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Financing and Advising: Optimal Financial Contracts with Venture Capitalists

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ABSTRACT

This paper analyses the joint provision of effort by an entrepreneur and by an advisor to improve the productivity of an investment project. Without moral hazard, it is optimal that both exert effort. With moral hazard, if the entrepreneur's effort is more efficient (less costly) than the advisor's effort, the latter is not hired if she does not provide funds. Outside financing arises endogenously. This explains why investors like venture capitalists are value enhancing. The level of outside financing determines whether common stocks or convertible bonds should be issued in response to incentives.

THE VENTURE CAPITAL INDUSTRY has grown dramatically over the last decade. In the United States, venture capital (hereafter VC) investments grew from $3.3 billion in 1990 to $100 billion in 2000. In Europe, funds invested in VC grew from $6.4 billion in 1998 to more than $10 billion in 1999. The success of VC is largely due to the active involvement of the venture capitalists. These so-called hands-on investors carefully select the investment projects they are proposed (Sahlman (1988, 1990)) and remain deeply involved in those projects after investment is realized. Their most recognized roles include the extraction of information on the quality of the projects (Gompers (1995)), the monitoring of the firms (Lerner (1995), Hellmann and Puri (2002)), and also the provision of managerial advice to entrepreneurs. This advising role has been extensively documented empirically by Gorman and Sahlman (1989), Sahlman (1990), Bygrave and Timmons (1992), Gompers and Lerner (1999), and more recently Hellmann and Puri (2002). Venture capitalists contribute to the definition of the firm's strategy and financial

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policy, to the professionalization of their internal organization, and to the recruitment of key employees.

This paper provides a theory for the dual (i.e., financing and advising) role of venture capitalists. Entrepreneurs endowed with the creativity and technical skills needed to develop innovative ideas may lack business expertise and need managerial advice. I analyze a model where, in the first best, some effort should be provided both by an entrepreneur and by an advisor. In line with the view that entrepreneurial vision is really key to the success of the venture, I assume that the entrepreneur's effort is more efficient (less costly) than the advisor's. I consider the case where advice can be provided by consultants or by venture capitalists. Quite plausibly, I assume that the level of effort exerted by the advisor, as well as by the entrepreneur, to develop the project is not observable. Consequently the entrepreneur and the advisor face a double moral-hazard problem. To induce them to provide effort, both the entrepreneur and the advisor must be given proper incentives through the cash-flow rights they receive over the outcome of the project. In addition to effort, the project requires financial investment. This can be provided by the entrepreneur, the advisor, or pure financiers.

The first question raised in the paper is: Why should the entrepreneur ask for advice from venture capitalists rather than from consultants? What makes VC advising different from consultant advising? I show that, even if the entrepreneur is not wealth constrained and could himself fund all the initial investment, he chooses to obtain funding from the advisor, thus relying on VC advising rather than on consultants.[1] To understand the intuition of the result, consider the extreme case where the advisor could not provide funds. In this case, although the project would be more profitable with external advice, the entrepreneur chooses not to hire a consultant. This is because the rent the entrepreneur would need to leave to the consultant (to motivate her) is too high. If, in contrast with the maintained hypothesis, the advisor's effort was more efficient than the manager's, (pure) consultants could be hired in equilibrium. This suggests that the relative roles of consultants and venture capitalists depend on how crucial their advice is to the success of the ventures. More drastic innovations that rely on the entrepreneur's human capital are more likely to rely on VC advising rather than consultant advising.

The model concludes that venture capitalists, through their financial participation, can provide advice that could not otherwise be provided by consultants. The second objective of the paper is to investigate the relative roles of external financing (venture capital) and internal financing (entrepreneurial financial participation). The result of the analysis is that some amount of external financing guarantees an optimal provision of effort by the venture capitalist and increases the value of the firm. Projects requiring a small initial investment compared to their expected cash flows are optimally financed by outside capital only. In that case, outside financing comes as a compensation for the agency rent left to the venture capitalist for incentive motive. The financial participation of

[1] Of course, when the entrepreneur is wealth constrained, VC financing is all the more desirable.

the entrepreneur is shown to be valuable for those projects where the initial investment is large compared to the expected cash flows. In that case, pure outside financing would produce too much advising effort and not enough entrepreneurial effort. This effect is corrected by the entrepreneur's financial participation. This implies a positive correlation between the level of entrepreneurial financial investment and the profitability of start-up firms, for the less profitable start-ups only.

The last question raised in the paper concerns the implementation of the contract between the entrepreneur and the venture capitalist. The way the financial agreement is designed must take into account the two agents' incentives. It must also provide them an expected return at least equal to their investment. Consequently, two regimes arise depending on the amount invested by the investor. When the amount invested by the venture capitalist is low, he receives common stocks, while the entrepreneur is given preferred equity. When the amount invested by the venture capitalist is high, he is given convertible bonds or preferred equity. The intuition of this result is that when the investment of one agent is low, she gets a small share of outcome. In order to motivate her, she must be given higher-powered incentives. In the first regime, the investor is given more powerful incentives to exert effort because her investment is low. The second regime corresponds to the symmetric case, where the entrepreneur must be given higher-powered incentives, since his investment is lower.

These results are consistent with the way venture capitalists structure their financial contracts. Fenn, Liang, and Prowse (1998) observe that business angels invest smaller amounts of money than venture capitalists and acquire common stocks. In contrast, venture capitalists acquire convertible bonds (see also Kaplan and Strömberg (2003)). The two regimes identified in my theoretical model can be interpreted respectively as business angel financing and venture capitalist financing. The present analysis can thus be viewed as a first step towards understanding the differences between business angels and venture capitalists. While both types of investors play a significant role in early stage financing, the analysis of their differences has not received, to my knowledge, much attention in the literature so far.

The present model offers a rationale for the use of convertible bonds or outside equity in the financing of start-ups to motivate the investor *and advisor*.[2] Other papers explain the use of convertible claims in VC financing by focusing on the incentives convertible claims provide to managers. For example, Green (1984) and Biais and Casamatta (1999) show that convertible bonds induce managers to exert effort while precluding inefficient risk taking. To the extent that the model derives the optimality of a mix of outside debt and outside equity, it is also related to the literature on optimal outside equity financing that includes Chang (1993), Dewatripont and Tirole (1994), or Fluck (1998, 1999) and that does not specifically focus on venture capital finance.

[2] An original approach is developed in Cestone and White (1998), who find that outside equity acts as a commitment device for the venture capitalist not to fund competing firms.

While the current paper focuses on how VC contracts deal with moral hazard issues, Cornelli and Yosha (1997), Bergemann and Hege (1998), Habib and Johnsen (2000), and Dessi (2001) analyze how financial contracts elicit information revelation, and are useful in discriminating across projects and taking efficient continuation or liquidation decisions.[3]

The special focus of the present model on the efficiency of the joint efforts of the manager and the investor is shared by a couple of recent papers.[4] In Repullo and Suarez (1999), unlike in the present paper, the entrepreneur does not have the option to implement the project alone. This makes my first question irrelevant in their setting. Schmidt (1999) also considers a double moral-hazard setting to explain the use of convertible bonds in VC financing. However, investment in his model is an unobservable variable, while the present model distinguishes between financial investment and effort. In contrast to these papers, I endogenize the level of financial investment by the venture capitalist, and study under which conditions consultants are not valuable for the entrepreneur.

The paper is organized as follows. The model and the assumptions are presented in Section I. The optimal contract is solved in Section II. Here I study why entrepreneurs are unwilling to hire pure consultants and analyze the optimal provision of effort and level of outside financing. Section III discusses how to implement the contracts between the VC and the entrepreneur with financial claims such as convertible bonds or stocks. Concluding remarks are made in Section IV. All proofs are in the Appendix.

I. The Model

Consider an entrepreneur endowed with an innovative investment project. The project requires three types of inputs: One contractible initial investment I (money) and two unobservable (and *a fortiori* noncontractible) investments denoted e and a, where e represents the innovative effort put into the project and a the management effort to run the project properly. The project is risky and generates a verifiable random outcome $\tilde{R}$. To keep things simple, assume that it can either succeed or fail. $\tilde{R}$ takes the value R^u in case of success and $R^d(<R^u)$ in case of failure. The probability of success is denoted p_u. The probability of failure is denoted $(1-p_u)$.

The production technology is the following: If I is not invested, p_u is equal to 0; if I is invested, $p_u = \min[e+a; 1]$[5] where e and a are continuous variables that take values between 0 and 1.

[3] Admati and Pfleiderer (1994) first studied the problem of acquisition of information in the context of stage financing. They argue that assigning a fixed claim to the venture capitalist prevents him from strategic trading and induces optimal continuation decisions.

[4] While not focusing on double moral-hazard problems, Renucci (2000) and Cestone (2001) analyze situations where the intervention of a venture capitalist may also be valuable.

[5] The assumption that unobservable effort increases the probability of success of the project is in line with Holmström and Tirole (1997). The additive specification implies that the two efforts are not complementary: Their joint realization is *not* required to implement the project. Instead, each effort contributes separately to improve the profitability of the project.

There is also a continuum of risk-neutral advisors and pure financiers. The different types of agents differ in their ability to provide the nonobservable efforts e and a. Specifically, e can only be provided by the entrepreneur while a must be provided by an outside advisor. Although the entrepreneur is endowed with the technical skills and creativity required to develop his idea, he lacks management expertise. Pure financiers cannot provide a or e.

Both efforts are costly. Let $c_E(\cdot)$ denote the entrepreneur's disutility of effort, and $c_A(\cdot)$ the advisor's disutility of effort. Assume

$$c_E(e) = \beta \frac{e^2}{2}, \tag{1a}$$

and

$$c_A(a) = \gamma \frac{a^2}{2}. \tag{1b}$$

Assume that for a given level of effort, the cost is lower for the entrepreneur than for the advisor: $\gamma > \beta$, that is, the effort of the entrepreneur is more efficient. It would be equivalent to consider that the two agents have the same cost function, and that the impact of each effort on p_u is weighted by $\frac{1}{\beta}$, and $\frac{1}{\gamma}$ respectively. This assumption captures the idea that the entrepreneur's contribution is more important for success than the managerial expertise of the advisor. The consequences of relaxing this assumption are discussed later.

Agents are not a priori wealth constrained. Any of them can provide the initial investment I. However, I assume that once the firm is created, agents are protected by limited liability. The only thing that can be shared is the outcome of the project.[6] All agents are risk neutral. Their opportunity cost of putting money into the firm is the riskless interest rate r, normalized to zero. Denote A_{VC} the amount of money provided by the advisor, A_F the money provided by the pure financier, and $I - A_{VC} - A_F$ the money provided by the entrepreneur.[7] If $A_{VC} = 0$, the advisor who exerts effort a will be called a consultant, while if $A_{VC} > 0$, she will be called a venture capitalist.

The social value of the project is

$$V(e, a) = \min[e + a; 1]R^u + \max[0; 1 - (e + a)]R^d - \beta \frac{e^2}{2} - \gamma \frac{a^2}{2} - I. \tag{2}$$

As a benchmark, let us determine the optimal levels of efforts when all inputs are contractible (i.e., when efforts are observable). This corresponds to the first-best solution that maximizes the social value of the project. It is straightforward to see that it is optimal to have both the entrepreneur and the advisor exert strictly positive levels of effort. When both efforts are observable, the optimal levels of

[6] This assumption is in the line of Innes (1990) and is meant to make the problem interesting under risk neutrality.

[7] Note that the amount of money the entrepreneur puts into the firm may be negative if $A_{VC} + A_F > I$, in which case he receives a strictly positive transfer when investment is made.

effort are given by the first-order conditions of the maximization of V:

$$e^{FB} = \frac{1}{\beta}(R^u - R^d) \tag{3}$$

and

$$a^{FB} = \frac{1}{\gamma}(R^u - R^d). \tag{4}$$

Assume $\left(\frac{1}{\beta}+\frac{1}{\gamma}\right)(R^u - R^d) < 1$, so that the constraint $\min[e+a;1] \leq 1$ is not binding at the first best. Note that as the effort of the entrepreneur is more efficient than the effort of the advisor, the optimal level of effort e^{FB} is larger than a^{FB}. The first-best value of the project is then given by

$$V^{FB} = \frac{1}{2}\left(\frac{1}{\beta}+\frac{1}{\gamma}\right)(R^u - R^d)^2 + R^d - I. \tag{5}$$

Assume that

$$I \leq \frac{1}{2}\left(\frac{1}{\beta}+\frac{1}{\gamma}\right)(R^u - R^d)^2 + R^d \equiv \bar{I} \tag{6}$$

so that, when the first-best levels of effort are provided, the project is profitable.

This first-best solution can be implemented in a number of ways. Efforts e and a must be provided by the entrepreneur and by the advisor, respectively, but the identity of the agent providing the financial investment I is irrelevant. Thus, the Modigliani and Miller theorem holds in the first best. Financial structure is indeterminate and real decisions do not depend on financial decisions. Participation is ensured as capital suppliers receive an expected income equal to the opportunity cost of their investment. This is always feasible since, by assumption, the NPV of the project is positive in the first best.

When there is no moral-hazard problem, it is always optimal for the entrepreneur to ask for the services of an advisor. Whether the advisor is a consultant or a venture capitalist is irrelevant: The same social value can be attained when a financier, an advisor, or the entrepreneur himself provides the financial investment I. We will see later that this contrasts sharply with the conclusions derived under moral hazard.

II. Optimal Contract with Moral Hazard

The timing of the game is as follows. First, the contract is signed and I is invested. Second, agents choose their level of effort. Third, the outcome of the project is realized. The two agents choose their effort level to maximize their expected utility, given the contract and given their rational expectation of the equilibrium level of effort of the other. This is a simultaneous move game. Assuming simultaneous moves is natural, since effort levels are not observable. As all agents are risk neutral, their expected utility is perfectly identified by their net expected payoffs. Those payoffs depend on the financial contract they agree on, which specifies the financial contribution of each party and the share of the revenue allocated to each party in each state of nature.

Denote α_E^θ (resp. α_A^θ) the share of the revenue accruing to the entrepreneur (resp. the advisor) in state $\theta \in \{u, d\}$. If a pure financier is included in the contract, she receives a share: $1 - (\alpha_E^\theta + \alpha_A^\theta)$ in state θ.

Contrary to the first-best case, the way the cash flow is shared determines how much effort will be provided. The level of effort chosen by the entrepreneur is given by his incentive compatibility condition, denoted $(IC)_E$:

$$e \in \arg\max_{\hat{e}} (\hat{e} + a)\alpha_E^u R^u + (1 - (\hat{e} + a))\alpha_E^d R^d - \beta\frac{\hat{e}^2}{2} - (I - (A_{VC} + A_F)), \tag{7}$$

which means that he chooses the level of effort that maximizes his expected profit, given the contract established, his rational expectation of the effort level of the other agent, and given his cost of effort.

Equivalently, the incentive compatibility condition of the advisor, denoted $(IC)_{VC}$, is given by:

$$a \in \arg\max_{\hat{a}} (e + \hat{a})\alpha_A^u R^u + (1 - (e + \hat{a}))\alpha_A^d R^d - \gamma\frac{\hat{a}^2}{2} - A_{VC}. \tag{8}$$

Assume $\frac{1}{\beta}R^u < 1$ (A.1). Assumption (A.1) simply ensures that we get an interior solution when one agent is given maximal incentives. In the remainder of the analysis, (A.1) will be assumed to hold. The following lemma states what levels of effort are chosen by the entrepreneur and by the advisor as a function of the parameters of the contract.

LEMMA 1: *The levels of effort e and a are given by the first order conditions of the incentive compatibility constraints* $(IC)_E$ *and* $_{(IC)VC}$:

$$e = \frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) \tag{9}$$

and

$$a = \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d). \tag{10}$$

For each agent, the level of effort increases in the difference between his profit in state u and his profit in state d. Indeed, e (resp. a) is increasing in α_E^u (resp. α_A^u), and decreasing in α_E^d (resp. α_A^d). Increasing the share of the final outcome given to one agent in case of success reduces the share left to the other agent and correspondingly his incentives. The optimal contract will reflect this trade-off.

The financial contract is chosen to maximize the expected utility of the entrepreneur. The underlying assumption is that the entrepreneur has a unique, innovative idea, and can ask for business advice and money from a large number of agents. The participation constraints of the advisor and of the financier, ensuring that they recoup their investment in expectations, must be included in the entrepreneur's program. The participation constraint of the advisor, denoted

$(PC)_{VC}$, is

$$(e+a)\alpha_A^u R^u + (1-(e+a))\alpha_A^d R^d - \gamma\frac{a^2}{2} \geq A_{VC}. \tag{11}$$

The participation constraint of the financier, denoted $(PC)_F$, is

$$(e+a)(1-(\alpha_E^u+\alpha_A^u))R^u + (1-(e+a))(1-(\alpha_E^d+\alpha_A^d))R^d \geq A_F. \tag{12}$$

Hence the program to be maximized is

$$\begin{aligned} \max_{\alpha_E^\theta,\alpha_A^\theta,A_{VC},A_F} \; & (e+a)\alpha_E^u R^u + (1-(e+a))\alpha_E^d R^d - \beta\frac{e^2}{2} - (I-(A_{VC}+A_F)), \\ \text{s.t. } & (PC)_{VC}, \\ & (PC)_F, \\ & (IC)_{VC}, \\ & (IC)_E, \end{aligned} \tag{13}$$

$$(\alpha_E^u, \alpha_E^d, \alpha_A^u, \alpha_A^d) \geq 0 \tag{14}$$

$$\alpha_E^u + \alpha_A^u \leq 1 \tag{15}$$

$$\alpha_E^d + \alpha_A^d \leq 1, \tag{16}$$

where $\theta \in \{u, d\}$ and the last three conditions are feasibility constraints ensuring limited liability holds for all agents.

A. Provision of Efforts and External Financing when the Advisor Is a Consultant

The previous section established that without moral-hazard problems, the entrepreneur was indifferent to whether he hires a consultant or contracts with a venture capitalist. Under moral hazard, however, the entrepreneur never chooses to hire a pure consultant, as stated in the next proposition.

PROPOSITION 1: *If $A_{VC} = 0$, the entrepreneur maximizes his expected utility by not hiring a consultant. The entrepreneur exerts his first-best level of effort e^{FB} if the amount of outside financing is not too large ($A_F \leq R^d$).*

The intuition of Proposition 1 is the following. To induce the consultant to exert effort, the entrepreneur needs to give her a strictly positive share of the final income in case of success. This affects the entrepreneur's own profit in three ways. The first one is a direct revenue effect: The entrepreneur's share of income is lower. The second one is an incentive effect: Having a lower share of income, the effort provided by the entrepreneur decreases and is not fully offset by the effort exerted by the consultant, because the consultant's effort is less efficient. Overall, the probability of success decreases. The third effect is a reduction in the entrepreneur's cost of effort, since his effort is lower. The first two effects affect negatively the entrepreneur's profit while the third effect is positive. However the cost effect is not high enough to compensate the first two, and the entrepreneur maximizes his profit by not hiring a consultant. This is, however, only a second-best optimum: Because the cost of effort is convex, it would be technologically efficient to split the provision of effort between the two agents, but this is suboptimal because of incentive considerations. Starting from the case presented in Proposition 1 where the entrepreneur does not hire an advisor, a small amount of business advice would increase the value of the project. The entrepreneur is not able to recoup the cost of this enhancement in social value, however. The rent he would have to surrender to the consultant would be too large compared to the increase in value the consultant's advice would induce.

The main result of Proposition 1 comes from the combination of two conditions. First, the consultant is less efficient, and second, he does not invest money into the project. If one of these assumptions is relaxed, it becomes optimal to hire an advisor. Consider the case where the entrepreneur's effort is less efficient. He would then find it optimal to hire a consultant. In the venture capital setting, however, the entrepreneur's specific expertise is key to the success of the venture. This prevents him from hiring a consultant. In the following section, we will see that one way to overcome this inefficiency is to ask the advisor to participate financially in the project, in the spirit of venture capital financing and advising. Intuitively, asking the advisor to contribute financially compensates the entrepreneur for granting the advisor a share of the proceeds and reduces the cost of getting business advice. This suggests that the relative roles of consultants and venture capitalists depend on how crucial their advice is to the success of the ventures. Pure consultants can be hired if their effort is more efficient than that of entrepreneurs. More drastic innovations that presumably rely on the entrepreneur's human capital are more likely to need VC advising.

The last part of Proposition 1 simply states when the first-best level of entrepreneurial effort is achieved. If A_{VC} is lower than R^d, the revenue promised to the financier is a constant, and the entrepreneur captures any increase in value induced by his effort. This gives rise to strong incentives to exert effort. This is reminiscent of the classical Harris and Raviv (1979) result. However, due to limited liability, if outside financing is higher than R^d, the first-best level of effort is infeasible because the difference between the revenue of the entrepreneur in the good and bad states is not large enough.

B. Provision of Efforts and External Financing when All Agents Can Invest

Let us now turn to the case where all agents can invest money into the firm, that is, when A_{VC} and A_F can both be positive. When A_{VC} and A_F are chosen to maximize the entrepreneur's expected payoff, the two participation constraints PC_{VC} and PC_F are obviously binding.[8] The program boils down to maximizing

[8] If they were not, increasing the financial participation of the advisor and of the financier would make the entrepreneur better off without affecting incentives.

the NPV of the project subject to the incentive compatibility conditions and the feasibility conditions described at the beginning of this section. From this section on, I restrict the analysis to the case where the revenue of the pure financier does not decrease with the project's income. As argued by Innes (1990), this assumption deters secret infusion of cash into the firm's accounts by insiders.[9] The nondecreasing condition thus generates more robust contracts.[10] To reflect this assumption, the condition

$$(1-(\alpha_E^u+\alpha_A^u))R^u \geq (1-(\alpha_E^d+\alpha_A^d))R^d, \tag{17}$$

is added to the program. The next proposition establishes that venture capital financing is desirable.

PROPOSITION 2: *When all agents can invest, it is optimal to ask for venture capital financing: $A_{VC}^* > 0$. The level of effort exerted by the VC a^* is strictly positive.*

Proposition 2 states that the entrepreneur is willing to hire an advisor who also invests a strictly positive amount of money into the project. Combined with Proposition 1, it implies that financing and advising must go hand in hand. The financial participation of the VC compensates the entrepreneur for conceding part of the project's income to motivate her. Optimally chosen, the VC's financial investment exactly offsets the agency rent he is given to be induced to work. The entrepreneur's objective turns out to be aligned with NPV maximization, which requires a positive effort a. The entrepreneur strictly prefers to have a financial partner investing in the project, even though he is wealthy enough to implement the project alone. A real *financial partnership* with the advisor arises endogenously.

This result provides a rationale for the commonly observed behavior of VC investors, or business angels. A distinctive feature is their personal involvement along with their financial investment to develop the projects they back. For instance, Gorman and Sahlman (1989) report that venture capitalists spend a great deal of time in the firms they invest in, providing advice and experience. Hellmann and Puri (2002) also document this "soft side" of venture capital. Less unanimity is found concerning the advising role of business angels. Although it is sometimes argued that they are less deeply involved in the projects they finance (see for instance Ehrlich et al. (1994)), many authors do find an important advising role in angels' financing.[11] Prowse (1998, p. 790) reports from interviews with business angels that "Active angels almost always provide more than money. Angels will often help companies arrange additional financing, hire top manage-

[9] Such a situation may occur if the monetary outcome is perfectly verifiable but not the origin of this outcome.

[10] This is at the expense of efficiency since those contracts provide less powerful incentives to exert effort. For the sake of completeness, I present in the Appendix the results when this condition does not hold. The main insights of this section concerning the role of venture capital financing are qualitatively unchanged.

[11] Other evidence is found in Freear, Sohl, and Wetzel (1994) or Mason and Harrison (2000). See also Berger and Udell (1998) and Lerner (1998) for a discussion on the different characteristics of angel investors.

ment, and recruit knowledgeable board members. Angels also help solve major operational problems… and develop the company's long-term strategy."

One of the insights of the model is that the level of effort provided by the advisor depends on the level of her financial contribution to the project. It is thus natural to investigate to what extent the financial participation of the entrepreneur is also desirable.

PROPOSITION 3: *There exists a threshold I^* such that the financial participation of the entrepreneur increases the NPV of the project if the initial investment I is large ($I > I^*$), while it is neutral if I is small ($I \leq I^*$)*

COROLLARY 1: *When $I > I^*$, the entrepreneur's effort e^* decreases with the amount of outside financing, while the VC's effort a^* increases with outside financing*

Proposition 3 states that the financial participation of the entrepreneur can enhance the value of the project if the initial investment needed is large. The intuition is that there is a maximal amount of outside financing (I^*) that can be raised while maintaining incentives for both agents to exert effort. As stated in Corollary 1, each extra dollar of outside financing above I^* affects negatively the entrepreneur's effort and reduces the project's value. The reason is the following. Increasing outside financing raises the share of the final income left to outside investors. This, in turn, destroys the entrepreneur's incentives to work. If the entrepreneur is wealthy enough, investing his own resources into the project reduces the amount of outside capital to be raised and preserves the entrepreneur's own incentives. The project's value consequently increases. If the level of investment is below I^*, it can be entirely financed by outside capital, for outside financing offsets the expected income left to the venture capitalist for incentive reasons. In that case, the NPV is maximal without the entrepreneur's financial participation.

The assumption of the model that no agent is wealth constrained is clearly an important one. The above result states that the entrepreneur's participation is efficient for some values of the parameters. It is likely though that some entrepreneurs have no cash to invest in their firm. I turn to the case where this assumption is relaxed. Suppose that the entrepreneur has no personal wealth. Proposition 3 shows that for those projects requiring a low initial outlay, the entrepreneur's wealth constraint has no bite. It can, however, be detrimental to the project's value if the initial investment required is large. Proposition 4 sheds light on the impact of the entrepreneur's wealth constraint.

PROPOSITION 4: *The maximal amount of outside financing (I_{max}) that the entrepreneur can raise under moral hazard is strictly lower than the maximal level of investment, such that the project is profitable in the first best ($\bar{I}$)*

Proposition 4 reflects the financial constraints faced by the entrepreneur because of moral-hazard problems. If the project requires an initial investment larger than I_{max} but lower than $\bar{I}$, it is, by assumption, potentially profitable.

However, if the entrepreneur has no personal wealth to invest, he is rationed on the capital market and cannot implement his project. If the level of outside financing that must be raised is above I_{max}, too large a share of profits must be left to the investors so that they recoup their investment. This, in turn, destroys the entrepreneur's incentives to exert effort and leads to a negative NPV project: Capital suppliers cannot recover the opportunity cost of their investment and refuse to invest.

The first part of Proposition 3 along with Proposition 4 illustrates the impact of agency costs on the firm's investment policy as well as the role of net worth or cash flows in mitigating these costs, as documented by Fazzari, Hubbard, and Peterson (1988), Gilchrist and Himmelberg (1995), or Lamont (1997). Raising external capital is expensive. It dilutes the entrepreneur's stake in the firm and discourages effort. This lowers the firm's value and reduces investment. However, Proposition 2 as well as the last part of Proposition 3 unveils another aspect of the role of external finance. In the specific venture capital setting, raising external capital is value enhancing, since it guarantees the involvement of the venture capitalist. Contrary to the traditional agency view of corporate finance,[12] projects financed by external capital can be more profitable than pure internally financed projects.

The above results delineate two types of situations. In the first one, projects should be entirely financed by external venture capital. This ensures that a sufficient level of effort a is exerted by the venture capitalist. This case arises when the initial investment is lower than I^*. Note that I^* increases with $(R^u - R^d)^2$. When I is small compared to $(R^u - R^d)^2$, projects exhibit high expected profitability. In the opposite case, projects with lower expected profitability benefit from the financial contribution of the entrepreneur. For those projects, the relation between the level of investment of the entrepreneur and the profitability of the project is expected to be positive.

This model explains why the joint provision of advice and money is so often observed in the case of start-ups. Although business expertise is not the exclusive property of VCs, it may sometimes be the only way for an entrepreneur to obtain efficient advice. The next section investigates which financial claims purchased by venture capitalists optimally cope with the double-sided moral-hazard problem studied here.

III. Optimal Financial Contracts between Venture Capitalists and Entrepreneurs

The previous section established the optimality of the venture capitalist's financial participation in the entrepreneur's project. This section aims at defining which financial claims will be optimally held by venture capitalists in response to their financial investment. The objective is to determine which financial claims will provide powerful incentives for both the venture capitalist and the entrepreneur. I restrict the analysis to the case where the only outside investor

[12] Surveys of this numerous literature include Harris and Raviv (1991) or Allen and Winton (1995).

is the VC. Such a restriction is harmless from an efficiency point of view. The presence of a pure financier along with the VC in the contract with the entrepreneur is irrelevant to the levels of effort exerted.[13] The following proposition states which financial claims are optimally issued, depending on the level of outside financing.

PROPOSITION 5: *There exists a threshold A^*_{VC}, strictly lower than I^*, such that*

- *When $A_{VC} \leq A^*_{VC}$,* the optimal contract can be implemented by giving common stocks to the VC and preferred stocks to the entrepreneur.
- *When $A_{VC} > A^*_{VC}$, the optimal contract can be implemented by giving preferred stocks or convertible bonds to the VC and common stocks to the entrepreneur*

Recall that I^* is the maximal amount of outside financing that can be raised while inducing optimal efforts for both agents. Proposition 5 states that within the optimal range of outside financing, incentive problems can be solved using different instruments. Two regimes arise. When the amount of outside financing is small, the VC's expected income is small, too. She must then be given higher-powered incentives to be induced to work. In that case, the entrepreneur is given preferred stocks that grant him a higher dividend than common stocks if the bad state of nature is realized. If the good state of nature is realized, the income is high enough so that common and preferred stocks give the same return. As a consequence, the VC who owns only common stocks is proportionally better remunerated in state R^u than in state R^d, which gives her more powerful incentives to exert effort. When the amount of outside financing is large, the VC must be pledged a large share of profits in order to recoup her investment. As there is little left for the entrepreneur, he is less prone to make an effort, and needs a higher-powered incentive scheme. When the VC is given convertible bonds or preferred stocks, she captures most of the income in state R^d. The common stocks held by the entrepreneur are only valuable in the good state of nature. The entrepreneur intensifies his effort to increase the probability of state R^u occurring.

The specific venture capital setting studied here provides a rationale for the use of convertible and equity-like claims as the optimal source of outside finance. These results contribute to the literature on the optimal capital structure of firms. The main insight is that outside equity, or equity-like claims, provide proper incentives to active investors such as venture capitalists. This is consistent with the empirical observation that convertible claims (bonds or preferred stocks) are extensively used in VC financing, as evidenced by Sahlman (1988, 1990) or Kaplan and Strömberg (2000).

These two regimes are also related to the findings of Fenn, Liang, and Prowse (1998). They compare empirically the financial claims used by business angels and venture capitalists. In their sample of 107 U.S. firms of high-tech sectors (medical

[13] This is true when the financial contract of the pure financier cannot decrease with the final outcome of the project. Otherwise it could improve incentives as mentioned in footnote 10.

equipment and software industry), they find that business-angel-backed firms obtain an average funding of U.S. \$1.5 million, while venture-capital-backed firms obtain an average funding of U.S. \$12 million. In addition, three-quarters of the business angels' deals involve the acquisition of common stock, while three-quarters of the venture capitalists' deals involve the acquisition of convertible claims. Quite consistently, Proposition 5 states that when the VC's financial participation is small, she purchases common stocks, while she obtains convertible bonds or preferred stocks when her financial contribution is large.

It is important to stress that the optimal financial claims in each investment regime are not unique. In the model, convertible bonds do just as well as preferred stocks, and both can be used indifferently. This indeterminacy is itself an important feature of real venture capital contracts. As noted by Kaplan and Strömberg (2003) "While VCs use convertible securities most frequently, they also implement the same allocation of rights using combinations of multiple classes of common stock and straight preferred stock."

What matters is how the cash-flow rights allocated to each party (entrepreneur and venture capitalist) vary with the firm's performance.[14] On this issue, Kaplan and Strömberg (2003) find that VCs' cash-flow rights tend to decrease with the firm's performance, while the founder's cash-flow rights tend to increase with performance. This is consistent with the second regime described in Proposition 5 where the VC's investment is high, and where she is given convertible bonds, while the entrepreneur is given common stocks. In this case, the VC's cash-flow rights decrease with the firm's performance, while the entrepreneur's rights increase with performance.

IV. Conclusion

This paper analyzes a double-moral hazard problem whereby two agents must exert effort to improve the profitability of a venture. Because of incentive considerations, the most efficient agent prefers not to hire the less efficient one if the latter does not invest money into the project. In the venture capital setting, this implies that entrepreneurs do not want to rely on consultant advising when their own expertise is key to the success of the venture. To enhance the profitability of their project, entrepreneurs must ask advisors to invest financially into the project, in the spirit of venture capital financing and advising. This determines an optimal amount of outside financing. Traditional corporate finance theory emphasizes the agency costs associated with external financing, while this model highlights the reduction in agency costs owing to external financing. The financial claims purchased by venture capitalists also respond to incentive considerations. Common stocks provide high-powered incentives to venture capitalists. In contrast, convertible bonds are given to the venture capitalists when strong incentives must be provided to entrepreneurs.

[14] Thus the present analysis determines the optimal allocation of shares between managers and investors according to performance. See Fluck (1999) for an analysis of the dynamics of the allocation of shares between managers and investors.

The analysis of the model yields the following empirical predictions.

- First, there should be a relationship between the level of the venture capitalist's financial participation and the type of financial claim that is issued by the firm. Common stocks should be associated with small financial investment, while convertible bonds should be associated with large financial investment. This is consistent with the empirical findings of Fenn, Liang, and Prowse (1998) and Kaplan and Strömberg (2003).
- Second, the model predicts that in very innovative lines of business venture capital-backed firms should be more profitable than non-VC-backed firms: For those projects, only VCs can provide business advice to improve the firm's profitability. This suggests that a variable indicating the presence of venture capital should be included in the regression explaining the profitability of very innovative firms.
- Third, consultant services should be more frequent in those start-ups where the entrepreneur's competencies are not unique or crucial. Less innovative firms should rely more on consultant advising. To test this hypothesis, one could identify the product market strategies of different start-ups, in the spirit of the analysis of Hellmann and Puri (2000), and compare the frequency of use of consultant services between groups of different innovativeness.
- Fourth, there should be a positive correlation between the level of entrepreneurial financial investment (expressed as a percentage of total investment) and the profitability of start-up firms. This effect should be stronger among groups of less profitable start-ups. In gathering firm-specific data on financing patterns of start-ups, one could add the level of entrepreneurial investment in the explanatory variables of the firms' profitability.

Appendix

Proof of Lemma 1: The levels of effort chosen by the entrepreneur and the investor, given by the FOCs of IC_E and IC_{VC}, are:

$$e = \max\left[0; \min[1; \frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d)]\right] \tag{A1}$$

and

$$a = \max\left[0; \min[1; \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)]\right]. \tag{A2}$$

Under assumption (A.1), $\frac{1}{\beta}R^u < 1$, which implies:

$$\begin{cases} \frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) < 1 \\ \text{and} \\ \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d) < 1 \end{cases} \tag{A3}$$

We next show that when $e=0$, the entrepreneur never chooses α_E^u and α_E^d such that:

$$\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) < 0. \tag{A4}$$

When $e=0$, a must be strictly positive (otherwise the project cannot be implemented); hence it is given by

$$a = \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d). \tag{A5}$$

The constraints $(PC)_{VC}$ and $(PC)_F$ are binding. If they were not, increasing A_F and A_{VC} would increase the entrepreneur's expected income without affecting the advisor's incentives. Replacing a, A_F, and A_{VC} by their value, the program defined in Section II becomes

$$\max_{\alpha_A^u, \alpha_A^d} (R^u - R^d)\tfrac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d) - \tfrac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + R^d - I \tag{A6}$$

$$\text{s.t. } \alpha_E^d R^d - \alpha_E^u R^u \geq 0, \tag{A7}$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u, \tag{A8}$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d \tag{A9}$$

Suppose equation (A7) is binding. Solving the program gives

$$\alpha_A^u R^u - \alpha_A^d R^d = R^u - R^d. \tag{A10}$$

Given that $e=0$, effort a is equal to $\frac{1}{\gamma}(R^u - R^d)$, which corresponds to its first-best value.

Suppose now that equation (A7) is not binding, that is $\alpha_E^d R^d - \alpha_E^u R^u = \varepsilon$, $\varepsilon > 0$. It is easy to see that the solution described in equation (A10) can still be attained. This is because when $\alpha_E^d R^d > \alpha_E^u R^u$, the share of outcome given to the financier can adjust to induce the first-best level of effort a.[15] The value of the objective function is then

$$\frac{1}{2\gamma}[R^u - R^d]^2 + R^d - I. \tag{A11}$$

Hence, when $e=0$, it is efficient for the entrepreneur to choose α_E^u and α_E^d such that equation (A7) is binding. With no loss of generality, equation (A1) can

[15] Note that this would not be true anymore if there was no pure financier. In that case, setting $\alpha_E^d R^d = \alpha_E^u R^u$ when $e=0$ would be the only way to induce the first best level of effort a. Equation (A7) would then have to be binding.

be replaced by

$$e = \frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d), \tag{A12}$$

which states the first part of Lemma 1.

Equivalently, let us show that when $a = 0$, the entrepreneur never chooses α_A^u and α_A^d such that

$$\frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d) < 0. \tag{A13}$$

By the same reasoning as before, when $a = 0$, the program solved by the entrepreneur is

$$\max_{\alpha_E^u, \alpha_E^d} (R^u - R^d)\tfrac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) - \tfrac{1}{2\beta} + (\alpha_E^u R^u - \alpha_E^d R^d)^2 + R^d - I \tag{A14}$$

$$\text{s.t.} \quad \alpha_A^d R^d - \alpha_A^u R^u \geq 0, \tag{A15}$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u, \tag{A16}$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d \tag{A17}$$

Because of the presence of the pure financier, the same solution can be attained whether equation (A15) is binding or not and is characterized by

$$\alpha_E^u R^u - \alpha_E^d R^d = R^u - R^d. \tag{A18}$$

Given that $a = 0$, effort e is set at its first-best value, that is, $e = \frac{1}{\beta}(R^u - R^d)$. The value of the objective function is then

$$\frac{1}{2\beta}[R^u - R^d]^2 + R^d - I. \tag{A19}$$

As a consequence, with no loss of generality, equation (A2) can be replaced by

$$a = \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d). \tag{A20}$$

Proof of Proposition 1: The first step of the proof is to show that Lemma 1 still holds when one imposes $A_{VC} = 0$ in the general program. The main difference with the case where A_{VC} can be optimally chosen is that $(PC)_{VC}$ may not be binding.

- Suppose first that $e = 0$. Effort a is given by equation (A20) and $(PC)_F$ is binding. The program solved by the entrepreneur is written

$$\max_{\alpha_A^u, \alpha_A^d} \tfrac{1}{\gamma}[R^u - R^d - (\alpha_A^u R^u - \alpha_A^d R^d)] + (\alpha_A^u R^u - \alpha_A^d R^d) + R^d - I - \alpha_A^d R^d \tag{A21}$$

$$\text{s.t.} \quad \alpha_E^d R^d - \alpha_E^u R^u \geq 0 \tag{A22}$$

$$\tfrac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + \alpha_A^d R^d \geq 0 \tag{A23}$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u, \tag{A24}$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d \tag{A25}$$

The optimal solution is to set $\alpha_A^d = 0$ and $\alpha_A^u R^u = \frac{1}{2}(R^u - R^d)$. For the reasons mentioned in the proof of Lemma 1, this solution is feasible whether equation (A22) is binding or not.

- Suppose next that $a = 0$. Effort e is given by equation (A12) and $(PC)_F$ is binding. $(PC)_{VC}$ is written

$$\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d)(\alpha_A^u R^u - \alpha_A^d R^d) + \alpha_A^d R^d \geq 0. \tag{A26}$$

If $\alpha_A^u R^u = \alpha_A^d R^d$, the optimal solution of the program is given by equation (A18). Effort e is set at its first-best level, given that $a = 0$.

If $\alpha_A^u R^u < \alpha_A^d R^d$ (for instance, $\alpha_A^u R^u = \alpha_A^d R^d - \varepsilon$, $\varepsilon > 0$) it is not possible anymore to induce the first-best level of effort e. Indeed, at the optimum, we have

$$\alpha_E^u R^u - \alpha_E^d R^d = R^u - R^d + \varepsilon, \tag{A27}$$

which induces too large a level of effort e compared to the first best. The value of the objective function is strictly lower than in the case where $\alpha_A^u R^u = \alpha_A^d R^d$. Hence, Lemma 1 still holds when there is no financial participation by the advisor.

The second step of the proof consists of solving the general program after replacing $(IC)_{VC}$ and $(IC)_E$ using the expressions in Lemma 1. Note that $(PC)_F$ is still binding and can also be replaced. After manipulations, the program to solve is the following:

$$\begin{aligned}\max_{\alpha_E^\theta, \alpha_A^\theta} & [\tfrac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \tfrac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)][R^u - R^d - (\alpha_A^u R^u - \alpha_A^d R^d)] \\ & - \tfrac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + R^d - I - \alpha_A^d R^d\end{aligned} \tag{A28}$$

$$\text{s.t. } \tfrac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d) + \tfrac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d)(\alpha_A^u R^u - \alpha_A^d R^d) + \alpha_A^d R^d \geq 0 \tag{A29}$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u, \tag{A30}$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d, \tag{A31}$$

where $\theta \in \{u, d\}$. Note that equation (A29) representing $(PC)_{VC}$ cannot be binding if $e > 0$ and $a > 0$. The constraint $(PC)_{VC}$ can only be binding if $a = 0$ and $\alpha_A^u = \alpha_A^d = 0$, $= 0$, which corresponds to the case where the entrepreneur does not hire a consultant. To establish Proposition 1, it will be demonstrated that the entrepreneur is strictly better off if $(PC)_{VC}$ is binding.

Setting $\alpha_A^d = 0$ is optimal since it lowers the expected outcome of the advisor, and increases the entrepreneur's profit without affecting the latter's incentives to exert effort. Define $X = \alpha_E^u R^u - \alpha_E^d R^d$ and $Y = \alpha_A^u R^u$. Equation (A29) states

$$\frac{1}{2\gamma} Y^2 + \frac{1}{\beta} XY \geq 0. \tag{A32}$$

As $X > 0$, it is automatically satisfied when $Y \geq 0$, which implies that it is redondant compared to the feasibility constraint. The program solved by the entrepreneur is:

$$\max_{X,Y} -\frac{1}{2\beta} X^2 + \left(\frac{1}{\beta} X + \frac{1}{\gamma} Y\right)(R^u - R^d - Y)$$

$$\text{s.t.} \quad Y \geq 0$$

The objective function is concave if $2\beta > \gamma$ and convex otherwise. The Lagrangian of the program is

$$L = -\frac{1}{2\beta} X^2 + \left(\frac{1}{\beta} X + \frac{1}{\gamma} Y\right)(R^u - R^d - Y) + \lambda Y. \tag{A33}$$

The solutions must verify

$$\frac{\partial L}{\partial X} = 0 \Leftrightarrow \; -X + (R^u - R^d - Y) = 0 \tag{A34}$$

$$\frac{\partial L}{\partial Y} = 0 \Leftrightarrow \; -\frac{1}{\beta} X + \frac{1}{\gamma}(R^u - R^d - 2Y) = 0 \tag{A35}$$

$$\lambda \geq 0, \quad Y \geq 0, \quad \lambda Y = 0$$

If $\lambda = 0$, equations (A34) and (A35) imply $Y = [(\gamma - \beta)/(\gamma - 2\beta)](R^u - R^d)$. Note, however, that this solution is not feasible if $2\beta > \gamma$ (since Y must be positive). In that case, we must have $Y = 0$ and $X = R^u - R^d$. If $2\beta < \gamma$, $Y = [(\gamma - \beta)/(\gamma - 2\beta)](R^u - R^d)$ is feasible, but recall that in that case, the objective function is convex, which means that Y defined above is a minimum. The maximum is then also defined by $Y = 0$ and $X = R^u - R^d$. To conclude, it is optimal for the entrepreneur to set $Y = 0$, that is, not to hire a consultant. The optimal level of effort of the entrepreneur is then $e = \frac{1}{\beta}(R^u - R^d) = e^{FB}$. Note that if $e = e^{FB}$, the expected income of the pure financier is at most equal to R^d, which means that this solution holds for $A_F \leq R^d$. In case the entrepreneur needs to borrow more than R^d (say, if he is wealth constrained), it can be shown (using the same methodology) that the result of the proposition goes through: The entrepreneur never hires a consultant. However, because outside financing is too large, he is induced to exert a level of effort strictly lower than the first best. More formal proof is available upon request. □

Optimal Contract when the Revenue of the Pure Financier Is Not Constrained to Be Nondecreasing

Using Lemma 1, the program of the entrepreneur becomes:

$$\max_{\alpha_E^\theta,\alpha_A^\theta,A_{VC},A_F} \tfrac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + \tfrac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)(\alpha_E^u R^u - \alpha_E^d R^d) + \alpha_E^d R^d - (I - (A_{VC} + A_F)) \quad (A36)$$

$$\text{s.t. } \tfrac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + \tfrac{1}{\beta}(\alpha_A^u R^u - \alpha_A^d R^d)(\alpha_E^u R^u - \alpha_E^d R^d) + \alpha_A^d)R^d \geq A_{VC} \quad (A37)$$

$$\left(\tfrac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \tfrac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)\right)(R^u - R^d - (\alpha_E^u R^u - \alpha_E^d R^d) - (\alpha_A^u R^u - \alpha_A^d R^d)) + R^d - (\alpha_A^d R^d + \alpha_E^d R^d) \geq A_F \quad (A38)$$

$$(\alpha_E^u, \alpha_E^d, \alpha_A^u, \alpha_A^d) \geq 0 \quad (A39)$$

$$1 - (\alpha_E^u + \alpha_A^u) \geq 0 \quad (A40)$$

$$1 - (\alpha_E^d + \alpha_A^d) \geq 0 \quad (A41)$$

where $\theta \in \{u, d\}$. The participation constraints are binding. If they were not, increasing A_F and A_{VC} would increase the entrepreneur's expected income without affecting the advisor's incentives. Replace then A_F and A_{VC} in the objective function. The program is written

$$\max_{\alpha_E^\theta,\alpha_A^\theta,} -\tfrac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + (R^u - R^d)[\tfrac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \tfrac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)] - \frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + R^d - I \quad (A42)$$

$$\text{s.t. } \alpha_A^\theta \geq 0; \quad \alpha_E^\theta \geq 0, \ 1 - (\alpha_A^u + \alpha_E^u) \geq 0; \quad 1 - (\alpha_A^d + \alpha_E^d) \geq 0 \quad (A43)$$

where $\theta \in \{u,d\}$. Consider first not taking into account the feasibility constraints, and define $X = \alpha_E^u R^u - \alpha_E^d R^d$ and $Y = \alpha_A^u R^u - \alpha_A^d R^d$. The objective function is concave since the Hessian is negative semidefinitive. First-order conditions of the maximization of the objective function give

$$X = Y = R^u - R^d.$$

It is straightforward to see that if feasible, this solution corresponds to the first-best levels of effort being exerted. Replacing X and Y by their value, and using the fact that $\alpha_E^u + \alpha_A^u \leq 1$, it follows that this solution is feasible if and only if

$$2(R^u - R^d) + \alpha_E^d R^d + \alpha_A^d R^d \leq R^u. \tag{A44}$$

Since the smallest possible value for α_E^d and α_A^d is 0, it follows that first-best levels of effort can be implemented if and only if $R^u \leq 2R^d$.

If $R^u > 2R^d$, one must write down the Lagrangian L of the program, including all the feasibility constraints described above:

$$L = -\frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d) + (R^u - R^d)\left[\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)\right]$$

$$-\frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + \lambda_1 \alpha_E^u R^u + \lambda_2 \alpha_E^d R^d + \lambda_3 \alpha_A^u R^u + \lambda_4 \alpha_A^d R^d$$

$$+\lambda_5(R^u - (\alpha_E^u R^u + \alpha_A^u R^u)) + \lambda_6(R^d - (\alpha_E^d R^d + \alpha_A^d R^d)) \tag{A45}$$

Straight application of the theorem of Kuhn-Tucker and tedious algebra give the following solution:

$$\begin{cases} \alpha_E^{d*} R^d = \alpha_A^{d*} R^d = 0 \\ \alpha_E^{u*} R^u = \frac{\gamma R^d + \beta(R^u - R^d)}{\gamma + \beta}, \\ \alpha_A^{u*} R^u = \frac{\beta R^d + \gamma(R^u - R^d)}{\gamma + \beta}. \end{cases} \tag{A46}$$

To conclude, note that $a^* > 0$ in both cases. Also, (PC_{VC}) binding implies that $A_{VC} > 0$ under the optimal contract: The results of Proposition 2 still hold.

The maximal amount of outside financing is given by $A_{VC}^* + A_F^*$. Replacing the parameters of the contract by their optimal value gives the following:

$$\text{if } R^u \leq 2R^d, \quad A_{VC}^* + A_F^* \leq R^d - \frac{1}{2\gamma}(R^u - R^d)^2; \tag{A47}$$

$$\text{if } R^u > 2R^d, \quad A_{VC}^* + A_F^* \leq \frac{(R^u - 2R^d)[3\gamma\beta R^u + 2\gamma R^d(\gamma - 2\beta)]}{2\beta(\gamma + \beta)^2} + R^d\left(1 - \frac{R^d}{2\gamma}\right). \tag{A48}$$

If the entrepreneur has to raise an amount of outside capital larger than the values defined above, the previously defined optimal contract cannot hold anymore and the value of the project decreases, which corresponds to the results of Proposition 3. The main differences with the case where the revenue of the financier is nondecreasing are that (1) efforts are higher and (2) the financier needs to invest a strictly positive amount of capital ($A_F^* > 0$) while her contribution is neutral when her revenue is nondecreasing. □

Proof of Proposition 2: The program to be solved is the same as in the previous section, except that equation (17) must be added to the program.

Note that the limited liability constraint represented in equation (A40) becomes redundant, as it is automatically satisfied when equation (17) holds. The new Lagrangian is the following:

$$\begin{aligned} L = & -\frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + (R^u - R^d)\left[\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)\right] \\ & - \frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + \lambda_1 \alpha_E^u R^u + \lambda_2 \alpha_E^d R^d + \lambda_3 \alpha_A^u R^u + \lambda_4 \alpha_A^d R^d \\ & + \lambda_5 (R^u - R^d - (\alpha_E^u R^u - \alpha_E^d R^d) - (\alpha_A^u R^u - \alpha_A^d R^d)) + \lambda_6 (R^d - (\alpha_E^d R^d + \alpha_A^d R^d)) \end{aligned} \tag{A49}$$

Again, straight application of the theorem of Kuhn-Tucker gives

$$\begin{aligned} \alpha_E^{u*} R^u - \alpha_E^{d*} R^d &= \tfrac{\gamma}{\gamma+\beta}(R^u - R^d), \\ \alpha_A^{u*} R^u - \alpha_A^{d*} R^d &= \tfrac{\beta}{\gamma+\beta}(R^u - R^d). \end{aligned} \tag{A50}$$

Replace α_E^u, α_A^u, α_E^d and α_A^d in (PC_F) and (PC_{VC}) to obtain

$$A_F^* = R^d - \alpha_E^d R^d - \alpha_A^d R^d \tag{A51}$$

$$A_{VC}^* = \frac{(R^u - R^d)^2}{(\gamma+\beta)^2}\left(\frac{\beta^2 + 2\gamma^2}{2\gamma}\right) + \alpha_A^d R^d \tag{A52}$$

Note that the solutions presented in equation (A50) imply that $a^* > 0$ and that the minimum value of A_{VC}^* is strictly positive, which concludes the proof of Proposition 2. □

Proof of Proposition 3: Define

$$I^* \equiv \frac{(R^u - R^d)^2}{(\gamma+\beta)^2}\left(\frac{\beta^2 + 2\gamma^2}{2\gamma}\right) + R^d, \tag{A53}$$

and use equations (A51) and (A52) to state that under the optimal contract $A_F^* + A_{VC}^* \leq I^*$.

If $I \leq I^*$, the project can entirely be financed by outside capital and the entrepreneur's participation is useless. In that case, the value of the project is

$$V^* = \frac{(\gamma^2 + \beta^2 + \beta\gamma)}{2\gamma\beta(\gamma+\beta)}(R^u - R^d)^2 + R^d - I. \tag{A54}$$

If $I > I^*$, either the entrepreneur is able to invest $I - I^*$ and the second-best outcome is feasible, that is, the value of the project is V^* defined above, or one must solve the general program adding the constraint

$$A_{VC} + A_F > I^*. \tag{A55}$$

Replace A_F and A_{VC} by their value in (PC_{VC}) and (PC_F), set $\alpha_E^d = 0$ (which is obviously optimal when equation (A55) holds) and use the fact that constraint (17) is binding to get

$$A_{VC} + A_F = -\left(\frac{1}{\beta} - \frac{1}{2\gamma}\right) Y^2 + \frac{1}{\beta}(R^u - R^d) Y + R^d, \tag{A56}$$

where Y stands for $\alpha_A^u \mathrm{R}^u - \alpha_A^d \mathrm{R}^d$. The determinant Δ is

$$\Delta = \frac{(R^u - R^d)^2}{\beta^2} - 2((A_{VC} + A_F) - R^d)\frac{2\gamma - \beta}{\gamma\beta}. \tag{A57}$$

The solution is readily computed and gives

$$Y = \frac{\gamma(R^u - R^d) - \gamma\beta\sqrt{\Delta}}{2\gamma - \beta}. \tag{A58}$$

Replacing Y by its value, and using equation (17) to find the expression of α_e^u gives, for $A_{VC} + A_F > I^*$

$$\begin{aligned} \alpha_E^{d*} &= 0, \\ \alpha_E^{u*} R^u &= \tfrac{(\gamma-\beta)(R^u - R^d) + \gamma\beta\sqrt{\Delta}}{2\gamma - \beta}, \\ \alpha_A^{u*} R^u - \alpha_A^{d*} R^d &= \tfrac{\gamma(R^u - R^d) - \gamma\beta\sqrt{\Delta}}{2\gamma - \beta}. \end{aligned} \tag{A59}$$

Check that the value of the project is then strictly lower than V^* defined in equation (A54). When the entrepreneur is forced to raise an amount of outside capital strictly larger than I^*, the value of the project decreases. Put differently, if $I > I^*$, the entrepreneur's financial participation increases the value of the project.

Proof of Corollary 1: Use Lemma 1 and the optimal contract derived in the proof of Proposition 3 to compute the optimal levels of effort when $A_{VC} + A_F > I^*$. Note that Δ decreases with $A_{VC} + A_F$. It follows immediately that e decreases with $A_{VC} + A_F$ and a increases with $A_{VC} + A_F$. □

Proof of Proposition 4: See that Δ, defined in the proof of Proposition 3, is positive if and only if

$$A_{VC} + A_F \leq R^d + \frac{\gamma}{2\beta}\frac{(R^u - R^d)^2}{2\gamma - \beta} \equiv I_{max}. \tag{A60}$$

Hence the maximal amount of outside financing is I_{max}. Simple comparison with the maximal level of initial investment defined in Section I yields the result of Proposition 4. □

Proof of Proposition 5: I first derive the conditions under which the investor acquires common stocks and the entrepreneur gets preferred stocks.

Preferred stocks ensure a minimum rate of return (dividend) to their owner before common stocks' returns are paid. When the outcome of the project is sufficiently high, both types of stocks give the same rate of return. Define $\underline{R}$ as the minimum dividend pledged on each preferred stock, multiplied by the number of preferred stocks. Let α be the fraction of preferred stocks in the firm's equity. The fraction of common stocks is $(1 - \alpha)$. To be able to distinguish between preferred and common stocks, assume that $\alpha R^d < \underline{R} \leq R^d$ and $\underline{R} < \alpha R^u$. Hence, when the income is low, it is impossible to remunerate common stocks with the same dividend as preferred stocks. When the income is high, both types of stocks generate the same dividend. Under these assumptions, the optimal contract can be implemented by giving common stocks to the investor and preferred stocks to the entrepreneur if and only if

$$(1 - \alpha_E^d)R^d = R^d - \underline{R}, \tag{A61}$$

$$(1 - \alpha_E^u)R^u = (1 - \alpha)R^u, \tag{A62}$$

$$\alpha \in]\frac{\underline{R}}{R^u}, \frac{\underline{R}}{R^d}], \tag{A63}$$

$$\underline{R} \leq R^d. \tag{A64}$$

When $A_{VC} \leq I^*$, (A61) and (A62) write

$$\underline{R} = I^* - A_{VC}, \tag{A65}$$

$$\alpha = \frac{\frac{\gamma}{\beta+\gamma}(R^u - R^d) - A_{VC} + I^*}{R^u}. \tag{A66}$$

It is easy to check that (A64) is satisfied if and only if $A_{VC} \geq I^* - R^d$. Besides, (A63) is satisfied if and only if

$$A_{VC} \leq I^* - \frac{\gamma}{\beta + \gamma}R^d \equiv A^*_{VC}. \tag{A67}$$

I next turn to the case where the investor acquires convertible bonds or preferred stocks.

In this stylized model, issuing convertible bonds or preferred stocks generates the same pattern of return for their owner: The face value of the bond corresponds to a minimum dividend pledged before common shareholders are remunerated. When the project's income is high, bonds are converted, and the return they generate is similar to the return of preferred (or common) stocks. Differences between these two types of claim usually concern the right to trigger bankruptcy, which is irrelevant in this setting. Convertible bonds are characterized by a face value D, and a fraction $1-\alpha$ of the firm's equity, such that if $(1-\alpha)R^{\theta} \leq D(\theta \in \{d; u\})$, the investor gets $\min[D; R^{\theta}]$; if $(1-\alpha)R^{\theta} > D$, the investor gets $(1-\alpha)R^{\theta}$.

To be able to distinguish between convertible bonds and common stocks, assume $(1-\alpha)R^d < D < (1-\alpha)R^u$.

Consider convertible bonds with $D \leq R^d$. Such a contract implies $A_{VC} \leq I^*$, since the investor's revenue must be lower than (or equal to) R^d in state d. The contract must verify

$$(1-\alpha_E^d)R^d = D, \tag{A68}$$

$$(1-\alpha_E^u)R^u = (1-\alpha)R^u, \tag{A69}$$

$$\alpha \in \left]\frac{R^d - D}{R^d}, \frac{R^u - D}{R^u}\right[, \tag{A70}$$

$$D \leq R^d. \tag{A71}$$

Replacing α_E^d and α_E^u by their values, (A68) and (A69) become

$$D = A_{VC} - I^* + R^d, \tag{A72}$$

$$1-\alpha = \frac{1}{R^u}\left[\frac{\beta}{\beta+\gamma}(R^u - R^d) + A_{VC} - I^* + R^d\right]. \tag{A73}$$

Condition (A70) implies $A_{VC} > A_{VC}^*$. It follows that issuing convertible bonds (as structured above) is possible if and only if $A_{VC} \in]A_{VC}^*, I^*]$. By the same reasoning, one can show that convertible bonds with $D > R^d$ can be issued when $A_{VC} > I^*$. □

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