which the contracts are sold at the inception of the hedge is 113, the gain or loss on the futures position is found as follows:

$$([113 - \text{final futures price}]/100) \times \$100,000 \times \text{number of futures contracts}$$

For example, for the first scenario in Exhibit 2, the futures price is 105.8551 and 112 futures contract were sold. Therefore,

$$([113 - 105.8551]/100) \times \$100,000 \times 112 = \$800,229$$

There is a gain from the futures position because the futures price is less than 113. Note that for all the final futures prices above 113 in Exhibit 2, there is a negative value which means that there is a loss on the futures position. For all futures prices below 113, there is a gain.

Finally, Column (8) shows the effective sale price for the FNMA bond. This value is found as follows:

$$\begin{aligned} \text{effective sale price for FNMA bond} &= \text{actual sale price of FNMA bond} \\ &+ \text{gain or loss on futures position} \end{aligned}$$

For the actual sale price of \$8 million, the gain is \$800,229. Therefore the effective sale price for the FNMA bond is

$$\$8,000,000 + \$800,229 = \$8,800,229$$

Looking at Column (8) of Exhibit 2 we see that if the simplifying assumptions hold, a futures hedge using the recommended number of futures contracts (112) very nearly locks in the target price for \$10 million par value of the FNMA bonds.

b. Refining for Changing Yield Spread Another refinement in the hedging strategy is usually necessary when hedging nondeliverable securities. This refinement concerns the assumption about the relative yield spread between the CTD issue and the bond to be hedged. In the prior discussion, we assumed that the yield spread was constant over time. Yield spreads, however, are not constant over time. They vary with the maturity of the instruments in question and the level of rates, as well as with many unpredictable and nonsystematic factors.

Regression analysis allows the manager to capture the relationship between yield levels and yield spreads and use it to advantage. For hedging purposes, the variables are the yield on the bond to be hedged and the yield on the CTD issue. The regression equation takes the form:

$$\text{yield on bond to be hedged} = a + b \times \text{yield on CTD issue} + \text{error} \quad (8)$$

The regression procedure provides an estimate of b , which is the expected relative yield change in the two bonds. This parameter b is called the **yield beta**. Our example that used constant spreads implicitly assumes that the yield beta, b , equals 1.0 and a equals 0.70 (because 0.70 is the assumed spread).

For the two issues in question, the FNMA bond and the CTD issue, suppose the estimated yield beta is 1.05. Thus, yields on the FNMA issue are expected to move 5% more than yields on the Treasury issue. To calculate the number of futures contracts correctly, this fact must be taken into account; thus, the number of futures contracts derived in our earlier example is multiplied by the factor 1.05. Consequently, instead of shorting 112 Treasury bond futures contracts to hedge \$10 million of the FNMA bond, the investor would short 118 (rounded up) contracts.

To incorporate the impact of a yield beta, the formula for the number of futures contracts is revised as follows:

$$\begin{aligned} \text{number of futures contracts} = & - \frac{\text{current dollar duration without futures}}{\text{dollar duration of the CTD issue}} \\ & \times \text{conversion factor for the CTD issue} \times \text{yield beta} \quad (9) \end{aligned}$$

The effect of a change in the CTD issue and the yield spread can be assessed before the hedge is implemented. An exhibit similar to Exhibit 2 can be constructed under a wide range of assumptions. For example, at different yield levels at the date the hedge is to be lifted (the second column in Exhibit 2), a different yield spread may be appropriate and a different acceptable issue will be the CTD issue. The manager can determine what this will do to the outcome of the hedge.

4. Monitoring and Evaluating the Hedge After a target is determined and a hedge is set, there are two remaining tasks. The hedge must be monitored during its life and evaluated after it is over. While hedges must be monitored, overactive management of a hedge poses more of a threat to most hedges than does inactive management. The reason for this is that the manager usually will not receive enough new information during the life of the hedge to justify a change in the hedging strategy. For example, it is not advisable to readjust the hedge ratio every day in response to new data and a possible corresponding change in the estimated value of the yield beta.

There are, however, exceptions to this general rule. As rates change, dollar duration changes. Consequently, the hedge ratio may change slightly. In other cases, there may be sound economic reasons to believe that the yield beta has changed. While there are exceptions, the best approach is usually to let a hedge run its course using the original hedge ratio with only slight adjustments.

A hedge can normally be evaluated only after it has been lifted. Evaluation involves, first, an assessment of how closely the hedge locked in the target rate—that is, how much error there was in the hedge. To provide a meaningful interpretation of the error, the manager

should calculate how far from the target the sale (or purchase) would have been, had there been no hedge at all. One good reason for evaluating a completed hedge is to ascertain the sources of error in the hedge in the hope that the manager will gain insights that can be used to advantage in subsequent hedges. A manager will find that there are three major sources of hedging errors:

1. The dollar duration for the hedged instrument was incorrect.
2. The projected value of the basis at the date the hedge is removed can be in error.
3. The parameters estimated from the regression (a and b) can be inaccurate.

Let's discuss the first two sources of hedging error. The third source is self-explanatory. Recall from the calculation of duration that interest rates are changed up and down by a small number of basis points and the security is revalued. The two recalculated values are used in the numerator of the duration formula. The first source of error listed above recognizes that the instrument to be hedged may be a complex instrument (i.e., one with embedded options) and that the valuation model does not do a good job of valuing the security when interest rates change.

The second major source of errors in a hedge—an inaccurate projected value of the basis—is a more difficult problem. Unfortunately, there are no satisfactory easy models like regression analysis that can be applied to the basis. Simple models of the basis violate certain equilibrium relationships for bonds that should not be violated. On the other hand, theoretically rigorous models are very unintuitive and usually solvable only by complex numerical methods. Modeling the basis is undoubtedly one of the most important and difficult problems that managers face when seeking to hedge.

III. CONTROLLING INTEREST RATE RISK WITH SWAPS

An interest rate swap is equivalent to a package of forward/futures contracts. Consequently, swaps can be used for controlling interest rate risk and hedging as we discussed earlier with futures.

A. Hedging Interest Rate Risk

The following illustration demonstrates how an interest rate swap can be used to hedge interest rate risk by altering the cash flow characteristics of an entity so as to better match the cash flow characteristics of assets and liabilities. In our illustration we will use two hypothetical financial institutions—a commercial bank and a life insurance company.

Suppose a bank has a portfolio consisting of 4-year commercial loans with a fixed interest rate. The principal value of the portfolio is \$100 million, and the interest rate on every loan in the portfolio is 11%. The loans are interest-only loans; interest is paid semiannually, and the principal is paid at the end of four years. That is, assuming no default on the loans, the cash flow from the loan portfolio is \$5.5 million every six months for the next four years and \$100 million at the end of four years. To fund its loan portfolio, assume that the bank can borrow at 6-month LIBOR for the next four years.

The risk that the bank faces is that 6-month LIBOR will be 11% or greater. To understand why, remember that the bank is earning 11% annually on its commercial loan portfolio. If

6-month LIBOR is 11% when the borrowing rate for the bank's loan resets, there will be no spread income for that 6-month period. Worse, if 6-month LIBOR rises above 11%, there will be a loss for that 6-month period; that is, the cost of funds will exceed the interest rate earned on the loan portfolio. The bank's objective is to lock in a spread over the cost of its funds.

The other party in the interest rate swap illustration is a life insurance company that has committed itself to pay an 8% rate for the next four years on a \$100 million guaranteed investment contract (GIC) it has issued. Suppose that the life insurance company has the opportunity to invest \$100 million in what it considers an attractive 4-year floating-rate instrument in a private placement transaction. The interest rate on this instrument is 6-month LIBOR plus 120 basis points. The coupon rate is set every six months.

The risk that the life insurance company faces in this instance is that 6-month LIBOR will fall so that the company will not earn enough to realize a spread over the 8% rate that it has guaranteed to the GIC policyholders. If 6-month LIBOR falls to 6.8% or less at a coupon reset date, no spread income will be generated. To understand why, suppose that 6-month LIBOR is 6.8% at the date the floating-rate instrument resets its coupon. Then the coupon rate for the next six months will be 8% (6.8% plus 120 basis points). Because the life insurance company has agreed to pay 8% on the GIC policy, there will be no spread income. Should 6-month LIBOR fall below 6.8%, there will be a loss for that 6-month period.

We can summarize the asset/liability problems of the bank and the life insurance company as follows.

Bank:

1. has lent long term and borrowed short term
2. if 6-month LIBOR rises, spread income declines

Life insurance company:

1. has lent short term and borrowed long term
2. if 6-month LIBOR falls, spread income declines

Now suppose the market has available a 4-year interest rate swap with a notional amount of \$100 million, with the following swap terms available to the bank:

1. every six months the bank will pay 9.50% (annual rate)
2. every six months the bank will receive LIBOR

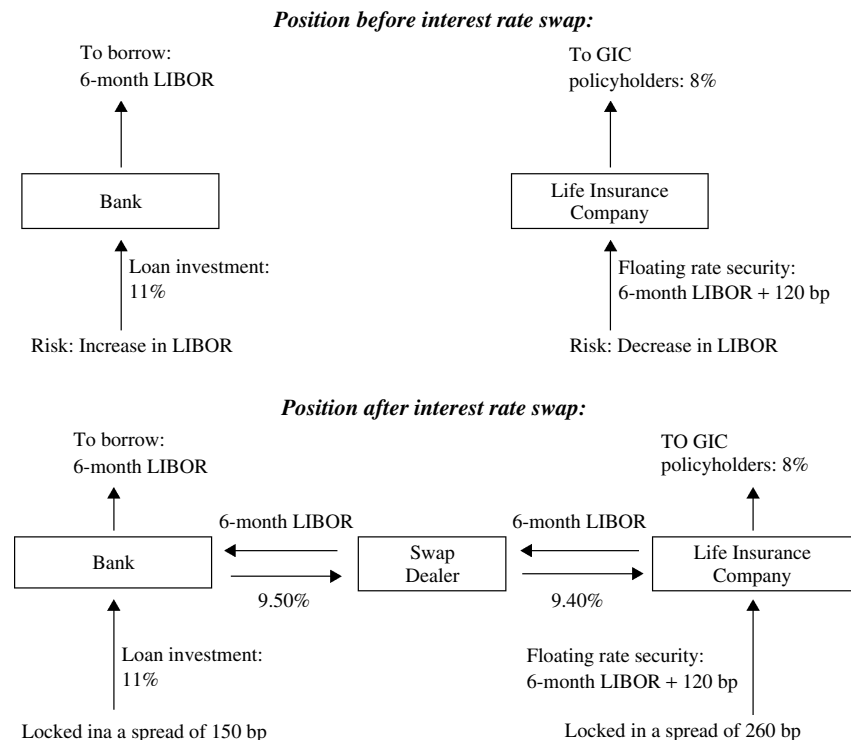
Suppose the swap terms available to the insurance company are as follows:

1. every six months the life insurance company will pay LIBOR
2. every six months the life insurance company will receive 9.40%

Now let's look at the positions of the bank and the life insurance company after the swap. Exhibit 3 summarizes the position of each institution before and after the swap. Consider first the bank. For every 6-month period for the life of the swap, the interest rate spread will be as follows:

Annual interest rate received:		
From commercial loan portfolio	=	11.00%
From interest rate swap	=	6-month LIBOR
Total	=	11.00% + 6-month LIBOR

EXHIBIT 3 Position of Bank and Life Insurance Company Before and After Swap



Annual interest rate paid:		
To borrow funds	=	6-month LIBOR
On interest rate swap	=	9.50%
Total	=	9.50% + 6-month LIBOR

Outcome:		
To be received	=	11.0% + 6-month LIBOR
To be paid	=	9.50% + 6-month LIBOR
Spread income	=	1.50% or 150 basis points

Thus, regardless of changes in 6-month LIBOR, the bank locks in a spread of 150 basis points assuming no loan defaults or payoffs.

Now let's look at the effect of the interest rate swap on the life insurance company:

Annual interest rate received:		
From floating-rate instrument	=	1.20% + 6-month LIBOR
From interest rate swap	=	9.40%
Total	=	10.60% + 6-month LIBOR

Annual interest rate paid:		
To GIC policyholders	=	8.00%
On interest rate swap	=	6-month LIBOR
Total	=	8.00% + 6-month LIBOR

Outcome:	
To be received	= 10.60% + 6-month LIBOR
To be paid	= 8.00% + 6-month LIBOR
Spread income	= 2.60% or 260 basis points

Regardless of what happens to 6-month LIBOR, the life insurance company locks in a spread of 260 basis points assuming the issuer of the floating-rate instrument does not default.

The interest rate swap has allowed each party to accomplish its asset/liability objective of locking in a spread.⁵ It permits the two financial institutions to alter the cash flow characteristics of its assets: from fixed to floating in the case of the bank, and from floating to fixed in the case of the life insurance company.

B. Dollar Duration of a Swap

Effectively, a position in an interest rate swap is a leveraged position. This agrees with the economic interpretations of an interest rate swap explained earlier. We know that futures/forwards are leveraged instruments. In the case of a package of cash instruments, it is a leveraged position involving either buying a fixed-rate bond and financing it on a floating-rate basis (i.e., fixed-rate receiver position) or buying a floating-rate bond on a fixed-rate basis (i.e., fixed-rate payer position). So, we would expect that the dollar duration of a swap is a multiple of the bond that effectively underlies the swap.

To see how to calculate the dollar duration, let's work with the second economic interpretation of a swap—a package of cash flows from buying and selling cash market instruments. From the perspective of the *fixed-rate receiver*, the position can be viewed as follows:

$$\text{long a fixed-rate bond} + \text{short a floating-rate bond}$$

The fixed-rate bond is a bond with a coupon rate equal to the swap rate, a maturity equal to the term of the swap, and a par value equal to the notional amount of the swap.

This means that the dollar duration of an interest rate swap from the perspective of a fixed-rate receiver is the difference between the dollar durations of the two bond positions that comprise the swap. That is,

$$\begin{aligned} &\text{dollar duration of a swap for a fixed-rate receiver} \\ &= \text{dollar duration of a fixed-rate bond} - \text{dollar duration of a floating-rate bond} \end{aligned}$$

Most of the swap's interest rate sensitivity results from the dollar duration of the fixed-rate bond since the dollar duration of the floating-rate bond will be small. The dollar duration of a floating-rate bond is smaller the closer the swap is to its reset date. If the dollar duration of the floating-rate bond is close to zero then:

$$\text{dollar duration of a swap for a fixed-rate receiver} \approx \text{dollar duration of a fixed-rate bond}$$

⁵Whether the size of the spread is adequate is not an issue to us in this illustration.

Thus, adding an interest rate swap to a portfolio in which the manager pays a floating-rate and receives a fixed-rate increases the dollar duration of the portfolio by roughly the dollar duration of the underlying fixed-rate bond. This is because it effectively involves buying a fixed-rate bond on a leveraged basis.

We can use the cash market instrument economic interpretation to compute the dollar duration of a swap for the fixed-rate payer. The dollar duration is:

$$\begin{aligned} &\text{dollar duration of a swap for a fixed-rate payer} \\ &= \text{dollar duration of a floating-rate bond} - \text{dollar duration of a fixed-rate bond} \end{aligned}$$

Again, assuming that the dollar duration of the floater is small, we have

$$\text{dollar duration of a swap for a fixed-rate payer} \approx -\text{dollar duration of a fixed-rate bond}$$

Consequently, a manager who adds to a portfolio a swap involving paying fixed and receiving floating decreases the dollar duration of the portfolio by an amount roughly equal to the dollar duration of the fixed-rate bond.

The dollar duration of a portfolio that includes a swap is:

$$\text{dollar duration of assets} - \text{dollar duration of liabilities} + \text{dollar duration of a swap position}$$

Let's look at our bank/life insurance illustration in terms of duration mismatch. The bank has a larger duration for its assets (the fixed-rate loans) than the duration for its liabilities (the short-term funds it borrows). Effectively, the position of the bank is as follows:

$$\text{bank's dollar duration} = \text{dollar duration of assets} - \text{dollar duration of liabilities} > 0$$

The bank entered into an interest rate swap in which it pays fixed and receives floating. The dollar duration of that swap position is negative, so adding the swap position moves the bank's dollar duration position closer to zero and, therefore, reduces interest rate risk.

For the life insurance company, the duration of the liabilities is long while the duration of the floating-rate assets is short. That is,

$$\begin{aligned} &\text{life insurance company's dollar duration} = \text{dollar duration of assets} \\ &- \text{dollar duration of liabilities} < 0 \end{aligned}$$

The life insurance company entered into an interest rate swap in which it pays floating and receives fixed. This swap position has a positive duration. By adding it to a portfolio it moves the duration closer to zero, thereby reducing interest rate risk.

IV. HEDGING WITH OPTIONS

Hedging strategies using options involve taking a position in an option and a position in the underlying bond in such a way that changes in the value of one position will offset any unfavorable price (interest rate) movement in the other position. We begin with the basic hedging strategies using options. Then we illustrate these basic strategies using futures options

to hedge the FNMA bond for which a futures hedge was used in Section II. Using futures options in our illustration is a worthwhile exercise because it shows the complexities of hedging with futures options and the key parameters involved in the process. We also compare the outcome of hedging with futures and hedging with futures options.

A. Basic Hedging Strategies

There are three popular hedging strategies: (1) a protective put buying strategy, (2) a covered call writing strategy, and (3) a collar strategy. We discuss each strategy below.

1. Protective Put Buying Strategy Consider a manager who has a bond and wants to hedge against rising interest rates. The most obvious options hedging strategy is to buy put options on bonds. This is referred to as a **protective put buying strategy**. The puts are usually out-of-the-money and may be puts on either cash bonds or interest rate futures. If interest rates rise, the puts will increase in value (holding other factors constant), offsetting some or all of the loss on the bonds in the portfolio.

This strategy is a simple combination of a long put option with a long position in a cash bond. Such a position has limited downside risk, but large upside potential. However, if rates fall, the total position value of the hedged portfolio is diminished in comparison to the unhedged position because of the cost of the puts. Exhibit 4 compares the protective put buying strategy to an unhedged position.

The protective put buying strategy is very often compared to purchasing insurance. Like insurance, the premium paid for the protection is nonrefundable and is paid before the coverage begins. The degree to which a portfolio is protected depends upon the strike price of the options; thus, the strike price is often compared to the deductible on an insurance policy. The lower the deductible (that is, the higher the strike price for the put), the greater the level of protection and the more the protection costs. Conversely, the higher the deductible (the lower the strike price on the put), the more the portfolio can lose in value; but the cost of

EXHIBIT 4 Protective Put Buying Strategy

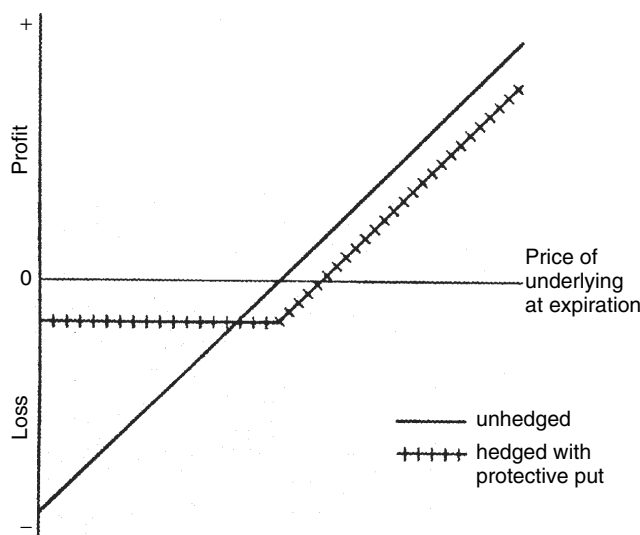
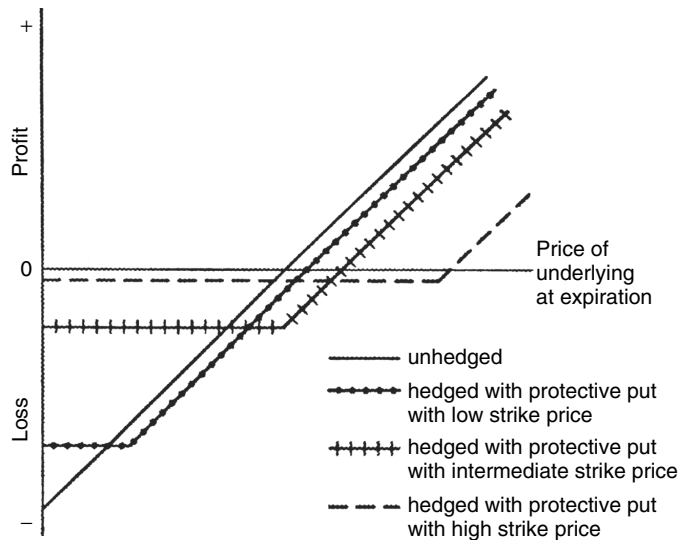


EXHIBIT 5 Protective Put Buying Strategy with Different Strike Prices

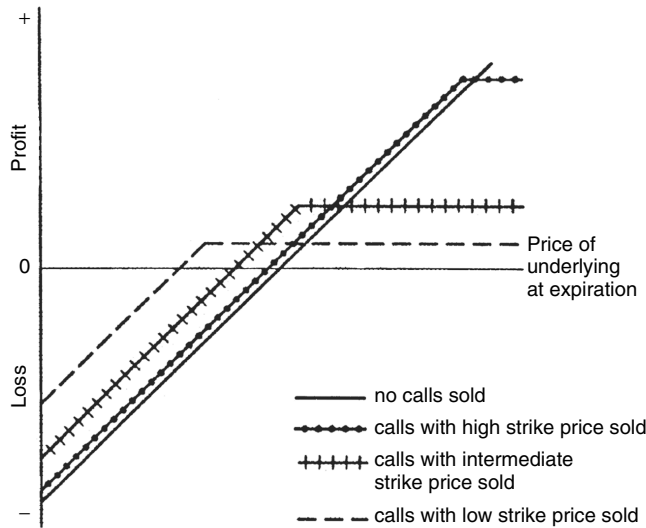


the insurance is lower. Exhibit 5 compares an unhedged position with several protective put positions, each with a different strike price, or level of protection. As the exhibit shows, no one strategy dominates any other, in the sense of performing better at all possible rate levels. Consequently, it is impossible to say that one strike price is necessarily the “best” strike price, or even that buying protective puts is necessarily better than doing nothing at all.

2. Covered Call Writing Strategy Another options hedging strategy used by many portfolio managers is to sell calls against the bond portfolio. This hedging strategy is called a **covered call writing strategy**. The calls are usually out-of-the-money, and can be either calls on cash bonds or calls on interest rate futures. Covered call writing is just an outright long position combined with a short call position. Obviously, this strategy entails much more downside risk than buying a put to protect the value of the portfolio. In fact, many portfolio managers do not consider covered call writing a hedge.

Regardless of how it is classified, it is important to recognize that while covered call writing has substantial downside risk, it has less downside risk than an unhedged long position alone. On the downside, the difference between the long position alone and the covered call writing strategy is the premium received for the calls that are sold. This premium acts as a cushion for downward movements in prices, reducing losses when rates rise. The cost of obtaining this cushion is the upside potential that is forfeited. When rates decline, the call option liability increases for the covered call writer. These incremental liabilities decrease the gains the manager would otherwise have realized on the portfolio in a declining rate environment. Thus, the covered call writer gives up some (or all) of the upside potential of the portfolio in return for a cushion on the downside. The more upside potential that is forfeited (that is, the lower the strike price on the calls), the more cushion there is on the downside. Exhibit 6 illustrates this point by comparing an unhedged position to several covered call writing strategies, each with a different strike price. Like the protective put buying strategy, there is no “right” strike price for the covered call writer.

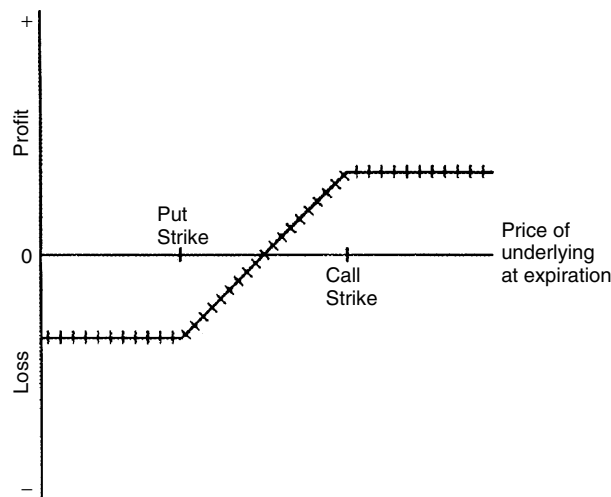
EXHIBIT 6 Covered Call Writing Strategy with Different Strike Prices



3. **Collar Strategy** Another hedging strategy frequently used by portfolio managers combines protective put buying and covered call writing. By combining a long position in an out-of-the-money put and a short position in an out-of-the-money call, the manager creates a long position in a *collar*. Consequently, this hedging strategy is called a **collar strategy**. The manager who uses the collar eliminates part of the portfolio's downside risk by giving up part of its upside potential. A long position hedged with a collar is shown in Exhibit 7.

The collar in some ways resembles the protective put, in some ways resembles covered call writing, and in some ways resembles an unhedged position. The collar is like the protective put

EXHIBIT 7 Long Position Hedged with a Collar



buying strategy in that it limits the possible losses on the portfolio if interest rates go up. Like the covered call writing strategy, the portfolio's upside potential is limited. Like an unhedged position, within the range defined by the strike prices the value of the portfolio varies with interest rates.

4. **Selecting the "Best" Strategy** Comparing the two basic strategies for hedging with options, one cannot say that the protective put buying strategy or the covered call writing strategy is necessarily the better or more correct options hedge. The best strategy (and the best strike price) depends upon the manager's view of the market and risk tolerance. Paying the required premium to purchase a put is appropriate if the manager is fundamentally bearish. If, instead, the manager is neutral to mildly bearish, it is better to receive the premium by selling covered calls. If the manager prefers to take no view on the market at all, and to assume as little risk as possible, then the futures hedge is the most appropriate. If the manager is fundamentally bullish, then an unhedged position is probably the best strategy.

B. Steps in Options Hedging

Like hedging with futures (described in Section II), there are several steps that managers should consider before implementing their hedges. These steps include:

Step 1: Determine the option contract that is the best hedging vehicle. The best option contract to use depends upon several factors. These include option price, liquidity, and price correlation with the bond(s) to be hedged. Whenever there is a possibility that the option position may be closed out prior to expiration, liquidity is an important consideration. If a particular option is illiquid, closing out a position may be prohibitively expensive, so that the manager loses the flexibility to close out positions early, or rolling into other positions that may become more attractive. Correlation with the price of the underlying bond(s) to be hedged is another factor in selecting the right contract. The higher the correlation, the more precisely the final profit and loss can be defined as a function of the final level of rates. Low correlation leads to more uncertainty.

While most of the uncertainty in an options hedge usually comes from the uncertainty of interest rates, the correlation between the prices of the bonds to be hedged and the instruments underlying the options contracts determines the extent to which the hedging objective is accomplished. Low correlation leads to greater uncertainty.

Step 2: Find the appropriate strike price. For a cross hedge, the manager converts the strike price on the options that are bought or sold into an equivalent strike price for the bonds being hedged.

Step 3: Determine the number of contracts. The hedge ratio is the number of options to buy or sell.

Steps 2 and 3 can best be explained with examples using futures options.

C. Protective Put Buying Strategy Using Futures Options

As explained above, managers who want to hedge bond positions against a possible increase in interest rates find that buying puts on futures is one of the easiest ways to purchase protection against rising rates. To illustrate a protective put buying strategy, we can use the same FNMA

bond that we used to demonstrate how to hedge with Treasury bond futures.⁶ In that example, a manager held \$10 million par value of a 6.25% FNMA bond maturing 5/15/29 and used September 1999 Treasury bond futures to lock in a sale price for those bonds on the futures delivery date. Now we want to show how the manager could use futures options instead of futures to protect against rising rates.

On 6/24/99 the FNMA bond was selling for 88.39 to yield 7.20% and the CTD issue's yield was 6.50%. For simplicity, it is assumed that the yield spread between the FNMA bond and the CTD issue remains at 70 basis points.

1. Selecting the Strike Price In our illustration, selecting the strike price means determining the minimum price that the manager wants to establish for the FNMA bonds. In our illustration we will assume that the minimum acceptable price (before the cost of the put options) is 84.453. This is equivalent to saying that the manager wants to establish a strike price for a put on the hedged bonds of 84.453. But, the manager is not buying a put on the FNMA bond. He is buying a put on a Treasury bond futures contract. Therefore, the manager must determine the strike price for a put on a Treasury bond futures contract that is equivalent to a strike price of 84.453 for the FNMA bond.

This can be done with the help of Exhibit 8. Notice that all the boxes are numbered and we shall refer to these numbered boxes as we illustrate the process. We begin at Box 1 of the exhibit. In our illustration, the portfolio manager set the strike price at 84.453 for the FNMA bond. Now we can calculate, given its coupon rate of 6.25%, its maturity, and the price of 84.453 (the strike price desired), that its yield is 7.573%. That is, setting a strike price of 84.453 for the FNMA bond is equivalent to setting a strike yield (or equivalently a maximum yield) of 7.573% in Box 2.

Now let's move to Box 3—the yield of the cheapest-to-deliver issue. To move from Box 2 to Box 3 we use the assumption that the spread between the FNMA bond and the cheapest-to-deliver issue is a constant 70 basis points. Since the strike yield for the FNMA bond is 7.573%, subtracting 70 basis points gives 6.873% as the strike yield (or maximum yield) for the cheapest-to-deliver issue.

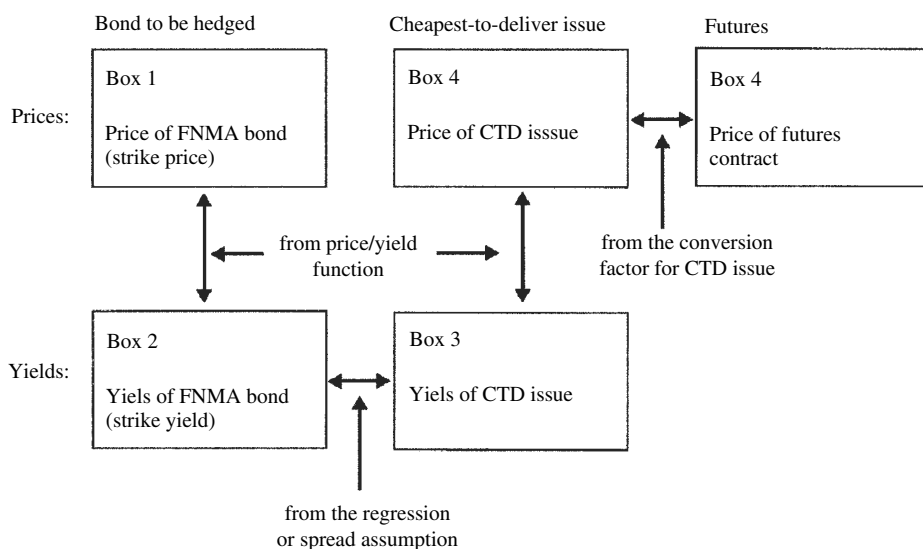
We again use the price/yield relationship to move from Box 3 to Box 4. The cheapest-to-deliver issue was the 11.25% of 2/15/15 issue. Given the maturity, the coupon rate, and the strike yield of 6.873%, the price is computed to be 141.136. This is the value that would go in Box 4.

Now for the final value we need—the strike price for the Treasury bond futures contract. We know that the converted price for any issue eligible for delivery on the bond futures contract is:

$$\text{converted price} = \text{futures price} \times \text{conversion factor}$$

⁶Futures options on Treasury bonds are more commonly used by institutional investors. The mechanics of futures options are as follows. If a put option is exercised, the option buyer receives a short position in the underlying futures contract and the option writer receives the corresponding long position. The futures price for both positions is the strike price for the put option. The exchange then marks the positions to market and the futures price for both positions is then the current futures price. If a call option is exercised, the option buyer receives a long position in the underlying futures contract and the option writer receives the corresponding short position. The futures price for both positions is the strike price for the call option. The exchange then marks the positions to market and the futures price for both positions is then the current futures price.

EXHIBIT 8 Calculating Equivalent Strike Prices and Yields for Hedging with Futures Options



In the case of the cheapest-to-deliver issue it is:

$$\text{converted price for CTD issue} = \text{futures price} \times \text{conversion factor for CTD issue}$$

The goal is to get the strike price for the futures contract. Solving the above for the futures price we get

$$\text{futures price} = \frac{\text{converted price for CTD issue}}{\text{conversion factor of CTD issue}}$$

Since the converted price for the CTD issue in Box 4 is 141.136 and the conversion factor is 1.283, the strike price for the Treasury bond futures contract is:

$$\text{futures price} = \frac{141.136}{1.283} = 110.0047 \text{ or } 110$$

A strike price of 110 for a put option on a Treasury bond futures contract is roughly equivalent to a put option on our FNMA bond with a strike price of 84.453.

The foregoing steps are always necessary to obtain the appropriate strike price on a futures put option. The process is not complicated. It simply involves (1) the relationship between price and yield, (2) the assumed relationship between the yield spread between the bonds to be hedged and the cheapest-to-deliver issue, and (3) the conversion factor for the cheapest-to-deliver issue. Once again, the success of the hedging strategy will depend on (1) whether the cheapest-to-deliver issue changes and (2) the yield spread between the bonds to be hedged and the cheapest-to-deliver issue.

2. Calculating the Number of Options Contracts Since we assume a constant yield spread between the bond to be hedged and the cheapest-to-deliver issue, the hedge ratio is

determined using the following equation, which is similar to equation (7):

$$\begin{aligned} \text{number of options contracts} &= \frac{\text{current dollar duration without options}}{\text{dollar duration of the CTD issue}} \\ &\quad \times \text{conversion factor for CTD issue} \end{aligned}$$

The current duration refers to the duration at the time the hedge is placed. Recall that the dollar durations are calculated as of the date that the hedge is expected to be removed using the target yield for the bond to be hedged. For the protective put buying strategy, we will assume that the hedge will be removed on the expiration date of the option (assumed to be the end of September 1999). To obtain the current dollar duration for the bond, the target yield is the 7.573% strike yield of the FNMA bond. The current dollar duration for the FNMA bond as of the end of September 1999 for a 50 basis point change in rates and for a target yield of 7.573% would produce a value of \$512,320. This value differs from the current dollar duration used to compute the number of contracts when hedging with futures. That value was \$545,300 because it was based on a target yield of 7.26%.

The dollar duration for the CTD issue is based on a different target price (i.e., the minimum price) than in the hedging strategy with futures. The target price in the futures hedging strategy was 113 and the dollar duration of the CTD issue was \$6,255. For the protective put buying strategy, since the strike price of the futures option is 110 (the target price in this strategy), it can be shown that the dollar duration for the CTD issue for a 50 basis point change in rates is \$6,021.

Therefore, we know that for a 50 basis point change in rates:

$$\text{current dollar duration without options} = \$512,320$$

$$\text{dollar duration for the CTD issue} = \$6,021$$

Substituting these values and the conversion factor for the CTD issue of 1.283 into the formula for the number of options contracts, we find that:

$$\text{number of options contracts} = \frac{\$512,320}{\$6,021} \times 1.283 = 109 \text{ put options}$$

Thus, to hedge the FNMA bond position with put options on Treasury bond futures, 109 put options must be purchased.

3. Outcome of the Hedge To create a table for the protective put hedge, we can use data from Exhibit 2. Exhibit 9 shows the scenario analysis for the protective put buying strategy. The first five columns are the same as in Exhibit 2. For the put option hedge, Column (6) shows that the value of the put position at expiration equals zero if the futures price is greater than or equal to the strike price of 110. If the futures price is below 110, then the options expire in the money and:

$$\begin{aligned} \text{value of put option position} &= ([110 - \text{futures price}]/100) \times \$100,000 \\ &\quad \times \text{number of put options} \end{aligned}$$

For example, for the first scenario in Exhibit 9, the corresponding futures price is 105.8551 and the value of the put options is

$$([110 - \text{futures price}]/100) \times \$100,000 \times 109 = \$451,794$$

EXHIBIT 9 Hedging a Nondeliverable Bond to a Delivery Date with Puts on Futures

Instrument to be hedged: \$10 million FNMA 6.25% of 05/15/29

Price of FNMA as of hedge date (6/24/99) = 88.39

Conversion factor = 1.283

Price of futures contract = 113

Target price per bond for FNMA bonds = 84.453

Effective minimum sale price = 83.908

Par value hedged = \$10,000,000

Strike price for put = 110

Number of puts on futures = 109

Price per contract = \$500.00

Cost of put position = \$54,500

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Actual sale price of FNMA bonds	Yield at sale	Yield of 11.25% Treasury bond	Price of 11.25% Treasury bond	Futures price ¹	Value of put options ²	Cost of put position	Effective sale price ³
8,000,000	8.027%	7.327%	135.812	105.85510	451,794	54,500	8,397,294
8,100,000	7.922%	7.222%	137.024	106.79953	348,851	54,500	8,394,351
8,200,000	7.818%	7.118%	138.226	107.73658	246,712	54,500	8,392,212
8,300,000	7.717%	7.017%	139.419	108.66640	145,362	54,500	8,390,862
8,400,000	7.617%	6.917%	140.603	109.58913	44,785	54,500	8,390,285
8,500,000	7.520%	6.820%	141.778	110.50490	—	54,500	8,445,500
8,600,000	7.424%	6.724%	142.944	111.41384	—	54,500	8,545,500
8,700,000	7.330%	6.630%	144.102	112.31608	—	54,500	8,645,500
8,800,000	7.238%	6.538%	145.251	113.21175	—	54,500	8,745,500
8,900,000	7.148%	6.448%	146.392	114.10096	—	54,500	8,845,500
9,000,000	7.059%	6.359%	147.524	114.98383	—	54,500	8,945,500
9,100,000	6.971%	6.271%	148.649	115.86047	—	54,500	9,045,500
9,200,000	6.886%	6.186%	149.766	116.73101	—	54,500	9,145,500
9,300,000	6.801%	6.101%	150.875	117.59554	—	54,500	9,245,500
9,400,000	6.719%	6.019%	151.977	118.45417	—	54,500	9,345,500
9,500,000	6.637%	5.937%	153.071	119.30702	—	54,500	9,445,500

¹These numbers are approximate because futures trade in 32nds.²From Maximum of $([110 - \text{Futures price}]/100) \times \$100,000 \times 109, 0]$ ³Does not include transaction costs or the financing of the options position.

The effective sale price for the FNMA bonds is then equal to

$$\text{effective sale price} = \text{actual sale price} + \text{value of put option position} - \text{option cost}$$

Let's look at the option cost. Suppose that the price of the put option with a strike price of 110 is \$500 per contract. The cost of the protection is \$54,500 ($109 \times \$500$, not including financing costs and commissions). This cost is shown in Column (7) and is equivalent to \$0.545 per \$100 par value hedged.

The effective sale price for the FNMA bonds in each scenario, shown in the last column of Exhibit 9, is never less than 83.902. This equals the price of the FNMA bonds equivalent to the futures strike price of 110 (i.e., 84.453), minus the cost of the puts (that is, \$0.545 per \$100 par value hedged). This minimum effective price is something that can be calculated before the hedge is initiated. (As prices decline, the effective sale price actually exceeds the target minimum sale price of 83.908 by a small amount. This is due only to rounding; the hedge ratio is left unaltered although the relative dollar durations used in the hedge ratio calculation change as yields change.) As prices increase, however, the effective sale price of

the hedged bonds increases as well; unlike the futures hedge shown in Exhibit 2, the options hedge protects the investor if rates rise, but allows the investor to profit if rates fall.

D. Covered Call Writing Strategy with Futures Options

Unlike the protective put buying strategy, the purpose of covered call writing is not to protect a portfolio against rising rates. The covered call writer, believing that the market will not trade much higher or much lower than its present level, sells out-of-the-money calls against an existing bond portfolio. The sale of the calls brings in premium income that provides partial protection in case rates increase. The premium received does not, of course, provide the kind of protection that a long put position provides, but it does provide some additional income that can be used to offset declining prices. If, instead, rates fall, portfolio appreciation is limited because the short call position constitutes a liability for the seller, and this liability increases as rates decline. Consequently, there is limited upside price potential for the covered call writer. Of course, this is not a concern if prices are essentially going nowhere; the added income from the sale of call options is obtained without sacrificing any gains.

To see how covered call writing with futures options works for the bond used in the protective put example, we construct a table much as we did before. We assume in this illustration that a sale of a call option with a strike price of 117 is selected by the manager.⁷ As in the futures hedging and protective buying strategies it is assumed that the hedged bond will remain at a 70 basis point spread over the CTD issue. We also assume that the price of each call option is \$500.

Working backwards from box 5 in Exhibit 8, a strike price for the futures option of 117 would be equivalent to the following at the expiration date of the call option:

$$\text{target price for CTD issue} = 117 \times 1.283 = 150.111 (\text{value at box 4})$$

$$\text{target yield for CTD issue} = 6.16\% (\text{from price/yield relationship, box 3})$$

$$\text{target yield for FNMA bond} = 6.86\% (\text{from 70 bp spread assumption, box 2})$$

$$\text{target price for FNMA bond} = 92.3104 (\text{from price/yield relationship, box 1})$$

Given the above information, the current dollar duration for the FNMA bond and the dollar duration of the CTD issue can be computed. The information above represents values as of the expiration date of the call option. It can be shown that the dollar durations based on a 50 basis point change in rates are:

$$\text{current dollar duration without options} = \$592,031$$

$$\text{dollar duration for the CTD issue} = \$6,573$$

Substituting these values and the conversion factor for the CTD issue of 1.283 into the formula for the number of options contracts, we find:

$$\text{number of options contracts} = \frac{\$592,031}{\$6,573} \times 1.283 = 115.6 (\text{rounded to 116})$$

The proceeds received from the sale of 116 call options are \$58,000 (116 calls $\times$ \$500) and proceeds per \$100 of par value hedged are \$0.580.

⁷Note that this is based on the risk-return profile the manager is willing to accept. It is not derived analytically in this illustration.

While the target price for the FNMA bond is 92.3104, the maximum effective sale price is determined by adjusting the target price by the proceeds received from the sale of the call options. The maximum effective sale price is 92.8904 ($= 92.3104 + 0.5800$).

Exhibit 10 shows the outcomes of the covered call writing strategy. The first five columns of the exhibit are the same as Exhibit 9. In Column (6), the liability resulting from the call option position is shown. The liability is zero if the futures price is less than the strike price of 117. If the futures price for the scenario is greater than 117, the liability is calculated as follows:

$$([\text{futures price} - 117]/100) \times \$100,000 \times \text{number of call options}$$

For example, consider a sale price for the FNMA bond of \$9.5 million. The corresponding futures price is 119.30702. The number of call options sold is 116. Therefore,

$$([119.30702 - 117]/100) \times \$100,000 \times 116 = \$267,614$$

That is,

$$\begin{aligned} \text{effective sale price} &= \text{actual sale price} + \text{proceeds from sale of the call options} \\ &\quad - \text{liability of call position} \end{aligned}$$

EXHIBIT 10 Writing Calls on Futures against a Nondeliverable Bond

Instrument to be hedged: \$10 million FNMA 6.25% of 05/15/29

Price of FNMA as of hedge date (6/24/99) = 88.39

Conversion factor for September 1999 = 1.283

Price of futures contract = 113

Target price for FNMA bonds per bond = 92.3104

Effective maximum sale price = 92.8904

Par value hedged = \$10,000,000

Strike price for call = 117

Number of calls on futures = 116

Price per contract = \$500.00

Value of call position = \$58,000

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Actual sale price of FNMA bonds	Yield at sale	Yield of 11.25% Treasury bond	Price of 11.25% Treasury bond	Futures price ¹	Liability of call options ²	Proceeds from call position	Effective sale price ³
8,000,000	8.027%	7.327%	135.812	105.85510	—	58,000	8,058,000
8,100,000	7.922%	7.222%	137.024	106.79953	—	58,000	8,158,000
8,200,000	7.818%	7.118%	138.226	107.73658	—	58,000	8,258,000
8,300,000	7.717%	7.017%	139.419	108.66640	—	58,000	8,358,000
8,400,000	7.617%	6.917%	140.603	109.58913	—	58,000	8,458,000
8,500,000	7.520%	6.820%	141.778	110.50490	—	58,000	8,558,000
8,600,000	7.424%	6.724%	142.944	111.41384	—	58,000	8,658,000
8,700,000	7.330%	6.630%	144.102	112.31608	—	58,000	8,758,000
8,800,000	7.238%	6.538%	145.251	113.21175	—	58,000	8,858,000
8,900,000	7.148%	6.448%	146.392	114.10096	—	58,000	8,958,000
9,000,000	7.059%	6.359%	147.524	114.98383	—	58,000	9,058,000
9,100,000	6.971%	6.271%	148.649	115.86047	—	58,000	9,158,000
9,200,000	6.886%	6.186%	149.766	116.73101	—	58,000	9,258,000
9,300,000	6.801%	6.101%	150.875	117.59554	69,083	58,000	9,288,917
9,400,000	6.719%	6.019%	151.977	118.45417	168,684	58,000	9,289,316
9,500,000	6.637%	5.937%	153.071	119.30702	267,614	58,000	9,290,386

¹These numbers are approximate because futures trade in 32nds.

²From Maximum of $([\text{Futures price} - 117]/100) \times \$100,000 \times 116, 0]$

³Does not include transaction costs or interest earned on the option premium.

Since the proceeds from sale of the call options is \$58,000, then

$$\text{effective sale price} = \text{actual sale price} + \$58,000 - \text{liability of call position}$$

The last column of Exhibit 10 shows the effective sale price for each scenario.

As Exhibit 10 shows, if the hedged bond trades at 70 basis points over the CTD issue as assumed, the maximum effective sale price for the hedged bond is, in fact, slightly over 92.8904. The discrepancies shown in the exhibit are due to rounding.

E. Comparing Alternative Strategies

In this chapter we reviewed three basic strategies for hedging a bond position: (1) hedging with futures, (2) hedging with out-of-the-money puts, and (3) covered call writing with out-of-the-money calls. Similar, but opposite, strategies exist for managers who are concerned that rates will decrease. As might be expected, there is no “best” strategy. Each strategy has advantages and disadvantages, and we never get something for nothing. To get anything of value, something else of value must be forfeited.

To make a choice among strategies, it helps to compare the alternatives side by side. Using the futures example from Section II B and the futures options examples from Section IV C, Exhibit 11 shows the final values of the portfolio for the various hedging alternatives. It is easy to see that no one strategy dominates the others in every scenario.

Consequently, we cannot conclude that one strategy is the best strategy. The manager who makes the strategy decision makes a choice among probability distributions, not usually among specific outcomes. Except for the perfect hedge, there is always some range of possible final values of the portfolio. Of course, exactly what that range is, and the probabilities associated with each possible outcome, is a matter of opinion.

EXHIBIT 11 Comparison of Alternative Strategies

(1)	(2)	(3)	(4)
Actual sale price of FNMA bonds	Effective sale price with futures hedge	Effective sale price with protective puts	Effective sale price with covered calls
8,000,000	8,800,229	8,397,294	8,058,000
8,100,000	8,794,453	8,394,351	8,158,000
8,200,000	8,789,503	8,392,212	8,258,000
8,300,000	8,785,363	8,390,862	8,358,000
8,400,000	8,782,017	8,390,285	8,458,000
8,500,000	8,779,451	8,445,500	8,558,000
8,600,000	8,777,650	8,545,500	8,658,000
8,700,000	8,776,599	8,645,500	8,758,000
8,800,000	8,776,284	8,745,500	8,858,000
8,900,000	8,776,693	8,845,500	8,958,000
9,000,000	8,777,812	8,945,500	9,058,000
9,100,000	8,779,627	9,045,500	9,158,000
9,200,000	8,782,127	9,145,500	9,258,000
9,300,000	8,785,300	9,245,500	9,288,917
9,400,000	8,789,132	9,345,500	9,289,316
9,500,000	8,793,614	9,445,500	9,290,386

F. Hedging with Options on Cash Instruments

Hedging a position with options on cash bonds is relatively straightforward. Most strategies, including the purchase of protective puts, covered call writing, and buying collars, are essentially the same whether futures options or options on physicals are used. As explained, an option on a physical is an option on a bond. In our illustrations we used options on futures because they are the more actively traded types of futures contracts and therefore the contracts used in hedging. While there are mechanical differences in the way the two types of option contracts are traded, the basic economics of the hedging strategies are virtually identical.

Using options on physicals frequently eliminates much of the basis risk associated with a futures options hedge. For example, a manager of Treasury bonds or notes can usually buy or sell options on the exact security held in the portfolio. Using options on futures, rather than options on Treasury bonds, introduces additional elements of uncertainty.

Given the illustration presented above, and given that the risk-return profile of options on physicals and options on futures are essentially identical, additional illustrations for options on physicals are unnecessary. The only important differences are the calculations of the hedge ratio and the equivalent strike price. To derive the hedge ratio, we always resort to an expression of relative dollar durations. Thus, assuming a constant spread, the hedge ratio for options on physicals is:

$$\frac{\text{current dollar duration without options}}{\text{dollar duration of underlying for option}}$$

If a relationship is estimated between the yield on the bonds to be hedged and the instrument underlying the option, the appropriate hedge ratio is:

$$\frac{\text{current dollar duration without options}}{\text{dollar duration of underlying for option}} \times \text{yield beta}$$

Unlike futures options, there is only one deliverable, so there is no conversion factor. When cross hedging with options on physicals, the procedure for finding the equivalent strike price on the bonds to be hedged is similar. Given the strike price of the option, the strike yield is determined using the price/yield relationship for the instrument underlying the option. Then given the projected relationship between the yield on the instrument underlying the option and the yield on the bonds to be hedged, an equivalent strike yield is derived for the bonds to be hedged. Finally, using the yield-to-price formula for the bonds to be hedged, the equivalent strike price for the bonds to be hedged can be found.

V. USING CAPS AND FLOORS

Interest rate caps can be used in liability management to create a cap for funding costs. Combining a cap and a floor creates a collar for funding costs. Floors can be used by buyers of floating-rate instruments to set a floor on the periodic interest earned. To reduce the cost of a floor, a manager can sell a cap. By doing so, the manager limits the upside on the coupon rate of a floating-rate instrument should rates rise, thereby creating a collar for the coupon rate.

To see how interest rate caps and floors can be used for asset/liability management, consider the problems faced by the commercial bank and the life insurance company discussed in Section III A. Recall that the bank's objective is to lock in a spread over its cost of funds.

Yet because the bank borrows short term, its cost of funds is uncertain. The bank may be able to purchase a cap, however, so that the cap rate plus the cost of purchasing the cap is less than the rate it is earning on its fixed-rate commercial loans. If short-term rates decline, the bank does not benefit from the cap, but its cost of funds declines. The cap therefore allows the bank to impose a ceiling on its cost of funds while retaining the opportunity to benefit from a decline in rates.

The bank can reduce the cost of purchasing the cap by selling a floor. In this case, the bank agrees to pay the buyer of the floor if the reference rate falls below the strike rate. The bank receives a fee for selling the floor, but it has sold off its opportunity to benefit from a decline in rates below the strike rate. By buying a cap and selling a floor, the bank has created a predetermined range for its cost of funds (i.e., a collar).

Recall the problem of the life insurance company that guarantees an 8% rate on a GIC for four years and is considering the purchase of a floating-rate instrument in a private placement transaction. The risk the company faces is that interest rates will fall so that it will not earn enough to realize the 8% guaranteed rate plus a spread. The life insurance company may be able to purchase a floor to set a lower bound on its investment return, yet retain the opportunity to benefit should rates increase. To reduce the cost of purchasing the floor, the life insurance company can sell an interest rate cap. By doing so, however, it forfeits the opportunity to benefit from an increase in the reference rate above the strike rate of the cap.

CHAPTER 23

HEDGING MORTGAGE SECURITIES TO CAPTURE RELATIVE VALUE*

I. INTRODUCTION

Because of the spread offered on residential agency mortgage-backed securities, they often outperform government securities with the *same interest rate risk* and therefore they can be used to generate enhanced returns relative to a benchmark when the yield advantage of mortgage securities is attractive. However, to execute this strategy successfully, the prepayment risk of mortgage securities must be managed carefully. In this chapter, we will see how this is done. Specifically, we will see how to “hedge” the interest rate risk associated with a fixed rate mortgage security in order to capture the spread over Treasuries.¹ Note that we use the terms mortgage-backed securities and mortgage securities interchangeably in this chapter. As explained earlier, the most basic form of mortgage-backed security is the mortgage passthrough security. Securities that are created from mortgage passthrough securities include collateralized mortgage obligations and mortgage strips (interest-only and principal-only securities).

II. THE PROBLEM

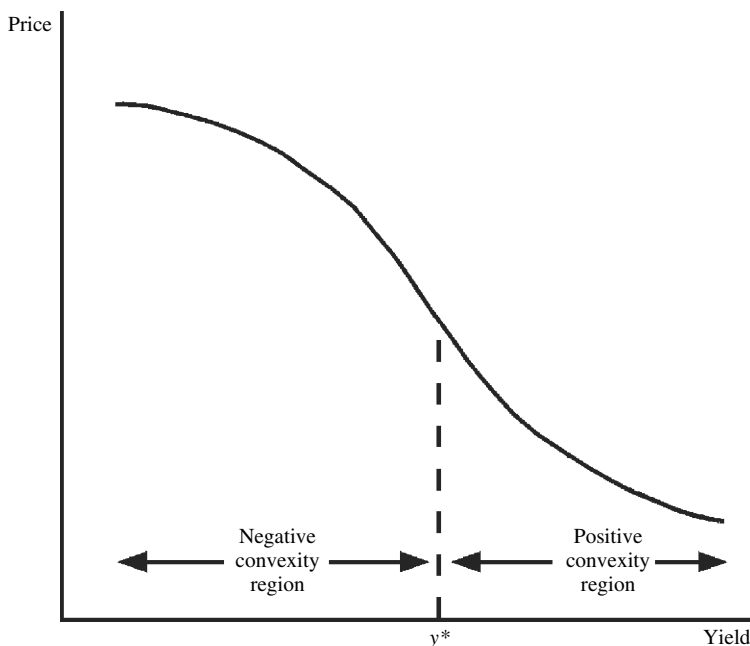
To illustrate the problem faced by a portfolio manager who believes that the spread offered on a mortgage security, is attractive and wants to hedge that spread, look at Exhibit 1. The exhibit shows the relationship between price and yield for a mortgage passthrough security. The yield for a mortgage security is the cash flow yield.² The price-yield relationship exhibits both positive and negative convexity. At yield levels above y^* , the mortgage security exhibits positive convexity; at yield levels below y^* , the mortgage security exhibits negative convexity.

*This chapter is authored by Kennath B. Dunn, PhD, Roberto M. Sella and Frank J. Fabozzi, PhD, CFA, CPA.

¹In European countries where mortgage-backed securities are issued, the coupon rate is typically a floating rate.

²The cash flow yield is the interest rate (properly annualized) that makes the present value of the projected cash flows from a mortgage-backed security equal to its price.

EXHIBIT 1 Price-Yield Relationship for a Mortgage Passthrough Security



It is important to understand this characteristic of a mortgage security. One way to understand it is to look at what happens to the change in price when interest rates move up and down by the same number of basis points. This can be seen in Exhibit 2 which shows the price-yield relationship for a security that exhibits positive convexity such as a Treasury security and a security that exhibits negative convexity. Let's look at what happens to the price change in absolute terms when interest rates change. From Exhibit 2 we observe the following property:

For a security that exhibits positive convexity, the price increase when interest rates decline is greater than the price decrease when interest rates rise.

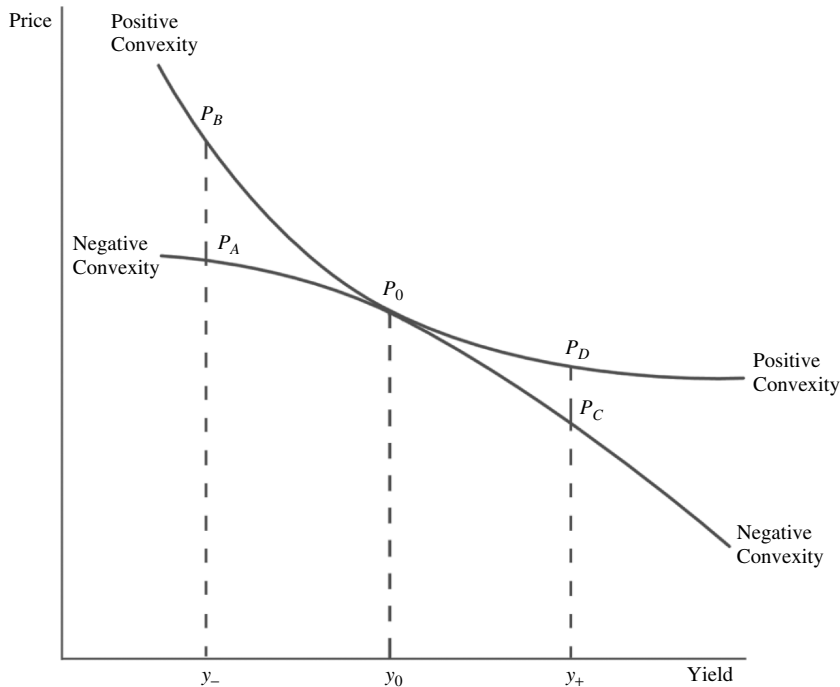
This is not the case for a security that exhibits negative convexity. Instead, we also observe from Exhibit 2 the following property:

For a security that exhibits negative convexity, the price increase when interest rates decline is less than the price decrease when interest rates rise.

Why will a mortgage security exhibit negative convexity at some yield level? The explanation is the homeowner's prepayment option. The value of a mortgage security declines as the value of the prepayment option increases. As mortgage rates in the market decline, the value of the prepayment option increases. As a result, the appreciation due to a decline in interest rates that would result if there had not been a prepayment option will be reduced by the increase in the value of the option.

We can see this by thinking about an agency mortgage security as equivalent to a position in a comparable-duration Treasury security and a call option. We can express the value of the mortgage security as follows:

EXHIBIT 2 Price Changes Resulting from Positive and Negative Convexity



Source:

P_0 = initial price

P_A = price if yield decreases from y_0 to y_- for the negative convexity bond

P_B = price if yield decreases from y_0 to y_- for the positive convexity bond

P_C = price if yield increases from y_0 to y_+ for the negative convexity bond

P_D = price if yield increases from y_0 to y_+ for the positive convexity bond

Implications:

For a given change in yield, for a bond with positive convexity:

$P_B - P_0 > |P_D - P_0|$ (i.e., gain is greater than the loss)

For a given change in yield, for a bond with negative convexity:

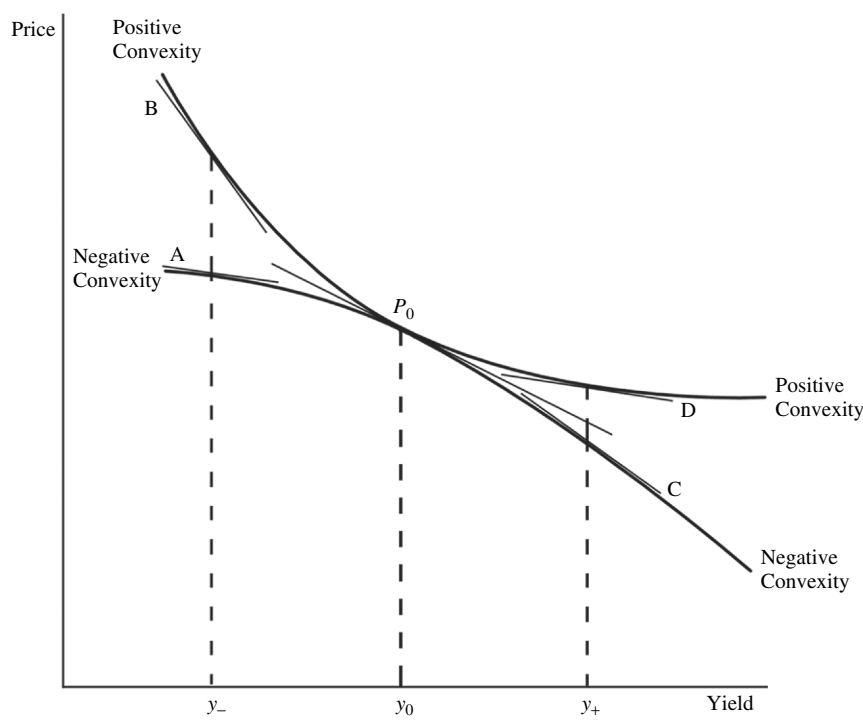
$|P_C - P_0| > P_A - P_0$ (i.e., loss is greater than the gain)

Value of mortgage security = Value of a Treasury security – Value of the prepayment option

The reason for subtracting the value of the prepayment option is that the investor in a mortgage security has sold a prepayment option. Consider what happens as interest rates change. When interest rates decline, the value of the Treasury security component of the mortgage security's value increases. However, the appreciation is reduced by the increase in the value of the prepayment option which becomes more valuable as interest rates decline. The net effect is that while the value of a mortgage security increases, it does not increase by as much as a same-duration Treasury security because the increase in the value of the prepayment option offsets part of the appreciation.

When interest rates rise, we see the opposite effect of the prepayment option. A rise in interest rates results in a decline in the value of the Treasury security component of the mortgage security's value. At the same time, the value of the prepayment option declines. The

EXHIBIT 3 Positive and Negative Convexity and Duration Changes



Duration at y_0 is the same for both the positive and negative convex securities. When interest rates decrease, duration for the positively convex security (tangent line B) increases (i.e., becomes steeper) while the duration for negatively convex security (tangent line A) decreases (i.e., becomes flatter). When interest rates increase, duration for the positively convex security (tangent line D) decreases (i.e., becomes flatter) while the duration for negatively convex security (tangent line C) increases (i.e., becomes steeper).

net effect is that while the value of a mortgage security decreases, it does not decline in value as much as a same-duration Treasury security because the decline in the value of the prepayment option offsets part of the depreciation.

Another way of viewing positive and negative convexity is how the duration changes when interest rates change. Exhibit 3 shows tangent lines to the price/yield relationship for two securities, one exhibiting positive convexity and the other negative convexity. The tangent line is related to the duration. The steeper the tangent line, the higher the duration. Notice in Exhibit 3 the following:

Convexity	Effect on duration when interest rates	
	fall	rise
Positive	increases	decreases
Negative	decreases	increases

That is, for a security that exhibits positive convexity, the duration changes in the desired direction; for a security that exhibits negative convexity, there is an adverse change in the duration.

While a mortgage security can exhibit both positive and negative convexity, a Treasury security exhibits only positive convexity. Look at the problem of hedging the interest rate risk associated with a mortgage security by either shorting Treasury securities or selling Treasury futures (i.e., hedging an instrument that has the potential for negative convexity with an instrument that only exhibits positive convexity). As explained in Chapter 22, the hedging principle is that the change in the value of the mortgage security position for a given basis point change in interest rates will be offset by the change in the value of the Treasury position for the same basis point change in interest rates. When interest rates decline, prepayments cause the value of a mortgage security to increase less in value than that of a Treasury position with the same initial duration. Thus, when interest rates decline, simply matching the dollar duration of the Treasury position with the dollar duration of the mortgage security will not provide an appropriate hedge when the mortgage security exhibits negative convexity.

For this reason, many investors consider mortgages to be market-directional investments that should be avoided when one expects interest rates to decline.³ Fortunately, when properly managed, mortgage securities are not market-directional investments. Proper management begins with separating mortgage valuation decisions from decisions concerning the appropriate interest-rate sensitivity of the portfolio. In turn, this separation of the value decision from the duration decision hinges critically on proper hedging. Without proper hedging to offset the changes in the duration of mortgage securities caused by interest rate movements, the portfolio's duration would drift adversely from its target duration. In other words, the portfolio would be shorter than desired when interest rates decline and longer than desired when interest rates rise.

III. MORTGAGE SECURITY RISKS

Proper hedging requires understanding the principal risks associated with investing in mortgage securities. There are five principal risks: spread, interest-rate, prepayment, volatility, and model risk.⁴ The yield of a mortgage security—the cumulative reward for bearing all five of these risks—has two components: the yield on equal interest-rate risk Treasury securities plus a spread. This spread is itself the sum of the option cost, which is the expected cost of bearing prepayment risk, and the option-adjusted spread (OAS), which is the risk premium for bearing the remaining risks, including model risk.

A. Spread Risk

A portfolio manager would want to invest in mortgage securities when their spreads versus Treasuries are large enough to compensate for the risk surrounding the homeowner's prepayment option. Because the OAS can be thought of as the risk premium for holding mortgage securities, *a portfolio manager does not seek to hedge spread risk*. If a portfolio manager hedges against spread widening, she also gives up the benefit from spread narrowing. Instead, a

³This perception is exacerbated by the common practice of comparing the returns of the mortgage index with the returns of the government and corporate indices without adjusting for differences in duration. Because the mortgage index typically has less duration than either the corporate or government index, it generally has better relative performance when interest rates rise than when interest rates fall.

⁴Model risk was discussed earlier.

portfolio manager seeks to capture the OAS over time by increasing the allocation to mortgage securities when yield spreads are wide and reducing exposure to mortgage securities when yield spreads are narrow.

To calculate the OAS for any mortgage security, a prepayment model is employed that assigns an expected prepayment rate every month—implying an expected cash flow—for a given interest rate path. These expected cash flows are discounted at U.S. Treasury spot rates to obtain their present value. This process is repeated for a large number of interest rate paths. Finally, the average present value of the cash flows across all paths is calculated. Typically, the average present value across all paths is not equal to the price of the security. However, we can search for a unique “spread” (in basis points) that, when added to the U.S. Treasury spot rates, equates the average present value to the price of the security. This spread is the OAS.

Historical comparisons are of only limited use for making judgments about current OAS levels relative to the past, because option-adjusted spreads depend on their underlying prepayment models. As a model changes, so does the OAS for a given mortgage security. There are periods where prepayment models change significantly, making comparisons to historical OASs tenuous. A portfolio manager should augment OAS analysis with other tools to help identify periods when spreads on mortgage securities are attractive, attempting to avoid periods when spread widening will erase the yield advantage over Treasuries with the same interest rate risk. The risk that the OAS may change, or **spread risk**, is managed by investing heavily in mortgage securities only when the initial OAS is large.

B. Interest Rate Risk

The interest rate risk of a mortgage security corresponds to the interest rate risk of comparable Treasury securities (i.e., a Treasury security with the same duration). This risk can be hedged directly by selling a package of Treasury notes or Treasury note futures. Once a portfolio manager has hedged the interest rate risk of a mortgage security, what can the manager earn? Recall that by hedging interest rate risk, a manager synthetically creates a Treasury bill and therefore earns the return on a Treasury bill. But what still remains after the interest rate risk is removed is the spread risk which, as just explained, is not hedged away. So, the portfolio manager after hedging interest rate risk can earn the Treasury bill return plus a spread over Treasuries. However, a portfolio manager cannot capture all of this spread because some of it is needed to cover the value of the homeowner’s prepayment option. After netting the value of the option, the portfolio manager earns the Treasury bill rate plus the potential to capture the OAS.

1. Yield Curve Risk Duration and convexity are measures of interest rate risk for “level” changes in interest rates. That is, if all Treasury rates shifted up or down by the same number of basis points, these measures do a good job of approximating the exposure of a security or a portfolio to a rate change. However, yield curves do not change in a parallel fashion. Consequently, portfolios with the same duration can perform quite differently when the yield curve shifts in a nonparallel fashion. **Yield curve risk** is the exposure of a portfolio or a security to a nonparallel change in the yield curve shape.

One approach to quantifying yield curve risk for a security or a portfolio is to compute how changes in a specific spot rate, holding all other spot rates constant, affect the value of a security or a portfolio. The sensitivity of the change in value of a security or a portfolio to a

particular spot rate change is called rate duration.⁵ In theory, there is a rate duration for every point on the yield curve. Consequently, there is not one rate duration, but a profile of rate durations representing each maturity on the yield curve. The total change in value if all rates change by the same number of basis points is simply the duration of a security or portfolio to a change in the level of rates. That is, it is the duration measure for level risk (i.e., a parallel shift in the yield curve).

Vendors of analytical system do not provide a rate duration for every point on the yield curve. Instead, they focus on key maturities of the spot rate curve. These rate durations are called key rate durations.

The impact of any type of yield curve shift can be quantified using key rate durations. A level shift can be quantified by changing all key rates by the same number of basis points and computing, based on the corresponding key rate durations, the affect on the value of a portfolio. The impact of a steepening of the yield curve can be found by (1) decreasing the key rates at the short end of the yield curve and determining the change in the portfolio value using the corresponding key rate durations, and (2) increasing the key rates at the long end of the yield curve and determining the change in the portfolio value using the corresponding key rate durations.

The value of an option-free bond with a bullet maturity payment (i.e., entire principal due at the maturity date) is sensitive to changes in the level of interest rates but not as sensitive to changes in the shape of the yield curve. This is because for an option-free bond whose cash flow consists of periodic coupon payments but only one principal payment (at maturity), the change of rates along the spot rate curve will not have a significant impact on its value. In contrast, while the value of a portfolio of option-free bonds is, of course, sensitive to changes in the level of interest rates, it is much more sensitive to changes in the shape of the yield curve than individual option-free bonds.

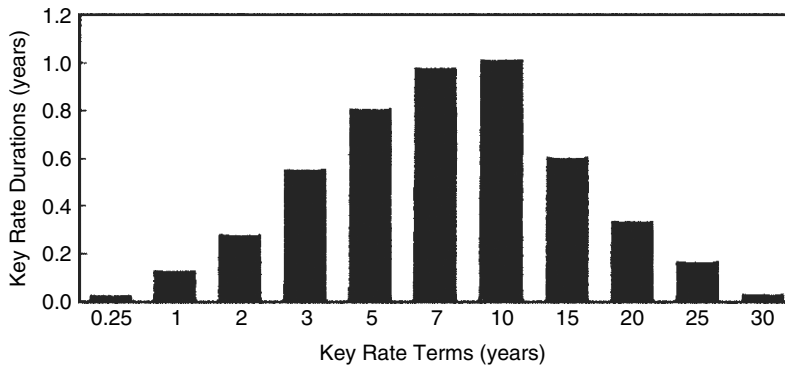
In the case of mortgage securities, the value of both an individual mortgage security and a portfolio of mortgage securities will be sensitive to changes in the shape of the yield curve, as well as changes in the level of interest rates. This is because a mortgage security is an amortizing security with a prepayment option. Consequently, the pattern of the expected cash flows for an individual mortgage security can be materially affected by the shape of the yield curve. To see this, look at Exhibit 4 which shows the key rate durations for a Ginnie Mae 30-year 10% passthrough, the current coupon passthrough at the time the graph was prepared. Exhibit 5 shows the key rate duration profile for a principal-only (PO) and an interest-only (IO) mortgage strip created from the Ginnie Mae passthrough whose key rate durations are shown in Exhibit 4.

From Exhibit 4 it can be seen that the passthrough exhibits a bell-shaped curve with the peak of the curve between 5 and 15 years. Adding up the key rate durations from 5 to 15 years (i.e., the 5-year, 7-year, 10-year, and 15-year key rate durations) indicates that of the total interest rate exposure, about 70% is within this maturity range. That is, the effective duration alone masks the fact that the interest rate exposure for this passthrough is concentrated in the 5-year to 15-year maturity range.

A PO will have a high positive duration. From the key rate duration profile for the PO shown in Exhibit 5 it can be seen that the key rate durations are negative up to year 7. Thereafter, the key rate durations are positive and have a high value. While the total risk exposure (i.e., effective duration) may be positive, there is exposure to yield curve risk. For example, the key rate durations suggest that if the long end of the yield curve is unchanged,

⁵Rate duration was discussed previously.

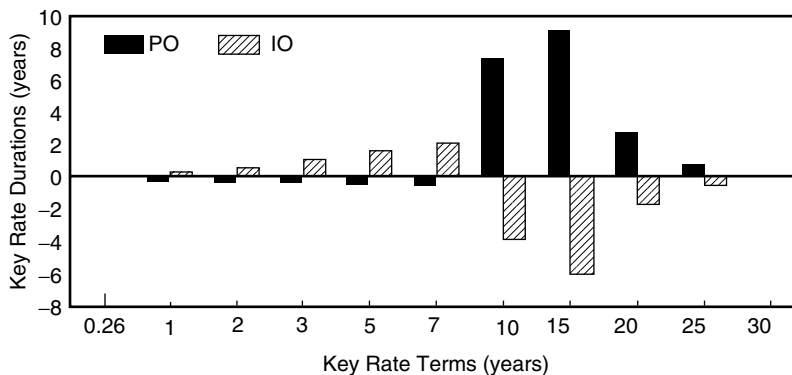
EXHIBIT 4 Key Rate Duration Profile for a 30-Year Ginnie Mae Passthrough



Source: Exhibit 12 in Thomas S.Y. Ho, "Key Rate Durations: Measures of Interest Rate Risks," *Journal of Fixed Income* (September 1992), p. 38.

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EXHIBIT 5 Key Rate Duration Profiles for a PO and an IO for Current Coupon Ginnie Mae



Source: Exhibit 13 in Thomas S.Y. Ho, "Key Rate Durations: Measures of Interest Rate Risks," *Journal of Fixed Income* (September 1992), p. 38.

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but the short end of the yield curve (up to year 7) decreases, the PO's value will decline despite an effective duration that is positive.

IOs have a high negative duration. However, from the key rate duration profile in Exhibit 5 it can be seen that the key rate durations are positive up to year 10 and then take on high negative values. As with the PO, this security is highly susceptible to how the yield curve changes.

While the key rate durations are helpful in understanding the exposure of a mortgage security or a portfolio to yield curve risk, we will present an alternative methodology for

assessing the yield curve risk of a mortgage security when we discuss the hedging methodology in Section V.

C. Prepayment Risk

When interest rates decline, homeowners have an economic incentive to prepay their existing mortgages by refinancing at a lower rate. As demonstrated earlier, because of the prepayment option the duration of mortgage securities varies in an undesirable way as interest rates change: extending as rates rise and shortening as rates fall. Therefore, the percentage increase in price of a mortgage security for successive 25 basis point declines in yield, for example, becomes smaller and smaller. Conversely, the percentage decline in price becomes greater as interest rates rise. Termed negative convexity, this effect can be significant—particularly for mortgage securities that concentrate prepayment risk such as interest-only strips.

When interest rates change we must offset the resultant change in mortgage durations in order to keep the overall interest rate risk of the portfolio at its desired target. Neglecting to do so would leave the portfolio with less interest rate risk than desired after interest rates decline and more risk than desired after rates increase. A portfolio manager should adjust for changes in durations of mortgage securities—or equivalently, manage negative convexity—either by buying options or by hedging dynamically.

Hedging dynamically requires lengthening duration—buying futures—after rates have declined, and shortening duration—selling futures—after rates have risen.⁶ Whether a portfolio manager employs this “buy high/sell low” dynamic strategy, or buys options, the portfolio’s performance is bearing the cost associated with managing negative convexity by foregoing part of the spread over Treasuries.

D. Volatility Risk

An investment characteristic of an option is that its value increases with expected volatility.⁷ In the case of an interest rate option, the pertinent volatility is interest rate volatility. The prepayment option granted to a homeowner is an interest rate option and therefore the homeowner’s prepayment option becomes more valuable when future interest rate volatility is expected to be high than when it is expected to be low. Because the OAS adjusts to compensate the investor for selling the prepayment option to the homeowner, OAS tends to widen when expected volatility increases and narrow when expected volatility declines.

A portfolio manager can manage volatility risk by buying options or by hedging dynamically. The selection depends on the following:

- When the volatility implied in option prices is high and the portfolio manager believes that future realized volatility will be lower than implied volatility, he should hedge dynamically.
- When implied volatility is low and the portfolio manager believes that actual future volatility will be higher than implied volatility, he should hedge by purchasing options.

Because it has been our experience that implied volatilities have tended to exceed subsequent realized volatility, we have generally hedged dynamically to a greater extent than we have hedged through the use of options.

⁶How the buying and selling of futures affects duration is explained in Chapter 22.

⁷This was explained earlier.

E. Model Risk

Mortgage prepayment models generate cash flows for a given set of interest rate paths. But what happens when the models are wrong? In the rally of 1993, premium mortgages prepaid at much faster rates than predicted by most prepayment models in use at that time. Investors who had purchased interest-only strips (IOs) backed by premium mortgages, and had relied on the prepayment predictions of those models, sustained losses. It is important to note that prior to the rally, the OAS on IOs seemed attractive on a historical basis. However, the model OAS assumed a conditional prepayment rate (CPR) of 40% for premium mortgages; the actual CPR for premium mortgages was as high as 60%, causing the realized OAS to be negative.

Current models calibrate to historical experience. Although a portfolio manager does not know the magnitude of model error going forward, he can measure sensitivity to model error by increasing the prepayment rate assumed by the model for mortgage securities that are hurt by faster-than-expected prepayments and decreasing the prepayment rate assumed by the model for mortgage securities that are hurt by slower-than-expected prepayments.

Over time it has become cheaper to refinance mortgages as technological improvements have reduced the costs associated with refinancing. We expect this type of prepayment innovation to continue in the years ahead. Models calibrated to past behavior will understate the impact of innovation. Therefore, when evaluating mortgage securities that are vulnerable to this type of risk, a portfolio manager should carefully consider the likelihood and the effect of prepayment innovation in determining the size of a portfolio's mortgage securities. Although a portfolio manager cannot hedge model risk explicitly, he can measure it and manage it by keeping a portfolio's exposure to this risk in line with that of the broad-based bond market indices.

IV. HOW INTEREST RATES CHANGE OVER TIME

While key rate duration is a useful measure for identifying the exposure of a portfolio to different potential shifts in the yield curve, it is difficult to employ this approach to yield curve risk in hedging a portfolio. An alternative approach is to investigate how yield curves have changed historically and incorporate typical yield curve change scenarios into the hedging process. This approach has been used by several firms that specialize in the management of mortgage-backed securities.⁸

Empirically, studies have found that yield curve changes are not parallel. Rather, when the level of interest rates changes, studies have found that short-term rates move more than longer-term rates. Some firms develop their own proprietary models that decompose historical movements in the rate changes of Treasury strips with different maturities in order to analyze typical or likely rate movements. The statistical technique used to decompose rate movements is principal components analysis.

⁸For example, the approach is used by Morgan Stanley as discussed in this chapter and by Smith Breeden Associates (see Michael P. Schumacher, Daniel C. Dektar, and Frank J. Fabozzi, "Yield Curve Risk of CMO Bonds," Chapter 15 in Frank J. Fabozzi (ed.), *CMO Portfolio Management* (New Hope, PA: Frank J. Fabozzi Associates, 1994).

Most empirical studies, published and proprietary, find that more than 95% of historical movements in rate changes can be explained by changes in (1) the overall level of interest rates and (2) twists in the yield curve (i.e., steepening and flattening). For example, Morgan Stanley's proprietary model of the movement of monthly Treasury strip rates finds the following "typical" monthly rate change in basis points for three maturities:

Years	Level		Change from	
	Up	Down	Flattening	Steepening
2	23.0	-23.0	17.2	-17.2
5	25.8	-25.8	11.2	-11.2
10	24.3	-24.3	3.4	-3.4

"Typical" means one standard deviation in the change in the monthly rate. The last two columns in the above table indicate the change in the monthly rate found by principal components analysis that is due to a flattening or steepening of the yield curve. From the above table, the impact on the yield curve for a typical rise in the overall level of interest rates and a flattening of the yield curve is found as follows. To find the typical change in the slope of the 10-year–2-year yield-curve segment, the difference between the 17.2 basis points at 2-year, and 3.4 basis points at 10 years is computed. The difference of 13.8 basis points means that the typical monthly flattening is 13.8 basis points. The typical monthly steepening is 13.8 basis points.

Because of the importance of yield curve risk for mortgage securities, a hedging methodology should incorporate this information about historical yield curve shifts. We will see how this is done in the next section.

V. HEDGING METHODOLOGY

As explained in Chapter 22, hedging is a special case of interest rate risk control where the portfolio manager seeks to completely offset the dollar price change in the instrument to be hedged by taking an opposite position in an appropriate hedging instrument that will produce the same dollar price change for the same change in interest rates. In the discussion in Chapter 22, the methodology presented for hedging a non-mortgage security addressed only a parallel shift in the yield curve.

To properly hedge the interest rate risk associated with a mortgage security, the portfolio manager needs to incorporate his knowledge of the following:

- how the yield curve changes over time
- the effect of changes in the yield curve on the homeowner's prepayment option

Using this information, a portfolio manager can estimate how mortgage security prices will change as interest rates change.

A. Interest Rate Sensitivity Measure

Scott Richard and Benjamin Gord introduced the concept of **interest-rate sensitivity** (IRS) and discussed why it is a better measure of interest rate risk than modified or effective

duration.⁹ IRS measures a security's or a portfolio's percentage price change in response to a shift in the yield curve.

Since two factors (the “level” and “twist” factors discussed in the previous section) have accounted for most of the changes in the yield curve, two Treasury notes (typically the 2-year and 10-year) can hedge virtually all of the interest rate risk in mortgage securities. Since two hedging instruments are used, the hedge is referred to as a **two-bond hedge**.

To create the two-bond hedge, we begin by expressing a particular mortgage security in terms of an “equivalent position” in U.S. Treasuries or “equivalent position” in Treasury futures contracts.¹⁰ We identify this equivalent position by picking a package of 2-year and 10-year Treasuries that—on average—has the same price performance as the mortgage security to be hedged under the assumed “level” and “twist” yield curve scenarios. For hedging purposes, the direction of the change—up or down in the case of the “level” factor, flattening or steepening in the case of the “twist” factor—is not known. (In calculating how the price will change in response to changes in the two factors, it is assumed that the OAS is constant.)

In this way, the portfolio manager can calculate the unique quantities of 2-year and 10-year Treasury notes or futures that will simultaneously hedge the mortgage security's price response to both “level” and “twist” scenarios. This combination is the appropriate two-bond hedge for typical yield curve shifts and therefore defines the interest rate sensitivity of the mortgage security in terms of 2-year and 10-year Treasury notes or futures.

B. Computing the Two-Bond Hedge

The steps to compute the two-bond hedge are as follows:

Step 1: For an assumed shift in the level of the yield curve, compute the following:

- price of the mortgage security for an assumed increase in the level of interest rates
- price for the mortgage security for an assumed decrease in the level of interest rates
- price of the 2-year Treasury note (or futures) for an assumed increase in the level of interest rates
- price of the 2-year Treasury note (or futures) for an assumed decrease in the level of interest rates
- price of the 10-year Treasury note (or futures) for an assumed increase in the level of interest rates
- price of the 10-year Treasury note (or futures) for an assumed decrease in the level of interest rates

Step 2: From the prices found in Step 1, calculate the price change for the mortgage security, 2-year Treasury note (or futures), and 10-year Treasury note (or futures) for the assumed shift in the level of interest rates. There will be two price changes for each of the mortgage security, 2-year hedging instrument, and 10-year hedging instrument.

⁹Scott F. Richard and Benjamin J. Gord, “Measuring and Managing Interest-Rate Risk,” Chapter 7 in Frank J. Fabozzi (ed.), *Perspectives on Interest Rate Risk Management for Money Managers and Traders* (New Hope, PA: Frank J. Fabozzi Associates, 1998).

¹⁰Treasury futures contracts were discussed at earlier.

Step 3: Calculate the average price change for the mortgage security and the two hedging instruments for the assumed shift in the level of interest rates assuming that the two scenarios (i.e., increase and decrease) are equally likely to occur. The average price change will be denoted as follows:

MBS price_L = average price change for the mortgage security for a level shift

2-H price_L = average price change for the 2-year Treasury hedging instrument
for a level shift

10-H price_L = average price change for the 10-year Treasury hedging instrument
for a level shift

Step 4: For an assumed twist (flattening and steepening) of the yield curve, compute the following:

- price of the mortgage security for an assumed flattening of the yield curve
- price of the mortgage security for an assumed steepening of the yield curve
- price of 2-year Treasury note (or futures) for an assumed flattening of the yield curve
- price of 2-year Treasury note (or futures) for an assumed steepening of the yield curve
- price of 10-year Treasury note (or futures) for an assumed flattening of the yield curve
- price of 10-year Treasury note (or futures) for an assumed steepening of the yield curve

Step 5: From the prices found in Step 4, calculate the price change for the mortgage security, 2-year Treasury note (or futures), and 10-year Treasury note (or futures) for the assumed twist in the yield curve. There will be two price changes for each of the mortgage security, 2-year hedging instrument, and 10-year hedging instrument.

Step 6: Calculate the average price change for the mortgage security and the two hedging instruments for the assumed twist in the yield curve assuming that the two scenarios (i.e., flattening and steepening) are equally likely to occur. The average price change will be denoted as follows:

MBS price_T = average price change for the mortgage security for a twist in the
yield curve

2-H price_T = average price change for the 2-year Treasury hedging instrument
for a twist in the yield curve

10-H price_T = average price change for the 10-year Treasury hedging instrument
for a twist in the yield curve

Step 7: Compute the change in value of the two-bond hedge portfolio for a change in the level of the yield curve. This is done as follows. Let

H_2 = amount of the 2-year hedging instrument per \$1 of market value of the mortgage security

H_{10} = amount of the 10-year hedging instrument per \$1 of market value of the mortgage security

Our objective is to find the appropriate values for H_2 and H_{10} that will produce the same change in value for the two-bond hedge as the change in the price of the mortgage security that the portfolio manager seeks to hedge.

The change in value of the two-bond hedge for a change in the level of the yield curve is:

$$H_2 \times (2-H\text{price}_L) + H_{10} \times (10-H\text{price}_L)$$

Step 8: Determine the change in value of the two-bond hedge portfolio for a twist in the yield curve. This value is

$$H_2 \times (2-H\text{price}_T) + H_{10} \times (10-H\text{price}_T)$$

Step 9: Determine the set of equations that equates the change in the value of the two-bond hedge to the change in the price of the mortgage security. To be more precise, we want the change in the value produced by the two-bond hedge to be in the opposite direction to the change in the price of the mortgage security. Using our notation, the two equations are:

$$\text{Level: } H_2 \times (2-H\text{price}_L) + H_{10} \times (10-H\text{price}_L) = -\text{MBS price}_L$$

$$\text{Twist: } H_2 \times (2-H\text{price}_T) + H_{10} \times (10-H\text{price}_T) = -\text{MBS price}_T$$

Step 10: Solve the simultaneous equations in Step 9 for the values of H_2 and H_{10} . Notice that for the two equations, all of the values are known except for H_2 and H_{10} . Thus, there are two equations and two unknowns.

In Step 9, a negative value for H_2 or H_{10} represents a short position and a positive value for H_2 or H_{10} represents a long position.

C. Illustrations of the Two-Bond Hedge

To illustrate the steps to compute the two-bond hedge, we will examine at two different dates: February 12, 2003 and March 4, 1997. Illustration 1 shows a combination of a long position and a short position in two hedging instruments to hedge the long position in a passthrough security. Illustration 2 involves a short position in two hedging instruments in order to hedge a long position in a passthrough security.

1. Illustration 1: Two-Bond Hedge with a Long and Short Position In this illustration we will see how to hedge a position in the Fannie Mae 5% coupon passthrough on February 12, 2003. The price of this mortgage security was 99.126. In our illustrations we will use the 2-year and 10-year Treasury note futures as the hedging instruments for the two-bond hedge. The prevailing price for the 2-year Treasury note futures was 107.75. The prevailing price for the 10-year Treasury note futures was 114.813.

a. Finding the Two-Bond Hedge *Step 1:* In computing the price resulting from a change in the level of interest rates, an increase and decrease of 24.3 basis points is used. (This is the typical monthly overall change in the level of rates.) The dollar price change per \$100 of par value is shown below:

Instrument	Price for	
	increase in yield	decrease in yield
Fannie Mae 5%	97.787	100.334
2-year Treasury note futures	107.333	108.168
10-year Treasury note futures	113.137	116.510

For the Fannie Mae 5%, the Monte Carlo simulation model is used to calculate the price after the change in yield. The OAS is held constant at its initial value in the valuation model. (The prices of the two futures instruments are based on the valuation methodology described previously.)

Step 2: From the prices found in Step 1, calculate the price changes:

Instrument	Price for	
	increase in yield	decrease in yield
Fannie Mae 5%	−1.339	1.208
2-year Treasury note futures	−0.417	0.418
10-year Treasury note futures	−1.676	1.697

Step 3: Calculate the average price change (using absolute values) for each instrument resulting from a level change:

$$\text{MBS price}_L = 1.274$$

$$\text{2-H price}_L = 0.418$$

$$\text{10-H price}_L = 1.687$$

Step 4: In computing the price resulting from a twist in the shape of the yield curve, the 2–10 slope is assumed to change by 13.8 bps. (Recall from Section IV that this is the typical monthly twist in the shape of the yield curve.) The dollar price per \$100 of par value is shown below:

Instrument	Price for	
	flattening	steepening
Fannie Mae 5%	98.89	99.363
2-year Treasury note futures	107.441	108.064
10-year Treasury note futures	114.342	115.285

Step 5: From the prices found in Step 4, calculate the price changes:

Instrument	Price for	
	flattening	steepening
Fannie Mae 5%	−0.236	0.237
2-year Treasury note futures	−0.309	0.314
10-year Treasury note futures	−0.471	0.472

Step 6: Calculate the average price change for each instrument resulting from a twist in the yield curve:

$$\text{MBS price}_L = 0.237$$

$$\text{2-H price}_L = 0.312$$

$$\text{10-H price}_L = 0.472$$

Step 7: The change in value of the two-bond hedge portfolio for a change in the level of the yield curve is:

$$H_2 \times (0.418) + H_{10} \times (1.687)$$

Step 8: The change in value of the two-bond hedge portfolio for a twist of the yield curve is:

$$H_2 \times (0.312) + H_{10} \times (0.472)$$

Step 9: The two equations that equate the change in the value of the two-bond hedge to the change in the price of the mortgage security are:

$$\text{Level: } H_2 \times (0.418) + H_{10} \times (1.687) = -1.274$$

$$\text{Twist: } H_2 \times (0.312) + H_{10} \times (0.472) = -0.237$$

Step 10: Solve the simultaneous equations in Step 9 for the values of H_2 and H_{10} . This is done as follows:

Solve for H_2 in the “Level” equation:

$$H_2 = (-1.274 - 1.687H_{10})/0.418 = -3.048 - 4.036H_{10}$$

Substitute the above for H_2 in the “Twist” equation:

$$\begin{aligned}
[-3.048 - 4.036H_{10}](0.312) + H_{10}(0.472) &= -0.237 \\
-0.950976 - 1.259232H_{10} + 0.472H_{10} &= -0.237 \\
-0.950976 - 0.787232H_{10} &= -0.237
\end{aligned}$$

Solve for H_{10} :

$$\begin{aligned}
0.787232H_{10} &= -0.713976 \\
H_{10} &= -0.906945
\end{aligned}$$

To obtain H_2 , we can substitute $H_{10} = -0.906945$ into the “Level” or the “Twist” equation and solve for H_2 . Substituting into the “Level” equation we get:

$$\begin{aligned}
H_2 \times (0.418) + (-0.906945) \times (1.687) &= -1.274 \\
H_2 \times (0.418) - 1.530016 &= -1.274 \\
H_2 &= 0.612478
\end{aligned}$$

Thus, $H_2 = 0.612478$ and $H_{10} = -0.906945$.¹¹

The value of 0.612478 for H_2 means that the par amount in the 2-year Treasury note futures will be 0.612478 per \$1 of par amount of the mortgage security to be hedged. So, if the par amount of the Fannie Mae 5% to be hedged against interest rate risk is \$1 million, then 2-year Treasury note futures with a notional value of \$612,478 ($= 0.612478 \times \$1$ million) should be long (i.e., buying 2-year Treasury note futures). Notice that we have an example here of going long in a futures contract despite that we are seeking to hedge a long position!

The value of -0.906945 for H_{10} means that the par amount in the 10-year Treasury note futures will be 0.906945 per \$1 of par amount of the mortgage security to be hedged. Assuming again that the par amount of the Fannie Mae 5% to be hedged is \$1 million, then 10-year Treasury note futures with a notional value of \$906,945 ($= 0.906945 \times \$1$ million) should be shorted.

b. Duration Hedge versus Two-Bond Hedge It is interesting to note the difference between the potential performance difference between the bond hedge using duration only and the two-bond hedge that takes into consideration changes in both level and twist changes.

At the time of the hedge, the Fannie Mae 5% passthrough had an effective duration of 5.5. Using only duration to obtain the hedge position, it can be demonstrated that if the yield curve shift is a level one, the following price changes would result:

	Fannie Mae 5%	Duration hedge	Error
Increase in rates/Level up	-1.339	1.360	0.021
Decrease in rates/Level down	1.208	-1.379	-0.171

¹¹The results can be verified by substituting these values into the “Level” or “Twist” equation.

The above results indicate that if the duration hedge is used, when interest rates increase, the Fannie Mae 5% will decline by 1.339 points but the gain from the duration hedge will be 1.36 points. Hence, using a duration hedge the gain will be greater than the loss, resulting in a profit on the hedge. As can be seen from the results above, the opposite occurs if interest rates decline. This is the reason there is a market belief that mortgage securities are “market-directional” investments.

However, evidence using the two-bond hedge suggests otherwise: because the yield curve seldom moves in a parallel fashion, properly hedged mortgage securities are not market-directional. The two-bond hedge would produce the following results:

	Fannie Mae 5%	Two-bond hedge	Error
Increase in rates/Level up	−1.339	1.263	−0.076
Decrease in rates/Level down	1.208	−1.284	−0.076

As can be seen from the above table, when “likely level and twist” changes in the yield curve are accounted for, virtually all of the market-directionality is removed. The “error” in the two-bond hedge is a measure of the negative convexity of the Fannie Mae 5% passthrough. For a 24 basis point move (one stand deviation of monthly level shift) in the 10-year—assuming no rebalancing of the hedge—the Fannie Mae 5% passthrough would underperform its two-bond hedge using Treasury note futures by 8 basis points (or eight cents per \$100 of par amount). This loss is more than offset by the carry advantage of a mortgage security over Treasuries (i.e., the higher interest earned on the mortgage security versus the cost of financing the Treasury positions). For example, the Fannie Mae 5% has a yield of 5.17% and a 200 basis point spread to Treasuries. This implies that every month this security has a carry advantage of 17 basis points (200 basis points/12), more than enough to offset the 8 basis point hedging loss.

Let’s take a look at the Fannie Mae 5% duration and the duration implied for the two-bond hedging package. The duration for the Fannie Mae 5% is 5.5. We can obtain the implied duration of the two-bond hedge by computing the dollar value of a basis point (DV01) for the 2-year and 10-year Treasury note futures contracts.¹² The DV01 for the 2-year Treasury note futures is 0.0186. For the 10-year Treasury note futures the DV01 is 0.067. So the hedging package (i.e., the two-bond hedge) has a DV01 that is the weighted average of the two dollar durations as shown below:

$$0.612478 \times 0.0186 + (-0.906945) \times 0.067 = -0.049373$$

The sign is negative because of the inverse relationship between price change and interest rates. This implies that the two-bond hedge for the Fannie Mae 5% has a duration of $(0.049373/99.126) \times 10,000 = 4.98$. This is about 9% less than the duration of 5.5 for the Fannie Mae 5%.

2. Illustration 2: Two-Bond Hedge with Two Short Positions The position to be hedged in Illustration 2 is the Freddie Mac 7.5% coupon passthrough on March 4, 1997. The price of this mortgage security was 99 25/32. As with Illustration 1, we will use the 2-year and 10-year Treasury note futures as the hedging instruments for the two-bond hedge. While

¹²The dollar value of a basis point, also called the price value of a basis point, is the change in the value of a position for a 1 basis point change in interest rates.

in Illustration 1 a long position was required in the 2-year Treasury note futures and a short position in the 10-year Treasury note futures, we will see that a short position is required in both hedging instruments in Illustration 2.

We will not go through all the steps but just provide the following basic information so that the positions in the hedging instruments can be computed:

$$\Delta \text{MBS price}_L = 1.22 \quad \Delta 2\text{-H price}_L = 0.62 \quad \Delta 10\text{-H price}_L = 1.69$$

$$\Delta \text{MBS price}_T = 0.25 \quad \Delta 2\text{-H price}_T = 0.01 \quad \Delta 10\text{-H price}_T = 0.55$$

Based on the above information, we can complete Steps 7 through 10 as shown below:

Step 7: The change in value of the two-bond hedge portfolio for a change in the level of the yield curve is found as follows:

$$H_2 \times (0.62) + H_{10} \times (1.69)$$

Step 8: The change in value of the two-bond hedge portfolio for a twist of the yield curve is found as follows:

$$H_2 \times (0.01) + H_{10} \times (0.55)$$

Step 9: The two equations that equate the change in the value of the two-bond hedge to the change in the price of the mortgage security are:

$$\text{Level: } H_2 \times (0.62) + H_{10} \times (1.69) = -1.22$$

$$\text{Twist: } H_2 \times (0.01) + H_{10} \times (0.55) = -0.25$$

Step 10: Solve the simultaneous equations in Step 9 for the values of H_2 and H_{10} . This is done as follows:

Solve for H_2 in the “Level” equation:

$$H_2 = \frac{-1.22 - 1.69 H_{10}}{0.62} = -1.967742 - 2.725806 H_{10}$$

Substitute the above for H_2 in the “Twist” equation:

$$[-1.967742 - 2.725806 H_{10}](0.01) + H_{10}(0.55) = -0.25$$

$$-0.019677 - 0.027258 H_{10} + 0.55 H_{10} = -0.25$$

$$-0.019677 + 0.522742 H_{10} = -0.25$$

Solve for H_{10} :

$$0.522742 H_{10} = -0.230323$$

$$H_{10} = -0.440605$$

To obtain H_2 , we can substitute $H_{10} = -0.440605$ into the “Level” or the “Twist” equation and solve for H_2 . Substituting into the “Level” equation we get:

$$H_2 \times (0.62) + (-0.440605) \times (1.69) = -1.22$$

$$H_2 \times (0.62) - 0.744622 = -1.22$$

$$H_2 = -0.766739$$

Thus, $H_2 = -0.766739$ and $H_{10} = -0.440605$.

These values indicate that a short position will be taken in both the 2-year and 10-year Treasury note futures. The value of 0.766739 for H_2 means that the par amount in the 2-year Treasury note futures will be 0.766739 per \$1 of par amount of the mortgage security to be hedged. So, if the par amount of the Freddie Mac 7.5% to be hedged against interest rate risk is \$1 million, then 2-year Treasury note futures with a par amount of \$766,739 ($= 0.766739 \times \$1 \text{ million}$) should be shorted. Similarly, the value of 0.440605 for H_{10} means that the par amount in the 10-year Treasury note futures will be 0.440605 per \$1 of market value of the mortgage security to be hedged.

Once again we see the value of using the two-bond hedge rather than using a duration hedge. At the time of the hedge, the Freddie Mac 7.5% passthrough had an effective duration of 4.4. Using only duration to obtain the hedge position, it can be demonstrated that if the yield curve shift is a parallel one, the following price changes would result:

	Freddie Mac 75%	Duration hedge	Error
Increase in rates/Level up	-1.27	1.36	+0.09
Decrease in rates/Level down	1.18	-1.34	-0.16

As in Illustration 1, using a duration hedge the gain will be greater than the loss, resulting in a profit on the hedge and suggesting that mortgage securities are “market-directional” investments. However, for the two-bond hedge we would find the following:

	Freddie Mac 75%	Two-bond hedge	Error
Increase in rates/Level up	-1.27	1.25	-0.02
Decrease in rates/Level down	1.18	-1.20	-0.02

As can be seen from the above table, when typical changes in the yield curve are accounted for, virtually all of the market-directionality is removed.

D. Underlying Assumptions

Now that the underlying principles and mechanics for hedging the interest rate risk of a mortgage security have been covered, let’s look at the underlying assumptions for the two-bond hedge. They are:

- the yield curve shifts used in constructing the two-bond hedge are reasonable
- the prepayment model used does a good job of estimating how the cash flows will change when the yield curve changes
- assumptions underlying the Monte Carlo simulation model are realized (e.g., the interest rate volatility assumption)

- the average price change is a good approximation of how the mortgage security's price will change for a small movement in interest rates

This last assumption may be unacceptable for certain types of mortgage securities as explained in the next section.

VI. HEDGING CUSPY-COUPON MORTGAGE SECURITIES

In many cases, the “average” price change is a good approximation of how a mortgage security's price will change for a small movement in interest rates. This can be seen by the tangent line labeled “Current Coupon” in Exhibit 6. However, some mortgage securities are very sensitive to small movements in interest rates. For example, a mortgage security whose coupon is 100 basis points higher than the current coupon could be prepaid slowly if rates rise by 25 basis points but prepaid very quickly if rates fall by 25 basis points. Small changes in interest rates have large effects on prepayments for such securities and hence on their prices.

A mortgage security with this characteristic is referred to as a “cuspy-coupon” mortgage security. For such mortgage securities, averaging the price changes is not a good measure of how prices will change. In other words, the tangent line (labeled “Cuspy Coupon” in Exhibit 6) is not a good proxy for the price/yield curve. At times, cuspy-coupon mortgage securities offer attractive risk-adjusted expected returns; however, they have more negative convexity than current-coupon mortgages.

Hedging cuspy-coupon mortgage securities only with Treasury notes or futures contracts may leave the investor exposed to more negative convexity than is desired. The negative convexity feature, as has been mentioned, is the result of the option that the mortgage security investor has granted to homeowners to prepay. That is, the investor has effectively sold an option to homeowners. To hedge the sale of an option (i.e., a short position in an option), an investor can buy an option. Thus, a portfolio manager can extend the two-bond hedge by buying interest rate options to offset some or all of a cuspy-coupon mortgage security's negative convexity.

We will use a Freddie Mac 8.5% passthrough, a cuspy-coupon mortgage security in February 1997, to illustrate how the two-bond hedging methodology is extended to include options. The price of the Freddie Mac 8.5% passthrough was 103.50.

Without going through the mechanics of how to construct the two-bond hedge, panel a in Exhibit 7 shows (1) the change in the price for the mortgage security and (2) the change in the value of the two-bond hedge for changes in the level and twist in the yield curve. The last column in panel a of the exhibit shows the error for the two-bond hedge. The error is due to the negative convexity characteristic of the Freddie Mac 8.5% passthrough (cuspy-coupon mortgage security).

Because the prepayment option of the Freddie Mac 8.5s is closer to the refinancing threshold than that of the Freddie Mac 7.5s at the time of the analysis, the two-bond hedge error is greater: -0.05 for the 8.5s versus -0.02 for the 7.5s. Buying calls and puts eliminates this drift. Specifically, we added the purchase of the following two option positions per \$100 of the Freddie Mac 8.5s to be hedged:

- \$18 6-month call option on a 10-year Treasury note with a strike price of 99.5
- \$17 6-month put option on a 10-year Treasury note with a strike price of 95

EXHIBIT 6 Mortgage Price/Yield Curve

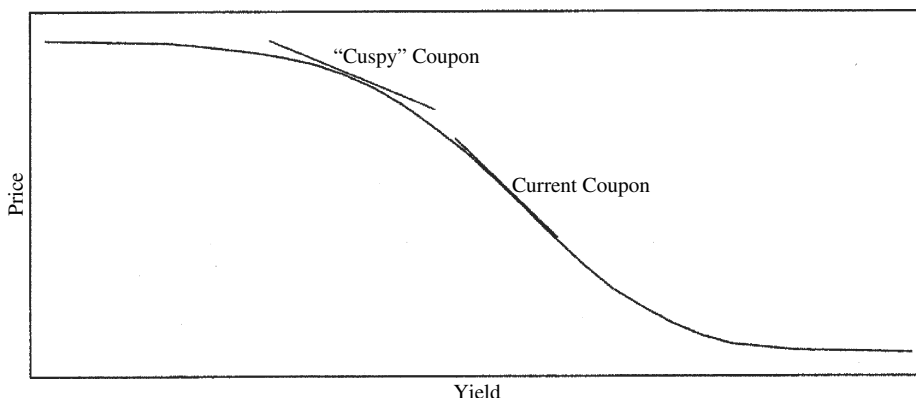


EXHIBIT 7 Alternatives for Hedging a Cuspy-Coupon Mortgage Security

Panel a: Two-Bond Hedge

Yield curve change	Price change for Freddie Mac 8.5%	Price change for two-bond hedge	Error for two-bond hedge
“Level” Up	−0.98	0.93	−0.05
“Level” Down	0.85	−0.90	−0.05
“Twist” Flattening	0.11	−0.12	−0.01
“Twist” Steepening	−0.12	0.11	−0.01

Panel b: Two-Bond Hedge Plus Options

Yield curve change	Price change for Freddie Mac 8.5%	Price change for two-bond hedge	Error options payoff	Two-bond hedge + options
“Level” Up	−0.98	0.93	0.05	0.00
“Level” Down	0.85	−0.90	0.05	0.00
“Twist” Flattening	0.11	−0.12	0.01	0.00
“Twist” Steep	−0.12	0.11	0.01	0.00

(At the time, the 10-year Treasury was priced at 97 15/32.) Determining how to obtain the positions in the options is beyond the scope of this chapter.

Panel b of Exhibit 7 shows that adding the two option positions on the 10-year note offsets the two-bond hedge error, making the total package (mortgage security + two-bond hedge + options) insensitive to likely interest rate movements. Of course, buying these options requires paying a premium which amounted to about 7 basis points per month. Since the yield advantage of the 8.5s versus Treasuries was about 11 basis points, the expected excess return over Treasuries was about 4 basis points per month after we hedge out the negative convexity.

CREDIT DERIVATIVES IN BOND PORTFOLIO MANAGEMENT*

I. INTRODUCTION

Derivatives are financial instruments designed to efficiently transfer some form of risk between two parties. Derivatives can be classified based on the type of risk that is being transferred. In the fixed-income market, derivatives include interest rate derivatives which transfer interest rate risk and credit derivatives which transfer credit risk. With credit derivatives, a portfolio manager can either acquire or reduce credit risk exposure. Many managers have portfolios that are highly sensitive to changes in the spread between riskless and risky assets and credit derivatives are an efficient way to manage this exposure. Conversely, other managers may use credit derivatives to target specific exposures as a way to enhance portfolio returns. In each case, the ability to transfer credit risk and return provides a tool for portfolio managers to improve performance.

Credit derivatives can be classified as follows:

- total return swaps
- credit default products
- credit spread options

Credit derivative products used in structured credit products include synthetic collateralized debt obligations and credit-linked notes. In this chapter we will review each type of credit derivative describing their structure and how they can be used by portfolio managers. After reviewing credit derivatives, we will explain synthetic credit debt obligations.

We begin this chapter with a short discussion about the participants in the credit derivatives market and why credit risk is important.

*This chapter is authored by Mark J.P. Anson, PhD, CFA, CDA, Esq., and Frank J. Fabozzi, PhD, CFA, CPA.

II. MARKET PARTICIPANTS

According to the British Bankers Association (BBA), the dominant center for the global credit derivatives market is London, which is well ahead of New York and Asia.¹ The credit derivatives market consists of three groups of players:²

- end-buyers of protection
- end-sellers of protection
- intermediaries

End-buyers of protection are entities that seek to hedge credit risk taken in other parts of their business. The predominate entity in this group is commercial banks for their loan portfolio. However, there are also insurance companies, pension funds, and mutual funds who seek protection for credits held in their portfolio. **End-sellers of protection** are entities that seek to diversify their current portfolio and can do so more efficiently with credit derivatives. An entity that provides protection is seeking exposure to a specific credit or a basket of credits.

Intermediaries include investment banking arms of commercial banks and securities houses. Their key role in the credit derivatives market is to provide liquidity to end-users. They trade for their own account looking for arbitrage opportunities and other profitable opportunities. In addition, some intermediaries will assemble, using credit derivatives, structured products which, in turn, they may or may not manage.

III. WHY CREDIT RISK IS IMPORTANT

A fixed-income instrument represents a basket of risks. There is (1) interest rate risk (as measured by duration and convexity), (2) call risk, and (3) credit risk. Credit risk includes the risk of defaults, downgrades, and widening credit spreads. The total return from a fixed-income instrument is the compensation for assuming all of these risks. Depending upon the rating on the underlying debt instrument, the return from credit risk can be a significant part of a bond's total return.

A. Types of Credit Risk

Credit risk may affect a portfolio in three ways: default risk, credit spread risk, and downgrade risk. Each can have a detrimental impact on the value of a fixed-income portfolio.

1. **Default Risk** **Default risk** is the risk that the issuer will default on its obligations. The default rate on credit risky bonds can be quite high. For example, estimates of the average default rates for high-yield bonds in the United States range from 3.17% to 6.25%.³ However, default rates increase significantly during periods of economic malaise. For example, during the recession of 2001, the default rate for high-yield bonds was 10.2%. All told, there were

¹British Bankers Association, *Credit Derivatives Report 2002*.

²David Rule, "The Credit Derivatives Market: Its Development and Possible Implications for Financial Stability," *Financial Stability Review* (June 2001), pp. 117–140.

³Edward Altman, "Measuring Corporate Bond Mortality and Performance," *The Journal of Finance* (June 1991), pp. 909-922; and Gabriella Petrucci, "High-Yield Review—First-Half 1997," Salomon Brothers Corporate Bond Research (August 1997).

a total of \$106 billion of corporate bond defaults in 2001.⁴ The greatest default rate in the United States for all corporate bonds (high yield and investment grade) was 9.2% during the Great Depression in the 1930s.⁵

There is default risk associated with investing in the foreign currency debt of another nation compared to an investor's home country. Most investors consider the sovereign debt of the G-7 countries (the largest industrialized nations in the world: The United States, the United Kingdom, France, Germany, Italy, Canada, and Japan) to be default free. The reason is that these countries have significant internal resources and borrowing capacity to support the prompt payment of their outstanding debt.

Instead, sovereign debt default risk is associated mainly with emerging economies. An emerging economy relies on two forms of cash flows to finance its government programs and to pay its debt obligations: taxes and revenues from state-owned enterprises. Taxes can come from personal income taxes, corporate taxes, import duties, and other excise taxes. State-owned enterprises can be oil companies, telephone companies, national airlines and railroads, and other manufacturing enterprises. In times of economic turmoil such as a recession, cash flows from state-owned enterprises decline along with the general malaise of the economy. Additionally, tax revenues decline as corporations earn less profits, as unemployment rises, and as personal incomes decline. Lastly, with a declining foreign currency value, imports decline, reducing revenue from import taxes.

The possibility of default is a significant risk for portfolio managers, and one that can be effectively hedged by shifting the credit exposure to another party. Credit derivatives therefore appeal to portfolio managers who invest in corporate bonds—particularly, high-yield corporate bonds—and sovereign bonds.

2. Credit Spread Risk **Credit spread risk** is the risk that the interest rate spread for a risky bond over a riskless bond will increase after the risky bond has been purchased. For instance, in the United States, U.S. Treasury securities are generally considered to be without credit risk (default free). Therefore, corporate bonds, agency debentures, and the debt of foreign governments are typically priced at a spread to comparable U.S. Treasury securities. Should this spread widen after purchase of the credit risky bond, the value of the bond would decline. Credit spreads can widen based on macroeconomic events in the domestic or global financial markets.

As an example, in October of 1997, a rapid decline in Asian stock markets spilled over into the U.S. stock markets, causing a significant decline in financial stocks. The turbulence in the financial markets, both domestically and worldwide, resulted in a flight to safety of investment capital. In other words, investors sought safer havens for their investment capital in order to avoid further losses and volatility. This flight to safety resulted in a significant increase in credit spreads of corporate bonds to U.S. Treasuries. For instance, on June 30, 1997, corporate bonds rated BB by Standard & Poor's were trading at an average spread over U.S. Treasuries of 215 basis points.⁶ However, by October 31, 1997, this spread had increased to 319 basis points. For a \$1,000 market value BB-rated corporate bond with a duration of 5, this resulted in a loss of value of about \$52.50 per bond.

⁴See "Default and Recovery Rates of Corporate Bond Issuers," Moody's Investors Services, February 2002.

⁵Moody's Investor Service Global Credit Research, Special Comment, "Historical Default Rates of Corporate Bond Issuers, 1920–1999," January 2000.

⁶"Financial Firms Lose \$8 Billion so Far," *The Wall Street Journal* (September 3, 1998), p. A2.

An estimate of this credit spread risk is spread duration, a measure discussed earlier. For credit-risky bonds, spread duration is the approximate percentage change in the bond's price for a 100 basis point increase in the credit spread (holding the Treasury rate constant). For example, a spread duration of 3 means that for a 100 basis point increase in the credit spread, the bond's price will decline by approximately 3%. The spread duration for a portfolio is found by computing a market weighted average of the spread duration for each bond. The same is true for a bond market index. Note, however, that the spread duration reported for a bond market index is not the same as the spread duration for estimating the credit spread risk of an index. For example, on April 25, 2003, the spread duration reported for the Lehman Brothers Aggregate Bond Index was 2.99. However, the spread duration for the index is computed by Lehman Brothers based on all non-Treasury securities. Some of these sectors offer a spread to Treasuries that reflects more than just credit risk. For example, the mortgage sector in the index offers a spread due to prepayment risk. The same is true for some sub-sectors within the ABS sector. Lehman Brothers does have a Credit Sector for the index. For that sector, the spread duration reflects the exposure to credit spreads in general. It was 1.49 on April 25, 2003 and is interpreted as follows: If credit spreads increase by 100 basis points, the approximate decline in the value of the index will be 1.49%.

3. Downgrade Risk **Downgrade risk** occurs when a nationally recognized statistical rating organization such as Standard & Poor's, Moody's Investors Services, or Fitch Ratings reduces its outstanding credit rating for an issuer based on an evaluation of that issuer's current earning power versus its capacity to pay its fixed income obligations as they become due.⁷

The rating agencies construct credit transition matrices. These matrices can be used to forecast the probabilities of credit upgrades or downgrades for a particular class of rated bonds. For example, a transition matrix might forecast the probability (on average) of a company rated "single A" being upgraded to "double A" to be 2%. Conversely, a transition matrix might forecast the probability of a company rated "single A" being downgraded to "triple B" to be 5%.

B. Reasons for Selling Credit Protection

Credit risk is not all one-sided. A market participant may be willing to be a seller of credit protection. This can be done in one of two ways. First, a market participant can sell contingent or insurance-type protection. That is, if the market participant believes that credit performance will be such that it will be unnecessary to make an insurance payment to a counterparty (i.e., the party buying credit protection), then the market participant earns the insurance premium received. Second, a market participant may want to take the opposite view of a credit protection buyer and in fact benefit from an improvement in a credit. This is done by selling protection.

There are at least three reasons why a portfolio manager may be willing to assume the credit risk of an underlying asset or issuer. First, there are credit upgrades as well as downgrades. A factor affecting credit rating upgrades is a strong stock market which encourages public offerings of stock by credit risky companies. Often, a large portion of these equity financings are used to reduce outstanding costly debt, resulting in improved balance sheets and credit ratings for the issuers.

A second reason why a portfolio manager may be willing to sell credit protection is that there is an expectation of other credit events which have a positive effect on credit risky bonds. Mergers and acquisitions, for instance, have been historically a frequent occurrence in the high-yield corporate bond market. Even though a credit risky issuer may have a low

⁷For a discussion of the factors considered by rating agencies, see Chapter 15.

debt rating, it may have valuable technology worth acquiring. High-yield corporate bond issuers tend to be small to mid-cap companies with viable products but nascent cash flows. Consequently, they make attractive takeover candidates for financially mature companies.

The third reason is that with a growing economy, banks may be willing to provide term loans to high-yield companies at more attractive rates than the bond market. Consequently, it has been advantageous for credit risky companies to redeem their high-yield bonds and replace an outstanding bond issue with a lower cost term loan. The resulting premium for redemption of high-yield bonds is a positive credit event which enhances portfolio returns.

IV. TOTAL RETURN SWAP

A **total return swap** in the fixed-income market is a swap in which one party makes periodic floating payments to a counterparty in exchange for the total return realized on an individual reference obligation or a basket of reference obligations. A total return payment includes all cash flows that flow from the reference obligations as well as any capital appreciation or depreciation. The party that agrees to make the floating payments and receive the total return is referred to as the **total return receiver**; the party that agrees to receive the floating payments and pay the total return is referred to as the **total return payer**. When the reference obligation is a sector of a bond index, it is called a **total return index swap**.

Notice that in a total return swap, the total return receiver is exposed to both credit risk and interest rate risk. For example, the credit risk spread can decline (resulting in a favorable price movement for the reference obligation), but this gain can be offset by a rise in the level of interest rates.

A portfolio manager typically uses a total return swap to increase credit exposure. A total return swap transfers all of the economic exposure of a reference obligation or reference obligations to the total return receiver. In return for accepting this exposure, the total return receiver makes a floating payment to the total return payer.

A. Illustration of a Total Return Swap

Consider a portfolio manager who believes that the fortunes of XYZ Mobile Corporation will improve over the next year, and that the company's credit spread relative to U.S. Treasury securities will decline. The company has issued a 10-year bond at par with a coupon rate of 8.5% and therefore the yield is 8.5%. Suppose at the time of issuance, the 10-year Treasury yield is 5.5%. This means that the credit spread is 300 basis points and the portfolio manager believes it will decrease over the year to less than this amount.

The portfolio manager can express this view by entering into a total return swap that matures in one year as a total return receiver with the reference obligation being the 10-year, 8.5% XYZ Mobile Corporate bond issue. Suppose (1) the swap calls for an exchange of payments semiannually and (2) the terms of the swap are such that the total return receiver pays the 6-month Treasury rate plus 140 basis points in order to receive the total return on the reference obligation. The notional amount for the contract is \$10 million.

Assume that over the one year, the following occurs:

- the 6-month Treasury rate is 4.6% initially
- the 6-month Treasury rate for computing the second semiannual payment is 5.6%
- at the end of one year the 9-year Treasury rate is 7%
- at the end of one year the credit spread for the reference obligation is 200 basis points

First let's look at the payments that must be made by the portfolio manager. The first semi-annual swap payment made by the portfolio manager is 3% (4.6% plus 140 basis points divided by two) multiplied by the \$10 million notional amount. The second swap payment made is 3.5% (5.6% plus 140 basis points divided by two) multiplied by the \$10 million notional amount. Thus,

$$\begin{array}{rcl} \text{First swap payment paid : } \$10 \text{ million} \times 3\% & = & \$300,000 \\ \text{Second swap payment paid : } \$10 \text{ million} \times 3.5\% & = & \underline{\$350,000} \\ \text{Total payments} & = & \underline{\$650,000} \end{array}$$

The payments that will be received by the portfolio manager are the two coupon payments plus the change in the value of the reference obligation. There will be two coupon payments. Since the coupon rate is 8.5%, the amount received for the coupon payments is \$850,000.

Finally, the change in the value of the reference obligation must be determined. At the end of one year, the reference obligation has a maturity of 9 years. Since the 9-year Treasury rate is assumed to be 7% and the credit spread is assumed to decline from 300 basis points to 200 basis points, the reference obligation will sell to yield 9%. The price of an 8.5%, 9-year bond selling to yield 9% is 96.96. Since the par value is \$10 million, the price is \$9,696,000. The capital loss is therefore \$304,000. The payment to the total return receiver is then:

$$\begin{array}{rcl} \text{Coupon payment} & = & \$850,000 \\ \text{Capital loss} & = & \underline{\$304,000} \\ \text{Swap payment} & = & \underline{\$546,000} \end{array}$$

Netting the swap payment made and the swap payment received, the portfolio manager must make a payment of \$104,000.

Notice that even though the portfolio manager's expectations were realized (i.e., a decline in the credit spread), the portfolio manager had to make a net outlay. This illustration highlights one of the disadvantages of a total return swap: the return to the investor is dependent on both credit risk (declining or increasing credit spreads) and market risk (declining or increasing market rates). Two types of market interest rate risk can affect the price of a fixed-income instrument. Credit independent market risk is the risk that the general level of interest rates will change over the term of the swap. This type of risk has nothing to do with the credit deterioration of a reference obligation. Credit dependent market interest rate risk is the risk that the discount rate applied to the value of an asset will change based on either perceived or actual default risk.

In the illustration, the reference obligation was adversely affected by credit independent market interest rate risk, but positively rewarded for accepting credit dependent market interest rate risk. To remedy this problem, a total return receiver can customize the total return swap transaction. For example, the portfolio manager could negotiate to receive the coupon income on the reference obligation plus any change in value due to changes in the credit spread. Now the portfolio manager has expressed a view exclusively of credit risk; credit independent market risk does not affect the swap outcome. In this case, in addition to the coupon income, the portfolio manager would receive the difference between the present value of the reference obligation at a current spread of 200 basis points and the present value of the reference obligation at a credit spread of 300 basis points.

B. Benefits of Total Return Swaps

There are several benefits in using a total return swap as opposed to purchasing reference obligations themselves. First, the total return receiver does not have to finance the purchase of the reference obligations. Instead, it pays a fee to the total return payer in return for receiving the total return on the reference obligations.

Second, the total return receiver can achieve the same economic exposure to a diversified basket of assets in one swap transaction that would otherwise take several cash market transactions to achieve. In this way a total return swap is much more efficient means for transacting than the cash market.

Finally, an investor who wants to short the corporate bond of one or more issuers will find it difficult to do so in the corporate bond market. An investor can do so efficiently by using a total return swap. In this case the investor will pay the total return and receive a floating payment.

V. CREDIT DEFAULT PRODUCTS

Credit default products fall into two categories:

- credit default swap
- credit default options on a credit-risky asset

By far, the most popular type of credit derivative is the credit default swap. An annual survey of the BBA finds that credit default swaps account for almost half of the credit derivatives market. Not only is this form of credit derivative the most commonly used stand-alone product, but it is also used extensively in structured credit products.

A. Credit Default Swaps

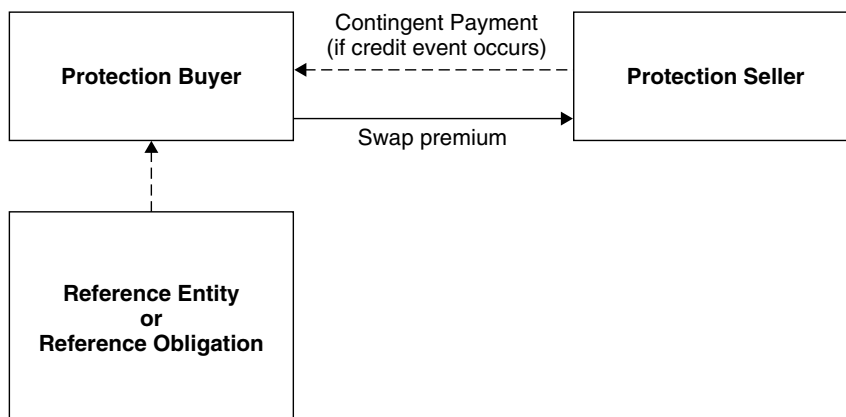
A **credit default swap** is probably the simplest form of credit risk transference among all credit derivatives. Credit default swaps are used to shift credit exposure to a credit protection seller. Their primary purpose is to hedge the credit exposure to a particular asset or issuer. In this sense, credit default swaps operate much like a standby letter of credit or insurance policy.

While credit default swaps are customized transactions, there has been standardization of contract terms and provisions for certain types of credit default swaps. Swaps are usually documented under a standard set of forms published by the International Swap and Derivatives Association (ISDA).

In a credit default swap, the documentation will identify the reference entity and the reference obligation. The **reference entity** is the issuer of the debt instrument. It could be a corporation or a sovereign government. The **reference obligation** is the particular issue for which the credit protection is being sought. For example, one of the most liquid reference entities traded in the credit default swap market is Ford Motor Credit Company. The reference obligation would be a specific Ford Motor Credit Company bond issue.

A credit default swap in which there is only one reference obligation is called a **single-name credit default swap**. When there is a portfolio of reference obligations (e.g., a high-yield corporate bond portfolio), it is referred to as a **basket default swap**. For a **credit default swap index** the underlying is a standardized basket of reference entities. The most actively traded credit default swap index is the North American Investment Grade Index. The index consists of 125 corporate names.

EXHIBIT 1 Credit Default Swap



In a single-name credit default swap, the protection buyer pays a fee to the protection seller in return for the right to receive a payment conditional upon the occurrence of a credit event by the reference obligation or the reference entity. Should a credit event occur, the protection seller must make a payment. The payments made by the protection buyer are called the **premium leg**; the contingent payment that might have to be made by the protection seller is called the **protection leg**. Exhibit 1 shows the payments in a credit default swap.

In a basket default swap, a credit event with respect to one reference obligation may or may not trigger a payment from the protection seller. The number of reference obligations for which a credit event must occur to obtain a payment is specified in the contract. In Section VIII, we will explain the different types of basket default swaps and credit protection they provide.

The interdealer market has evolved to where single-name credit default swaps for corporate and sovereign reference entities are standardized. While trades between dealers have been standardized, there are occasional trades in the interdealer market where there is a customized agreement. For portfolio managers seeking credit protection, dealers are willing to create customized products.

In the interdealer market, the tenor, or length of time of a credit default swap, is typically five years. Portfolio managers can have a dealer create a tenor equal to the maturity of the reference obligation or to a shorter time period to match the portfolio manager's investment horizon.

1. Settlement Methods Credit default swaps can be settled in cash or physically. In the interdealer market, single-name credit default swaps are typically settled physically. **Physical delivery** means that if a credit event as defined by the documentation occurs, the reference obligation is delivered by the protection buyer to the protection seller in exchange for a cash payment. Because physical delivery does not rely upon obtaining market prices for the reference obligation in determining the amount of the payment in a single-name credit default swap, this method of delivery is more efficient. When a credit default swap is cash settled, there is a netting of payment obligations with the same counterparty.

If no credit event has occurred by the maturity of the swap, both sides terminate the swap agreement and no further obligations are incurred.

2. Determination of the Payment Obligation The methods used to determine the amount of the payment obligation of the protection seller under the swap agreement can vary greatly. For instance, a credit default swap can specify at the contract date the exact amount of payment that will be made by the protection seller should a credit event occur. Conversely, the credit default swap can be structured so that the amount of the swap payment by the seller is determined after the credit event. Under these circumstances, the amount payable by the protection seller is determined based upon the observed prices of similar debt obligations of the reference entity in the market. Finally, the swap can be documented much like a credit put option (discussed later) where the amount to be paid by the protection seller is an established strike price less the current market value of the reference obligation.

The cash payment by the credit protection seller if a credit event occurs may be a predetermined fixed amount or it may be determined by the decline in value of the reference asset. In the interdealer market, the standard single-name credit default swap when the reference obligation is a corporate bond or a sovereign bond is fixed based on a notional amount. When the cash payment is based on the amount of asset value deterioration, this amount is typically determined by a poll of several dealers.

In a typical credit default swap, the protection buyer pays for the protection premium over several settlement dates rather than upfront. A standard credit default swap in the interdealer market specifies quarterly payments.

3. Credit Event Definitions The most important section of the documentation for a credit default swap is what the parties to the contract agree constitutes a credit event for a credit default payment. Definitions for credit events are provided by the ISDA. First published in 1999, there have been periodic updates and revisions of these definitions.

The *1999 ISDA Credit Derivatives Definitions* (referred to as the “1999 Definitions”) provides a list of eight credit events:

1. bankruptcy
2. credit event upon merger
3. cross acceleration
4. cross default
5. downgrade
6. failure to pay
7. repudiation/moratorium
8. restructuring

These eight events attempt to capture every type of situation that could cause the credit quality of the reference entity to deteriorate, or cause the value of the reference obligation to decline.

Bankruptcy is defined as a variety of acts that are associated with bankruptcy or insolvency laws. **Failure to pay** results when a reference entity fails to make one or more required payments when due. When a reference entity breaches a covenant, it has defaulted on its obligation.

When a default occurs, the obligation becomes due and payable prior to the scheduled due date had the reference entity not defaulted. This is referred to as an **obligation acceleration**. A reference entity may disaffirm or challenge the validity of its obligation. This is a credit

event that is covered by **repudiation/moratorium**. In the U.S. credit default swap market, obligation acceleration and repudiation/moratorium were dropped as standard credit events in November 2002. In April 2003, these two credit events were dropped in the standard credit default swap in the European market.

A **restructuring** occurs when the terms of the obligation are altered so as to make the new terms less attractive to the debt holder than the original terms. The terms that can be changed would typically include, but are not limited to, one or more of the following: (1) a reduction in the interest rate, (2) a reduction in the principal, (3) a rescheduling of the principal repayment schedule (e.g., lengthening the maturity of the obligation) or postponement of an interest payment, or (4) a change in the level of seniority of the obligation in the reference entity's debt structure.

Restructuring is the most controversial credit event that may be included in a credit default swap. The reason why it is so controversial is easy to understand. A protection buyer benefits from the inclusion of a restructuring as a credit event and feels that eliminating restructuring as a credit event will erode its credit protection. The protection seller, in contrast, would prefer not to include restructuring since even routine modifications of obligations that occur in lending arrangements would trigger a payout to the protection buyer.

Moreover, if the reference obligation is a loan and the protection buyer is the lender, there is a dual benefit for the protection buyer to restructure a loan. The first benefit is that the protection buyer receives a payment from the protection seller. Second, the accommodating restructuring fosters a relationship between the lender (who is the protection buyer) and its customer (the corporate entity that is the obligor of the reference obligation).

Because of this problem, the *Restructuring Supplement to the 1999 ISDA Credit Derivatives Definitions* (the Supplement Definition) issued in April 2001 provided a modified definition for restructuring. The modified definition includes a provision for the limitation on reference obligations in connection with restructuring of loans made by the protection buyer to the borrower that is the obligor of the reference obligation.⁸

Consequently, in the credit default swap market, until 2003, the parties to a trade had the following three choices for restructuring:

1. no restructuring
2. restructured based on the 1999 Definition for restructuring, referred to as "old restructuring" or "full restructuring"
3. restructuring as defined by the Restructuring Supplement Definition, referred to as "modified restructuring."

Modified restructuring is typically used in North America while old restructuring is used in Europe. When the reference entity is a sovereign, restructuring is often old restructuring.

Whether restructuring is included and, if it is included, whether it is old restructuring or modified restructuring, affects the swap premium. Specifically, all other factors constant, it is more expensive if restructuring is included. Moreover, old restructuring results in a larger swap premium than modified restructuring.

⁸This provision requires the following in order to qualify for a restructuring: (1) there must be four or more holders of the reference obligation and (2) there must be a consent to the restructuring of the reference obligation by a supermajority (66 2/3%). In addition, the supplement limits the maturity of reference obligations that are physically deliverable when restructuring results in a payout triggered by the protection buyer.

EXHIBIT 2 Credit Event Selection Portion of “Exhibit A to 2003 ISDA Credit Derivatives Definitions”

Credit Events: The following Credit Event[s] shall apply to this Transaction:

[Bankruptcy]
 [Failure to Pay]
 [Grace Period Extension Applicable]
 [Grace Period:]
 Payment Requirement: []
 [Obligation Default]
 [Obligation Acceleration]
 [Repudiation/Moratorium]
 [Restructuring]
 [[Restructuring Maturity Limitation and Fully Transferable Obligation: [Applicable]]
 [[Modified Restructuring Maturity Limitation and Conditionally Transferable Obligation: [Applicable]]
 [[Multiple Holder Obligation: [Applicable]]
 [Default Requirement: []]

Note: Footnotes have been deleted

Source: International Swaps and Derivatives Association, Inc.

As the credit derivatives market developed, market participants learned a great deal about how to better define credit events, particularly with the record level of high-yield corporate bond default rates in 2002 and the sovereign defaults, experienced with the 2001–2002 Argentina debt crisis. In January 2003, the ISDA published its revised credit events definitions in the *2003 ISDA Credit Derivative Definitions* (the “2003 Definitions”).

The revised definition reflected amendments to several of the definitions for credit events set forth in the 1999 Definitions. Specifically, there were amendments for bankruptcy, repudiation, and restructuring. The major change was to restructuring whereby the ISDA allows parties to a given trade to select from among the following four definitions:

1. no restructuring
2. “full” restructuring, with no modification to the deliverable reference obligations aspect
3. “modified restructuring”
4. “modified modified restructuring”

The last choice is a new one and was included to address issues that arose in the European market.

The ISDA’s confirmation form for credit derivative transactions, “Exhibit A to 2003 ISDA Credit Derivatives Definitions,” sets forth the terms and conditions for the transaction. The definitions for a credit event are in “check box” format. Exhibit 2 shows the portion of the confirmation form where the two parties select the credit events that are to be included in the trade.

4. Illustration of a Standard Single-Name Credit Default Swap Let’s illustrate the mechanics of a standard single-name credit default swap where the reference entity is a corporation. We will use an actual trade on November 26, 2002 as reported by *creditex*, a major broker in the credit default swap market, where the reference obligation is a specific senior bond issue of Ford Motor Credit Company. The scheduled term for the trade is November 26, 2007. That is, it was a 5-year scheduled term—the typical tenor in the interdealer market. The tenor of a swap is referred to as “scheduled” because a credit event will result in a payment by the protection seller, resulting in the credit default swap being terminated. The swap premium—the payment made by the protection buyer to the protection seller—was 410 basis points. If a credit event occurs, the protection seller pays the protection

buyer the notional amount of the contract. In our illustration, we will assume that the notional amount is \$10 million.

The notional amount is not the par value of the reference obligation. For example, suppose that a bond issue is trading at 80 (par value being 100). If a portfolio manager owns \$12.5 million par value of the bond issue and wants to protect the current market value of \$10 million (= 80% of \$12.5 million), then the portfolio manager will want a \$10 million notional amount. If a credit event occurs, the portfolio manager will deliver the \$12.5 million par value of the bond and receive a cash payment of \$10 million.

The standard contract for a single-name credit default swap in the interdealer market calls for a quarterly payment of the swap premium. The quarterly payment is determined using one of the day count conventions in the bond market. A day count convention indicates the number of days in the month and the number of days in a year that will be used to determine how to prorate the swap premium to a quarter. The possible day count conventions are (1) actual/actual, (2) actual/360, and (3) 30/360. The day count convention used in the U.S. government bond market is actual/actual, while the convention used in the corporate bond market is 30/360. The day count convention used for credit default swaps is actual/360. This is the same convention used in the interest rate swap market. A day convention of actual/360 means that to determine the payment in a quarter, the actual number of days in the quarter are used and 360 days are assumed for the year. Consequently, the

quarterly swap premium payment

$$= \text{notional amount} \times \text{swap premium (in decimal)} \times \frac{\text{actual number of days in quarter}}{360}$$

For example, if the notional amount is \$10 million and there are 92 actual days in a quarter, then if the swap rate is 410 basis points (0.0410), the quarterly swap premium payment made by the protection buyer would be:

$$\$10,000,000 \times 0.0410 \times \frac{92}{360} = \$104,777.80$$

In the absence of a credit event occurring in any quarter, the protection buyer will continue to make a quarterly swap premium payment. If a credit event occurs, the protection seller pays the protection buyer the notional amount, \$10 million in our illustration, and receives from the protection buyer the Ford Motor Credit Company senior bonds (i.e., the reference obligation).

5. Market Terminology Newcomers to the credit default swap market sometimes get confused regarding market terminology. The first potential source of confusion arises because market participants attempt to relate a position in the derivative market to a position in the cash market. The second potential source has to do with the use of the term “swap” to describe the transaction when the payment is contingent on the a credit event occurring.

a. Cash versus Credit Default Swap Market Terminology Participants in derivatives markets find it helpful to compare their exposure (long or short) in the derivative market to that of an exposure in the cash market. Sometimes the relationship is straightforward. For example, as explained in Chapter 22, a long position in a Treasury bond futures contract is equivalent to a long position in the Treasury bond market; a short position in a Treasury

bond futures contract is equivalent to a short position in the Treasury bond market. In other cases, the relationship is not straightforward. For example, in a generic interest rate swap, the fixed-rate payer is said to be “short the bond market” and the fixed-rate receiver is said to be “long the bond market.” This is because for the fixed-rate payer, the value of an interest rate swap increases when interest rates increase. A position in the cash market whereby the value of the position increases when interest rates increase is a short bond position. Similarly, for the fixed-rate receiver, the value of an interest rate swap increases when interest rates decrease and therefore is equivalent to being long a bond.

The terminology of the position of the parties in a credit default swap can be confusing. To “go long” an instrument generally is to purchase it. In the cash market, going long a bond means one is buying a bond and so receiving the coupon payments; the bond buyer has therefore taken on credit risk exposure to the issuer. In a credit default swap, going long is to buy the swap, but the buyer is purchasing protection and therefore paying the swap premium; the buyer has no credit exposure on the reference entity and has in effect “gone short” on the reference obligation (the equivalent of shorting a bond in the cash market and paying coupons). So buying a credit default swap (buying protection) is frequently referred to in the credit derivatives market as “shorting” the reference obligation.

b. Swap versus Option Nomenclature The protection seller has literally “insured” the protection buyer against any credit losses on the reference obligation. While the term “swap” is used to describe this credit derivative, it should be clear that it has an option-type payoff. That is, it does not have the characteristics of the typical swap found in the derivatives market. For example, in a plain vanilla or generic interest rate swap, two parties swap payments periodically. One of the counterparties pays a fixed rate (called the “swap rate”) and the other party pays a floating rate. The payments are made by both parties over the term of the swap agreement. Moreover, the payments are not contingent on some event and the occurrence of any event does not terminate the swap agreement. This is not a characteristic of a credit default swap.

The question is then: Why is the transaction referred to as a *swap*? The reason has to do with the way one characterizes an option. There are two attributes for characterizing a derivative as an option. The first attribute is that there is an asymmetric payoff. The second attribute involves the price performance feature. While a credit default swap does have an asymmetric payoff, its price performance is like that of a swap rather than an option. The price performance of an option depends on the price of the underlying. When a credit-risky bond is the underlying, it is the credit spread that affects the price of the bond. So the price performance mechanism for an option is as follows: changes in the credit spread affect the price of the underlying bond which, in turn, changes the price of the option. In the case of a credit default swap, the change in the credit spread directly affects the price of the transaction rather than through its effect on the reference obligation (i.e., underlying bond). This is a characteristic of a swap such as an interest rate swap where the price of a swap depends directly on interest rates. It is for this reason that a credit default swap is referred to as a swap.

B. Default Options on a Credit Risky Asset

A **default option** on a credit risky asset is another form of credit default product. These options have not been nearly as popular as credit default swaps. Consequently, we will only provide a brief review of them.

In a **binary credit option** the option seller will pay out a fixed sum if and when a default event occurs with respect to a reference obligation or reference entity. Therefore, a binary

option represents two states of the world: default or no default. It is the clearest example of credit protection. At maturity of the option, if the reference obligation or reference entity has defaulted, the option holder receives a predetermined payout. If there is no default at maturity of the option, the option buyer receives nothing.

A binary credit option could also be triggered by a rating downgrade. We'll illustrate a binary credit option (both a put and a call) based on a credit rating. Consider the situation where a portfolio manager holds an investment-grade bond but is concerned that the credit rating for the issuer will be lowered and that the bond will decline in value. The portfolio manager can purchase a binary credit put option which allows putting the bond back to the option seller at the bond's par value if the bond issuer is downgraded below investment grade.

As an example, assume that the portfolio manager purchased at par \$1 million of Company W bonds, currently rated AA. The portfolio manager purchases a put option where he can sell the bonds at par value to the put option seller should the credit rating for Company W fall below investment grade (below BBB). The payoff to this binary put option can be described as:

$$\text{payout} = \begin{cases} \$1,000,000 - \text{market value of the bonds, if the credit rating of} \\ \text{Company W falls below a BBB rating} \\ \text{or} \\ \$0 \text{ if the credit rating of Company W remains investment grade} \end{cases} \quad (1)$$

Equation (1) is called a binary credit option because of its "either or" payout structure. Either Company W's credit rating is below investment grade or it is not. Therefore, the portfolio manager receives a payout on the credit put option only in one state of the world: Company W is downgraded to below investment grade; otherwise, the portfolio manager receives nothing.

Let's now consider a binary credit call option. Suppose that instead of purchasing a binary credit put option on the par value of the bonds to protect her investment, the portfolio manager purchases a series of call options that provide her with additional income should Company W be downgraded. In other words, whenever Company W is downgraded, the portfolio manager gets to call for a payment that will compensate her for the greater credit risk associated with her bond holdings. This is like receiving additional coupon income to reflect the higher credit risk associated with Company W's bonds.

Consider the example where the portfolio manager gets to call for an additional 25 basis points of income should Company W be downgraded one credit rating, 50 basis points of income should Company W be downgraded two steps, and so forth. The pay out to this credit call option may be described as:

$$\text{payout} = \begin{cases} \$2,500 \text{ if the credit rating of Company W declines by one credit rating;} \\ \text{or} \\ \$5,000 \text{ if the credit rating of Company W declines by two credit rating;} \\ \text{or} \\ \$0 \text{ if the credit rating of Company W is not downgraded.} \end{cases} \quad (2)$$

$$\text{where } \$2,500 = 0.25\% \times \$1,000,000$$

$$\text{and } \$5,000 = 0.50\% \times \$1,000,000$$

Equation (2) is different from equation 1 in that the payout to the binary credit call option is not a function of the bonds' market value.

VI. CREDIT SPREAD PRODUCTS

The third category of credit derivatives is credit spread products which include:

- credit spread options
- credit spread forward

We describe each below. However, it should be noted that the market for credit spread options has not developed as rapidly as market participants had anticipated.

A. Credit Spread Options

A **credit spread option** is an option whose value/payoff depends on the change in credit spreads for a reference obligation. It is critical in discussion of credit spread options to define what the underlying is. The underlying can be:

- a reference obligation with a fixed credit spread
- the level of the credit spread for a reference obligation

1. **Underlying is a Reference Obligation with a Fixed Credit Spread** When the underlying is a reference obligation with a fixed credit spread, then a credit spread option is defined as follows:

Credit spread *put* option: An option that grants the option buyer the right, but not the obligation, to sell a reference obligation at a price that is determined by a strike credit spread over a referenced benchmark at the exercise date.

Credit spread *call* option: An option that grants the option buyer the right, but not the obligation, to buy a reference obligation at a price that is determined by a strike credit spread over a referenced benchmark at the exercise date.

A credit spread option can have any exercise style: only at the exercise date (European), at any time prior to the exercise date (American), or only on specified dates by the exercise date (Bermudean).

The price for the reference obligation (i.e., the credit-risky bond) is determined by specifying a strike credit spread over the referenced benchmark, typically a default-free government security. For example, suppose that the reference obligation is an 8% 10-year credit-risky bond selling to yield 8%. The price of this bond is 100. Suppose further that the referenced benchmark is a same maturity U.S. Treasury bond that is selling to yield 6%. Then the current credit spread is 200 basis points. Assume that a strike credit spread of 300 basis points is specified and that the option expires in six months. At the end of six months, suppose that the 9.5-year Treasury rate is 6.5%. Since the strike credit spread is 300 basis points, then the yield used to compute the strike price for the reference obligation is 9.5% (the Treasury rate of 6.5% plus the strike credit spread of 300 basis points). The price of a 9.5-year 8% coupon bond selling to yield 9.5% is \$90.75 per \$100 par value.

The payoff at the expiration date would then depend on the market price for the reference obligation. For example, suppose that at the end of six months, the reference obligation is trading at 82.59. This is a yield of 11% and therefore a credit spread of a 450 basis points over the 9.5-year Treasury yield of 6.5%. For a credit spread put option, the buyer can sell the

reference obligation (selling at 82.59) for the strike price of 90.75. The payoff from exercising is 8.16. This payoff is reduced by the cost of the option. For a credit spread call option, the buyer will not exercise the option and will allow it to expire worthless. There is a loss equal to the cost of the option.

There is one problem with using a credit spread option in which the underlying is a reference obligation with a fixed credit spread. The payoff is dependent upon the value of the reference obligation's price, which is affected by both the change in the level of the interest rates (as measured by the referenced benchmark) and the change in the credit spread. For example, suppose in our illustration that the 9.5-year Treasury at the exercise date is 4.5% (instead of 6.5%) and the credit spread increases to 450 basis points. This means that the reference obligation is trading at 9% (4.5% plus 450 basis points). Since it is an 8% coupon bond with 9.5-years to maturity selling at 9%, the price is 93.70. In this case, the credit spread put option would have a payoff of \$965 because the price of the reference obligation is 93.70 and the strike price is 103.35. Thus, there was no protection against credit spread risk because interest rates for the referenced benchmark fell enough to offset the increase in the credit spread.

Notice the following payoff from the perspective of the owner of the option before taking into account the option cost when the underlying for the a credit spread option is the reference obligation with a fixed credit spread:

Type of option	Positive payoff if at expiration
Put	credit spread at expiration > strike credit spread
Call	credit spread at expiration < strike credit spread

Consequently, to protect against credit spread risk, an investor can buy a credit spread *put* option where the underlying is a reference obligation with a fixed credit spread.

2. Underlying is a Credit Spread on a Reference Obligation When the underlying for a credit spread option is the credit spread for a reference obligation over a referenced benchmark, then the payoff of a call and a put option if exercised are as follows:

Credit spread call option:

$$\text{payoff} = (\text{credit spread at exercise} - \text{strike credit spread}) \times \text{notional amount} \times \text{risk factor}$$

Credit spread put option:

$$\text{payoff} = (\text{strike credit spread} - \text{credit spread at exercise}) \times \text{notional amount} \times \text{risk factor}$$

where the strike credit spread (in decimal form) is fixed at the outset of the option; the credit spread at exercise (in decimal form) is the credit spread over a referenced benchmark at the exercise date; and, the risk factor is based on the interest rate sensitivity of the debt instrument.

Notice that when the underlying for the credit spread option is the credit spread for a reference obligation over a referenced benchmark, a credit spread *call* option is used to protect against an increase in the credit spread. In contrast, when the underlying for the credit spread option is the reference obligation, a credit spread *put* option is used to protect against an increase in the credit spread.

The **risk factor** is determined by the sensitivity of the reference obligation to changes in the credit spread. The risk factor can be computed in one of several ways. One way is to compute the percentage change in the price of the reference obligation to a 100 basis point

change in interest rates. Since the percentage price change will differ depending on whether we are looking at an increase or decrease in the credit spread, the change used will be dictated by the circumstances. For example, if the option involved a credit spread widening, an increase in interest rates is used to determine the price decline. Once the percentage change in price is determined for a 100 basis point change in rates, then it is divided by 100 to get the approximate percentage price change for a 1 basis point change in rates. That is,

$$\begin{aligned} & \text{approximate percentage price change per 1 bp change in rates} \\ &= \frac{\text{percentage price change for a 100 basis point change in rates}}{100} \end{aligned}$$

To scale the risk factor in terms of how the credit spread at expiration and the strike credit spread are defined, the following calculation is performed:

$$\text{risk factor} = \text{percentage price change for a 1 basis point change in rates} \times 10,000$$

By including the risk factor, this form of credit spread option overcomes the problem we identified with the credit spread option in which the underlying is a reference obligation: the payoff depends on both changes in the level of interest rates (the yield on the referenced benchmark) and the credit spread. Instead, it is only dependent upon the change in the credit spread. Therefore, fluctuations in the level of the referenced benchmark's interest rate will not affect the value of the options.

To illustrate a credit spread call option, consider the BB rated, 7.75 Niagara Mohawk Power bond due in 2008. In September 1998, this bond was trading at a price of \$104.77 with a yield to maturity around 7.08%. The risk factor is determined using the percentage change in price for a 100 basis point *increase* in interest rates.⁹ For the Niagara Mohawk bond, there would be a percentage price change of 6.65% for a 100 basis point increase in rates. So, for each 1 basis point increase in the credit spread, there would be approximately a 0.0665% price change. The risk factor is then:

$$0.000665 \times 10,000 = 6.65$$

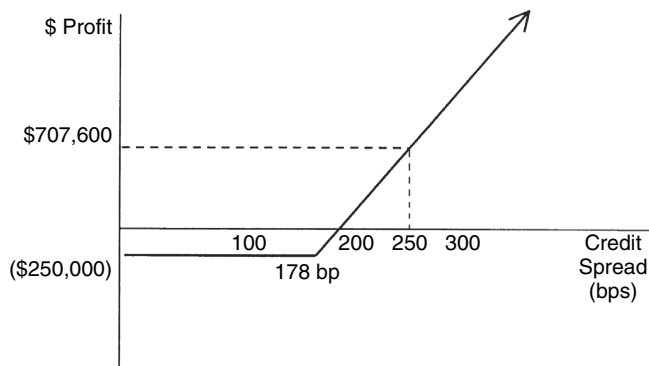
At the time that this bond was offering a yield of 7.08%, the 10-year Treasury note was yielding about 5.3% for a credit spread of 178 basis points. At the time this was a very narrow spread considering Niagara Mohawk Power's BB credit rating. Perhaps the market was implying that the credit risk of Niagara Mohawk was closer to BBB than BB.

Alternatively, it could be that the market overvalued the bond. If a portfolio manager believed that the bond was overvalued, the portfolio manager could purchase a credit spread option struck at 178 basis points. This is the same as the portfolio manager expressing a view that the price of the reference obligation is inflated at the prevailing credit spread, and expecting the credit spread to widen out to more normal levels.

Suppose that the manager believes that the credit spread for this bond will increase to 250 basis points in one year. The portfolio manager can purchase a \$20 million notional at-the-money call option on the credit spread between the debt of Niagara Mohawk Power

⁹An increase in interest rates is used because we are looking at the price sensitivity to an increase in the credit spread.

EXHIBIT 3 Niagara Mohawk Power Option Struck at 178 Basis Points.



and U.S. Treasuries. The tenor of the option is one year, the premium costs 125 basis points, and the risk factor is 6.65.

At the maturity of the option, if the credit spread is 250 basis points (i.e., the credit spread at expiration) the portfolio manager will receive:

$$\text{payoff} = (0.025 - 0.0178) \times \$20,000,000 \times 6.65 = \$957,600$$

The amount earned by the portfolio manager is the amount received less the cost of the option. Since the option cost is 125 basis points for a notional amount of \$20 million, the option cost is \$250,000. The portfolio manager's profit is \$707,600 ($= \$957,600 - \$250,000$).

This profit/loss profile is demonstrated in Exhibit 3.

B. Credit Spread Forwards

Credit spread forward requires an exchange of payments at the settlement date based on a credit spread. As with a credit spread option, the underlying can be the value of the reference obligation with a fixed credit spread or the credit spread. The payoff depends on the credit spread at the settlement date of the contract. The payoff is positive (i.e., the party receives cash) if the credit spread moves in favor of the party at the settlement date. The party makes a payment if the credit spread moves against the party at the settlement date.

For example, suppose that a manager has a view that the credit spread will increase (i.e., widen) to more than the current 250 basis points in one year for a credit-risky bond. Then the payoff function for this credit spread forward contract would be:

$$(\text{credit spread at settlement date} - 250) \times \text{notional amount} \times \text{risk factor}$$

Assuming that the notional amount is \$10 million and the risk factor is 5, then if the credit spread at the settlement date is 325 basis points, then the amount that will be received by the portfolio manager is:

$$(0.0325 - 0.025) \times \$10,000,000 \times 5 = \$375,000$$

Instead, suppose that the credit spread at the settlement date decreased to 190 basis points, then the portfolio manager would have to payout \$300,000 as shown below:

$$(0.019 - 0.025) \times \$10,000,000 \times 5 = -\$300,000$$

In general, if a portfolio manager takes a position in a credit spread forward to benefit from an increase in the credit spread, then the payoff would be as follows:

$$(\text{credit spread at settlement date} - \text{contracted credit spread}) \times \text{notional amount} \times \text{risk factor}$$

For a portfolio manager taking a position that the credit spread will decrease, the payoff is:

$$(\text{contracted credit spread} - \text{credit spread at settlement date}) \times \text{notional amount} \times \text{risk factor}$$

VII. SYNTHETIC COLLATERALIZED DEBT OBLIGATIONS

Credit derivatives are used to create debt instruments with structures whose payoffs are linked to, or derived from, the credit characteristics of a reference obligation or entity or a basket of reference obligations or entities. These products are called **structured credit products**. We will focus on one of these products: synthetic collateralized debt obligations.

Previously, collateralized debt obligations were explained. A collateralized debt obligation (CDO) is backed by a diversified pool of one or more of the following types of debt obligations: investment-grade corporate bonds, high-yield corporate bonds, emerging market bonds, bank loans, asset-backed securities, residential and commercial mortgage-backed securities, and real estate investment trusts. In a CDO structure, there is a collateral manager responsible for managing the collateral assets. The funds to purchase the collateral assets are obtained from the issuance of bonds. Typically, there are senior bonds and mezzanine bonds. There is also a subordinate/equity tranche.

A CDO is classified as either a **cash CDO** or a **synthetic CDO**. In a cash CDO the collateral manager purchases the collateral assets. It was the cash CDO that was discussed. Our focus in this section is on synthetic CDOs, the fastest growing sector of the CDO market. A synthetic CDO is so named because the collateral manager does not actually own the pool of assets on which it has the credit risk exposure. Stated differently, a synthetic CDO absorbs the credit risk, but not the legal ownership, of its reference credit exposures. A basket credit default swap is typically employed to allow institutions to transfer the credit risk, but not the legal ownership, of underlying assets.

The basic structure of a synthetic CDO is as follows. As with a cash CDO, there are liabilities issued. The proceeds received from the bonds sold are invested by the collateral manager in assets with low risk. At the same time, the collateral manager enters into a basket credit default swap with a counterparty in which it will provide credit protection (i.e., the collateral manager is a protection seller) for the reference obligations that have credit risk exposure. Because it is selling credit protection, the collateral manager will receive the credit default swap premium.

On the other side of the basket credit default swap is a protection buyer who will be paying the swap premium to the collateral manager. This entity will be a financial institution seeking to shed the credit risk of assets that it owns and are the reference obligations for the credit default swap. If a credit event does not occur, the return realized by the collateral

manager that will be available to meet the payment to the CDO bondholders is the return on the collateral consisting of low risk assets plus the credit default swap premium. If a credit event occurs for any of the reference obligations, the collateral manager must make a payment to the counterparty. This reduces the return available to meet the payments to CDO bondholders.

Recently, a new form of synthetic CDO was traded: Standard tranches of credit default swap indices.¹⁰

VIII. BASKET DEFAULT SWAPS

Because of the growing importance of the synthetic CDO market and the role of basket default swaps in these structures and other types of credit structured products that may be created in the market, in this section we will take a closer look at the different types of basket default swaps. In a basket default swap, there is more than one reference entity. Typically, in a basket default swap, there are three to five reference entities. There are different types of basket default swaps. They are classified as follows:

- *N*th-to-default swaps
- Subordinate basket default swaps
- Senior basket default swaps

In this section we describe each type.

A. *N*th-to-Default Swaps

In an ***N*th-to-default swap**, the protection seller makes a payment to the protection buyer only after there has been a default for the *N*th reference entity and no payment for default of the first (*N* − 1) reference entities. Once there is a payout for the *N*th reference entity, the credit default swap terminates. That is, if the other reference entities that have not defaulted subsequently do default, the protection seller does not make any payout.

For example, suppose that there are five reference entities. In a **first-to-default basket swap**, a payout is triggered after there is a default for only one of the reference entities. There are no other payouts made by the protection seller even if the other four reference entities subsequently have a credit event. If a payout is triggered only after there is a second default from among the reference entities, the swap is referred to as a **second-to-default basket swap**. So, if there is only one reference entity for which there is a default over the tenor of the swap, the protection seller does not make any payment. If there is a default for a second reference entity while the swap is in effect, there is a payout by the protection seller and the swap terminates. The protection seller does not make any payment for a default that may occur for the three remaining reference entities.

B. Subordinate and Senior Basket Credit Default Swaps

In a **subordinate basket default swap** there is (1) a maximum payout for each defaulted reference entity and (2) a maximum aggregate payout over the tenor of the swap for the basket of reference entities. For example, assume there are five reference entities and that (1) the maximum payout is \$10 million for a reference entity and (2) the maximum aggregate payout

¹⁰See Chapter 15 in Douglas J. Lucas, Laurie S. Goodman, and Frank J. Fabozzi, *Collateralized Debt Obligations: Structures and Analysis*, Second Edition (Hoboken, NJ: John Wiley & Sons, 2006).

is \$15 million. Also assume that defaults result in the following losses over the tenor of the swap:

Loss resulting from default of first reference entity = \$6 million

Loss result from default of second reference entity = \$10 million

Loss result from default of third reference entity = \$16 million

Loss result from default of fourth reference entity = \$12 million

Loss result from default of fifth reference entity = \$15 million

When there is a default for the first reference entity, there is a \$6 million payout. The remaining amount that can be paid out on any subsequent defaults for the other four reference entities is \$9 million. When there is a default for the second reference entity of \$10 million, only \$9 million will be paid out. At that point, the swap terminates.

In a **senior basket default swap** there is a maximum payout for each reference entity, but the payout is not triggered until after a specified dollar loss threshold is reached. To illustrate, again assume there are five reference entities and the maximum payout for an individual reference entity is \$10 million. Also assume that there is no payout until the first \$40 million of default losses (the threshold). Using the hypothetical losses above, the payout by the protection seller would be as follows. The losses for the first three defaults is \$32 million. However, because of the maximum loss for a reference entity, only \$10 million of the \$16 million loss is applied to the \$40 million threshold. Consequently, after the third default, \$26 million (\$6 million + \$10 million + \$10 million) is applied toward the threshold. When the fourth reference entity defaults, only \$10 million is applied to the \$40 million threshold. At this point, \$36 million is applied to the \$40 million threshold. When the fifth reference entity defaults in our illustration, only \$10 million is relevant since the maximum payout for a reference entity is \$10 million. The first \$4 million of the \$10 million is applied to cover the threshold. Thus, there is a \$6 million payout by the protection seller.

C. Comparison of Riskiness of Different Default Swaps¹¹

Let's compare the riskiness of each type of default swap from the perspective of the protection seller. This will also help reinforce an understanding of the different types of swaps. We will assume that for the basket default swaps there are the same five reference entities. Four credit default swaps, ranked by highest to lowest risk for the reasons to be explained, are:

1. *Subordinate basket default swap*: the maximum for each reference entity is \$10 million with a maximum aggregate payout of \$10 million.
2. *First-to-default swap*: the maximum payout is \$10 million for the first reference entity to default.
3. *Fifth-to-default swap*: the maximum payout for the fifth reference entity to default is \$10 million.
4. *Senior basket default swap*: there is a maximum payout for each reference entity of \$10 million, but there is no payout until a threshold of \$40 million is reached.

¹¹The illustration and discussion in this section draws from "Nth to Default Swaps and Notes: All About Default Correlation," *CDO Insight* (May 30, 2003) UBS Warburg.

All but the senior basket default swap will definitely require the protection seller to make a payout by the time the fifth loss reference entity defaults (subject to the maximum payout on the loss for the individual reference entities). Consequently, the senior basket default swap exposes the protection seller to the least risk.

Now let's look at the relative risk of the other three default swaps with a \$10 million maximum payout: subordinate basket default swap, first-to-default swap, and fifth-to-default swap. Consider first the subordinate basket swap versus first-to-default swap. Suppose that the loss for the first reference entity to default is \$8 million. In the first-to-default swap the payout required by the protection seller is \$8 million and then the swap terminates (i.e., there are no further payouts that must be made by the protection seller). For the subordinate basket swap, after the payout of \$8 million of the first reference entity to default, the swap does not terminate. Instead, the protection seller is still exposed to \$2 million for any default loss resulting from the other four reference entities. Consequently, the subordinate basket default swap has greater risk than the first-to-default swap.

Finally, the first-to-default has greater risk for the protection seller than the fifth-to-default swap because the protection seller must make a payout on the first reference entity to default.



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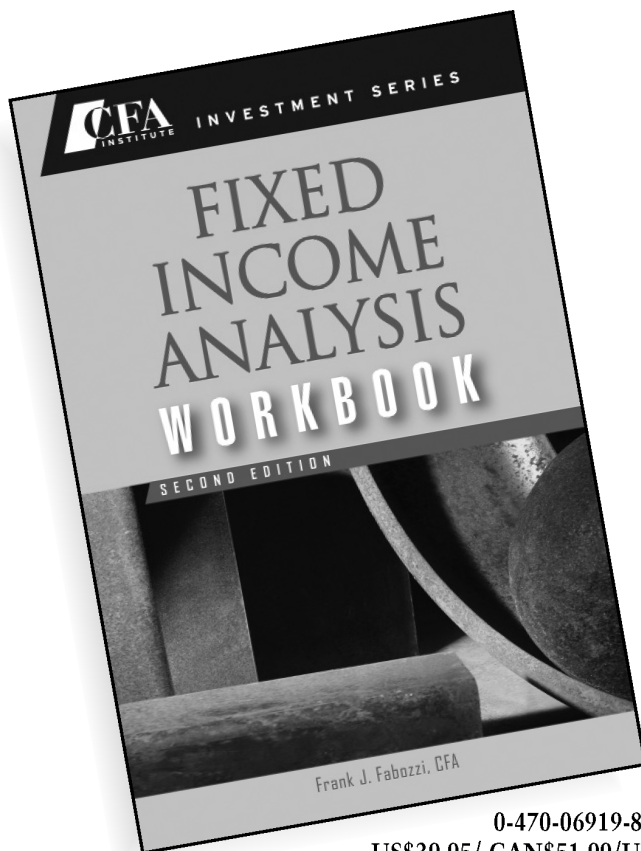
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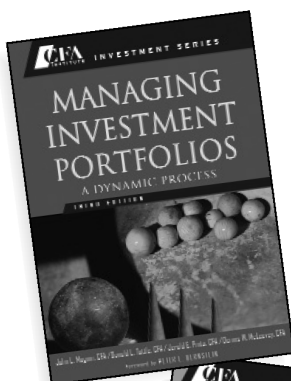
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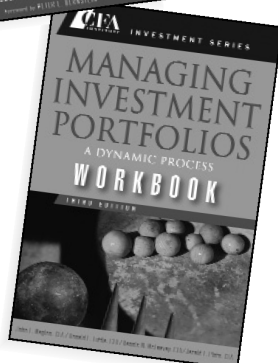
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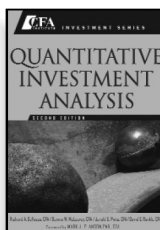
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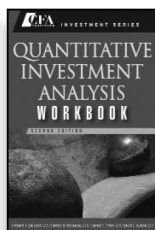
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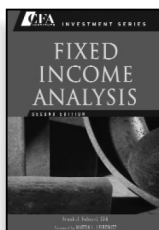
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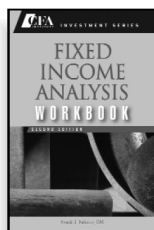
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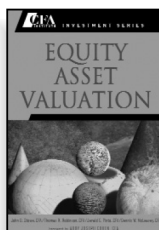
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